# 984: Some Letters to and by Gerbert d'Aurillac Dealing with "Arabic" Mathematics and Astronomy

Samer Sayed Qandil, in cooperation with Daniel G. König



Samer Sayed Qandil, in cooperation with Daniel G. König, 984: Some Letters by Gerbert d'Aurillac Dealing with "Arabic" Mathematics and Astronomy, in: *Transmediterranean History* 3.1 (2021).

DOI: https://doi.org/10.18148/tmh/2021.3.1.63.

**Abstract**: In 984, Gerbert d'Aurillac exchanged three letters with fellow churchmen, which mention manuscripts dealing with arithmetic and astronomy. On this basis, this contribution traces a broader transfer of mathematical and astronomical knowledge from the Arabic-Islamic to the Latin-Christian sphere in the late- tenth century. Addressing the so-called "dark legend" of Gerbert, the earliest corpus of Latin works on the astrolabe, the abacus, and Hindu-Arabic numerals, as well as evidence for exchange between scholarly milieus in Umayyad al-Andalus and Catalonia, it discusses Gerbert's direct involvement in this transfer.

#### Source

Gerbert d'Aurillac, Epistolae 17, 25, 24, ed./trans. Pierre Riché, in: Gerbert d'Aurillac, *Correspondance*, ed. Pierre Riché, vol. 1: Lettres 1 à 129, Paris: Les Belles Lettres, 1993, pp. 36-37, 50-51, 48-49; alternatively in: *Lettres de Gerbert* (983-997), ed. Julien Havet, Paris: Alphonse Picard, 1889, pp. 13–14, 19–20. Translation: Gerbert of Aurillac, Letters 25, 33, 32, in: *The Letters of Gerbert: With his Papal Privileges as Sylvester II*, trans. Harriet Pratt Lattin, New York: Columbia University Press, 1961, pp. 63–64, 69–70.)

[Geraldo abbati Aureliacensi] De multiplicatione et divisione numerorum libellum a Joseph Ispano editum abbas Warnerius penes vos reliquit; ejus exemplar in commune rogamus. Si limina beatorum Remigii et Dionisii datur vobis copia videndi, nuntio praemisso, vestris alloquiis poterimus codelectari.

[Bonfilio Gerundensi episcopo] Multa quidem auctoritas vestris nominis me movet, cum ad videndum et alloquendum, tum etiam ad obtemperandum, et hoc diu negatum distulit negata libertas. Ea cum dolore domino concessa, meo Ottone Caesare jam non superstite, fas et amicis loqui, et eorum imperiis obsequi. Si qua nobis significare voletis, usque ad kl. novenbr. Remis, VIII kl. janr. Romae dicetis, si pace uti poterimus. De multiplicatione et

[To abbot Gerald of Aurillac, spring 984] Abbot Warnerius left with you a book "On Multiplication and the Division of Numbers" (*De multiplicatione et divisione numerorum*) written by Joseph the Spaniard, and we both would like a copy of it. If an opportunity of visiting St. Rémy or St. Denis presents itself to you, and if you send a messenger ahead, we will be able to enjoy some conversations with you.

[To bishop Bonfill of Girona, spring 984] The great reputation of your name, indeed moves me not only to see and speak with you, but also to comply with your orders. It is only due to the fact that liberty has been denied me that I have so long put off this compliance. Now I that have my liberty, though only through sorrow, since my lord Otto the Caesar no longer lives, it is right for me both to talk to friends and to obey their commands. If you wish to make known anything to us, you can communicate with us at Reims until 1 November, or at Rome on 25 December, if we are able to avail ourselves of peace. Joseph the Wise edited certain

divisione numerorum Joseph sapiens sententias quasdam edidit, eas pater meus Adalbero Remorum archiepiscopus vestro studio habere cupit.

[Lupito Barchinonensi] Licet apud te nulla mea sint merita, nobilitas tamen ac affabilitas tua me adducit in te confidere, de te praesumere. Itaque librum de astrologia translatum a te michi petenti dirige, et si quid mei voles in reconpensationem indubitate reposce. sentences "On Multiplication and the Division of Numbers" (*De multiplicatione et divisione numerorum*), and my father Adalbero, archbishop of Reims, wishes to have these through your efforts.

[To Lupitus of Barcelona, April 984] Although I have no claim on you, still your renown and your courtesy lead me to trust in you, in fact to presume on you. And, so, I am asking you to send me the book "On Astrology" (*De Astrologia*), transferred by you, and if you desire anything from me in return, ask for it unhesitatingly.

### **Authorship & Work**

[§1] Gerbert of Aurillac (d. 1003) is one of the most well-known intellectuals of the tenth century. One of the most educated men in the Latin West of his time, he significantly broadened the intellectual horizon of several students by means of various new teaching methods and devices. As a politician and statesman, he played a significant role in the rise of the Capetian Dynasty in the Western Frankish kingdom and exerted decisive influence over the young Emperor Otto III (r. 983/996–1002). Last but not least, he ranks as the first "French" pope who, under the name Sylvester II (sed. 999–1003), presided as pope over the turn of the first to the second millennium CE.<sup>1</sup>

[§2] The essential facts of Gerbert's earlier life are known from his pupil Richer of Reims (d. after 998), who dedicated several enthusiastic pages of his "Four Books of Histories" (Historiarum libri quattuor) to his teacher.<sup>2</sup> Much of Gerbert's later life can be reconstructed on the basis of his letters.<sup>3</sup> Gerbert was born in Aquitaine around the years 945–950. Since neither his exact place of birth nor his parents are known, it seems likely that he was of low birth (humili genere).<sup>4</sup> In his childhood, he entered the monastery of St-Géraud d'Aurillac where he studied Latin Grammar (grammatica edoctus est). Since Gerbert proved to be an apt student, the abbot promoted his study of the quadrivium, i.e. arithmetic, geometry, music, and astronomy. When, in 967, Count Borrell II of Barcelona (r. 947–992) visited the monastery, the abbot asked the latter to take Gerbert along to Catalonia. There, Gerbert studied mathematics for around three years under Bishop Atto of Vich (sed. 957–971). When he accompanied the count and the bishop on a pilgrimage to Rome in 970, his extensive knowledge brought him to the attention of Pope John XIII (sed. 965–972) and Emperor Otto I (r. 936/951/962–973). With the latter's support, he entered the service of Archbishop Adalbero of Reims (sed. 969-989), under whose tutelage he assumed the role of secretary to Adalbero and teacher of various subjects in the local cathedral school. Gerbert's success in an official disputation with Otric of Cologne on philosophical questions in the presence of Otto II (r. 973–983) in Ravenna in 980

<sup>&</sup>lt;sup>1</sup> See the comprehensive biography by Riché, *Gerbert d'Aurillac*.

<sup>&</sup>lt;sup>2</sup> See Richerius Remensis, *Historiarum libri quattuor*, ed. Hoffmann (MGH SS 38), lib. 3, cap. 43–65, pp. 191–205. On Richer see Bur, Richer v. Reims, cols 830–831.

<sup>&</sup>lt;sup>3</sup> Riché, Gerbert d'Aurillac révélé par sa correspondance, pp. 85–92.

<sup>&</sup>lt;sup>4</sup> The *Liber Pontificalis*, ed. Duchèsne, vol. 2, p. 263, mentions an otherwise unknown father named Agilbert of Aquitanian origin: "patre Agilberto natione Aquitaniorum." On the discussion about Gerbert's origins, see Darlington, Gerbert "Obscuro loco natus," pp. 509–520; Juillet, Du lieu de naissance de Gerbert, pp. 146–150; Charbonnel, La ville de Gerbert, pp. 58–65.

seems to have earned him the position of abbot in the North Italian monastery of Bobbio, renowned for its huge library, in 982. In Bobbio, however, Gerbert suffered from repeated attacks on the abbey and its possessions by local nobles.<sup>5</sup> When Otto's II death in December 983 made matters worse, Gerbert fled to the imperial palace at Pavia<sup>6</sup>, and left for Reims early in 984, where he reassumed his functions under Archbishop Adalbero. After the latter's death, Gerbert rose swiftly in the ecclesiastical hierarchy: he became archbishop of Reims (sed. 991–997), then—significantly supported by the young Emperor Otto III (r. 983/996–1002)—archbishop of Ravenna (sed. 998–999), finally bishop of Rome (sed. 999–1003).<sup>7</sup>

[§3] Gerbert's influence is based on the fact that he taught a number of illustrious scholars, and even kings and emperors. In and outside the cathedral school of Reims, Gerbert had many students, some of whom became remarkable scholars, clerics, and statesmen. These include the historian Richer, the abbot of Micy known as Constantine of Fleury (sed. 1011–1021)<sup>8</sup>, bishop Fulbert of Chartres (sed. 1006–1028) who contributed to a revival of the bishopric's cathedral school<sup>9</sup>, the future French King Robert II the Pious (r. 987/996–1031), and Emperor Otto III.<sup>10</sup>

[§4] Contemporary and later observers of Gerbert had difficulties to explain why a person of such humble birth had been able to acquire such a high degree of knowledge and rise to such important public and political positions. Already in the eleventh century, a "dark legend" emerged, which depicted Gerbert as a magician who had made a pact with the devil. This legend was then repeated and embellished in the twelfth century.<sup>11</sup>

[§5] Gerbert seems to have compensated his involvement in the political affairs of his day by seeking and finding consolation in the quest for knowledge and intensive study.<sup>12</sup> In his writings, he devoted much attention to the classical heritage, as is clear in his many quotes of Roman authors, including Cicero, Terentius, Pliny, Seneca etc.<sup>13</sup> He expresses his attitude towards classical works and philosophies by stating that "in our opinion, nothing in human affairs is preferable to a knowledge of the most distinguished men which, assuredly, is unfolded in the numerous volumes of their works."<sup>14</sup> Gerbert's works include a treatise on logic entitled

<sup>&</sup>lt;sup>5</sup> Riché, Gerbert d'Aurillac et Bobbio, pp. 49–64, on the basis of Gerbert's letters. See: Gerbert d'Aurillac, Correspondance, ed./trans. Riché, vol. 1, ep. 5, 14, 16, 18, 19, 83, pp. 10–11, 28–29, 32–33, 36–39, 38–41, 186–187.

<sup>&</sup>lt;sup>6</sup> Gerbert, *Letters*, trans. Lattin, p. 9.

<sup>&</sup>lt;sup>7</sup> Riché, Gerbert d'Aurillac.

<sup>&</sup>lt;sup>8</sup> Werner, Constantin v. Fleury, cols 169–170.

<sup>&</sup>lt;sup>9</sup> DeMayo, Fulbert of Chartres; see also Faulk, *Bishop Fulbert of Chartres*.

<sup>&</sup>lt;sup>10</sup> Lindgren, *Gerbert von Aurillac und das Quadrivium*, pp. 40–48; DeMayo, Students of Gerbert, pp. 97–117.

