

A variable force analysis of positive polarity neg-raising modals¹

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Abstract. I develop a new analysis of “weak necessity” modals *should* and *supposed to* that is motivated by novel observations of extra-weak meanings in some environments. I argue that this evidence of weak readings suggests that these modal verbs exhibit a type of variable force. Sentences with weak necessity modals express universal force in positive sentences and existential force under negation. The analysis will build on an analogy with free-choice disjunction that assumes a basic weak meaning that strengthens in upward-entailing environments (Fox, 2007; Bassi and Bar-Lev, 2016). I hypothesize that the precise distribution of the strengthened readings is governed by the polarity-sensitive nature of the modals (cf. Iatridou and Zeijlstra, 2013; Homer, 2015). In particular, I argue that the polarity sensitivity of the modals is the result of the association of their domains with a covert *even* (cf. Lahiri, 1995; Crnič, 2014, 2019 for NPI *any*). This hypothesis makes intricate predictions about the range of readings that should be observed in various logical environments that I show to be borne out. I also argue that the analysis provides a natural link between neg-raising and weak necessity.

Keywords: Neg-raising, modals, polarity sensitivity, variable force, free choice

1. Introduction

Weak necessity (WN) modals in English include *should*, *ought*, *supposed to*. (discussed in various subgroups by Sloman, 1970; Horn, 1972; von Fintel and Iatridou, 2008; Homer, 2015, a.o.). They are called “weak necessity” because they are intuitively weaker than “strong necessity” *must*, *have to*, *required to*. For example, a sequence of an asserted WN modal with a negated strong necessity modal is not contradictory, while it is with two strong necessity modals.

- (1) a. You ought to do the dishes, but you don't have to.
- b. #You must do the dishes, but you don't have to.
- (von Fintel and Iatridou 2008)

Also, sequences like the following suggest that the strong necessity *have to* is more informative than the WN *should* / *supposed to*.

- (2) a. {You should / are supposed to} go, in fact you have to.
- b. #You have to go, in fact {you should / are supposed to.}
- (Homer, 2015 building on Horn, 1972)

Despite the weakness, a predominant view in recent work analyzes WN modals as expressing universal quantificational force (von Fintel and Iatridou, 2008; Rubinstein, 2012; Iatridou and Zeijlstra, 2013; Homer, 2015).² The relative weakness is then attributed to universal quantification over a smaller set of worlds (von Fintel and Iatridou, 2008 building on Sloman, 1970).

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²See von Fintel and Iatridou 2008 and Copley (2006) for arguments against a possible analysis (attributed to Horn (1972)) that analyzes weak necessity modals as expressing the quantificational force of the quantifier *most*.

1.1. Weak necessity modals and neg-raising

It has been noted that negated WN modals give rise to a meaning that is stronger than what is expected from the negation of a universally quantified statement. As indicated in the margins, some authors describe this as universal force taking obligatory wide scope over negation, analyzing these modals as positive-polarity items (Iatridou and Zeijlstra, 2013; Homer, 2015).

- (3) a. Amanda shouldn't / isn't supposed to leave, # but it's okay if she does.³ $\square > \neg$
 ($\approx$ *Amanda should / is supposed to not leave.*)
 b. Amanda isn't required / doesn't have to leave, but it's okay if she does. $\neg > \square$
 (*There's no requirement for Amanda to leave*)

WN modals even appear to take semantic scope over negation in a higher clause (provided the matrix verb is neg-raising (NR)) These 'cyclic neg-raised' readings in (4) are taken as evidence for the generalization that weak necessity modals are also neg-raising predicates (NRPs). (Horn, 1978, 1989; Homer, 2015; Rubinstein, 2014).

- (4) a. Hana doesn't think that Tom is supposed to leave *think* $> \square > \neg$
 b. Hana doesn't think that Tom should leave.
 c. *Paraphrasable as:* Hana thinks that Tom [%]is supposed to / should not leave.
 (5) a. Hana doesn't think that Tom is required to / ? must leave. *think* $> \neg > \square$
 b. *NOT paraphrasable as:* Hana thinks that Tom is required to / must not leave.

The goal of this article is to present novel data that motivates a new analysis of the properties of weak necessity modals described above. I will argue for a characterization of these modals as expressing a type of variable force. Sentences with *should* and *supposed to* express universal force in positive sentences and existential force under negation. The analysis will integrate recent work on free-choice phenomenon and negative polarity.

2. New Data

The following shows that *should* and *supposed to* can give rise to extra-weak existential-like readings.

- (6) **Context:** Walking through tunnels to a talk on campus, we find ourselves in a basement area among potentially dangerous supplies (obviously not the best place to be).
 a. Should we be down here?
 b. Are we supposed to be down here?

Intuitively, the speaker of (6) is not asking if this is the optimal / required place to be. Instead the modal statement can be paraphrased with '*is it okay / appropriate to be down here?*'. This appears to only work with the NR modals, as the following don't exhibit the same weak reading:

- (7) a. Are we required to be down here?
 b. Must we be down here?

While *should/supposed to* are considered 'weak necessity' modals, as described in section 1, the readings above are much weaker than normally attested, as shown by the infelicity of the

³As noted in Homer (2015) testing for contradictions in the sentences in (3) require the particular accessibility relation to be strictly constant across conjuncts.

positive answers to the questions in the context.

- (8) a. # Yeah, we should / we're supposed to (be here).
 b. ✓ Yeah, It's fine / okay (to be here)
 c. ✓ No, we shouldn't / aren't supposed to (be here)

In order to delineate these super weak readings attested in the questions in (6) from basic weak necessity readings in positive declaratives like (8a) , I will refer to them as “existential readings.”⁴ Existential readings are also attested in embedded questions, and under some negated non-NR universal attitude verbs.⁵

- (9) **Context:** My housemate wants me to pick something up at the office, but...
 a. It's not clear whether we're supposed to be on campus without a special permit.
 b. I'm not sure that we should be on campus without a special permit.

