Scaleless Implicatures, blocked by Actuality Entailments[1]

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Abstract. The French necessity modals *falloir* and *devoir* can produce wide scope interpretations with respect to negation. However, when they are marked by the perfective aspect, this ability disappears, and they must take narrow scope. In this paper, I present a novel analysis of these modals' wide scope interpretation, as a 'scaleless implicature' – a strengthening phenomenon arising from these necessity modals' lack of a possibility scalemate. This strengthening, I argue, is blocked by the actuality entailment triggered by the perfective aspect.

Keywords: actuality entailments, scaleless implicatures, modals, scope, positive polarity, exhaustivity, Innocent Exclusion

1. Introduction

In this paper, I address a previously unanalyzed puzzle raised by the scopal interaction of negation and perfective-marked *falloir* and *devoir*, French modal verbs that express root necessity meanings. When negated, in simple contexts (unembedded, not perfective marked), *falloir* takes apparent wide scope with respect to negation, as in (1a), and *devoir* is ambiguous between a wide and narrow scope, as in (1b).

(1) a. Il **ne** ***faut*** **pas** sortir.
it NEG must NEG go.out
We must not go out. (□¬);[2]
*We don't have to go out. (¬□)

b. On **ne** ***doit*** **pas** sortir.
we NEG must NEG go.out
We must not go out. (□¬);
We don't have to go out. (¬□)

However, in the perfective aspect, the modals unexpectedly take narrow scope only.

(2) a. Il **n'** a **pas** ***fallu*** sortir.
it NEG AUX NEG must.PF go.out
*We had to not go out. (□¬)
We didn't have to go out. (¬□)

b. On **n'** a **pas** ***dû*** sortir.
we NEG AUX NEG must.PF go.out
*We had to not go out. (□¬)
We didn't have to go out. (¬□)

When perfective-marked like in (2), in non-negated sentences, French modals license Actuality Entailments (AEs), i.e. non-cancellable inferences that the prejacent is true, as in (3a); when negated, they license anti-AEs, i.e. non-cancellable inferences that the prejacent is false, as in (3b) (Hacquard, 2006; Homer, 2011: a.o.).

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[2]I use standard modal logic symbols: □/◇ are necessity/possibility modals, ¬ is negation; order indicates scope.

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(3) a. {Il a ***fallu***, On a ***dû***} sortir. ... *#but we didn't go out.*
it AUX NEC.PF we AUX NEC.PF go.out
We had to go out.

b. {Il **n'** a **pas** ***fallu***, On **n'** a **pas** ***dû***} sortir. ... *#but we did.*
it NEG AUX NEG NEC.PF we NEG AUX NEG NEC.PF go.out
We didn't have to go out.

In this paper, I argue that the exceptional narrow scope of *falloir* and *devoir* in (2) is not a result of the syntax or semantics of the perfective-marking, but rather of the anti-Actuality Entailment triggered by it. I analyze their apparent wide scope interpretations in (1) as scaleless implicatures – a grammaticalized strengthening from a narrow scope necessity interpretation $\neg\Box$ – the base interpretation – to a wide scope necessity interpretation $\Box\neg$. I show how a scaleless implicature analysis of this strengthening predicts its blocking by (anti)-Actuality Entailments.

I have organized the paper as follows. In section 2, I present the phenomenon of scaleless implicature (2.1), and give an analysis of wide scope interpretations of *falloir* and *devoir* as scaleless implicatures (2.2); in 2.3, I provide support for this analysis and against previous accounts that derive wide scope from interpretable syntactic movement of the modal above negation, driven by the modal's positive polarity (Iatridou and Zeijlstra, 2013; Homer, 2011, 2015; Zeijlstra, 2017). In section 3, I derive the blocking of the scaleless implicature by AEs (3.1), and then provide support for the claim that AEs are the source of the blocking (3.2). In section 4, I provide typological evidence that scaleless implicatures in general are blocked by (anti-)AEs.

2. Wide scope necessities as scaleless implicatures

2.1. Scaleless implicatures

Let's begin with the better-known phenomenon of scalar implicature. A standard example of a scalar implicature is the strengthening from an existential claim $\exists$ to its conjunction with a corresponding negated universal claim $\exists \wedge \neg\forall$, as in example (4).

(4) Some of the students left. $\rightsquigarrow$ *'Some but not all of the students left.'*

Scalar implicatures have been argued to arise due to the presence of a stronger alternative utterance obtained by replacing a word in the utterance with its Horn scalemate (Horn, 1972, 1989). For example, in (4), the alternative utterance is obtained from replacing *some* with *all*.

In contrast, a *scaleless* implicature arises when there is no such stronger scalemate, and corresponds to a strengthening to a universal interpretation. For example, imagine a language English' that differs from English in that it lacks the word *all*, and *some* does not form a Horn scale with a universal quantifier. As a result, under the right conditions, *some* is strengthened to a universal interpretation:

(5) Some of the students left. $\rightsquigarrow$ *'All of the students left.'* [English']

Scaleless implicatures have been observed with a variety of expressions. For example, Walpiri's connective *manu* is analyzed as a disjunction strengthened to a conjunction, due to the lack of a conjunctive item in the lexicon (Bowler, 2014). Hebrew *kol* acts as a free choice item, negative polarity item, and a universal quantifier, and is taken to be an existential quantifier strengthened to a universal in appropriate environments (Bar-Lev and Margulis, 2014). Another example

comes from an analysis by Bassi and Bar-Lev (2016) of conditionals as underlyingly existential quantifiers over worlds. Finally, I have given an account of Ecuadorian Siona modal *ba'iji* as a possibility modal strengthened to a necessity, due to the lack of a dedicated necessity modal in the lexicon (Jeretič, to appear).

An item that triggers a scaleless implicature can be identified by its characteristic distribution (see previous work on scaleless implicatures, Jeretič (to appear) in particular). In unembedded contexts, the scaleless implicature is obligatory, making the expression unambiguously strong. In non-upward-entailing contexts, the expression allows both a weak and a strong interpretation.[3]

These strengthening phenomena have been captured using a standard theory of grammaticalized scalar implicature, as the one proposed by Fox (2007), developed to account for embedded scalar implicatures and Free Choice effects. The strengthening is a consequence of the fact that the item projects subdomain alternatives, but no scalar alternatives, due to the lack of a scalemate in the lexicon. The subdomain alternatives are exhaustified with respect to each other before they are excluded from the original utterance – this exclusion corresponds to a strengthening to the equivalent of a universal interpretation. This exhaustification procedure is obligatory when it strengthens the utterance, and optional when it does not, thus capturing the characteristic distribution of items triggering scaleless implicatures.