<sup>&</sup>lt;sup>11</sup> The earliest traces of the legend of Gerbert can be found in Beno, *Gesta Romanae aecclesiae contra Hildebrandum*, ed. Francke (MGH Libelli de lite, 2), lib. 2, cap. 3, p. 376, and cardinal to the antipope Guibert during the so-called investiture controversy between Pope Gregory VII and Emperor Henry IV. Beno claimed that Pope Gregory VII had been educated in "demonic doctrines," taught to him by Benedict IX and archbishop Lawrence of Amalfi (*doctrinis demoniorum quas accepit Hildebrandu magister eorum a magistris suis*), one or both of whom were pupils of Gerbert. According to Beno, *Gesta*, ed. Francke, lib. II, cap. 4, p. 377, Gerbert asked the Demon about the date of his death and received the response that he would die after having celebrated mass in Jerusalem. Later, Gerbert was also depicted as a magician in the works of Ordericus Vitalis and William of Malmesbury etc. See Flusche, *Life and Legend*, pp. 82–84; Ricklin, Philosoph als Nekromant, pp. 238–247. Gerbertus, *Opera mathematica*, ed. Bubnov, pp. 376–393, contains a list of all Latin sources which mention Gerbert's mathematical skills in praise and polemics under the chapter heading "Testimonia de Gerberto mathematico."

<sup>&</sup>lt;sup>12</sup> See, for example, Gerbert, *Correspondance*, ed./trans. Riché, ep. 152, 153, pp. 372–379; Gerbert, *Letters*, trans. Lattin, ep. 160, 161, pp. 188–191.

<sup>&</sup>lt;sup>13</sup> See, for example, Gerbert, *Letters*, trans. Lattin, ep. 175, 182, 201, pp. 205, 210, 237; Gerbert, *Correspondance*, ed./trans. Riché, ep. 167, 173, 584–585.

<sup>&</sup>lt;sup>14</sup> Gerbert, *Letters*, trans. Lattin, ep. 175, p. 205; Gerbert, *Correspondance*, ed./trans. Riché, vol. 2, ep. 167, pp. 418–419: "Nichil enim nobis antiquius in humanis rebus clarissimorum hominum scientia, quae utique multiplicibus librorum voluminibus explicatur." On Gerbert's use of classical authors, see Passalaqua, Gerberto di

"On the Rational and the Use of Reason" (*De rationali et ratione uti*), which is the product of his disputation with Otric of Cologne in Ravenna 980 and dedicated to Otto III.<sup>15</sup>

[§6] More relevant in this context are his writings on various mathematical topics which, within the framework of the quadrivium, included deliberations on arithmetics, music, geometry, and astronomy.<sup>16</sup> Gerbert's pupil, Richer of Reims, does not list Gerbert's writings. He explains that Gerbert taught dialectics and rhetorics on the basis of several ancient authors<sup>17</sup>, and mentions that Gerbert constructed a celestial globe, two armillary spheres as well as an abacus to be able to explain certain facets of astronomy and arithmetics to his students.<sup>18</sup> It is in connection with the abacus that Richer makes his only reference to a mathematical treatise written by Gerbert, a book on arithmetics dedicated to Constantine of Fleury, which Nicolaus Bubnov has published under the title "Rules for operating with the numbers of the abacus" (Regulae de numerorum *abaci rationibus*).<sup>19</sup> This and Gerbert's other mathematical writings have been conveniently edited in one volume by Nicolaus Bubnov, which, however, presents us with several difficulties. Most of Gerbert's mathematical deliberations that Bubnov considered genuine, take on the form of short letters on various mathematical subjects. Aside from these letters, Bubnov only counts a book on geometry (*Geometria Gerberti*) among his genuine works<sup>20</sup>, whereas a "Book on the Astrolabe" (Liber de astrolabio) figures among the works of doubtful authorship.<sup>21</sup> Additional writings on the abacus consist either in commented excerpts of Boethius, or in later references to ideas and rules said to have been laid down by Gerbert:<sup>22</sup> A "Book on the Abacus" (Liber abaci) by a certain Bernelinus, for example, claims to explain mathematical operations on the basis of Gerbert's teachings, but was not written by Gerbert himself.<sup>23</sup> This constellation of sources makes it extremely difficult to understand precisely what Gerbert knew about certain astronomical and mathematical subjects and which sources he drew on.

[§7] Among Gerbert's writings, his 220 letters stand out as authentic documents of Gerbert's interests as a scholar, teacher, politician, and churchman. All the original letters he wrote before he became pope were lost. However, he regularly began to keep copies of his letters after he became abbot of St. Columban of Bobbio in 982. The existing collections of Gerbert's letters were copied by his students and can be divided into two groups: the first one consists of miscellaneous letters that were either found in different manuscripts or included in the acts of certain church councils.<sup>24</sup> The second group is known through two different sets of manuscripts and is based on copies which Gerbert himself kept in one or more registers. The reasons that motivated Gerbert to collect his letters are multiple. Whereas Oscar Darlington claimed very

Aurillac corretore di manoscritti classici, pp. 317–320; DeMayo, Ciceronian Amicitia, pp. 319–337; Lindgren, Ptolemée chez Gerbert, pp. 619–644; Moehs, Gerbert of Aurillac as Link, pp. 331–350.

<sup>&</sup>lt;sup>15</sup> Gerbertus Aureliacensis, *De rationali et ratione uti*, ed. Stoppacci, pp. 173–193. Gerbert of Aurillac, *De Rationale et Ratione Uti*. trans. Ronk.

<sup>&</sup>lt;sup>16</sup> See Lindgren, *Gerbert von Aurillac und das Quadrivium*, pp. 13–39.

<sup>&</sup>lt;sup>17</sup> Richerius Remensis, *Historiarum libri quattuor*, ed. Hoffmann (MGH SS 38), lib. 3, cap. 46–47, pp. 193–194.

<sup>&</sup>lt;sup>18</sup> Richerius Remensis, *Historiarum libri quattuor*, ed. Hoffmann (MGH SS 38), lib. 3, cap. 49–54, pp. 195–198.

<sup>&</sup>lt;sup>19</sup> Richerius, *Historiarum libri quattuor*, ed. Hoffmann (MGH SS 38), lib. 3, cap. 54, p. 198: "eius librum quem scribit ad C[onstantinum] grammaticum"; Gerbertus, *Opera mathematica*, ed. Bubnov, pp. 1–22.

<sup>&</sup>lt;sup>20</sup> Gerbertus, *Opera mathematica*, ed. Bubnov, pp. 46–97.

<sup>&</sup>lt;sup>21</sup> Gerbertus, *Opera mathematica*, ed. Bubnov, pp. 109–147. On this work see Darby, A Note, p. 179.

<sup>&</sup>lt;sup>22</sup> Gerbertus, *Opera mathematica*, ed. Bubnov, pp. 155–196, for excerpts from Boethius, and pp. 197–245, 376–393, for later references to Gerbert's mathematical teachings.

<sup>&</sup>lt;sup>23</sup> Bernelinus, Liber abaci, in: Gerbert, *Oeuvres*, ed. Olleris, p. 357: "Cogis enim et crebris pulsas precibus, ut tibi multiformes abaci rationes persequar diligenter, negligentia quidem apud nos jam pene demersas, sed a domino papa Gerberto quasi quaedam seminaria breviter et subtilissime seminatas."; Gerbertus, *Opera mathematica*, ed. Bubnov, p. 383. See also the more recent edition: Bernelin, élève de Gerbert, *Libre d'abaque*, ed. Béatrice Bakhouche, Jean Cassinet, Pau: Princi Néguer, 2000.

<sup>&</sup>lt;sup>24</sup> Gerbert, *Letters*, trans. Lattin, pp. 20–21; Gerbert, *Correspondance*, ed./trans. Riché, pp. xi-xxxii.

generally that Gerbert collected these letters to demonstrate his loyalty to the Ottonian dynasty,<sup>25</sup> Harriet Pratt Lattin assigned Gerbert's initial impetus to collect his letters to the period, in which Gerbert tried to rescue the monastery of Bobbio from encroachment by local nobles. In this situation, Gerbert had sent many letters to Emperor Otto II urging him to support the monastery and its abbot against these assaults. Since Gerbert waited in vain for an adequate imperial response, Lattin argued, Gerbert collected the letters as evidence of his attempts to protect the monastery's property.<sup>26</sup> In the later period, Gerbert's collection may have also had an educational purpose, e.g. in the training of Emperor Otto III, who employed Gerbert as a tutor and counsellor from 997 onwards. Since these letters were not only written by Gerbert on behalf of himself, but also on behalf of archbishops and rulers, they fulfilled certain formal and stylistic requirements and could thus serve as models of the epistolary genre within the field of political communication.<sup>27</sup> Relevant in this context is that these letters often represent short treatises on scientific subjects and thus serve as a testimony to the first steps of the reception of the so-called Arabic sciences in Catalonia in the last quarter of the tenth century.

# **Content & Context**

[§8] The excerpts quoted above are taken from three letters written to and by Gerbert in spring 984. Gerbert had just fled Bobbio and had reassumed his position as secretary to the archbishop and teacher at the cathedral school of Reims. As he claims himself, this was a period of his life, in which he was seeking solace in his intellectual studies.<sup>28</sup> Although a negative experience in administrative and political terms, Gerbert's stay at Bobbio had enhanced his knowledge and enlarged his collection of manuscripts. It should be borne in mind that Bobbio's importance as a centre of study had been one of the main reasons that had motivated Gerbert to accept the position as abbot in the first place.<sup>29</sup> From the monastery's library, he had obtained two important mathematical works, i.e. Boethius' *De astronomia* and *De geometria*.<sup>30</sup> Obviously inspired by these books, Gerbert reactivated his acquaintances in Aquitaine and Catalonia to acquire two additional mathematical treatises by writing three letters.

[§9] All three letters mention churchmen in Catalonia and document Gerbert's efforts to receive access to mathematical and astronomical manuscripts from this region. The first letter (ep. 17 or 25), dated between February and March 984 by Lattin and to spring by Riché, was written to abbot Gerald of Aurillac. It mentions Warnerius (or Guarin), abbot of St Michel de Cuxá who was a close friend of Bishop Miró Bonfill of Girona (sed. 971–984). The latter was the son of Count Miro II of Cerdanya and Besalù (r. 897/911–927) and the addressee of the second letter (ep. 25 or 33), dated March 984 by Lattin, March-April by Weigle and spring by Riché. Finally, the last letter (ep. 24 or 32), assigned to March 984 by Lattin, April by Riché, was sent to Lupitus of Barcelona (d. after 984), who has been identified with the archdeacon Seniofred of Barcelona.<sup>31</sup>

[§10] The first two letters (nos. 17 or 25 and 25 or 33) mention an arithmetic treatise "On Multiplication and the Division of Numbers" (*De multiplicatione et divisione numerorum*)

<sup>&</sup>lt;sup>25</sup> Darlington, Gerbert the Teacher, p. 468.