In addition to matrix and embedded questions, existential readings can also be detected in the scope of the downward-entailing (DE) presupposition trigger *no longer* (c.f. Staniszewski, 2019 building on observations in Homer, 2015). To see how this works, I assume that *no longer* presupposes that the clause in its scope denotes a proposition that used to be true (for example '*I no longer live in Cambridge*' presupposes that *I used to live in Cambridge*). This being the case, the WN modal in the scope of *no longer* in (10a) presupposes the weaker (10b), as opposed to (10c).

- (10) a. You are no longer supposed to be on campus without a special permit.
 b. **Presupposed:** It used to be that it was okay to be on campus without a permit.
 c. **Not presupposed:** It used to be that you were supposed to be on campus without a permit.

I take this, as well as the data in (6) and (9) above, to show that existential readings are attested for *should* and *supposed to* in a variety of non upward-entailing (UE) environments.

2.1. Connection to negation data

The core idea of the present analysis is that the negation data and the new data all follow from assuming a basic weak existential meaning for *should* / *supposed to*. The apparent wide scope of these modals over negation is in fact negation taking wide scope over this weak basic meaning (as $\neg \Diamond \Leftrightarrow \Box \neg$).

- (11) a. Amanda shouldn't / isn't supposed to leave. $\neg > \Diamond$
 b. Hana doesn't think that Tom should / is supposed to leave. **think** $> \neg > \Diamond$

Given the assumption of a basic existential meaning revealed by the data above, developing a full account will require answering the following questions:

⁴There appears to be variation among languages w.r.t. whether or not these existential readings are available for *should* / *supposed to* equivalents. An exploration of the cross-linguistic patterns is a topic for future work.

⁵An anonymous SuB reviewer notes that the existential readings appear to require some type of rhetorical effect (as not all polar questions with WN modals give rise to existential readings.) I discuss these considerations and how to account for them within the proposed analysis in section 6.1.

Question1 : How do we generate the attested stronger meanings in positive sentences that express universal force?

- **Proposal:** The stronger meaning is an enriched meaning generated by exhaustification. (Following work on free-choice disjunction (Fox, 2007 a.o.) and an extensions of FC to a modal domain (Bassi and Bar-Lev, 2016)).

Question 2: How is the system constrained in a way that generates the precise distribution of the various readings?

- **Proposal:** I assume that these modals are polarity sensitive items (cf. Iatridou and Zeijlstra, 2013; Homer, 2015). The particular view of polarity-sensitivity will be adopted from Crnič (2014, 2019)’s analysis of PSI *any* in which the domain of the modals associate with a covert ‘even.’

Section 3 introduces the basic ingredients of the analysis and addresses the first question above. Section 4 addresses the second question and the technical implementation of polarity sensitivity. Sections 5 and 6 discuss a range of predictions that the analysis makes for WN modals in various logical environments.

3. Analysis

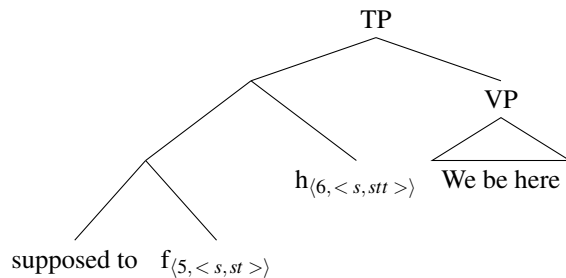
3.1. Basic existential meaning

I assume a Kratzerian framework in which modals are quantifiers over sets of worlds. The set they quantify over is determined by a modal base and an ordering source. The ordering source (h) is a function that takes an evaluation world and returns a set of propositions, and the modal base (f) takes an evaluation world and returns a set of worlds. The ordering source is used to rank the worlds in the modal base, and the *max* function picks out a subset that contains the most highly ranked worlds given the ordering (which are referred to as the favored worlds).^{6,7} In (12), I assume that *should* / *supposed to* existentially quantify over the favored worlds.

$$(12) \quad \llbracket \text{supposed} \rrbracket^{g,w} = \lambda f_{\langle s, st \rangle}. \lambda h_{\langle s, stt \rangle}. \lambda p_{\langle s, t \rangle}. \exists w' \in (\max_{h(w)} f(w)) [p(w')=1]$$

I assume that the modal base and ordering source functions are represented in the syntax by silent pronouns as in (13). Given this LF, the context in (14) generates the meaning in (15).

(13) We are supposed to be here



⁶For any set of propositions P , we define a strict partial order $<_p$:

$\forall w', w'' : (w' <_p w'' \text{ iff } \forall p \in P (w' \in p \rightarrow w'' \in p) \text{ and } \exists p \in P (w' \in p \ \& \ w'' \notin p))$

⁷For a given strict partial order $<_p$ on worlds, define the selection function $\max_p$ that selects the set of $<_p$ -best worlds from any set X : $\forall x \subseteq W : \max_p(X) = \{w \in X : \neg \exists w' \in X : w' <_p w\}$.

- (14) Let c be a context in which:
- a. $g(5) = f$ = the modal base function that assigns to any world w the set of worlds compatible with the relevant circumstances in w
 - b. $g(6) = h$ = the ordering source function that assigns to any world w the set of propositions representing the rules / ideals in w
- (15) $\wedge \llbracket (13) \rrbracket^{g,w} = \lambda w. \exists w' \in (\max_{g(6)(w)} g(5)(w)) [\text{We are here in } w']$

A paraphrase of (15) given context c says that ‘*we are here*’ is true in at least one world in the set of worlds maximally consistent with the rules / ideals. This weak meaning is intuitively correct for WN modals in the negated sentences and the new data in the previous section.

3.2. Strengthening by exhaustification and Innocent Inclusion

To derive the attested strengthened meanings in UE sentences, I adopt a grammatical exhaustification mechanism that generates scalar implicatures, which has been motivated in particular as an analysis of free-choice disjunction (Fox, 2007). A growing body of work has generalized the logic of this free-choice strengthening to different domains (Bowler, 2014; Meyer, 2015; Singh et al., 2016; Bassi and Bar-Lev, 2016; Bar-Lev, 2018, a.o.).