Evidence for the empirical facts and details of the corresponding scaleless implicature analysis are given in the following section for the case at hand – negated necessity modals.

2.2. Wide scope necessities as scaleless implicatures

In this section, I lay out my analysis for the apparent wide-scoping of necessity modals by means of scaleless strengthening, rather than syntactic movement. The negated necessity expression $\neg\Box$ corresponds to the base interpretation for *falloir* and *devoir*, given by the order of projections in the syntax, where root modals are in the head position of the ModP, merged to the vP, below the NegP (Homer, 2011; Hacquard, 2006; Iatridou and Zeijlstra, 2013: a.o.). This expression is strengthened to the equivalent of a wide scope interpretation $\Box\neg$ as a result of the fact that *falloir* lacks a formal scalar alternative, and *devoir* is optionally associated with one.

This scaleless implicature analysis differs from previous scaleless implicature analyses cited in the section above in two major ways. The first is that instead of being scaleless existential operators, *falloir* and *devoir* are scaleless universal operators. In unembedded contexts, a universal quantifier's lack of a scalemate does not affect the interpretation, as the expression is strong. In contrast, a negated universal quantifier is weak. Without a possibility scalemate, a negated necessity expression $\neg\Box$ does not have a stronger $\neg\Diamond$ counterpart, and is thus strengthened to the equivalent of a wide scope necessity expression $\neg\Diamond/\Box\neg$. The second main difference between this analysis and previous ones is that the scalelessness of *falloir* and *devoir* does not strictly come from the lack of a scalemate in the lexicon of the language. I discuss in 2.2.3 how their scalelessness arises instead.

[3]Except in specific cases where there is no scaleless implicature at all, namely under sentential negation. In Jeretič (to appear), I argue that these are the result of the lack of a position for the exhaustification operator to apply in between sentential negation and the scaleless item. However, such cases are not relevant to the present object of study, as negated modals already contain a sentential negation and therefore cannot be embedded under another (clausemate) sentential negation.

2.2.1. A scaleless implicature pattern

Negated *falloir* follows a characteristic scaleless implicature distribution: it is unambiguously strong when unembedded; and ambiguous in non-upward-entailing contexts. We have seen the behavior of unembedded negated *falloir* in (1a). I give examples of the ambiguous behavior of negated *falloir* under non-upward-entailing contexts – conditional antecedents, the restrictor of a universal quantifier, and under negated *think*.

(6) *Conditional antecedents*

a. S'il **ne** ***faut*** **pas** y aller, je préfère rester.
if'it NEG must NEG there go I prefer stay
If we don't have to go, I prefer staying. ¬□

b. S'il **ne** ***faut*** **pas** y aller, il n'y a rien à faire, on reste.
if'it NEG must NEG there go it neg'there have nothing to do we stay
If we can't go, there's nothing we can do, we'll stay. □¬

(7) *Restrictor of universal*

Il n'y a pas de place dans les tiroirs, donc...
'There is no space in the drawers, so...'

a. ... j'ai décidé de jeter tous les papiers qu'il **ne** ***faut*** **pas** que je garde
I'have decided of throw all the papers that'it NEG must NEG that I keep
(même si je pourrais les garder).
even if I could them keep
(i) '... I decided to throw all the papers that I shouldn't keep.' □¬
(ii) '... I decided to throw all the papers that I don't have to keep, even if I could keep them.' ¬□

b. *Compare: restrictor of existential*
... j'ai décidé de jeter des papiers qu'il **ne** ***faut*** **pas** que je garde
I'have decided of throw some papers that'it NEG must NEG that I keep
(#même si je pourrais les garder).
even if I could them keep
(i) '... I decided to throw some papers that I shouldn't keep, #even if I could keep them.' □¬
(ii) *'... I decided to throw some papers that I don't have to keep, even if I could keep them.' *¬□

(8) *Under negated think*

a. Je ne pense pas qu'il **ne** ***faut*** **pas** y aller.
I NEG think NEG that'it NEG must NEG there go
'I don't think that we must not/don't have to go.' □¬, ¬□

b. *Compare: under non-negated think*
Je pense qu'il **ne** ***faut*** **pas** y aller.
I think that'it NEG must NEG there go
'I think we must not go.' □¬, *¬□

As for *devoir*, we observe the same ambiguous behavior in non-UE environments. However, because its interpretation is ambiguous in unembedded contexts as well, there is no apparent

change in behavior across contexts. While its polarity-sensitivity is opaque, *devoir*'s distribution is compatible with it triggering an optional scaleless implicature. However, the behavior of *falloir* and *devoir* converges when they take obligatory narrow scope in anti-AE sentences, which indeed suggests a unified analysis for the mechanism underlying their wide scope interpretations.

2.2.2. Formalization of the scaleless implicature analysis

As in previous work on scaleless implicatures, I analyze the strengthening of negated *falloir* and *devoir* with the recursive application of Fox's (2007) exhaustification operator EXH to the modal expression and its associated alternatives. The set of assumptions and derivations below are almost identical to those present in analyses of scaleless implicatures referenced in 2.1.

I assume that *falloir* and *devoir* are universal quantifiers over a set of accessible worlds (adopting standard (simplified) modal semantics (Kratzer, 1981, 1991)):

(9) $\llbracket falloir \rrbracket^{w_0} = \lambda p.\forall w \in Acc(w_0).p_w$
$= \lambda p.\Box_{Acc(w_0)}p$ (notation used in the rest of this paper)

As argued above, *falloir* lacks a possibility scalemate, and thus does not project scalar alternatives. Instead, I assume it projects subdomain alternatives. This assumption has been used in previous work to account for scaleless implicatures, but follows from proposals to account for a variety of phenomena: implicatures triggered by scalar items inside a disjunction (Sauerland, 2004), Free Choice (Fox, 2007), negative polarity (Chierchia, 2013), a.o. The alternative set of a *falloir* expression is formed by replacing the original modal base by members of its power set (the set of all its subsets).

(10) $Alt(\textbf{falloir}_C) = \{\textbf{falloir}_B : B \subseteq C\}$

I assume an operator EXH that negates all alternatives that can be negated without leading to inconsistency. EXH can be defined as in (11) (from Fox (2007)), where it negates all 'innocently excludable' (IE) alternatives, defined in (12).