<sup>&</sup>lt;sup>26</sup> Gerbert, *Letters*, trans. Lattin, p. 20.

<sup>&</sup>lt;sup>27</sup> Gerbert, *Correspondance*, ed./trans. Riché, pp. 32–34. See also Gerbert of Aurillac, *De Rationale et Ratione Uti*, trans. Ronk, p. 52.

<sup>&</sup>lt;sup>28</sup> Gerbert, *Correspondance*, ed./trans. Riché, ep. 152, 153, pp. 372–379; Gerbert, *Letters*, trans. Lattin, ep. 160, 161, pp. 188–191.

<sup>&</sup>lt;sup>29</sup> Gerbert, *Correspondance*, ed./trans. Riché, ep. 45, pp. 110–113; Gerbert, *Letters*, trans. Lattin, ep. 51, pp. 91–92.

<sup>&</sup>lt;sup>30</sup> Gerbert, *Correspondance*, ed./trans. Riché, ep. 8, pp. 16–19; Gerbert, *Letters*, trans. Lattin, ep. 15, p. 54.

<sup>&</sup>lt;sup>31</sup> On this identification, see Lattin, Lupitus Barchinonensis, pp. 58–64; Lindgren, Gerbert von Aurillac und das Quadrivium (1971), pp. 178–180.

attributed to a certain "Joseph the Spaniard" (*Joseph Ispanus*) or "Joseph the wise" (*Joseph sapiens*). The third letter (no. 24 or 32) to Lupitus of Barcelona mentions a treatise on astrology (*librum de astrologia*), to be understood here as a book of astronomical content. Its authorship is unclear, but Gerbert explains that Lupitus had "translated" this work in one way or another (*translatum a te*). The past participle of the Latin verb *transferre* does not automatically imply a translation from one language to another. Both in ancient and medieval Latin, the verb can also mean "to copy," "to transcribe," or simply to "transfer."<sup>32</sup> However, the use of this verb does not exclude a translation, e.g. from Arabic to Latin, at the hands of Lupitus either. This has led some scholars to claim that Gerbert's letter to Lupitus documents one of the earliest translations of a scholarly work from Arabic to Latin. Others, more cautious in identifying a proper translation, have nonetheless taken these excerpts either as the proof or as the starting point for a more intensive investigation of Arabic influences on the study of mathematics and astronomy in the Latin West.<sup>33</sup>

# **Contextualization, Analysis & Interpretation**

[§11] The following paragraphs try to explain why and how the three cited letters can be seen as documentary evidence for a larger process of astronomical and mathematical knowledge transfer from the Arabic-Islamic to the Latin-Christian sphere. This is done by dealing step by step with the available evidence for Gerbert's direct involvement in this process. This includes the so-called "dark legend," the corpus of the earliest Latin works on the astrolabe, the corpus of the earliest Latin writings explaining and depicting the abacus, finally contemporary evidence for the existence of a lively scholarly milieu in Umayyad al-Andalus as well as intensive relations between the latter and Catalonia. This evidence does not allow us to define the exact role and position of Gerbert within the larger process of knowledge transfer. It proves, however, that this process took place and that Gerbert was connected to it in terms of his geographical itinerary, his personal network, and his thematical interests.

[§12] Medieval historiography already characterized Gerbert as a scholar who profited intellectually from scientific knowledge available in the Arabic-Islamic sphere. However, if Gerbert is presented as someone influenced by the latter, it is always in the context of the aforementioned "dark legend," which depicts Gerbert as a magician inspired by evil forces, rather than as a respectable scholar making use of hitherto unknown foreign knowledge. Richer of Reims (d. after 998), Gerbert's admiring pupil, mentions that Gerbert went to Catalonia, where he encountered men proficient in the liberal arts, but does not refer to any Arabic influences.<sup>34</sup> In his short biographical notes on Gerbert, Adémar of Chabannes' (d. 1034) mentions a visit to Córdoba "for the sake of wisdom," but no concrete engagement with Arabic sciences.<sup>35</sup> The Anglo-Norman historiographer William of Malmesbury (d. 1143), in turn, mentions such influences, but contextualizes them negatively. In his depiction, Gerbert was a man "seized by lust of glory" who "fled by night into Spain, chiefly designing to learn astrology and other sciences of that description from the Saracens."<sup>36</sup>

http://ducange.enc.sorbonne.fr/TRANSFERSIO; http://ducange.enc.sorbonne.fr/TRANSFERITOR.

<sup>&</sup>lt;sup>32</sup> See <u>http://www.perseus.tufts.edu/hopper/morph?l=translatum&la=la#lexicon;</u>

<sup>&</sup>lt;sup>33</sup> E.g. Darlington, Gerbert, p. 461; Chandler, *Charlemagne's Last March*, pp. 169–174; Stock, Science, Technology, p. 37; Hugonnard-Roche, Influence of Arabic Astronomy, p. 288.

<sup>&</sup>lt;sup>34</sup> Richerius Remensis, *Historiae*, ed. Hoffmann (MGH SS 38), lib. 3, cap. 43, pp. 191–192.

<sup>&</sup>lt;sup>35</sup> Ademarus Cabannensis, *Chronicon*, ed. Bourgain, lib. 3, cap. 31, p. 154: "causa sophiae primo Franciam deinde Cordobam lustrans."

<sup>&</sup>lt;sup>36</sup> Willelmus Malmesbiriensis, *Gesta regum Anglorum*, ed. Hardy, lib. 2, § 167, pp. 271–272: "gloriae cupiditate captus, nocte profucit Hispaniam, animo praecipue intendens ut astrologiam et caeteras id genus artes a Saracenis edisceret."; William of Malmesbury, *Chronicle of the Kings of England*, trans. Giles, lib. 2, cap. 10, p. 172.

"There he surpassed Ptolemy with the astrolabe, and Alcandraeus in astronomy, and Julius Firmicus in judicial astrology; there he learned what the singing and the flight of birds portended, there he acquired the art of calling up spirits from hell: in short, whatever, hurtful or salutary, human curiosity has discovered. There is no necessity to speak of his progress in the lawful sciences of arithmetic and astronomy, music, and geometry, which he imbibed so thoroughly as to show they were beneath his talents, and which, with great perseverance, he revived in Gaul, where they had for a long time been wholly obsolete. Being certainly the first who seized on the abacus from the Saracens, he gave rules which are scarcely understood even by laborious computers."<sup>37</sup>

[§13] William then relates that Gerbert was associated with a certain "Saracen"<sup>38</sup>, who "would talk with him of matters at times serious, at others trivial, and lend him books to transcribe."<sup>39</sup> This friendship came to an end, however, when Gerbert stole a book "containing the knowledge of his whole art" and even made a pact with the devil to escape the furious Saracen.<sup>40</sup> Equipped with this knowledge, Gerbert returned to Gaul, where he taught many students, influenced rulers and applied his knowledge in various ways that astonished his contemporaries.<sup>41</sup> In this context, William also mentions that Gerbert dedicated a book on the rules of the abacus to Constantine of Fleury, abbot of the monastery of St. Maximin near Orléans.<sup>42</sup>

[§14] Since the influence of the so-called "Arabic sciences" is only mentioned explicitly in a polemical context, we must ask ourselves, if more neutral evidence is available to prove that Gerbert engaged with mathematical ideas received from Arabic sources. Turning to Gerbert's own works we are confronted with the fact that Gerbert's letters and those writings that are generally considered authentic, mention several ancient Greek and Latin scholars, but not a single Arabic name.<sup>43</sup>

[§15] In writings assigned to Gerbert, explicit references to "Arabs" (*Arabes*) or the Arabic language (*lingua Arabica*) are only found in the *Liber de astrolabio*, which sometimes also figures under the title of its first chapter, i.e. *De utilitatibus astrolabii*. In this work we find justifications for using Arabic terms<sup>44</sup>, Latin translations of Arabic terminology<sup>45</sup> as well as

<sup>&</sup>lt;sup>37</sup> Willelmus Malmesbiriensis, *Gesta regum Anglorum*, ed. Hardy, lib. 2, § 167, p. 272–273: "Ibi vicit scientia Ptholomaeum in astrolabio, Alandraeum in astrorum interstitio, Julium Firmicum in fato. Ibi quid cantus et volatus avium portendat, didicit; ibi excire tenues ex inferno figuras; ibi postremo quicquid vel noxium vel salubre curiositas humana deprehendit: nam de licitis artibus, arithmetica, musica, et astronomia, et geometria, nihil attinet dicere; quas ita ebibit ut inferiores ingenio suo ostenderet, et magna industria revocaret in Galliam omnino ibi jam pridem obsoletas. Abacum certe primus a Saracenis rapiens, regulas dedit quae a sudantibus abacistis vix intelliguntur."; William of Malmesbury, *Chronicle*, trans. Giles, lib. 2, cap. 10, pp. 173–174.

<sup>&</sup>lt;sup>38</sup> On the term "Saracens," see König, 621: Isidore.

 <sup>&</sup>lt;sup>39</sup> Willelmus Malmesbiriensis, *Gesta regum Anglorum*, ed. Hardy, lib. 2, § 167, p. 273: "Nec deerat Saracenus quin scientiam venditaret; assidere frequenter, nunc de seriis, nunc de nugis colloqui, libros ad scribendum praebere."; William of Malmesbury, *Chronicle*, trans. Giles, lib. 2, cap. 10, p. 174.
<sup>40</sup> Willelmus Malmesbiriensis, *Gesta regum Anglorum*, ed. Hardy, lib. 2, § 167, p. 273: "codex, totius artis

<sup>&</sup>lt;sup>40</sup> Willelmus Malmesbiriensis, *Gesta regum Anglorum*, ed. Hardy, lib. 2, § 167, p. 273: "codex, totius artis conscius"; ibid., pp. 273–274; William of Malmesbury, *Chronicle*, trans. Giles, lib. 2, cap. 10, p. 174.

<sup>&</sup>lt;sup>41</sup> Willelmus Malmesbiriensis, *Gesta regum Anglorum*, ed. Hardy, lib. 2, § 167–168, pp. 274–277; William of Malmesbury, *Chronicle*, trans. Giles, lib. 2, cap. 10, pp. 175–181.

<sup>&</sup>lt;sup>42</sup> Willelmus Malmesbiriensis, *Gesta regum Anglorum*, ed. Hardy, lib. 2, § 168, p. 275; William of Malmesbury, *Chronicle*, trans. Giles, lib. 2, cap. 10, p. 175.

<sup>&</sup>lt;sup>43</sup> See indexes in Gerbert, *Correspondance*, ed. Riché, vol. 2, pp. 709–715; Gerbertus, *Opera mathematica*, ed. Bubnov, pp. 563–574.

<sup>&</sup>lt;sup>44</sup> Liber de astrolabio, in: Gerbertus, *Opera mathematica*, ed. Bubnov, cap. III,1, p. 124: "ita in Arabico habetur"; cap. XVIII,1, p. 140: "cujus sententiam et Arabicam non est inutile scribere."