For concreteness, I adopt Bar-Lev and Fox’s (2017) EXH operator below, which generates free-choice inferences in a similar manner to the one proposed in Fox (2007). The addition of the notion of innocent inclusion, however, allows it to generate these inferences directly with a single application of EXH. This contrasts with the recursive application that is required for an EXH operator that utilizes only innocent exclusion.⁸

- (16) a. $\llbracket \text{EXH} \rrbracket^{g,w} = \lambda C_{\langle st,t \rangle}. \lambda p_{\langle s,t \rangle}. \forall q \in \text{IE}(p, C) [\neg q(w)]$
 $\wedge \forall r \in \text{II}(p, C) [r(w)]$
- b. Given a sentence p and a set of alternatives C :
- (i) $\text{IE}(p, C) = \bigcap \{C' \subseteq C : C' \text{ is a maximal subset of } C, \text{ s.t. } \{\neg q : q \in C'\} \cup \{p\} \text{ is consistent}\}$
 - (ii) $\text{II}(p, C) = \bigcap \{C'' \subseteq C : C'' \text{ is a maximal subset of } C, \text{ s.t. } \{r : r \in C''\} \cup \{p\} \cup \{\neg q : q \in \text{IE}(p, C)\} \text{ is consistent}\}$

In (16), EXH takes as arguments a prejacent (p) and set of alternatives (C) and returns the negation of all Innocently Excludable (IE) alternatives, as well as the assertion of all the Innocently Includable (II) alternatives. The IE alternatives are those that can be negated consistently without contradicting the prejacent, and without making arbitrary choices (thus each IE alternative must be in all the maximal sets). The II alternatives are those that can be asserted without contradicting the prejacent and without contradicting the negated IE alternatives (and also each must be in all the maximal sets).

To see how this works, consider what happens if EXH is applied to the sentence in (13), resulting in (17). The intension of TP2, which will be the propositional argument of EXH is given in (18).

- (17) We are supposed to be here.
- a. LF: $[\text{TP}_1 \text{ EXH}_C [\text{TP}_2 \text{ supposed } f h [\text{we be here}]]]$

⁸See Bar-Lev and Fox (2017) for additional motivation and discussion of innocent inclusion.

$$(18) \quad \wedge \llbracket TP2 \rrbracket^{g,w} = \lambda w. \exists w' \in (\max_{h(w)} f(w)) [\text{We are here in } w']$$

3.3. Alternatives and strengthening

I assume that the alternatives for WN modals are built from alternative values for the ordering source h that are supersets of the original $h(w)$ given a world w . These are all the possible refinements of the ordering source, and they contain every proposition in the original ordering source, plus some other proposition(s). The alternative set C that EXH takes as an argument is then the set in (19).

$$(19) \quad \text{Alt}(TP2) = \{ \lambda w. \exists w' \in (\max_{h'(w)} f(w)) [\text{We are here in } w'] \mid \forall w \ h'(w) \supseteq h(w) \}$$

Since each refinement (superset) of $h(w)$ will return a subset of the original domain of quantification (favored worlds), the alternatives that are generated will be equivalent to subdomain alternatives, in Chierchia's (2013) analysis of polarity-sensitive items.

$$(20) \quad \text{Alt}(TP2) = \{ \lambda w. \exists w' \in (\text{Fav}') [\text{We are here in } w'] \mid \forall w \ \text{Fav}' \subseteq (\max_{h(w)} f(w)) \}$$

An additional assumption is that *supposed* / *should* has no universal scalar alternative (c.f. Deal, 2011; Bassi and Bar-Lev, 2016, a.o.). This means that the space of alternatives is not closed under conjunction, which is the crucial property necessary to generate free-choice type strengthening in UE environments (Fox, 2007).⁹ Now the arguments of EXH in (17) are the prejacent in (21a) and the alternative set in (21b).

$$(21) \quad \begin{array}{ll} \text{a.} & \wedge \llbracket TP2 \rrbracket^{g,w} = \lambda w. \exists w' \in (\max_{h(w)} f(w)) [\text{We are here in } w'] \\ \text{b.} & \text{Alt}(TP2) = \{ \lambda w. \exists w' \in (\text{Fav}') [\text{We are here in } w'] \mid \forall w \ \text{Fav}' \subseteq (\max_{h(w)} f(w)) \} \end{array}$$

Because they express existential quantification over smaller domains, each alternative in (21b) will entail the original assertion in (21a). Although the alternatives are all stronger, they are not innocently excludable. They are, however, innocently includable. EXH applied to TP2 then results in the inclusion (assertion) of all of the alternatives in (21b). This exhaustified meaning is equivalent to (22). I will refer to this as the “inclusion implicature.”

$$(22) \quad \lambda w. \forall w' \in (\max_{h(w)} f(w)) [\text{We are here in } w']$$

To illustrate the logic of the procedure described above, consider the following extremely simple toy model in which the following holds for any world w :

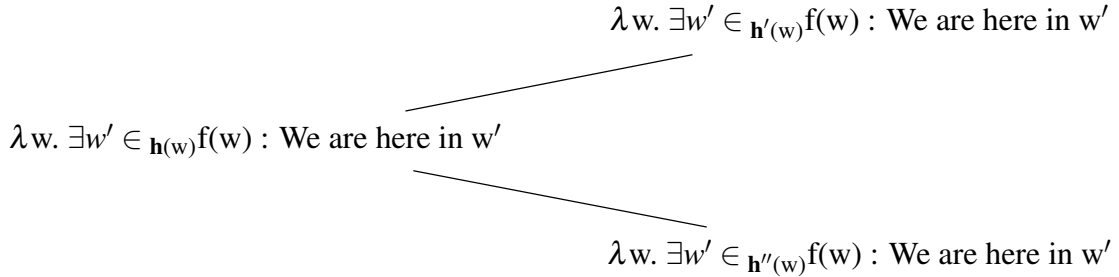
$$(23) \quad \begin{array}{ll} \text{a.} & h(w)f(w) = \{w1, w2\} \\ \text{b.} & h'(w)f(w) = \{w1\} \\ \text{c.} & h''(w)f(w) = \{w2\} \end{array}$$

Considering again the sentence in (17), the meaning of TP2 is the one in above in (18). This will be the prejacent to exhaust. But given the assumption in (23), the space of alternatives will consist of only three alternatives which correspond to the assumed different choices of ordering source function in (23a-c). There is the one equivalent to the prejacent, which uses h , and the ones that quantify over smaller domains, which are generated by h' , and h'' .