(11) $\llbracket \text{EXH} \rrbracket(Alt(p)_{<st,t>})(p_{st})(w)$ iff $p(w) \wedge \forall q \in Alt_{IE}(p,Alt(p)))[\neg q(w)]$

(12) $Alt_{IE}(p,Alt(p)) = \bigcap\{Alt(p)' \subseteq Alt(p) : Alt(p)'$ is a maximal set in $Alt(p)$, s.t. $\{\neg q : q \in Alt(p)'\} \cup \{p\}$ is consistent$\}$
"elements of the intersection of all maximal sets of alternatives whose negation is consistent with the prejacent"

In addition, I assume that if EXH application makes the whole utterance stronger, then it is obligatory. Also, EXH can apply at any clause boundary.

A sentence like (1a) starts with the LF in (13), with *falloir* below negation; I assume a toy modal base $\{w_1, w_2\}$, and p denotes the prejacent.

(13) $S = \neg\Box_{\{w_1,w_2\}}p$

The set of alternatives for S is as follows.

(14) $Alt(S) = \{\neg\Box_{\{w_1,w_2\}}p, \neg\Box_{\{w_1\}}p, \neg\Box_{\{w_2\}}p\}$

None of these alternatives can be excluded non-arbitrarily from S. In particular, the maximal sets of alternatives that can be excluded from S without yielding a contradiction are $\{\neg\Box_{\{w_1\}}p\}$ and $\{\neg\Box_{\{w_2\}}p\}$. Their intersection – corresponding to the set of IE alternatives – is the empty set. Thus, when EXH applies to S, there is no truth-conditional effect.

(15) $\quad S' = \textsc{exh}\ [Alt(S)][S] = \neg\Box_{\{w_1,w_2\}}p$

EXH can apply again, this time to S', the once exhaustified proposition. The set of alternatives of S' is formed by exhaustifying each member of $Alt(S)$ with respect to $Alt(S)$. The result is that all the alternatives that are the not the prejacent itself are strengthened. This results in alternatives that are IE, as indicated below.

(16) $\quad Alt(S') = \{\ \textsc{exh}[Alt(S)][\neg\Box_{\{w_1,w_2\}}p]\ ,\ \textsc{exh}[Alt(S)][\neg\Box_{\{w_1\}}p]\ ,\ \textsc{exh}[Alt(S)][\neg\Box_{\{w_2\}}p]\ \}$
$= \{\ \neg\Box_{\{w_1,w_2\}}p\ ,\ \underbrace{\neg\Box_{\{w_1\}}p \wedge \Box_{\{w_2\}}p\ ,\ \neg\Box_{\{w_2\}}p \wedge \Box_{\{w_1\}}p}_{Alt_{IE}(S')}\ \}$

The exclusion of these IE alternatives from the prejacent yields strengthening to the equivalent of a wide scope interpretation, as shown in the derivation below.

(17) $\quad S'' = \textsc{exh}[Alt(S')][S']$
$\equiv \neg\Box_{\{w_1,w_2\}}p \wedge \neg(\neg\Box_{\{w_1\}}p \wedge \Box_{\{w_2\}}p) \wedge \neg(\neg\Box_{\{w_2\}}p \wedge \Box_{\{w_1\}}p)$
$\equiv \neg\Box_{\{w_1,w_2\}}p \wedge (\Box_{\{w_1\}}p \leftarrow \Box_{\{w_2\}}p) \wedge (\Box_{\{w_2\}}p \leftarrow \Box_{\{w_1\}}p)$
$\equiv \neg\Box_{\{w_1,w_2\}}p \wedge (\Box_{\{w_2\}}p \leftrightarrow \Box_{\{w_1\}}p) \equiv \Box_{\{w_1,w_2\}}\neg p$

EXH EXH S is stronger than S, so the operation is obligatory, correctly predicting the obligatory strong reading of *falloir* in unembedded contexts.

Under a DE operator, call it D, there are two possibilities for the location of exhaustification, represented by the following LFs:

(18) a. $\quad \textsc{exh}\ \textsc{exh}\ (D\neg\Box p)$
b. $\quad D(\textsc{exh}\ \textsc{exh}\ \neg\Box p)$

The LF in (18a) is equivalent to $(D\neg\Box p)$. This is because the alternatives to this expression are all weaker than the expression, and therefore not IE. Therefore, EXH application in this case is trivial. For any non-UE operator, the LF in (18b) is equivalent to $D(\Box\neg p)$, since EXH applies directly to the negated modal proposition, which as shown above, is strengthened to a wide scope necessity reading when exhaustified. However, since D is non-UE, the interpretation $D(\Box\neg p)$ is not stronger than the non-exhaustified utterance $D(\neg\Box p)$. Therefore, its application is not obligatory, and a strong reading of the negated modal is predicted to be optional.

2.2.3. Scalelessness of *falloir* and *devoir*

Other accounts of scaleless implicatures rely on items that are scaleless because of the absence of scalemates in the lexicon. This is not strictly the case for *falloir* and *devoir*, because French has a root possibility modal verb, namely *pouvoir*. In this section I give support for the claim that *falloir* cannot associate with *pouvoir*, and that *devoir* optionally does.

Substitution predicted impossible with *falloir*. While there are root possibility modals in the French lexicon, *falloir* is unique in requiring an expletive subject *il*, as shown in (19a), in contrast with *pouvoir* that requires a non-expletive subject, as shown in (19b). In addition, *falloir* can select for finite complement clauses, while *pouvoir* can't.

(19) a. {Il, *je, *tu, ... } faut partir.
exp *1sg, *2sg ... must leave
One must leave. (cannot encode explicit attitude holder)
b. {Je, tu, il, ... } peux/t partir.
1sg, 2sg, 3sg/it ... can leave
{I, you, s/he/it, ...} can leave.