<sup>&</sup>lt;sup>45</sup> Liber de astrolabio, in: Gerbertus, *Opera mathematica*, ed. Bubnov, cap. V,1, p. 128: "altitudinem solis, quam Arabice Ertifa Aschemaz [cf. *irtifā* ' *aš-šams*] appellant"; cap. VIII,1, p. 132: "horas aequinoctiales et inaequales,

transcribed Arabic names of stars.<sup>46</sup> Bubnov had included the *Liber de astrolabio*in his collection of Gerbert's mathematical works, but classified it as being a work of uncertain authorship produced either by Gerbert or in his orbit. All extant manuscripts date from the eleventh and twelfth century.<sup>47</sup>

[§16] Initially, scholarship assumed Gerbert's authorship of the Liber de astrolabioand managed to reconstruct a link between this workand a manuscript from Catalonia. Josep Maria Millàs i Vallicrosa proved that parts of the text of the Liber de astrolabio are contained in MS 225 from the Catalonian monastery of Santa María de Ripoll. An analysis of the contents of MS 225 then revealed a close thematical relationship to two later Latin translations of originally Arabic works written by the Middle Eastern Jewish astronomer Mašā'allāh (d. after 193/809), which probably reached MS 225 in elaborations produced by the Andalusian astronomer Maslama al-Mağrītī (d. 398/1007) and his pupil Ibn al-Saffār (d. 426/1035). Against this backdrop, MS 225 constituted a kind of Latin paraphrase or translation of one or several works of Arabic astronomy. Since Millàs i Vallicrosa dated MS 225 to a period prior to Gerbert's request of an astronomical work from Lupitus of Barcelona, MS 225 seemed to represent Lupitus' paraphrase or translation of one or several Arabic texts on astronomy mentioned in Gerbert's letter. The Liber de astrolabio, in turn, seemed to represent Gerbert's commented copy of MS 225.<sup>48</sup> Uta Lindgren then elaborated upon this hypothesis by proposing that the Liber de astrolabio came into being, when Gerbert set out to describe the physical model of an astrolabe in his own words and then inserted certain passages from the work, which he had demanded from Lupitus of Barcelona, i.e. MS 225 Santa María de Ripoll.<sup>49</sup>

[§17] This hypothesis was already questioned by André Van de Vyver, a contemporary of Millas Villacrosa, who dated the MS 225 to the eleventh century, thus rebutting the supposition that MS 225 can be equated with the astronomical work demanded by Gerbert from Lupitus of Barcelona.<sup>50</sup> In the past three decades, a large number of additional manuscripts, texts, and testimonies have been brought into play by an increasing number of scholarly studies on the earliest Latin literature dealing with the astrolabe. In 2007, David Juste summarized the state of research as follows:

"Whoever wishes to synthesize what has been written will mainly find material to tear one's hair out since the questions raised most frequently—related for example to the dating and the localization of the different phases, in which the corpus was elaborated, to the filiation of texts and manuscripts, the modalities of transmission from Arabic to Latin, or the role assumed by Gerbert and other known scholars—are far from being answered unanimously."<sup>51</sup>

quos Arabes Muzzewihae [cf. *mustawiyya*], id est aequales, dicunt, et Ezemenie [cf. *zamāniyya*], id est inaequales (...)." See Kunitzsch, Glossar, pp. 542, 561.

<sup>&</sup>lt;sup>46</sup> Liber de astrolabio, in: Gerbertus, *Opera mathematica*, ed. Bubnov, cap. XVII,1, p. 138: "Sunt praeter has duae Ganamalgurab, Alcasal vel Alhimech in Centauro, quibus Chaldei statis ad discernendas horas utuntur (...)."

<sup>&</sup>lt;sup>47</sup> *Liber de astrolabio*, in: Gerbertus, *Opera mathematica*, ed. Bubnov, pp. 109–147. Bubnov's position on the doubtful authenticity of this work is questioned by Lindgren, *Gerbert von Aurillac und das Quadrivium*, p. 32 FN 157, p. 35.

<sup>&</sup>lt;sup>48</sup> Millàs i Vallicrosa, *Assaig d'Història*, pp. 190–192; Lattin, Review of Assaig d'Història, pp. 436–437. This hypothesis is still repeated in Lindberg, Transmission of Greek and Arabic Learning, pp. 60–62.

<sup>&</sup>lt;sup>49</sup> Lindgren, Gerbert von Aurillac und das Quadrivium, p. 35; Lattin, Review, pp. 436–437.

<sup>&</sup>lt;sup>50</sup> Van de Vyver, Les premières traductions, p. 275 and FN 38.

<sup>&</sup>lt;sup>51</sup> Juste, *Les Alchandreana primitifs*, p. 3: "quiconque désireux de faire la synthèse de ce qui a été écrit trouvera surtout matière à s'arracher les cheveux car les questions les plus souvent posées, relatives par exemple à la datation et à la localisation des différentes phases d'élaboration du corpus, à la filiation des textes et des manuscrits, aux modalités de transmission de l'arabe au latin ou au rôle de Gerbert et d'autres savants connus sont loin de faire l'unanimité."

[§18] Notwithstanding this not very encouraging state of research, Juste managed to re-establish a certain degree of order. According to Juste, MS 225 Santa Maria de Ripoll is now generally dated to the eleventh century and thus eliminated from the chain of manuscripts that supposedly connected Reims and Catalonia in the lifetime of Gerbert. In addition, this new dating has again raised the question of Gerbert's authorship of the aforementioned book on the astrolabe.<sup>52</sup> This, however, Juste claims, does not compromise the role of Catalonia as the most important zone of Arabic science in the Latin West of the tenth century: in this region, he confirms, we find the earliest conserved Latin astrolabe as well as the Codex Vigilanus and the Codex Emilianus, the earliest Latin manuscripts to boast Hindu-Arabic numerals. Juste defines a "Corpus primitif" of Latin treatises on the astrolabe, which are clearly influenced by Arabic works, as is manifest, for example, in their use of transliterated Arabic terminology. This corpus consists of four works entitled De mensura astrolapsus, Sententie astrolabii, De mensura astrolabii, and De utilitatibus astrolabii. He assigns this corpus to the period between approximately 980 and the beginning of the eleventh century. The terminus post quem is defined by the fact that they draw on passages from al-Hwārizmī's (d. c. 232/847) treatise on the use of the astrolabe as well as on Ptolemy's Planispherium, both of which were quoted and used by the most important contemporary Arabic-Islamic astronomer in al-Andalus, i.e. Maslama al-Mağrītī (d. 398/1007). The latter carried out astronomical observations in 369/979, which also figure in some of these works. The *terminus ante quem* is defined by the fact that excerpts of these works are cited in writings which are generally assumed to have been produced in the first half of the eleventh century by Fulbert of Chartres (d. 1028), Ascelinus of Augsburg, and Hermann of Reichenau (d. 1054).<sup>53</sup> Although it is not possible to define the exact position of Gerbert and his engagement with astronomy and mathematics in relation to this corpus, it is clear that his efforts were affected by the same flow of information from al-Andalus via Catalonia to northern France and southwestern Germany. It is also clear that he personally contributed to this flow, first by studying in Catalonia between 967 and 970, then by demanding mathematical and astronomical works from Catalonian colleagues.

[§19] A further indication that Gerbert engaged at least indirectly with Latin versions of Arabic texts is found in Richer's description of an abacus used by Gerbert in his teaching at Reims. According to Richer,

"Gerbert employed the offices of a shield maker to make an abacus, i.e., a board fitted with divisions of measurement. He divided its length into 27 columns of which he marked nine with a numeral, these signifying all possible numbers. In the likeness of which [i.e., the columns marked with Arabic numerals] he made 1000 counters of horn, which, by being moved around through the 27 columns of the abacus, could show the multiplication or division of any number you like. This instrument enabled performing very quickly some mathematical operations such as division and multiplication. Whoever wishes to receive more instruction on this, may read the treatise which he addressed to the grammatician Constantine."<sup>54</sup>

[§20] Most important in this quote is Richer's reference to "nine numerals which signify all possible numbers" (*novem numero notas omnem numerum significantes*). What Richer seems

<sup>&</sup>lt;sup>52</sup> Juste, Les Alchandreana primitifs, pp. 4–5.

<sup>&</sup>lt;sup>53</sup> Juste, *Les Alchandreana primitifs*, pp. 5–6; Burnett, King Ptolemy, pp. 329–368.

<sup>&</sup>lt;sup>54</sup> Richerius, *Historiae*, ed. Hoffmann, lib. 3, cap. 54, p. 198: "abacum id est tabulam dimensionibus aptam opere scutarii effecit. Cuius longitudini, in XXVII partibus diductę, novem numero notas omnem numerum significantes disposuit. Ad quarum etiam similitudinem, mille corneos effecit caracteres, qui per XXVII abaci partes mutuati, cuiusque numeri multiplicationem sive divisionem designarent. Tanto compendio numerorum multitudinem dividentes vel multiplicantes, ut pre nimia numerositate potius intellegi quam verbis valerent ostendi. Quorum scientiam qui ad plenum scire desiderat, legat eius librum quem scribit ad C(onstantinum) grammaticum."; translation adapted from Zuccato, Arabic Singing Girls, p. 108.

to suggest here, is that Gerbert did not use Roman numbers composed of one or more individual signs (e.g. the numbers 4, 6–9, i.e. IV, VI, VII, VIII, IX). Rather, he used individual signs to represent the basic numerals that make up the decimal system, i.e. one individual sign for each number from 1 to 9. Gerbert was not the first to substitute Roman numerals with alternative signs. So far, however, Greek letters had been used as substitutes.<sup>55</sup> As already mentioned above, the *Codex Vigilanus*, a manuscript copied in the Catalonian monastery of Albelda in the year 976 contains the earliest depiction of Hindu-Arabic numerals in a Latin manuscript. These signs represent a western variant used on the Iberian Peninsula that are classified as "dust" (*gubār*) numerals (See Fig. 1).<sup>56</sup> Some of the names of the *gubār*-numerals are clearly of Arabic origin: this applies to the numbers four (*arbas < arba*), five (*quimas < hamsa*), and eight (*temenias < tamaniya*).<sup>57</sup>

Dof jubal acu tectomoracu sc mumit n allon mu

**Figure 1**: Codex conciliorum Albeldensis seu Vigilanus, Madrid: El Escorial, MS D.I.2, fol. 12v.<sup>58</sup>

[\$21] Based on the research of Menso Folkerts, Charles Burnett, Paul Kunitzsch, and others<sup>59</sup>, Marek Otisk has listed the early pictorial and textual evidence for *gubār*-numerals in Latin manuscripts produced east of the Iberian Peninsula. The earliest evidence dates from the late tenth and early eleventh centuries and includes

1. Bernelinus' *Liber abaci*, which—because it refers to "pope Gerbert"—is dated to the years 999–1003. According to its author, it replicates Gerbert's teaching on the abacus.<sup>60</sup>

<sup>&</sup>lt;sup>55</sup> Folkerts, Names and Forms, p. 246, 248.