⁹There is a question of why *must* / *required to* are not universal alternatives for the existential *should* / *supposed to* in this system. For now this is stipulated, but I note that this assumption conforms to the independently noted puzzle that ‘weak necessity’ modals have no dual $\langle \text{can}, \text{must} \rangle$, $\langle \text{should}, \text{???} \rangle$ (Kratzer, 2013)

The alternative set C that EXH takes as an argument then are those in the diagram below in (24), which represents entailment relationships. Alternatives connected by lines to the right entail those to the left.

(24) Alt([We are supposed to be here])



It is easy to see now that no subdomain alternative is IE, as excluding one entails that the other must be included (none can be in all maximal sets whose negation is consistent with the prejacent). All alternatives, however, are II, since they all can be asserted without contradicting the prejacent. The conjunction of the II alternatives in (25a) results in the strengthening that is equivalent to universal quantification over the original domain in (25b).

- (25) a. $\lambda w. \exists w' \in h(w) f(w) : \text{We are here in } w'$
 $\wedge \lambda w. \exists w' \in h'(w) f(w) : \text{We are here in } w'$
 $\wedge \lambda w. \exists w' \in h''(w) f(w) : \text{We are here in } w'$
 b. $\lambda w. \forall w' \in h(w) f(w) : \text{We are here in } w'$

3.4. Neg raising

When EXH scopes over a negated WN modal, the entailment relationships are reversed. Assuming the LF of (26a) in (26b), the basic meaning of TP2 in (27a) entails all of the alternatives in (27b). EXH applies vacuously deriving the globally strong, “neg-raised” meaning.

- (26) a. We aren’t suppose to be here
 b. EXH [TP2 NEG [supposed f h [we are here]]]
- (27) a. $\wedge [[TP2]]^{g,w} = \lambda w. \neg \exists w' \in (\max_{h(w)} f(w)) [\text{We are here in } w']$
 b. $\text{Alt}(TP2) = \{ \lambda w. \neg \exists w' \in (Fav') [\text{We are here in } w'] \mid \forall w Fav' \subseteq (\max_{h(w)} f(w)) \}$

3.5. Connection between weak necessity and neg-raising

As discussed in section 1, *should* / *supposed* to are intuitively weaker than *must* / *have to*. The following example from Homer (2015) further illustrates this point.

- (28) Context: If you want to run the marathon. . . (Homer, 2015)
- a. You should train every day.
 b. You’re supposed to train every day.
 c. You must train every day.
 d. You have to train every day.

Sentences (28a-b) are intuitively weaker in the sense that training every day can be skipped while still meeting the goal, where in (28c-d) it is unavoidable. In von Fintel and Iatridou (2008), this weakness is attributed to universal quantification over a smaller set of worlds. They propose that WN modals involve promotion of a **secondary ordering source** that picks out the very best of the best worlds quantified over by the strong necessity modal. For the example above, the relevant ordering sources could be the ones in (29).

- (29) a. **Primary ordering source (given explicitly):** *You run the marathon.*
 b. **Possible secondary ordering source (from context):** *You feel good.*

Given this, *must* / *have to* say that the prejacent is true in all worlds in a domain generated with primary ordering, and *should* / *supposed to* say that the prejacent is true in all worlds in a domain generated by the primary and secondary ordering. This means that WN modal statements in (28a-b) (assuming the secondary ordering in (29b)) express something roughly equivalent to ‘*If you want to run the marathon **and feel good**, you have to train every day.*’

I propose that something similar can be implemented within the current proposal by an independently motivated procedure designed to weaken implicatures by ignoring (pruning) some alternatives that aren’t relevant. Recall that the universal meaning of *should* / *supposed to* is derived as an implicature, and it is known that implicatures are subject to relevance. An effect similar to the one described above, for example, could be generated by ignoring all alternatives that quantify over worlds in which you don’t feel good. A system of this sort, in fact, has been worked out in Bar-Lev (2018)’s analysis of ‘non-maximal’ readings within an implicature account of homogeneity in definite plurals. This generates the weakened (less than universal) readings that are known to be attested for definite plurals in sentences like (30).

- (30) Context: 8 out of the 10 kids laughed.
 a. The kids laughed. (TRUE)

While I cannot provide a detailed implementation of this for reasons of space, I tentatively suggest that a system like the one in Bar-Lev (2018) can be adopted for WN modals given the present account. I also point out that viewing weak necessity in this way provides a natural link to neg-raising, as they both follow from a core feature of the analysis: The strengthened (necessity) meaning is generated as an implicature. This means that the implicature disappears under negation which leads to neg-raising, and that the implicature can be weakened if not all alternatives are relevant, which leads to weak necessity.

4. Polarity-sensitivity

Having introduced the basic meaning of WN modals and the way that it can be strengthened by exhaustification, this section introduces a proposal that governs when exhaustification is obligatory and when it is optional in sentences that contain these modals. This then determines the range of meanings that they can express in various environments. Following work by Iatridou and Zeijlstra (2013); Homer (2015), I propose that WN modals are polarity-sensitive, and that this polarity-sensitivity is the result of a silent *even* associating with their domain (c.f. Crnič, 2014, 2019 building on Krifka, 1995; Lahiri, 1995). The logic of this account closely parallels the *even* account of NPI *any* as detailed in Crnič (2019), but the consequences will be different in some key ways that I will point out in the following discussion. It predicts that the modals should receive obligatorily strengthened interpretations in positive sentences, but remain weak

in negative sentences. For environments that contain both negative and positive components, such as the question in (6), two possible readings are predicted: one in which the positive component strengthens, and the negative component remains weak, and another in which both components remain weak, and the contribution of the covert *even* results in strong constraints on the discourse context. I provide an analysis in which the particular rhetorical effect observed in the polar questions in (6) can be generated with independently-motivated mechanisms proposed in Iatridou and Tatevosov (2016) to explain the effects of *even* in questions.