Due to these facts, replacing *falloir* with *pouvoir* is semantically odd unless the expletive is also changed; however it isn't clear what subject can take the place of expletive "il" and still produce the desired meaning. So maybe this is reason why *falloir* has no formal scalar alternative.[4]

In the case of *devoir*, I assume it is optionally associated with a scalemate. I leave for further research the exact mechanism that captures this optionality. One possibility is that the scalar alternative can be pruned, just as alternatives may be pruned if irrelevant, following Katzir (2014). This assumption would be in line with the discussion in Bar-Lev and Fox (2017) (section 4) in which innocently excludable alternatives such as scalar ones are prunable, but innocently includable ones are not (see the paper for the notion of innocently includable alternatives; in the present case, they would refer to subdomain alternatives). However, this assumption makes the prediction that any item that triggers a scalar implicature can also trigger a scaleless implicature. This prediction is at least at first glance incorrect. In order to avoid such overgeneration, one can assume that items differ in whether they project subdomain alternatives (see Jeretič (to appear) for a discussion of this question), or that prunability of alternatives could be encoded in the lexical entry. Another possibility is that there are two *devoir*'s: one projecting a scalar alternative and one not. I leave this question open for further investigation.

Lack of scalar implicatures. We find support for the obligatory scalelessness of *falloir* in environments in which the modal does not take wide scope, and a scalar implicature would be expected if the item were associated with a scalemate. These are environments involving syntactically extra-clausal negation. For example, I show that for universal quantifiers known to be scalar, like *every*, a scalar implicature can be derived under negated *think*. The derived scalar implicature is targeted by a disagreeing response to it.

(20) a. – Marie ne pense pas avoir vu tous les étudiants.
– Non, moi ce que j'ai compris c'est qu'elle pense avoir vu *aucun* étudiant.
b. – Mary doesn't think she has seen all the students.
– No, what I understood is that she thinks she saw *no* student.

(21) – Mary doesn't think she has to talk to John for things to calm down.
– No, what I understood is that Mary thinks she *shouldn't* talk to John for things to calm down.

[4]This emerging empirical picture calls for a typological investigation: do all items that do not have an appropriate scalemate in the lexicon (with the possible presence of a non-substitutable one) trigger a scaleless implicature?

In contrast note that correcting an expression with one that properly entails it is infelicitous.

(22) – Mary thinks she saw an animal.
– ??No, what I understood is that she thinks she saw a cat.

If the scalar implicatures in (20) and (21) were not computed, corrections would entail the utterances, and therefore be odd, like in (22). And indeed, a word-for-word translation of (21) to French, using *falloir* in the place of *have to*, is infelicitous.

(23) – Marie ne pense pas qu'il **faut** parler avec Jean pour calmer la situation.
– ??Non, moi ce que j'ai compris c'est que Marie pense qu'elle ne devrait pas parler avec Jean pour calmer la situation.

However, with a translation using *devoir*, the response is felicitous again.

(24) – Marie ne pense pas qu'elle **doit** parler avec Jean pour calmer la situation.
– Non, moi ce que j'ai compris c'est Marie pense qu'elle ne devrait pas parler avec Jean pour calmer la situation.

The infelicity of the example with *falloir* in (23) is expected if *falloir* does not associate with a scalemate.[5] In contrast, the example with *devoir* in (24) is expected to be felicitous give that *devoir* can associate with a scalemate.

These examples have shown that *falloir* cannot associate with a scalemate. I now give examples that suggest that *devoir* (and *falloir*) does not necessarily associate with a scalemate. As argued by Magri (2009), a scalar implicature is obligatory if the scalar alternative is contextually equivalent to the assertion. For example, the sentence 'Some Italians come from a warm country' is judged odd because it is contextually equivalent to its *all*-alternative. In a parallel fashion, I create a context in which $\neg\Diamond p$ is true, and check the felicity of utterances of type $\neg\Box p$. If a given $\Box$ item is associated with a $\Diamond$ scalemate, then $\neg\Box p$ should trigger an obligatory scalar implicature (since the alternative $\neg\Diamond p$ is contextually equivalent to $\neg\Box p$). This scalar implicature is equivalent to $\Diamond p$, it contradicts the context, and should therefore be judged infelicitous. If $\Box$ is not associated with a scalemate, no scalar implicature is triggered, and the utterance should be judged felicitous.

(25) *Context: I'm pretty sure one is not allowed to pass at the red light.*
a. Ce n'est pas le cas/Je ne pense pas {qu'il **faut**/qu'on **doit**} passer au feu rouge.
It's not the case/I don't think we **must** pass at the red light. $\not\rightsquigarrow$ *I think we can.*
b. #Ce n'est pas le cas/Je ne pense pas qu'on soit **obligés** de passer au feu rouge.
#It's not the case/I don't think we **have to** pass at the red light. $\rightsquigarrow$ *I think we can.*

Both *falloir* and *devoir* pass the test, while *être obligé* and *have to* don't. This lends support to the claim that *falloir* and *devoir*, in contrast with *être obligé* and *have to*, can lack a scalemate.[6]

[5]Note that this configuration does not trigger a scaleless implicature either.

(i) EXH EXH $\neg think\ \Box_{\{w_1,w_2\}} p \equiv \neg think\ \Box_{\{w_1,w_2\}} p \wedge (think\ \Box_{\{w_1\}} p \leftrightarrow think\ \Box_{\{w_2\}} p)$
$\equiv \neg think\ \Box_{\{w_1\}} p \wedge \neg think\ \Box_{\{w_2\}} p$

If we assume *think*'s neg-raisingness does not apply in this derivation, the effect of applying the exhaustifier is an inference that Marie does not have an opinion about which worlds in the modal base are p-worlds.

[6]This results suggests that the mechanism underlying the optionality of *devoir*'s scalar alternative cannot be equivalent to the pruning mechanism that makes scalar alternatives optional.

2.3. Arguments against an alternative analysis relying on syntactic movement

The scaleless implicature analysis captures the distribution of *falloir* and *devoir* in unembedded, non-UE environments and in AE sentences, as shown above. In this section, I present arguments favoring a scaleless implicature analysis over a PPI-raising analysis.

Previous accounts have argued for wide scope interpretations of modals, including *falloir* and *devoir*, as raising past negation, due to positive polarity (Iatridou and Zeijlstra, 2013; Homer, 2015; Zeijlstra, 2017). The main difference between the present analysis and previous ones is syntactic movement: with scaleless implicatures, the modal stays in situ, while in a PPI-raising analysis, the modal raises above negation to avoid a polarity clash. I refer the reader to Jeretič and Thoms (2020) for a discussion of the syntactic issues raised by this type of movement. In this section, I give additional, interpretation-based arguments that favor an *in situ* analysis of the modal versus a movement analysis.

Ellipsis. We can diagnose the LF position of an element using ellipsis, where there is a requirement for the position of the elided element to match that of its antecedent, dubbed as Scope Parallelism (Sag, 1976; Fox, 2000: a.o.).[7] We can observe this requirement in action in the following sentence, in both its French and English counterparts. The first sentence contains two scope taking elements, 'no doctor' and 'two patients', in subject and object positions; either scope order, reflected in (a) and (b), is possible.