<sup>&</sup>lt;sup>56</sup> See Kunitzsch, Transmission of Hindu-Arabic Numerals, pp. 11–14.

<sup>&</sup>lt;sup>57</sup> Buddhue, Origin of Our Numerals, pp. 256–267; Ball, *Short History of Mathematics*, p. 115; L'Huillier, Regards sur la formation progressive, p. 544; Allard, L'influence des mathématiques arabes, p. 200.

<sup>&</sup>lt;sup>58</sup> <u>https://upload.wikimedia.org/wikipedia/commons/3/3b/Codex\_Vigilanus\_Primeros\_Numeros\_Arabigos.jpg.</u> For a printed reproduction, see Szpiech, From Mesopotamia to Madrid, p. 44. The text runs: "Scire debemus in Indos subtilissimum ingenium habere. et ceteras gentes eis in arithmetica et geometrica. et ceteris liberalibus disciplinis concedere. Et hoc manifestum est in nobem figuris quibus designant unumquemque gradum. Cuiuslibet gradus quarum hec sunt forma. 987654321," i.e. "We should know that we find subtle inventiveness among the Indians and among several peoples, which they have given to them in arithmetics and geometry and other liberal disciplines. This becomes manifest in nine figures, each of which designate a specific [numeric] position. These are the forms of each position: 987654321." Note that the list of numbers still lacks zero.

<sup>&</sup>lt;sup>59</sup> Folkerts, Frühe Darstellungen, pp. 23–43; Folkerts, Names and Forms, pp. 245–265; Burnett, Abacus at Echternach, pp. 91–108; Kunitzsch, Transmission of Hindu-Arabic Numerals, pp. 3–22.

<sup>&</sup>lt;sup>60</sup> Bernelinus, Liber abaci, in: Gerbert, *Oeuvres*, ed. Olleris, p. 357; Gerbertus, *Opera mathematica*, ed. Bubnov, p. 383.

It contains an extensive description of the  $\dot{g}ub\bar{a}r$ -numbers and equates them to the Greek letters used hitherto as a substitute for Roman numerals.<sup>61</sup>

- 2. two eleventh-century manuscripts, which contain depictions of the abacus of Echternach, the latter dated to the 990s. These depictions contain  $gub\bar{a}r$ -numbers as well.<sup>62</sup>
- 3. a mathematical and computation manuscript from Bern from the late tenth century depicting an abacus. It does not contain *gubār*-numbers, but is very similar to the Echternach depictions. Moreover, it features the heading "Gerbertus Latio numeros abacique figuras," i.e. "Gerbert [gave] the numbers and the figures of the abacus to Latium," i.e. the Latin sphere.<sup>63</sup>
- 4. the depiction of the so-called Paris abacus from Fleury in a manuscript from the early eleventh century, which also features the *gubār*-numbers.<sup>64</sup>
- 5. the first pictorial depiction of an abacus in a mathematical manuscript, the so-called pseudo-Boethius *Geometria II*, from the first half of the eleventh century.<sup>65</sup>
- 6. the depiction of the so-called Vatican abacus, which seems inspired by the abacus from *Geometria II*. Whereas the manuscript dates from the late tenth century, the depiction of the abacus seems to have been inserted at the beginning of the eleventh century. It contains the signs and names of the *gubār*-numerals and again features the inscription "Gerbertus Latio numeros abacique figuras."<sup>66</sup>

[§22] Against this backdrop, it seems very plausible that Gerbert became acquainted with the original manuscript that served as the basis for the copy in the *Codex Vigilanus* (dated 976) during his period of study in Catalonia between 967 and 970, regarded the Western Arabic numerals as useful, and later used them to mark his particular form of abacus after he had returned to Reims.<sup>67</sup> His choice seems to have had an impact on several manuscripts of the early eleventh century, which all substituted the system of using Greek letters with the Western form of Hindu-Arabic numerals.<sup>68</sup>

[§23] Although Hindu-Arabic numerals had thus been introduced to the Latin West, their widespread use in arithmetic education did not become common until the twelfth century. During his studies in France, Adelard of Bath (d. ca. 1152) seems to have become acquainted with Gerbert's numeral system, before he turned to translating the astronomical tables of al-Hwārizmī from Arabic to Latin.<sup>69</sup> In calculations, Hindu-Arabic numerals were only used after the Latin translation of al-Hwārizmī's works by Gerald of Cremona (d. 1187) and Robert of Chester (fl. 1140s). In his *Liber abaci*, the Pisan Leonardo Fibonacci (d. after 1220) introduced a wider audience to this system, which, however, only became established in accounting and other pragmatic mathematical occupations in the course of the thirteenth to fourteenth centuries.<sup>70</sup>

<sup>&</sup>lt;sup>61</sup> Bernelinus, Liber abaci, ed. Olleris, *Oeuvres de Gerbert*, p. 361: "His igitur expeditis, ad ipsos caracteres veniamus, et quibus figuris praenotentur, ascribere properemus. Unitas, quae primus caracter dicitur, sic figuratur (...)."; Bernelin, *Libre d'abaque*, ed. Bakhouche and Cassinet, p. 25. See Otisk, Descriptions and Images, pp. 17– 19; Folkerts, Names and Forms of Numerals, pp. 246, 248.

<sup>&</sup>lt;sup>62</sup> Otisk, Descriptions and Images, pp. 19–20; Burnett, Abacus at Echternach, pp. 91–108.

<sup>&</sup>lt;sup>63</sup> Otisk, Descriptions and Images, pp. 20–22; Folkerts, Frühe Darstellungen, p. 28.

<sup>&</sup>lt;sup>64</sup> Otisk, Descriptions and Images, pp. 22–23; Folkerts, Names and Forms, pp. 252–253.

<sup>&</sup>lt;sup>65</sup> Otisk, Descriptions and Images, pp. 22–23; Folkerts, "Boethius" Geometrie II, pp. 83–94.

<sup>&</sup>lt;sup>66</sup> Otisk, Descriptions and Images, pp. 24–25.

<sup>&</sup>lt;sup>67</sup> Burnett, Semantics of Indian Numerals, p. 17; Qandīl, *Ğīrbirt "al-Bābā Sīlfastar al-<u>t</u>ānī,"* pp. 147–154.

<sup>&</sup>lt;sup>68</sup> That century witnessed the spread of around thirty-five arithmetic manuscripts, two of which seem to have been written in late tenth-century Lorraine and later attributed to Boethius, see Folkerts, Names and Forms, pp. 246, 248.

<sup>&</sup>lt;sup>69</sup> On Adelard's references to Gerbert, see Evans, A Note on the Regule abaci, pp. 33–35.

<sup>&</sup>lt;sup>70</sup> Folkerts, Zahlsysteme, cols 458–460.

[§24] Having established that the astronomical and mathematical sciences practised by Gerbert and in Gerbert's orbit boasted influences that can be classified as "Arabic," we must account for the possible sources in Muslim al-Andalus as well as the potential channels of transmission. This first takes us into the wider context of mathematical study in Muslim al-Andalus of the later tenth century.

[§25] In the tenth century, Umayyad Córdoba boasted one of the most developed intellectual climates in the early medieval period. In the ruling periods of 'Abd al-Rahmān III (r. 300–350/912–961) and his son al-Hakam II (r. 350–366/961–976), "ancient" and "modern" sciences began to flourish in al-Andalus as is amply documented in a history of the sciences (*Kitāb tabaqāt al-umam*) written around one century later by Ṣāʿid al-Andalusī (d. 462/1070), the  $q\bar{a}d\bar{i}$  of Toledo.<sup>71</sup> He reports the massive import of books from the Abbasid Middle East, provides lists of scholars working on various subjects, mentions translation initiatives, and highlights the supporting role of al-Hakam II, who pursued scientific interests himself, collected books and, according to various Arabic-Islamic sources, built up a library that was one of the largest and most impressive of his time.<sup>72</sup> Highly conducive to scholarly pursuits, this environment produced a number of respectable mathematicians, among others a certain Yaḥyā bin Yaḥyā, also known as Ibn al-Sumayna (d. 315/927), a specialist in arithmetics and astronomy, or Muḥammad b. Ismāʿīl (d. 330/943), also known as "the Wise" (*al-ḥakīm*).<sup>73</sup> Neither the Anglo-Saxon, Frankish, Lombard, nor the Iberian realms of the medieval Latin West boasted a capital or a scientific culture comparable to Umayyad Córdoba in the tenth century.

[§26] Engagement with the scientific culture of al-Andalus necessitated some kind of contact. While this was most easily available in the neighbouring Iberian realms, the Latin textual tradition suggests that Gerbert may have not only visited neighbouring Catalonia, but even Córdoba or places farther south. According to Adémar of Chabannes (d. 1034), Gerbert, "for the sake of wisdom, traversed (*lustrans*) first France, then Córdoba."<sup>74</sup> William of Malmesbury (d. 1143), in turn, speaks of "Spain" generally, but highlights the importance of Seville, wrongly identifying it as the capital of al-Andalus in Gerbert's time.<sup>75</sup> Again, we can assume that Gerbert visited Córdoba, the capital of Umayyad al-Andalus, but cannot provide any evidence.

[§27] We should consider, however, that the second half of the tenth century witnessed the establishment of direct diplomatic relations between the Ottonian court and Umayyad Córdoba: 'Abd al-Raḥmān III had sent a delegation to Otto I in 953, which was reciprocated by an Ottonian delegation headed by John, the latter abbot of Gorze in Lotharingia. He spent three years in Córdoba, interacting, among others with two important functionaries at the Umayyad court: 'Abd al-Raḥmān's Jewish confidant Ḥasdāy b. Šaprūt, and a certain Recemund, a Christian functionary capable of speaking Latin and Arabic who was responsible for accepting petitions to the caliph.<sup>76</sup> Recemund, possibly connected to the production of the bilingual Latin-Arabic "Calendar of Córdoba" (dated 961), was later sent as an envoy to the Ottonian court, befriended the Ottonian historiographer and bishop Liutprand of Cremona (sed. 961–*c*. 972),

<sup>&</sup>lt;sup>71</sup> Sāʿid al-Andalusī, *Kitāb ṭabaqāt al-umam*, ed. Cheikho, pp. 62–67.

<sup>&</sup>lt;sup>72</sup> Wasserstein, Library of al-Hakam II, pp. 375–382.

<sup>&</sup>lt;sup>73</sup> Qandīl, *Ğīrbirt "al-Bābā Sīlfastar al-<u>t</u>ānī,"* p. 160, on the basis of Ṣāʻid al-Andalusī, *Ṭabaqāt al-umam*, ed. Mu'nis, p. 87.

<sup>&</sup>lt;sup>74</sup> Ademarus Cabannensis, *Chronicon*, ed. Bourgain, lib. 3, cap. 31, p. 154.