4.1. Association with covert *even*

To capture their polarity sensitivity I assume that *should* / *supposed to* are the spellout of a weak existential modal (as proposed in section 3), and a covert *even* (c.f. Crnič, 2019, for NPIs). I further assume that the associate of *even* is the ordering source argument *h*.¹⁰

$$(31) \quad [\text{supposed}] = [\text{supposed } [h_F \text{ even}]]$$

I will assume a meaning for *even* that has a scalar presupposition and an additive presupposition (Karttunen and Peters, 1979). *Even* takes clausal scope and takes as arguments the prejacent proposition in its scope (*p*), and a set of alternatives (*C*). This set is a salient subset of the focus alternatives of the prejacent, built from substituting the focused constituent with elements of the appropriate type (Rooth, 1985). *Even* presupposes that the prejacent *p* is less likely in the context than all of the non-equivalent alternatives, and that at least one of the non-equivalent alternatives is true. It asserts that the prejacent is true.

$$(32) \quad \llbracket \text{even} \rrbracket^{g,w,c} = \lambda C_{\langle st,t \rangle}. \lambda p_{\langle s,t \rangle} : \forall q \in C [q \neq p \rightarrow p <_c q] \text{ and } \exists q \in C [q \neq p \wedge q(w)]. p(w)$$

I also assume that *even* can take scope at various scope sites, possibly outside of the clause that it appears in on the surface (Karttunen and Peters, 1979, a.o.).¹¹

$$(33) \quad \text{even}_C [_{TP1} \text{ supposed } h_F \text{ even}_C [VP]]$$

Like the alternatives quantified over by EXH in section 3, I assume that the alternatives that *even* quantifies over are generated with all of the different possible refinements (supersets) of the ordering source *h*. Again, each refinement of the ordering source *h* will return a subset of the favored worlds that the modal quantifies over in the original assertion. This makes them equivalent to the subdomain alternatives in Krifka (1995) and Chierchia (2013).

$$(34) \quad \text{Alternatives induced by focus on ordering source of } \text{supposed} \\ \text{a. } \text{Alt}([\text{supposed } h_F [VP]]) = \{ [\text{supposed } h' [VP]] \mid \forall w [[h'](w) \supseteq [h](w)] \}$$

The basic meaning of TP1 in (33) will say that the prejacent proposition denoted by the VP is true in at least one world in the set of worlds maximally consistent with the rules / ideals. This set of worlds represents those consistent with just the basic standards.

¹⁰In the following LF representations the F subscript on the ordering source pronoun *h* represents focus. I omit the modal base pronoun *f* for ease of presentation.

¹¹Following Crnič (2014), I adopt an implementation of the scope theory of *even* in which *even* moves at LF and doesn't leave a trace. See Nakanishi (2012) and Erlewine (2018) for recent discussion of covert movement of *even*.

Each subdomain alternative will say that prejacent proposition is true in at least one world in the set of worlds consistent with the basic rules / ideals in addition to some other proposition(s). These sets of worlds represents the basic standards in addition to some other more ambitious ideals, goals, or desires. As discussed before, all of these alternatives entail the basic meaning.

There is also a relationship between ordering by likelihood and entailment that comes from general principles of probability. That is if a proposition p entails a proposition q , p is at most as likely as q .

(35) Likelihood-ordering and entailment

For any propositions p , q , and any context c , if $p \Rightarrow q$, then $p \leq_c q$.

The assumptions above will have consequences for the distribution of readings that are associated with WN modals that closely parallel the constraints on polarity-sensitive *any*.

5. Consequences of *even* associating with the ordering source of *should* / *supposed to*

5.1. Upward-entailing environments

If every constituent is UE wrt the domain of *supposed to*, any scope site of *even* will generate an inconsistent presupposition.

(36) * even_C [supposed h_F [VP]] (inconsistent presupposition)

This is because every alternative entails the sister of *even*, and thus can be at most as likely as it, which contradicts the scalar presupposition stating that all alternatives must be more likely. Parallel logic explains the ungrammaticality of NPI *any* in *even* approaches to NPIs (Lee and Horn, 1994; Lahiri, 1995; Crnič, 2014, 2019).

5.1.1. Rescuing *should* / *supposed to* in UE environments by exhaustification

The state of affairs above, however, can be remedied if exhaustification applies under the scope of *even*, generating an LF with the structure in (37a).

- (37) a. $\text{even}_{C'} [TP1 \text{ EXH}_C [\text{supposed } h_F [\text{VP}]] - (\text{consistent, near tautological presupp.})$
b. $\wedge [[TP1]]^{g,w} = \lambda w. \forall w' \in \max_{h(w)}(f(w)). [[VP]](w') = 1$
c. $\text{Alt}(TP1) = \{ \lambda w. \forall w' \in \max_{h'(w)}(f(w)). [[VP]](w') = 1 \mid \forall w h'(w) \supseteq h(w) \}$

Following the strengthening procedure in section 3.3, this LF gives rise to the strong “universal” meaning of the modal sentences as a result of the inclusion implicature. In this case, the scalar presupposition of *even* will be consistent, since the constituent that is the sister of *even* is now DE on the domain of *supposed to*. This is because the sister of *even* is exhaustified, thus both the prejacent and the alternatives that *even* quantifies over will be exhaustified meanings. Specifically, the prejacent of *even* in (37a) has the exhaustified meaning in (37b), and the alternatives are the set of exhaustified meanings in (37c).¹²

It is now the case that the prejacent of *even* in (37b) entails each of the subdomain alternatives in (37c), as the prejacent universally quantifies over a larger set of favored worlds. This means

¹²Here I am assuming that two scalar operators can focus associate with the same constituent (c.f. Crnič, 2014, 2019).

that it can be at most as likely as the alternatives. Thus, the presupposition of *even* is a near tautology in the sense that it will be satisfied in any context in which the alternatives aren't contextually equivalent to the basic meaning.