(26) Aucun docteur n'a examiné deux patients, une infirmière – si.
'No doctor examined two patients, but a nurse did.'
a. no doctor $>$ 2 patients; one nurse $>$ 2 patients
b. 2 patients $>$ no doctor; 2 patients $>$ one nurse (covarying nurses possible)
c. *no doctor $>$ 2 patients; 2 patients $>$ one nurse
d. *2 patients $>$ no doctor; one nurse $>$ 2 patients

The scope order of the subject of the second sentence, 'a nurse', must match that of the subject of the first sentence 'no doctor'. Given the possibility of covariation in (b), it appears that the scopal readings can indeed be a result of QR movement. The scopal configurations in (c) and (d) are not available, meaning no mismatch is possible, i.e. if there is movement in the antecedent, there is movement from the ellipsis site.

Based on this data, one would expect that the scope of a subject and the modal will follow the same requirement. Given this reasonable assumption, I show that an apparent scope mismatch is observed with necessity modal *devoir*, that lends support to the scaleless implicature theory, in which the modal stays *in situ*, below negation at LF.

We take the sentence pair in (27), in which there is an apparent scope mismatch of the necessity modal, elided in the second sentence, and the subjects.

(27) Aucun enfant ne doit aller en prison. Au moins un adulte dans cette ville – si.
'No child must go to prison. At least one adult in this city – yes.'
interpretation: **must$>$no child; at least one adult$>$must**

[7] See Bassi and Bar-Lev (2016) for similar detection of a scaleless implicature using ellipsis of *devoir*.

Crucially, the antecedent sentence contains a negated subject, 'no child'. The sentence alone can be interpreted with the modal scoping above the negation of the subject.

(28) a. EXH^2 [no child] [must] (scaleless implicature)
b. [must] [no child] ~~[must]~~ (movement of 'must')
c. <*no child*> [must] [no child] (reconstruction of 'no child')

Option (c) is presumably available under both theories of wide scope interpretations of negated modals, since it is independent of them. Options (a) and (b) are dependent on whether scaleless implicatures or modal movement are available options.

However, if Scope Parallelism holds for these modals, as we assume, (a) is the only viable option, i.e. it is the only one that allows the modal to scope below the subject at LF in both the antecedent and elided sentence. I thus take this to be an argument for a theory of wide scope necessity modals in which the modal stays *in situ*, such as the one presented in this paper.

Negative connectives. *Falloir* and *devoir* can be embedded in a proposition coordinated by the negative connective *ni*, and still give a wide scope interpretation.

(29) a. Jean ne mange pas, ni ne faut-il qu'il mange.
Jean neg eat neg, nor neg must-it that'he eat
Jean does not eat, nor must he. $\Box > \neg$
b. Tu ne peux ni ne dois sortir.
you neg can.2sg nor neg must.2sg go.out
You cannot nor must go out. $\Box > \neg$

In these examples, the necessity modal in the second coordinand can be interpreted above above the negative operator introduced by the negative coordination. Following Gonzalez (2020), I take the negative connective *ni* in French to be a disjunction that scopes under a negation. Let p and $\Box q$ be the coordinated propositions as in the examples given in (29). In a disjunction analysis, negation scopes above the coordination. Therefore, if the modal is to take wide scope with respect to negation, it must raise above the entire coordination, resulting in an LF of the type $\Box\neg(p \vee q)$. This LF, however, does not correspond to the intended reading, since the modal only applies to one of the disjuncts. Thus, there is simply no way of achieving the desired reading by any movement of the modal, whether it be head movement or something else. This means that under a disjunctive analysis of French *ni*, a movement approach to derive a wide scope interpretation is not viable. Note that the scaleless implicature analysis, in contrast, can account for the wide scope interpretation of the modal in a negative disjunction, as shown in the derivation below.

(30) $S = \neg(q \vee \Box_{\{w_1,w_2\}} p)$

(31) $Alt(S) = \{\neg(q \vee \Box_{\{w_1,w_2\}} p), \neg(q \vee \Box_{\{w_1\}} p), \neg(q \vee \Box_{\{w_2\}} p)\}$

The set of alternatives of the negative coordination contains no IE alternative. I show this by contradiction below:

(32) $\neg(q \vee \Box_{\{w_1,w_2\}} p) \wedge (q \vee \Box_{\{w_1\}} p) \wedge (q \vee \Box_{\{w_2\}} p)$
$\equiv \neg(q \vee \Box_{\{w_1,w_2\}} p) \wedge \Box_{\{w_1\}} p \wedge \Box_{\{w_2\}} p$ (because q is false)
$\equiv \bot$ (because $\Box_{\{w_1,w_2\}} p \equiv \Box_{\{w_1\}} p \wedge \Box_{\{w_2\}} p$ is false)

(33) $S' = \text{EXH}\ [Alt(S)][S] = \neg(q \vee \Box_{\{w_1,w_2\}}p)$

(34) $Alt(S') = \{\neg(q \vee \Box_{\{w_1,w_2\}}p),$
$\neg(q \vee \Box_{\{w_1\}}p) \wedge (q \vee \Box_{\{w_2\}}p)$
$\neg(q \vee \Box_{\{w_2\}}p)\} \wedge (q \vee \Box_{\{w_1\}}p)\}$

The second EXH application, in contrast, yields strengthening.

(35) $S'' = \text{EXH}[Alt(S')][S']$
$\equiv \neg(q \vee \Box_{\{w_1,w_2\}}p) \wedge \neg(\neg(q \vee \Box_{\{w_1\}}p) \wedge (q \vee \Box_{\{w_2\}}p)) \wedge \neg(\neg(q \vee \Box_{\{w_2\}}p) \wedge (q \vee \Box_{\{w_1\}}p))$
$\equiv \neg(q \vee \Box_{\{w_1,w_2\}}p) \wedge ((q \vee \Box_{\{w_2\}}p) \leftrightarrow (q \vee \Box_{\{w_1\}}p))$
$\equiv \neg(q \vee \Box_{\{w_1,w_2\}}p) \wedge \Box_{\{w_2\}}p \leftrightarrow \Box_{\{w_1\}}p$ (because q is false)
$\equiv \neg q \wedge \Box_{\{w_1,w_2\}}\neg p$ (because $\Box_{\{w_1,w_2\}}p$ is false)

Typological predictions. The scaleless implicature analysis makes better typological predictions than the movement analysis of wide scope necessity modals. Cross-linguistically, many necessity modals can take wide scope, in contrast with possibility modals that never do (De Haan, 1997; Iatridou and Zeijlstra, 2013). A movement theory does not predict the asymmetry. A scaleless implicature theory, in which the modal cannot move, does: negated possibility expressions are already strong, thus a wide scope interpretation is underivable. Instead, a scaleless implicature theory predicts that there exist existential modals that are strengthened to necessity modals. This prediction appears to be borne out, as discussed in section 4.