<sup>&</sup>lt;sup>75</sup> Willelmus Malmesbiriensis, *Gesta regum Anglorum*, ed. Hardy, lib. 2, § 167, p. 272; William of Malmesbury, *Chronicle*, trans. Giles, lib. 2, cap. 10, p. 173. Seville only became a Muslim capital, in this case the Andalusian headquarters of the Almohad dynasty, in the twelfth and thirteenth century.

<sup>&</sup>lt;sup>76</sup> Johannes von St. Arnulf, *Vita Iohannis abbatis Gorziensis*, ed. Jacobsen (MGH SS rer. Germ. in us. School. 81), cap. 115–136, pp. 414–467; Jean, abbé de Saint-Arnoul, *La vie de Jean*, trans. Parisse, § 115–136, pp. 143–161; Walther, Der gescheiterte Dialog, pp. 20–44. Without producing evidence, Thompson, Introduction of Arabic Science into Lorraine, pp. 185–186, claims that the abbot of Gorze carried with him Arabic writings into Lorraine after returning from Córdoba.

and was later appointed bishop of Seville.<sup>77</sup> In addition to this, several Andalusian sources document the travels of the Andalusian Jew Ibrāhīm b. Yaʿqūb al-Isrāʾīlī to the Ottonian court and various regions of central Europe in the 960s or 970s.<sup>78</sup> Last but not least, the city of Córdoba sets the stage for one of the martyr dramas written by the Ottonian poetess Hrotsvit of Gandersheim (d. 1002).<sup>79</sup> All this shows that Umayyad Córdoba did not lie beyond the geographical, political, and imaginary horizon of Latin Christians from Central Europe, including Gerbert.<sup>80</sup>

[§28] What is also clear, is that the region of Catalonia, the transit region connecting Umayyad al-Andalus and Central Europe, boasted such intensive and largely peaceful relations with Córdoba in the second half of the tenth century that it can almost be classified as a client state of Umayyad al-Andalus.<sup>81</sup> According to the Andalusian historiographer Ibn Hayyān (d. 469/1076), William of Septimania, count of Toulouse (r. 844–850) and of Barcelona (r. 848-850) had already made an alliance with 'Abd al-Raḥmān II (r. 206–238/821–852) around 234/848–849, which, however, did not lead to the establishment of regular peaceful relations. A turning point seems to have occurred in 328/940, in which a peace treaty between 'Abd al-Raḥmān III and Count Sunyer of Barcelona, Girona, and Ausona (r. 911–947), renewed in the following year, was signed.<sup>82</sup> In this context, the count sent a delegation to the Umayyad court. One of the envoys, bishop Godemar of Girona (sed. 944–954) presented a book of Frankish history to the future caliph al-Hakam II.<sup>83</sup> Concerning the latter's reign, Ibn Hayyān reports frequent diplomatic dealings of Count Borrell II with Umayyad Córdoba in the years 360–364/970–974, i.e. the years immediately following Gerbert's stay in Catalonia.<sup>84</sup>

[§29] This shows that exchanges were particular intensive in the years preceding and immediately following Gerbert's visit to Catalonia.<sup>85</sup> Unfortunately, however, we have no concrete historiographical evidence for a transfer of astronomical or mathematical manuscripts from Córdoba to Catalonia in this period. This has given rise to several speculations about the possible transmitters of such knowledge to the scholarly circles among Catalonia's clergy. Generalizing to a high degree, some scholars have defined unnamed Mozarabs, i.e. Arabicized Christians under Muslim rule, as possible transmitters. Without providing any evidence, Margarita López Gómez invites us to "assume that Mozarabs dominated the *scriptoria* of Christian monasteries (San Millán de la Cogolla, Albelda, Silos, Santa María de Ripoll, and so on); we can imagine them seated there at their desks (...) painting miniatures or translating Arabic manuscripts into Latin or Romance, in order to impart the various Arab sciences to the world of Latin Christianity: the arithmetic of al-Khwārizmī, or books on astronomy like Mashallah's treatise on the astrolabe, in the monastery of Ripoll in the mid-tenth century."<sup>86</sup>

<sup>&</sup>lt;sup>77</sup> On the discussion of authorship, see *Le Calendrier de Cordoue de l'année 961*, ed. Dozy, pp. i–viii; Christys, *Christians in al-Andalus*, pp. 108–135, with the chapter "Recemund and the Calendar of Córdoba".

<sup>&</sup>lt;sup>78</sup> Jacob, *Berichte*, pp. 3–5, argues for 973; Miquel, L'Europe, pp. 1048–1064, esp. 1059, opts for 965; Engels, Reisebericht, pp. 413–422, for 961 or 966.

<sup>&</sup>lt;sup>79</sup> Hrotsvit, *Passio Pelagii*, ed. Winterfeld (MGH SS rer. Germ. in us. schol 34), pp. 52–62.

<sup>&</sup>lt;sup>80</sup> Darlington, Gerbert the Teacher, pp. 461–462.

<sup>&</sup>lt;sup>81</sup> Qandīl, *Ğīrbirt "al-Bābā Sīlfastar al-<u>t</u>ānī,"* pp. 34–35; Zuccato, Nobisque potius obaedire delegistis quam Hismahelitis, pp. 1–18.

<sup>&</sup>lt;sup>82</sup> Ibn Hayyān, *al-Muqtabas V*, ed. Chalmeta and Corriente, AH 328, pp. 454–455.

<sup>&</sup>lt;sup>83</sup> Al-Mas<sup>°</sup>ūdī, *Murūģ al-dahab / Les Prairies d'Or*, ed. Barbier de Meynard, Pavet de Courteille, Pellat, § 914– 916, pp. 147–148 (AR), pp. 344–345 (FR); König, *Arabic-Islamic Views*, pp. 194–195; Gümpel, Gerbert, p. 84.

<sup>&</sup>lt;sup>84</sup> Ibn Hayyān, al-Muqtabas fī ahbār balad al-Andalus [al-muqtabis VII], ed. al-Hağğī, AH 360–363, pp. 20–23, 168–169, 182.

<sup>&</sup>lt;sup>85</sup> Vgl. Juste, *Les Alchandreana primitifs*, pp. 234–235, König, *Arabic-Islamic Views*, pp. 301–302.

<sup>&</sup>lt;sup>86</sup> López Gómez, Mozarabs, p. 172. Many scholars assume without further ado that Catalonian monasteries contained Arabic manuscripts, e.g. Gainet, Voyage de Gerbert, pp. 218–232; Picavet, *Gerbert un pape philosophe*, pp. 30–31; Leflon, *Gerbert*, pp. 22–23; Riché, *Gerbert*, pp. 25–26.

aforementioned bishop Godmar of Girona, who could have carried manuscripts along with him on his way back from Córdoba to Catalonia in 328/940, especially if we surmise that bishop Atto of Vich and his successor Miró Bonfill (sed. 970–984) accompanied him.<sup>87</sup> Related to this, Marco Zuccato has proposed that the "erudite Joseph" mentioned in two of Gerbert's letters should be identified with Hasdāy b. Šaprūt alias "Abū Yūsuf," the erudite confidant of 'Abd al-Raḥmān III, who also played a role in bringing the astronomical works of the North African Jewish astronomer Dunāš b. Tamīm al-Qarawī (d. *c.* 360/971?) to al-Andalus.<sup>88</sup>

[§30] What is disconcerting in this context is that it is not possible to prove without doubt whether the libraries of Vich and Santa María de Ripoll contained works related to the quadrivium or Arabic sciences at the end of the tenth century. With regard to Vich, Karl Werner Gümpel proposed that such works may have existed as the private property of bishop Atto and were therefore not recorded in the library's catalogue.<sup>89</sup> With respect to Santa María de Ripoll, Zuccato emphasized that the monastery's booklists date from the eleventh century and therefore do not allow us to reconstruct the library's contents at the end of the tenth century.<sup>90</sup> Zuccato, who claims that "none of the astronomical texts included in the tenth-century manuscripts can explain Gerbert's astronomical teaching or his mastery of the mathematical sciences" has therefore resorted to the hypothesis that the Catalonian clergy as well as Gerbert mainly relied on "a non-written Arabic tradition of practical astronomy (...)."<sup>91</sup>

[§31] Whatever the case may be, it is difficult to deny that Gerbert of Aurillac—teacher of Reims, abbot of Bobbio, archbishop of Reims and Ravenna, pope—was involved in the transmission of astronomical and mathematical knowledge from Muslim al-Andalus to central European regions of the Latin-Christian sphere at the end of the tenth century in one way or another. At the same time, it is impossible to define, when, how, and under which conditions this transmission took place and to explain the exact role Gerbert played in it. However, contemporary letters and historiography, mathematical and astronomical works produced during and in the immediate aftermath of Gerbert's intellectual activities, and—last but not least—the later reception of his intellectual endeavours within the "dark legend" all point into the same direction: they all suggest intensive geographical, personal, and thematical links between Gerbert on the one, concrete documentary evidence for the transfer of astronomical and mathematical knowledge on the other side. In a certain way, the case of Gerbert thus epitomizes the still rather obscure early beginnings of an increasingly growing transfer of so-called "Arabic" knowledge to intellectual circles of the Latin-Christian sphere, that would really begin to bloom in the twelfth century.<sup>92</sup>

## Edition(s) & Translation(s)

*Historiae Francorum scriptores coaetanei*, ed. Andrea Du Chesne, vol. 2, Paris: Sébastien Cramoist, 1636, pp. 789–844 [edited on the basis of a lost manuscript], URL: <u>http://mdz-nbn-resolving.de/urn:nbn:de:bvb:12-bsb10327778-4</u>.

<sup>&</sup>lt;sup>87</sup> Gümpel, Gerbert, p. 84; Zuccato, Gerbert, p. 752.

<sup>&</sup>lt;sup>88</sup> Zuccato, Gerbert, pp. 754–755, also highlights that the similarities between Abu Yūsuf Hasdāy b. Ishāq b. Šaprūt and the biblical Joseph are striking: first, they were both Jews who achieved a primary political role (in Egypt and al-Andalus); second, they both became chief advisors of kings (Pharaoh and the caliph); third, they were both renowned for their wisdom and knowledge.

<sup>&</sup>lt;sup>89</sup> Gümpel, Gerbert, pp. 84–85; Zuccato, Gerbert, p. 748.

<sup>&</sup>lt;sup>90</sup> Zuccato, Gerbert, pp. 748–750.

<sup>&</sup>lt;sup>91</sup> Zuccato, Arabic Singing Girls, the Pope, pp. 99–120.

<sup>&</sup>lt;sup>92</sup> Burnett, Translation from Arabic to Latin, pp. 49–56.

Silvester II (Gerbertus), *Opera*, ed. Jean Paul Migne (Patrologia latina 139), Paris: Garnier, 1880, cols 202-287 [based on the edition of Duchesne)

Gerbert, *Oeuvres*, ed. Alexandre Olleris, Clermont-Ferrand: Thibaud, 1867, pp. 1–154, URL: <u>http://mdz-nbn-resolving.de/urn:nbn:de:bvb:12-bsb10685835-5</u>.