Now it can be understood why strengthening is obligatory in UE environments. There is no need to stipulate that EXH is generally obligatory. **The non-vacuous application of EXH is required as a mechanism to fulfill the needs of the presupposition of *even*.**

This is parallel to how free-choice strengthening is necessary to rescue polarity-sensitive *any* in UE environments (Crnič, 2019), but with an important difference. For *any*, free-choice readings must be “licensed” by being embedding under an existential modal. This follows from the assumption that *any* activates a universal scalar alternative in addition to subdomain alternatives. This makes embedding under an existential modal necessary to generate a space of alternatives that gives rise to free-choice strengthening (see Fox, 2007; Bar-Lev, 2018 for discussion of how the existence of a universal / conjunctive alternative blocks free-choice strengthening in unembedded environments).

5.2. DE environments

With *even* scoping above negation, entailment relationships are reversed with respect to the UE sentences. The prejacent of *even* entails all of its subdomain alternatives, as shown in (38).

- (38) a. $\text{even}_C [\text{TP1 NEG} [\text{supposed } h_F [\text{VP}]]] - (\text{consistent, near tautological presupp.})$
 b. $\wedge [[\text{TP1}]^{g,w} = \lambda w. \neg \exists w' \in \max_{h(w)}(f(w)). [[\text{VP}]](w') = 1$
 c. $\text{Alt}(\text{TP1}) = \{ \lambda w. \neg \exists w' \in \max_{h'(w)}(f(w)). [[\text{VP}]](w') = 1 \mid \forall w h'(w) \supseteq h(w) \}$

As discussed in section 3.4, this gives rise to the “neg-raised’ meaning. In this configuration, *even*’s scalar presupposition is consistent and a near tautology. It will be satisfied in any context in which the alternatives aren't contextually equivalent to the basic meaning.

There is, however, another parse that leads to a consistent and near tautological presupposition. This is one in which exhaustification applies under the negation.

- (39) a. $\text{NEG} [\text{even}_{C'} [\text{EXH}_C [\text{supposed } h_F [\text{VP}]]]]$
 consistent and near tautological presupposition.

Here, the presupposition of *even* is the same as in (37a), but the inclusion implicature is calculated under negation, which gives rise to the globally weaker “non neg-raised’ meaning. This parse, however, is generally disfavored by economy conditions, as it is globally weakening (Fox and Spector, 2018)

- (40) Economy Condition on Exhaustification (Fox and Spector, 2018 simplified):
 An occurrence of EXH in a sentence S is not licensed if this occurrence of EXH is globally weakening – if eliminating it does not alter or strengthens truth conditions, i.e., if $S(A)$ entails $S(\text{EXH}(A))$

I suggest that the marked status of this parse coincides with the available, but marked status of non neg-raised readings. For example, it is known that non neg-raised meanings are attested, but generally require special pitch accent with focus on the predicate (Gajewski, 2005, a.o.):¹³

¹³See also Homer (2015) for data that suggests that non neg-raised readings for *supposed* are available under

(41) Bill isn't SUPPOSED to come, but it's okay if he does. ¬ > □

See Fox and Spector (2018) for a discussion of the pitch accent pattern that licenses a parse with EXH embedded under negation given the condition in (40) (and also Meyer (2016) for a discussion of similar facts with an alternate account).

5.3. Non-monotone environments

In non-monotonic (NM) environments, there are generally two grammatical parses that will generate a consistent presupposition for *even*. One involves exhaustification above the NM operator:

(42) **option 1:** $\text{even}_{C'} [\text{TP}_1 \text{EXH}_C [\text{OP}^{\text{NM}} [\text{supposed } h_F [\text{VP}]]]]$
(scalar presupposition consistent and near-tautology)

This will give rise to readings that I will refer to as bearing a scalar implicature (SI) signature. This is one in which the meaning is strengthened in the UE component of meaning, and remains weak in the DE component (c.f. Bassi and Bar-Lev, 2016 for bare conditionals and free-choice disjunction). This will create a constituent (TP1 in (42)) that is DE on the domain of the modal, and thus the scalar presupposition will be a near tautology. Although this is difficult to show in the general case, the intuition is that in the UE component, the meaning expresses universal quantification, which is DE on the domain, and in the negative component, it expresses negated existential quantification, which is also DE on the domain (examples in section 6 will illustrate this more concretely). There is also a second option without exhaustification:

(43) **option 2:** $\text{EVEN} [\text{OP}^{\text{NM}} [\text{supposed}_{h_F} [\text{VP}]]]$
(scalar presupposition consistent but contingent – **generates context sensitivity**)

In the option in (43), since there is no logical entailment on the domain, the scalar presupposition of *even* is neither inherently inconsistent, nor a near tautology. It will be consistent, but contingent, which generates context-sensitivity. This follows the logic of Crnič (2019)'s analysis of the context-sensitivity of NPIs *any* in non-monotonic environments. With *should* and *supposed to*, however, the availability of strengthening in the parse in (42) (which is not available for *any* without embedding under an existential modal) gives it a unique profile.

(44) Effect of polarity-sensitivity for *should* / *supposed* in NM environments:
a. There is a **strengthened parse** that is **not context-sensitive** (nearly tautological presupposition), **and**
b. There is a **non-strengthened parse** that is **context-sensitive** (consistent but contingent presupposition).

It follows then that the parse in (44a) may be favored without a rich context, since it is much easier to imagine a context in which the presupposition is satisfied. The parse in (44b), however is available, and generates a presupposition that can be systematically derived given the hypothesized alternatives. The following section examines the predicted readings in NM environments.

certain structural and pragmatic conditions. Although reasons of space prevent a full discussion of these cases, the logic of the present account suggests that they will require an analysis in terms of the licensing of embedded EXH. I leave this to future work.

6. Predicted readings in non-monotonic environments

6.1. Polar questions

Here I group polar questions with non-monotonic environments in that they denote a set of two propositions of opposite polarity: (the positive *yes* answer and the negative *no* answer). When *should* or *supposed to* appear in polar questions, then, there will be two possible parses that yield consistent presuppositions, as described above. This section expands on the abstract schema in the previous section, and analyzes the readings that are generated in polar questions.