Previous accounts leave scope facts with AEs unexplained. Finally, previous accounts of modals as PPIs do not explain the scope facts in AE sentences. Homer (2015) says PPIs are 'anti-licensed' under negation, leaving open why they must remain below negation in anti-AE sentences. In Zeijlstra's (2017) account, he assumes that *'must'* invokes a covert exhaustifier, that negates all stronger domain alternatives (and not just IE alternatives), yielding ungrammaticality if the result is inconsistent. Under negation, *must* is ungrammatical, since it yields an inconsistent result, as shown in (36). Therefore, *'must'* must raise above negation, where it is grammatical.

(36) $\text{EXH}\ \neg\ must_{\{w_1,w_2\}}p \equiv \neg\Box_{\{w_1,w_2\}}p \wedge \neg\Box_{\{w_1\}}p \wedge \neg\Box_{\{w_2\}}p \equiv \bot$

Extending this analysis to *falloir*, it yield the wrong results with AEs. The alternatives $\neg\check{\Box}_{\{w_1\}}p$ and $\neg\check{\Box}_{\{w_2\}}p$ are stronger than $\neg\check{\Box}_{\{w_1,w_2\}}p$, therefore negated. The result, shown in (37), is again inconsistent, incorrectly predicting ungrammaticality of the modal below negation. (I adopt the notation $\check{\Box}$ used by Alxatib (2019) to represent an AE-triggering modal).

(37) $\text{EXH}\ \neg\ fallu_{\{w_1,w_2\}}p \equiv \neg\check{\Box}_{\{w_1,w_2\}}p \wedge \neg\check{\Box}_{\{w_1\}}p \wedge \neg\check{\Box}_{\{w_2\}}p \equiv \bot$

3. Blocking by Actuality Entailments

3.1. Deriving the blocking

In this section I address *falloir* and *devoir*'s narrow scope interpretations observed in the presence of anti-AEs. I argue that (anti-)AEs are predicted to block scaleless implicatures as analyzed in the previous section: the alternatives of an AE utterance all have the same AE, and therefore can never be Innocently Excludable with respect to each other. I detail how this works below.

While I remain neutral with respect to the details of a theory of actuality entailments, I adopt the assumption that the alternatives of an expression that triggers an (anti-)AE also trigger (anti-)AEs, as shown below for a necessity modal expression.

(38) $Alt(\neg\check{\Box}_{\{w_1,w_2\}}p) = \{\neg\check{\Box}_{\{w_1,w_2\}}p, \neg\check{\Box}_{\{w_1\}}p, \neg\check{\Box}_{\{w_2\}}p\}$

This assumption is in line with previous work: I follow Homer (to appear) who analyzes AEs as part of the semantic contribution of a perfective modal utterance, due to the fact that they are not cancellable. For this reason, they also should be part of the semantic contribution of the alternative utterances.

The inferences observed in (3a) and (3b) are summarized in (39a) for perfective-marked modals, and in (39b) for negated perfective marked modals.

(39) a. $\check{\Box}p \equiv \Box p \wedge p$
b. $\neg\check{\Box}p \equiv \neg\Box p \wedge \neg p$

Note that the conjunctive inference obtained with negated perfective-marked modals is stronger than the negation of the conjunctive inference $\Box p \wedge p$ obtained with non-negated perfective-marked modals. A theory of AEs should capture this. One way of doing so is to posit a biconditional requirement between the prejacent and the modalized expression (for versions of such a requirement, see Alxatib (2019), Homer (to appear)). For our purposes, it is important to note that this biconditional requirement continues to hold when an anti-AE expression is itself negated. In other words, the negation of a negated anti-AE expression is truth-conditionally equivalent to an AE expression, as expressed in (40), and confirmed empirically in (41).

(40) $\neg\neg\check{\Box}p \equiv \Box p \wedge p$

(41) Ce n'est pas le cas qu'il n'a pas fallu y aller.
it NEG'be NEG the case that'it NEG'have NEG must.PF there go
It is not the case that we didn't have to go.
a. #... mais on n'y est pas allés.
but we NEG'there be NEG went
... but we didn't go. → AE
b. #... mais on ne devait pas y aller.
but we NEG must.IPF NEG there go
... but we didn't have to go. → modal inference

This means that if two modal expressions have the same prejacent, and they license an (anti-)AE, they have the same (anti-)AE, therefore conjoining one and the negation of the other yields a contradiction:

(42) For any two modal bases X and Y:
$\neg\check{\Box}_X p \wedge \neg\neg\check{\Box}_Y p \equiv \neg\Box_X p \wedge \neg p \wedge \Box_Y p \wedge p \equiv \bot$

As far as our modal expressions are concerned, the (anti-)AE is the same between an utterance and its alternatives. This is because the modals, by hypothesis, project alternatives that vary only by modal base, while their prejacent stays constant. Therefore, the blocking arises from the fact that the alternatives of an AE utterance cannot be exhaustified with respect to each other, because their AEs conflict. Therefore, the alternatives are never IE with respect to each other,

which makes EXH application trivial at any point of the derivation, preventing strengthening. The blocking can be seen in the derivation below, for an AE-triggering *falloir* sentence $\check{S}$.

(43) a. $\check{S} = \neg\check{\Box}_{\{w_1,w_2\}}p$
b. $Alt(\check{S}) = \{\neg\check{\Box}_{\{w_1,w_2\}}p, \neg\check{\Box}_{\{w_1\}}p, \neg\check{\Box}_{\{w_2\}}p\}$

The first step of the derivation looks similar to a non-AE sentence; again, no alternative is IE (adding an AE does not change this fact), therefore there is no effect on truth conditions.

However, the second step of the derivation looks different. No member of $Alt(\check{S})$ can be excluded from another, since they all have the same AE. This means that the alternatives of the once exhaustified sentence remain the same, as shown below.