Gerbert, *Lettres de Gerbert (983-987)*, ed. Julien Havet (Collection de textes pour servir à l'étude et à l'enseignement de l'histoire, 6), Paris: Alphonse Picard, 1889, URL: https://gallica.bnf.fr/ark:/12148/bpt6k556198.texteImage.

Gerbertus, *Opera mathematica (972-1003)*, ed. Nicolaus Bubnov, Hildesheim: Olms, 1963 [reprint of Berlin: Friedländer & Sohn, 1899], pp. 101–102, URL: https://archive.org/details/operamathematic00bubngoog.

Gerbert, *The Letters of Gerbert with his Papal Privileges as Sylvester II*, trans. Harriet P. Lattin, New York: Columbia University Press, 1961, URL: https://archive.org/details/lettersofgerbert0000sylv/page/n5/mode/2up.

Gerbert von Reims, *Briefsammlung*, ed. Fritz Weigle (MGH Briefe d. dt. Kaiserzeit, 2),, Weimar: Hermann Böhlaus Nachfolger, 1966, pp. 23–261, URL: <u>https://www.dmgh.de/mgh\_briefe\_d\_dt\_kaiserzeit\_2/index.htm</u>.

Gerbert d'Aurillac, *Correspondance*, ed./trans. Pierre Riché, 2 vols, Paris: Les Belles Lettres, 1993.

Gerbert d'Aurillac, *Lettere (983-997)*, ed. Paolo Rossi, Pisa: Edizioni Plus, Pisa University Press, 2009.

### **Cited Primary Sources**

Ademarus Cabannensis, Chronicon, ed. P. Bourgain, Turnhout: Brepols, 1999.

Al-Masʿūdī, *Les Prairies d'Or*, trans. Charles Barbier de Meynard, Abel Pavet de Courteille, Charles Pellat, 5 vols, Paris: Société Asiatique, 1962–1997.

Al-Mas ʿūdī, *Murūğ al-dahab wa-ma ʿādin al-ǧawhar*, ed. Charles Barbier de Meynard, Abel Pavet de Courteille, Charles Pellat, 7 vols, Beirut: Publications de l'Université Libanaise, 1965–1979.

Beno, *Gesta Romanae aecclesiae contra Hildebrandum*, ed. Karl Francke (MGH Libelli de lite, 2), Hanover: Hahn, 1892.

Gerbert of Aurillac, *De Rationale et Ratione Uti. Translation and Context*, trans. Suzanne Ronk, MA-thesis, University of Oklahoma, 2003.

Gerbertus Aureliacensis, *De rationali et ratione uti*, ed. Patrizia Stoppacci, *Clavis Gerbertiana*, Florence: SISMEL, 2016.

Hrotsvit, *Passio Pelagii*, ed. Paul von Winterfeld (MGH SS rer. Germ. in us. schol 34), Berlin: Weidmann, 1965, pp. 52–62.

Ibn Hayyān, *al-Muqtabas fī ahbār balad al-Andalus [al-muqtabis VII]*, ed. 'Abd al-Rahmān 'Alī al-Haǧǧī, Beirut: Dār al-Taqāfa, 1965.

Ibn Hayyān, *al-Muqtabas li-Ibn Hayyān al-Qurṭubī (al-ǧuz ʾ al-ḫāmis) [al-muqtabis V]*, ed. Pedro Chalmeta and Federico Corriente, Madrid: al-maʿhad al-isbānī al-ʿarabī, 1979.

Jean, abbé de Saint-Arnoul, *La vie de Jean, abbé de Gorze*, trans. Michel Parisse, Paris: Picard, 1999.

Johannes von St. Arnulf, *Vita Iohannis abbatis Gorziensis*, ed. Peter Christian Jacobsen (MGH, SS rer. Germ. in us. School. 81), Wiesbaden: Harrassowitz, 2016.

*Le Calendrier de Cordoue de l'année 961*, ed. Reinhart Dozy, Leiden: Brill, 1873, URL: <u>http://mdz-nbn-resolving.de/urn:nbn:de:bvb:12-bsb11156573-1</u>

Liber Pontificalis, ed. Louis Duchèsne, 2 vols, Paris: Ernest Thorin, 1886 / 1892.

Richerius Remensis, *Historiarum libri quattuor*, ed. Hartmut Hoffmann (MGH SS 38), Hanover: Hahn, 2000.

Șā'id al-Andalusī, *Kitāb țabaqāt al-umam*, ed. Louis Cheikho, Beirut: Imprimérie catholique, 1912.

Ṣāʿid al-Andalusī, *Ṭabaqāt al-umam*, ed. Husayn Mu'nis, Cairo: Dār al-maʿārif, 1993.

Willelmus Malmesbiriensis, *Gesta regum Anglorum*, ed. Thomas Duffus Hardy, London: Samuel Bentley, 1840.

William of Malmesbury, *Chronicle of the Kings of England*, trans. J. A. Giles, London: Bohn, 1847.

#### **Cited & Additional Literature**

Allard, A.: L'influence des mathématiques arabes dans l'Occident médiéval, in: Roshdi Rashed, Régis Morelon (eds), *Histoire des sciences arabes*, vol. 2.: *Mathématiques et physique*, Paris: Le Seuil, 1997, pp. 199–229.

Ball, W. W. Rouse: A Short Account of the History of Mathematics, London: Dover Publishers, 1908.

Beaujouan, Guy: Étude paléographique sur la rotation des chiffres et l'emploi des apices du X<sup>e</sup> au XII<sup>e</sup> siècle, in: *Revue d'histoire des sciences et de leurs applications* 1/4 (1948), pp. 301–313, URL: <u>https://www.jstor.org/stable/23904137</u>.

Buddhue, John Davis: The Origin of Our Numerals, in: *The Scientific Monthly* 52 (1941), pp. 256–267.

Bur, M.: Richer v. Reims, in: Lexikon des Mittelalters 7 (1995), cols 830-831.

Burnett, Charles: The Abacus at Echternach in ca. 1000 AD, in: Charles Burnett (ed.), *Numerals and Arithmetic in the Middle Ages*, Aldershot: Variorum, 2010, pp. 91–108.

Burnett, Charles: The Semantics of Indian Numerals in Arabic, Greek and Latin, in: *Journal of Indian Philosophy* 34/1-2 (2006), pp. 15–30, URL: <u>https://www.jstor.org/stable/23496952</u>.

Burnett, Charles: Translation from Arabic to Latin in the Middle Ages, in: Harald Kittel (ed.), *Übersetzung: Ein internationales Handbuch zur Übersetzungsforschung*, vol. 2, Berlin: De Gruyter, 2007, pp. 49–56.

Cammarosano, Paolo: Gerbert et l'Italie de son temps, in: Nicole Charbonnel, Jean-Eric Iung (eds), *Gerbert l'Européen*, Aurillac: Société des lettres, sciences et arts La Haute Auvergne, 1997, pp. 103–111.

Chandler, Cullen J.: *Charlemagne's Last March: The Political Culture of Carolingian Catalonia*, 778-987, PhD thesis, Purdue University, 2003.

Charbonnel, Nicole: La ville de Gerbert: les origines, in: Nicole Charbonnel, Jean-Eric Iung (eds), *Gerbert l'Européen*, Aurillac: Société des lettres, sciences et arts de La Haute Auvergne, 1997, pp. 58–65.

Charles Burnett, King Ptolemy and Alchandreus the Philosopher: The Earliest Texts on the Astrolabe and Arabic Astrology at Fleury, Micy, and Chartres, in: *Annals of Science* 55 (1998), pp. 329–368.

Christys, Ann: Christians in al-Andalus (711-1000), Abingdon: Routledge, 2007.

Darby, H. C.: A Note on an Early Treatise on the Astrolabe, in: *The Geographical Journal* 85 (1935), pp. 179–181.

Darlington, Oscar G.: Gerbert the Teacher, in: *American Historical Review* 52 (1947), pp. 456–479.

Darlington, Oscar G.: Gerbert, "Obscuro Loco Natus," in: *Speculum* 11 (1936), pp. 509–520, URL: <u>https://www.jstor.org/stable/2848543</u>.

David Juste, Les Alchandreana primitifs: Étude sur les plus anciens traités astrologiques latins d'origine arabe ( $X^e$  siècle), Leiden: Brill, 2007.

De Mayo, Courtney: The Students of Gerbert of Aurillac's Cathedral School at Reims: An Intellectual Genealogy, in: *Medieval Prosopography*, 27 (2012), pp. 97–117.

DeMayo, Courtney: Ciceronian *amicitia* in the Letters of Gerbert of Aurillac, in: *Viator* 38/2 (2007), pp. 319–337.

DeMayo, Courtney: Fulbert of Chartres, bishop, 960/970–1028, in: *International Encyclopaedia for the Middle Ages-Online. A Supplement to LexMA-Online*. Turnhout: Brepols Publishers, 2012, URL: <u>http://apps.brepolis.net/lexiema/test/Default2.aspx</u>.

Engels, Peter: Der Reisebericht des Ibrahim ibn Ya'qub (961/966), in: Anton von Euw, Peter Schreiner (eds): *Kaiserin Theophanu. Begegnung des Ostens und Westens um die Wende des ersten Jahrtausends*, Cologne: Schnütgen Museum, 1991, pp. 413–422.

Evans, Gillian: A Note on the *Regule abaci*, in: Charles Burnett (ed.), *Adelard of Bath. An English Scientist and Arabist of the Twelfth Century*, London: Warburg Institute, 1987, pp. 33–35

Faulk, Josephine M.: *Bishop Fulbert of Chartres (1006-1028): A Political Biography*, PhD thesis, University of Illinois, Chicago, 1991.

Flusche, Anne Marie: *The Life and Legend of Gerbert of Aurillac: The Organ Builder Who Became Pope Silvester II*, Lewiston: Edwin Mellen Press, 2005.

Folkerts, Menso: Zahlsysteme, -zeichen, in: Lexikon des Mittelalters 9 (1998), cols 457-460.

Folkerts, Menso (ed.), "Boethius" Geometrie II: ein mathematisches Lehrbuch des Mittelalters, Wiesbaden: Steiner, 1970.

Folkerts, Menso: Frühe Darstellungen des Gerbertschen Abakus, in: Raffaella Franci, Paolo Pagli, Laura Toti Rigatelli (eds), *Itinera mathematica: Studi in onore di Gino Arrighi*, Siena: Centro studi sulla matematica medioevale, 1996, pp. 23–43.

Folkerts, Menso: Remarks on Mathematical Terminology in Medieval Latin: Greek and Arabic Influences, in: *Bulletin Du Cange* 63 (2005), pp. 149–160.