I adopt a semantics for polar questions that assumes that they contain a silent *whether* (Guerzoni, 2004 and references therein) that is treated like a *wh*-word with a Karttunen-style semantics. With this approach, *whether* is an existential quantifier that quantifies over a set of two type $\langle \text{st}, \text{st} \rangle$ functions which represent the possible answers to the question. One is the identity function $(\lambda p.p)$ and the other is the negation function $(\lambda p.\neg p)$. In this setup, polar questions can be paraphrased as asking *which of yes or no*.

While I skip many details of the composition of polar questions for reasons of space, the important point in the current discussion is that the trace of *whether* in the LFs below denotes the variable over the propositional functions that generate the *yes* or *no* answer. This means that the relative scope of additional propositional operators (like *even* or EXH) with this trace will determine its relative scope with negation in the *no* answer.¹⁴

Based on this, I suggest that a parse with EXH embedded under the trace of *whether* will produce a meaning for the *no* answer that is marked for the same general reasons that parses with EXH embedded under negation are marked (as discussed in section 5.2). This means that the preferred exhaustified parse of a polar question will be the one in (45) (corresponding to option 1 in the discussion above).

Option 1: *even* scoping over EXH in the question nucleus - consistent, nearly tautological presupposition - strengthened modal in YES, locally weak “neg-raised” reading in NO:

- (45) $\text{Whether}_1 [Q [\text{even}_{C'} [\text{EXH}_C [t_1 [\text{Supposed } h_F [\text{VP}]]]]]]$
a. YES: $\text{even}_{C'} [\text{EXH}_C [\text{supposed } h_F [\text{VP}]]]$
b. NO: $\text{even}_{C'} [\text{EXH}_C [\text{NEG } \text{supposed } h_F [\text{VP}]]]$

This LF produces the readings for both the positive and negative answers in which *even* has a nearly tautological presupposition. They correspond respectively to the readings in the positive and negative declaratives discussed above in (37a) and (38a). This means that this reading places no strong constraints on the context in which they can be used. An example of a question for which this LF is appropriate is the following. Uttered out of the blue, the question is most saliently understood as not having a weak existential reading of the modal.¹⁵

- (46) Am I supposed to smoke?

For (46), the most prominent reading is one for which a positive answer means that there is an obligation or recommendation to smoke, while a negative answer means that there is an

¹⁴This is worked out in detail in Guerzoni, 2004

¹⁵Thank you to a reviewer for pointing out this as an example of a question that appears not to have an existential reading similar to ‘is it okay if I smoke?’.

obligation or recommendation not to smoke (consistent with the neg-raised reading).

For the questions that give rise to existential readings, I propose that there is a different LF. This is one without exhaustification, and with *even* taking scope outside of the question nucleus.

Option 2: *even* outside of question nucleus and no EXH – consistent, but contingent presupposition - weak existential modal in YES, locally weak “neg-raised” reading in NO:

- (47) $\text{even}_C [\text{Whether}_1 [Q [t_1 [\text{Supposed } h_F [\text{VP}]]]]]$
 a. YES: [supposed h [VP]]
 b. NO: NEG [supposed h [VP]]

This configuration has been discussed in Iatridou and Tatevosov (2016)’s analysis of *even* in questions. They propose that the scalar presupposition generated by *even* scoping over a question can be analyzed with a particular notion of likelihood of question denotations. While “garden variety” *even* picks out the proposition that is least likely to be true, *even* applied to questions picks out the question that interlocutors are **the least likely to be ignorant about**.

For polar questions, this gives rise to what they call a “prerequisite effect.” This is an inference that “the speaker does not know if the most basic prerequisite of the topic under discussion holds (Iatridou and Tatevosov, 2016, p.322)” For example, B’s question below is a more basic “prerequisite” question than the salient alternative raised by A (as mammals are a proper subset of warm-blooded creatures).

- (48) a. A: Is this creature a mammal, you think? (Iatridou and Tatevosov, 2016)
 b. B: Is it even warm-blooded?

It is precisely this effect that I suggest is generated when *should* or *supposed to* receive existential readings in polar questions. The following section will show how this inference follows from the previously made assumptions about the alternatives generated by the modals. To see this, consider the following example which gives rise to an existential reading of *supposed*.

- (49) a. A: Can you pick up a book for me at the office?
 b. B: I don’t know. Are we supposed to be back on campus without a special permit?

There is a suggestive parallel between (48) and (49). In (49), B’s question (with the existential interpretation of the modal) is a prerequisite for A’s question. In order to know whether B can pick up the book at the office, it first must be determined if it is okay to be on campus at all.¹⁶

This effect can be derived with the LF below (*option 2* configuration in (47)).

- (50) $\text{even}_C [\text{CP}_1 \text{Whether}_1 [Q [t_1 [\text{Supposed } h_F [\text{we are on campus without a permit}]]]]]$
 a. Paraphrase: Is being on campus w/o a permit okay according to the basic rules?

The alternatives to CP1 in (50) will have the following form, as they are generated with supersets (refinements) of the original ordering source.

¹⁶The following with overt *even* are also felicitous in this discourse, which is also suggestive of the analysis:

- (i) a. B: Are we even supposed to be back on campus without a special permit?
 b. B’: Are we even allowed to be back on campus without a special permit?

- (51) $\{ [\text{Whether}_1 [Q [t_1 [\text{Supposed } h' [\text{we are on campus without a permit}]]]] \mid \forall w$
 $[[h]]'(w) \supseteq [[h]](w) \}$
 a. Paraphrase: $\{ \text{Is being on campus without a permit okay according to the basic rules and } p \mid p \text{ are additional goal(s), preference(s), ideal(s), etc } \}$

Following Iatridou and Tatevosov (2016)'s analysis, the scalar presupposition contributed by *even* in (50) is that the question denoted by CP1 is less likely for participants to be ignorant about than the alternatives in (51). In terms of the *prerequisite effect*, this means that the question in CP1 in (50) is the most basic prerequisite to the salient alternative(s) in (51). In the discourse in (49), it seems reasonable to assume that A's question raises to salience a member of the set in (51) that could be paraphrased in (52).