(44) $\check{S}' = \text{EXH}\ [Alt(\check{S})][\check{S}] = \neg\check{\Box}_{\{w_1,w_2\}}p$

(45) $Alt(\check{S}') = \{\ \text{EXH}[Alt(\check{S})][\neg\check{\Box}_{\{w_1,w_2\}}p]\ ,\ \text{EXH}[Alt(\check{S})][\neg\check{\Box}_{\{w_1\}}p]\ ,\ \text{EXH}[Alt(\check{S})][\neg\check{\Box}_{\{w_2\}}p]\ \}$
$= \{\ \neg\check{\Box}_{\{w_1,w_2\}}p\ ,\ \neg\check{\Box}_{\{w_1\}}p\ ,\ \neg\check{\Box}_{\{w_2\}}p\ \}$

Therefore, the alternatives of the once exhaustified sentence are not IE, and no strengthening is derived, as desired.

(46) $\check{S}'' = \text{EXH}\ [Alt(\check{S}')][\check{S}'] \equiv \neg\check{\Box}_{\{w_1,w_2\}}p$

Note that the EXH operator proposed by Bar-Lev and Fox (2017, 2020) that relies on the notion of Innocent Inclusion yields different results here, and incorrectly predicts strengthening with Actuality Entailments. In particular, this EXH operator 'includes' without contradiction the alternatives $\neg\check{\Box}_{\{w_1\}}p$ and $\neg\check{\Box}_{\{w_2\}}p$, yielding strengthening: $\neg\check{\Box}_{\{w_1,w_2\}}p \wedge \neg\check{\Box}_{\{w_1\}}p \wedge \neg\check{\Box}_{\{w_2\}}p \equiv \check{\Box}_{\{w_1,w_2\}}\neg p$. I leave this concern for further work.

3.2. Support for AEs – and not perfective aspect – as the source of the blocking

In this section, I provide support for the claim that (anti-)AEs are indeed the source of the blocking, and not the syntax or semantics of perfective aspect. I first present data in which wide scope interpretations are observed when the modal is perfective-marked, but AEs don't arise. I then argue that neither the particular syntax of the French perfective, nor its semantics, is likely to be the source of the blocking.

3.2.1. Neutralized anti-AE

Apparent wide scope of perfective-marked *falloir* and *devoir* can be observed in particular cases in which an (anti-)AE is not triggered, despite the presence of the perfective. Homer (2011, to appear) argues that AEs are a result of aspectual coercion, where the modal that is inherently stative is coerced into an eventive interpretation, namely the event described by the prejacent. The aspectual clash between the requirement imposed by the perfective and the stativity of the modal can be resolved in other ways, namely if the context introduces a bound time interval during which the modal semantics applies. In these cases, Homer reports that AEs are optional, as in the following examples:

(47) a. Olga a soudain **pu** soulever un frigo, mais ne l'a pas fait.
Olga suddenly became able to lift a fridge, but didn't do it.
b. Entre 15h et 17h, Olga a **pu** soulever un frigo, mais ne l'a pas fait.
In between 3 and 5pm, Olga was able to lift a fridge, but didn't do it.

In such cases, therefore, a wide scope reading of a necessity modal is predicted to be available. And it seems like this is the case. Below is an attested example[8] (continuation added) in which negated perfective *falloir* is coerced into an activity.[9]

(48) Cétait la dernière nuit pendant laquelle **il n'a pas fallu** qu'il relâche sa vigilance (mais il l'a malheureusement relâché vers 3h du matin.)
It was the last night during which he couldn't (#didn't have to) relax his vigilance (but he unfortunately relaxed it around 3am).

The context prefers a wide scope reading of the necessity modal. This contrasts with a context in which no activity is coerced, and an actuality entailment is derived.

(49) #**Il n'a pas fallu** qu'il relâche sa vigilance (#qu'il a malheureusement relâché vers 3h du matin.)
intended: He couldn't relax his vigilance.
actual: #He didn't have to relax his vigilance (#which he unfortunately relaxed around 3am).

These examples strongly suggest that the actuality entailment is indeed the source of the narrow scope interpretations, lending support for the proposal in this paper.

3.2.2. Perfective semantics is not the source of the blocking

An alternative analysis could take the semantics of the perfective aspect as the source of the modal's narrow scope. It is conceivable that perfective acts as a 'shielder', in the same way that PPIs can be shielded from the anti-licensing effect of negation (Homer, 2011; Szabolcsi, 2004; Nicolae, 2017). However, shielding is usually observed with universal quantifiers, and perfective is not universal. It is typically analyzed as an existential quantifier (Klein, 1994: a.o.):

(50) $[\![\text{PERF}]\!] = \lambda P \lambda t \exists e . \tau(e) \subseteq t \wedge P(e)$

Existential quantifiers are not known to block wide scope of modals. Moreover, imperfective is generally analyzed in parallel with perfective, only differing from it in encoding a different relationship between event time and topic time. Why would narrow scope be observed with perfective but not imperfective?

Perfective could also act as a 'flattener' by removing active alternatives. This is especially interesting as AE sentences also block Free Choice inferences (Alxatib, 2019), that can similarly be analyzed as an implicature (Fox, 2007; Bar-Lev and Fox, 2020):

[8] `https://infocapagde.com/index.php?op=newindex&catid=5&marqeur=805`

[9] Other examples in which the modal is coerced into punctual events (using adverbials such as *soudain* 'suddenly', *à un moment* 'at some point') are harder to construct, because they strongly prefer the negation *plus* 'no longer', that brings in its own semantic baggage and interacts with scope in non-trivial ways.

(51) a. Je pouvais rendre visite à Léa ou Zoé.
I could.ipf of give visit to Léa or Zoé
I could go visit Léa or Zoé.
→ *I could go visit Léa, and I could go visit Zoé.* (Free Choice Inference)

b. J'ai pu rendre visite à Léa ou Zoé.
I'have can.pf of give visit to Léa or Zoé
I could go visit Léa or Zoé.
↛ *I could go visit Léa, and I could go visit Zoé.* (no Free Choice Inference)

However, perfective does not always flatten alternatives. For example, a perfective-marked sentence with the modal expression *avoir la permission* ('have permission') does not trigger an AE (presumably because of its inherent eventive interpretation), but does license a Free Choice inference:

(52) J'ai eu la permission de rendre visite à Léa ou Zoé.
I'have have.pf the permission of give visit to Léa or Zoé
I got permission to go visit Léa or Zoé. → *I got permission to go visit Léa, and I got permission to go visit Zoé.* (Free Choice Inference)

Therefore, since perfective does not block strengthening in general, it is likely not the source (in itself) of the narrow scope of the modal.