Folkerts, Menso: The Names and Forms of the Numerals on the Abacus in the Gerbert Tradition, in: Flavio Nuvolone (ed.), *Gerberto d'Aurillac da Abate di Bobbio a Papa dell'Anno 1000*, Bobbio: Associazione culturale Amici di Archivum Bobiense, 2001, pp. 245–265.

Gainet, M.: Le Voyage de Gerbert en Espagne, in: *Travaux de l'Académie de Reims* 15 (1851), pp. 218–232.

Gümpel, Karl Werner: Gerbert von Aurillac und Spanien, in: Archivum Bobiense 24 (2002), pp. 79–96.

Hugonnard-Roche, Henri: The Influence of Arabic Astronomy in the Medieval West, in: Roshdi Rashed (ed.), *Encyclopedia of the History of Arabic Science*, vol. 1: *Astronomy – Theoretical and Applied*, London: Routledge, 1996, pp. 287–308.

Jacob, Georg: Arabische Berichte von Gesandten an germanische Fürstenhöfe aus dem 9. und 10. Jahrhundert, Berlin: De Gruyter, 1927.

Juillet, Jacques: Du lieu de naissance de Gerbert, pape Sylvestre II de l'an Mil (938-1003), in: *Lemouzi* 161 (2002), pp. 146–150.

König, Daniel G.: 621: Isidore of Seville on the Origins of the Term "Saracens," in: *Transmediterranean History* 1.1 (2019), DOI: <u>https://doi.org/10.18148/tmh/2019.1.1.8</u>.

Kunitzsch, Paul: Glossar der arabischen Fachausdrücke in der mittelalterlichen europäischen Astrolabliteratur, in: *Nachrichten der Akademie der Wissenschaften in Göttingen. I: Philologisch-historische Klasse* 11 (1982), S. 459–571.

Kunitzsch, Paul: Les relations scientifiques entre l'Occident et le monde arabe à l'époque de Gerbert, in: Nicole Charbonnel, Jean-Eric Iung (eds), *Gerbert l'Européen*, Aurillac: Société des lettres, sciences et arts La Haute Auvergne, 1997, pp. 193–203.

Kunitzsch, Paul: The Transmission of Hindu-Arabic Numerals Reconsidered, in: Jan P. Hogendijk and Abdelhamid I. Sabra (eds), The Enterprise of Science in Islam: New Perspectives, Cambridge/MA: MIT Press, 2003, pp. 3–22.

L'Huillier, Hervé: Regards sur la formation progressive d'une langue pour les mathématique dans l'Occident médiéval, in: Danielle Jacquart (ed.), *Comprendre et maîtriser la nature au moyen age*, Geneva: Droz, 1994, pp. 541–555.

Lattin, Harriet Pratt: Lupitus Barchinonensis, in: *Speculum* 7/1 (1932), pp. 58–64, URL: <u>https://www.jstor.org/stable/2848321</u>.

Lattin, Harriet Pratt: Review of Assaig d'Història de les Idees Fisiques I Matematiques a la Catalunya Medieval by J. Millàs Vallicrosa, in: *Speculum* 7 (1932), pp. 436–438.

Lauranson-Rosaz, Christian: Entre deux mondes: L'Auvergne de Gerbert, in: Nicole Charbonnel, Jean-Eric Iung (eds), *Gerbert l'Européen*, Aurillac: Société des lettres, sciences et arts La Haute Auvergne, 1997, pp. 33–52.

Leflon, Jean: *Gerbert: humanisme et chrétiené au X<sup>e</sup> siécle*, Saint-Wandrille: Éditions de Fontenelle, 1946.

Lindberg, David C.: The Transmission of Greek and Arabic Learning to the West, in: David C. Lindberg (ed.), *Science in the Middle Ages*, Chicago: University of Chicago Press, 1978, pp. 52–90.

Lindberg, David C.: The Transmission of Greek and Arabic Learning to the West, in: David C. Lindberg (ed.), *Science in the Middle Ages*, Chicago: University of Chicago Press, 1978, pp. 52–90.

Lindgren, Ptolemée chez Gerbert, in: Michele Tosi (ed.), *Gerberto. Scienza, storia e mito*, Bobbio: Associazione culturale Amici di Archivum Bobiense, 1985, pp. 619–644.

Lindgren, Uta: Gerbert von Aurillac und das Quadrivium, in: *Spanische Forschungen der Görresgesellschaft zur Kulturgeschichte Spaniens* 26 (1971), pp. 151–200.

Lindgren, Uta: Gerbert von Aurillac und das Quadrivium. Untersuchungen zur Bildung im Zeitalter der Ottonen, Wiesbaden: Steiner, 1976.

López Gómez, Margarita: The Mozarabs: Worthy Bearers of Islamic Culture, in: Salma Khadra Jayyusi (ed.), *The Legacy of Muslim Spain*, Leiden: Brill, 1994, pp. 171–175.

Millàs i Vallicrosa, Josep Maria: Assaig d'Historia de les Idees Fisiques i Matematiques a la Catalunya Medieval, Barcelona: Institucio Patxot, 1931.

Miquel, André: L'Europe occidentale dans la relation arabe d'Ibrahim b. Yaʿqub (X<sup>e</sup> siècle), in: *Annales* 21/5 (1966), pp. 1048–1064.

Moehs, Teta Ernestine: Gerbert of Aurillac as Link Between Classicism and Medieval Scholarship, in: Michele Tosi (ed.), *Gerberto. Scienza, storia e mito*, Bobbio: Associazione culturale Amici di Archivum Bobiense, 1985, pp. 331–350

Nuvolone, Flavio G.: Influenze islamiche nell'astronomia di Gerberto?, in: *Archivum Bobiense* 24 (2004), pp. 599–655.

Otisk, Marek: Descriptions and Images of the Early Medieval Latin Abacus, in: *Średniowiecze Polskie i Powszechne* 11 (2015), pp. 13–35, URL: <u>https://www.ceeol.com/search/article-detail?id=330164</u>.

Passalaqua, Gerberto di Aurillac corretore di manoscritti classici, in: Pierre Lardet (ed.), *La tradition vive: Mélanges d'histoire des textes en l'honneur de Louis Holtz*, Turnhout: Brepols, 2003, pp. 317–320.

Picavet, François: *Gerbert un pape philosophe: d'après l'histoire et d'après la légende*, Paris: Ernest Leroux, 1897.

Qandīl, Sāmir Sayyid Muḥammad Husayn: *Ğīrbirt "al-Bābā Sīlfastar al-<u>t</u>ānī" wa-dawruhu fī l-ḥayāt al-siyāsiyya wa-l-<u>t</u>aqafiyya fī ġarb Ūrubbā (945-1003 m.) [Pope Sylvester II and his Role in the Political and Cultural Life in Medieval Europe], MA Thesis, Faculty of Arts, Ain Shams University, Cairo, 2009.* 

Riché, Pierre: Gerbert d'Aurillac et Bobbio, in: Flavio G. Nuvolone (ed.), *Gerberto d'Aurillac da abate di Bobbio a papa dell'anno 1000*, Bobbio: Associazione culturale Amici di Archivum Bobiense, 2001, pp. 49–64.

Riché, Pierre: Gerbert d'Aurillac révélé par sa correspondance, in: Pierre Riché (ed.), Les lumières de l'an mille, Paris: CNRS, 2014, pp. 85–92.

Ricklin, Thomas: Der Philosoph als Nekromant. Gerbert von Aurillac (Silvester II.) und Vergil im europäischen Hochmittelalter, in: *Interfaces* 1 (2015), pp. 236–264, URL: <u>https://riviste.unimi.it/interfaces/article/view/4933/5063</u>.

Ronk, Suzanne van: *Gerbert of Aurillac and "De Rationale et Ratione Uti": Translation and Context*, MA. thesis, University of Oklahoma, 2003.

Sigismondi, Costantino (ed.), *Culmina Romulea. Fede e Scienza in Gerberto, Papa Filosofo*, Rome: Ateneo Pontificio Regina Apostolorum, 2008.

Stock, Brian: Science, Technology, and Economic Progress in the Early Middle Ages, in: David C. Lindberg (ed.), *Science in the Middle Ages*, Chicago: University of Chicago Press, 1978, pp. 1–51.

Szpiech, Ryan: From Mesopotamia to Madrid: The Legacy of Ancient and Medieval Science in Early Modern Spain, in: Enrique Garcá Santo-Tomás (ed.), *Science on Stage in Early Modern Spain*, Toronto: University of Toronto Press, pp. 25–57.

Thompson, James Westfall: The Introduction of Arabic Science into Lorraine in the Tenth Century, in: *Isis* 12 (1929), pp. 184–193, URL: <u>https://www.jstor.org/stable/224784</u>.

Uhlirz, Mathilde: Studien zu Gerbert von Aurillac, in: Archiv für Urkundenforschung 11 (1930), pp. 391–422.

Uhlirz, Mathilde: Studien zu Gerbert von Aurillac, in: Archiv für Urkundenforschung 13 (1935), pp. 437–474.

Van de Vyver, André: Les premières traductions latines (X<sup>e</sup>-XI<sup>e</sup> s.) de traités arabes sur l'astrolabe, in: F. Quicke (ed.), *Premier Congrès International de Géographie Historique, vol.* 2: *Mémoires*, Brussels: Falk, 1931, pp. 266–290, URL: <u>http://www.mgh-bibliothek.de/dokumente/a/a147234.pdf</u>.

Walther, Helmut G.: Der gescheiterte Dialog. Das Ottonische Reich und der Islam, in: Albert Zimmermann and Ingrid Craemer-Ruegenberg (eds), *Orientalische Kultur und europäisches Mittelalter*, Berlin and New York: Walter de Gruyter, 1985, pp. 20–44.

Wasserstein, David: The Library of al-Hakam II al-Mustansir and the Culture of Islamic Spain, in: *Manuscripts of the Middle East* 5 (1990-1991), pp. 99–105.

Werner, Karl Ferdinand: Constantin v. Fleury, in: *Lexikon des Mittelalters*, Stuttgart: Metzler, 1977-1999, vol. 3, cols 169–170.

Zimmermann, Michel: La Catalogne de Gerbert, in: Nicole Charbonnel, Jean-Eric Iung (eds), *Gerbert l'Européen*, Aurillac: Société des lettres, sciences et arts La Haute Auvergne, 1997, pp. 79–101.

Zuccato, Marco: "Nobisque potius obaedire delegistis quam Hismahelitis": The Political Predicament of Tenth-century Catalonia", in: *Journal of Medieval Iberian studies* 11 (2019), pp. 1–18.

Zuccato, Marco: Arabic Singing Girls, the Pope, and the Astrolabe. Arabic Science in Tenth-Century Latin Europe, in: *Viator* 45/1 (2014), pp. 99–120.

Zuccato, Marco: Gerbert of Aurillac and a Tenth-Century Jewish Channel for the Transmission of Arabic Science to the West, in: *Speculum* 80 (2005), pp. 742–763, URL: <u>https://www.jstor.org/stable/20463383</u>.