- (52) Paraphrase of a member of the alt. set in in (51) made salient by A in (49a):
 Is being on campus without a permit okay according to the basic rules **and B's ability / desire to pick up a book at the office?**

In summary, the rhetorical effect that comes along with existential readings for WN modals can be understood as the contribution of the silent *even* in its LF. The existential reading means that exhaustification has not applied, which necessitates a parse in which *even* scopes outside of the question nucleus. This generates the 'prerequisite effect' that has been independently observed and analyzed in Iatridou and Tatevosov (2016).

6.2. Exactly 'n'

The analysis also predicts that WN modals in the scope of the non-monotonic quantifier *exactly n*, should be ambiguous between a non context-sensitive parse with exhaustification, and a context-sensitive parse without exhaustification. The following indicates that this is borne out.

For example, assuming an analysis for *exactly n* that means '*at least n and and no more than n*' the following sentence most saliently gives rise to a reading in which the modal has a strengthened universal reading in the UE component of the meaning (*at least n*), and a locally weak existential interpretation in the DE component (*no more than n*).

- (53) Exactly eighteen students are supposed to be in the office today.
 a. Eighteen students are supposed to be there,
 b. No more than eighteen are allowed.

While I cannot provide a full derivation of the reading in (53) for reasons of space, I tentatively suggest that it can be generated with the LF in (54) given assumptions made in Gotzner et al. (2018)'s analysis of free-choice readings in the scope of NM quantifiers.

- (54) $\text{even}_C [\text{EXH}_C [\text{Exactly eighteen students}_1 [\text{supposed } h_F [t_1 \text{ be here}]]]]$

The more interesting parse from the perspective of the current analysis, however, is the one that gives rise to existential reading of *supposed to* in (55).

- (55) Context: I was surprised. All twenty students showed up on campus today. And given the new regulations...
 Exactly two of them were (even) supposed to be there.
 a. Two students were allowed / okay to be there.

- b. No more than two were allowed / okay to be there.

Invoking the more rich context in (55), the existential reading becomes available and is accompanied by a rhetorical effect.¹⁷ The rhetorical effect, in fact, is similar to the one that accompanies NPIs in non-monotonic environments discussed in Crnič (2014):

- (56) Exactly four people in the whole world have ever read that dissertation: Bill, Mary, Tom, and Ed. (Linebarger, 1987)

The analogy to the rhetorical effect that accompanies the NPI in (56) is expected under the current analysis (which invokes similar mechanisms). To see how it is generated, the LF for (55) is provided below, along with a paraphrase of the existential meaning of the modal.

- (57) $\text{even}_C [\text{TP}_1 \text{ Exactly two of them}_1 [\text{supposed } h_F [\text{t}_1 \text{ be here }]]]$
 a. Paraphrase of TP1: Exactly two of them are okay to be there according to the basic rules.

The alternatives are in (58), again quantifying over refinements of the original ordering source.

- (58) $\text{Alt}(\text{TP}_1) = \{ [\text{Exactly two of them}_1 [\text{supposed } h'_F [\text{t}_1 \text{ be here }]]] \mid \forall w [[h]]'(w) \supseteq [[h]](w) \}$
 a. Paraphrase: { Exactly two of them are okay to be there according to the basic rules **and** $p \mid p$ are **additional goal(s), preference(s), ideal(s), etc.** }

The scalar presupposition of *even* will be satisfied in contexts in which (57a) is less likely than all of the salient alternatives in (58). These would be contexts in which it is expected that a relatively large number of students (perhaps all 20) would have the basic permission to be on campus, and smaller subgroups would be okay to be there according to the rules and other more ambitious goals (such as helping in the lab or picking up important items). This means that it is unlikely for exactly two (a relatively small number) to be okay to be there according to the basic rules, and more likely that exactly two would be okay to be there according to the various other more refined orderings.

Also extending the analogy with Crnič's analysis of NPIs, this predicts that the existential readings should be harder to get with larger numerals. This too, appears to be borne out. The following strongly favors the strengthened interpretation of the modal in the UE component.

- (59) Context: All twenty students showed up today. And given the new regulations...
 Exactly nineteen of them were (# even) supposed to be there.
 a. ??Nineteen students were allowed / okay to be there.
 (c.f. Nineteen students were supposed to be there.)
 b. No more than nineteen were allowed / okay to be there.

Although a more detailed technical discussion is warranted, the pattern of available readings in the scope of non-monotonic quantifiers discussed here supports the proposed analysis.

7. Conclusion and open issues

This article presents novel data that I argue supports a view of weak necessity modals as having an underlying existential meaning that strengthens in UE environments due to a polarity-

¹⁷I also note here that the inclusion of overt *even* helps to bring out the existential reading, possibly by filtering out the exhaustified parse. The mechanism behind this is a topic for future work.

sensitive ordering source argument. I discuss the consequences of the analysis for the range of readings that are available in various logical environments. Left out of the discussion, however, are Strawson DE environments such as conditional antecedents and restrictors of universal quantifiers. Existential readings are predicted to be available in these environments, but it is not clear if this is borne out in the data. Further investigation of these environments is required.

There are also questions about how the current analysis fits in typologically with variable-force modals in languages like Nez Perce (Deal, 2011) and St’át’imcets (Rullmann et al., 2008), for which both existential and universal meanings are attested in unembedded sentences. Perhaps there is a rich typology, and modals in these languages have a similar underlying semantics to English WN modals, but lack the silent *even* that makes the strengthening obligatory. This is essentially Deal’s proposal for Nez Perce variable-force modals as existentials without a universal alternative. Another possibility, perhaps simpler from a learnability standpoint, is that all existential modals that lack a universal alternative are polarity-sensitive and must strengthen in UE environments. The difference then would be that additional language-specific factors could disrupt the monotonicity of sentences (possibly due to parametric variation in how presuppositions are integrated into meaning (c.f. Matthewson, 2006)), which would facilitate existential readings in unembedded sentences. Research into these issues is a topic for future work.

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