3.2.3. Perfective syntax is not the source of the blocking

One could argue that the differences in the syntax of (1) and (2) are the source of the narrow scope. After all, the French perfective form 'passé composé' is constructed using an auxiliary and a non-finite form of the modal verb, displaying a suggestive surface order relative to negation, and potentially reflecting a different order at LF. However, I argue these facts do not generalize. For example, French near future tense is formed from the auxiliary *va* 'go' and the infinitive form of the modal verb, yielding a word order parallel to that of passé composé. However, wide scope interpretations are available, paralleling those of the basic case, as shown below.

(53) a. Il **ne** va **pas** ***falloir*** sortir.
it NEG go NEG must.INF go.out
We will have to not go out. (□¬, *¬□)

b. Tu **ne** vas **pas** ***devoir*** sortir.
2s NEG go NEG must.INF go.out
You will have to not / won't have to go out. (□¬, ¬□)

In addition, we can find examples of other types of perfective-marking that are realized as verbal inflection, thus resembling the basic case in syntax and word order, but still yield narrow scope interpretations of the modal. One example is French archaic 'passé simple', a form encoding perfective past where the verb is directly inflected and appears before the negation marker *pas*, as shown in (54). Another example is found with Spanish necessity modals *tener que* and *hay que*, which have the same basic behavior as *devoir*, as shown in (55), but whose perfective form is a typically inflected modal verb, akin to French 'passé simple'.

(54) Il **ne** ***fallut*** **pas** /nous **ne** ***dûmes*** **pas** sortir. ... *#but we did.*
it NEG must.PS NEG we NEG must.PS NEG go.out
We didn't have to go out. (¬□, *□¬)

(55) a. **No** ***tengo/hay que*** ir.
NEG must go
I don't have to/must not go. (¬□, □¬)
b. **No** ***tuve/hubo que*** ir. # ... *but I didn't.*
NEG must.PF go
I didn't have to go. (¬□, *□¬)

4. Typological evidence for other implicatures blocked by AEs

As mentioned in section 2.1, we can also find existential items strengthened to universal interpretations via scaleless implicature. The analysis presented in this paper predicts that modal items of this sort will also have their implicatures blocked by AEs.[10] I present some data that appear to bear out those predictions with Slovenian modal *moči* and French modal *avoir à*.

Slovenian has a modal *moči*, interpreted as *must* in unembedded contexts, but as *can* under negation and in the perfective aspect.[11,12] Unembedded, in its past, imperfective form, *moči* is unambiguously interpreted as universal (56). Under negation, it is unambiguously interpreted as existential (57). In the perfective aspect, it triggers an actuality entailment, and it is unambiguously interpreted as existential (58).

(56) ***Mogla*** sem dvigniti tega kamna.
MOD AUX.1SG lift this stone
I had to lift this stone.
*I could lift this stone.

(57) **Ni**-sem ***mogla*** dvigniti tega kamna.
NEG-AUX.1SG MOD lift this stone
I wasn't able to lift this stone.
*I didn't have to lift this stone.

(58) ***Z-mogla*** sem dvigniti tega kamna. ... *#but I didn't lift it.*
PF-MOD AUX.1SG lift this stone
I was able to lift this stone, #but I didn't lift it.
*I had to lift this stone.

This data is consistent with an analysis of *moči* as triggering a scaleless implicature, that is blocked by the actuality entailment of the modal.

Similarly, French has a modal *avoir à* that is unambiguously interpreted as a necessity modal in unembedded contexts, but has a possibility reading that emerges in DE contexts and with AEs, as shown in the examples below.

[10] In fact, the analysis presented in this paper also makes the prediction that scalar implicatures are blocked. I leave this investigation for future work.

[11] Thanks to Zala Mojca Jerman Kuželički and Maša Močnik for judgments.

[12] Interestingly, in its archaic meaning, *moči* was only interpreted as possibility, while a now lesser-used form *morati* was used to express necessity (current prescriptive grammar align with these archaic forms).

(59) J'***ai*** ***à*** lui parler.
I'have to pro talk
I {have to, ****can***} talk to her.

(60) Je **n'*ai*** **pas** *à* lui parler.
I neg'have neg to pro talk
I {don't have to, can't} talk to her.

(61) J'ai ***eu*** ***à*** lui parler ... *# but I didn't talk to her*.
I'have have.pf to pro talk
I {had to, **could/got to**} talk to her.

An additional puzzle emerges for *avoir à*, for which I suggest a solution here. We observe ambiguous force interpretations with negation and when marked by the perfective, in contrast with unambiguous necessity force when unembedded. A source of this ambiguity might come from the particular syntax of *avoir à*. This modal expression is formed from a possessive construction, which according to Bhatt (1998), involves a silent modal operator. It may be that in contrast with typical modal heads, the *avoir à* construction introduces a silent possibility modal, and has a spot for the EXH operator to apply before negation or the perfective operator, allowing for the following configurations in (i), in addition to the expected ones in (ii):

(62) a. (i) $\neg \text{EXH}^2 \Diamond \equiv \neg\Box$
(ii) $(\text{EXH}^2)\ \neg\Diamond \equiv \neg\Diamond$
b. (i) $Pf\ \text{EXH}^2 \Diamond \equiv \check{\Box}$
(ii) $(\text{EXH}^2)\ Pf \Diamond \equiv \check{\Diamond}$

No matter what the analysis for *avoir à* is, the data for both *moči* and *avoir à* is quite striking: in each case, a modal with an unambiguous necessity interpretation reveals its underlying possibility reading with an AE and under negation, as is predicted from a scaleless implicature analysis. This borne-out prediction lends additional support that the process observed with negated necessity modals is indeed a scaleless implicature, instead of movement.

5. Conclusion

In this work, I have presented a proposal for wide scope interpretations of French necessity modals, in which negated modal expressions are strengthened via scaleless implicature. This analysis, contrary to previously proposed movement proposals, accounts for the narrow scope interpretations with Actuality Entailments – which block strengthening by rendering alternatives non-innocently excludable. The scaleless implicature theory makes typological predictions that are borne out, namely that there exist scaleless implicatures of existential modals strengthened to necessities that are also blocked by Actuality Entailments.

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