Part-Whole-Modifiers and the *-Operator[1]

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Abstract. In this paper, I present an analysis of German *'ganz'* (≈ *whole*), which I see as evidence for the existence of the plural operator * in the syntax as well as its ability to enter into scope relationships with other operators at LF. I also elaborate on more general ideas of 'wholeness' in natural language.

Keywords: Plural, Semantics, Theoretical Linguistics, German, Part/Whole Distinction, Scope Ambiguity

1. Introduction

In this paper, I present an interesting ambiguity in some German sentences containing the word *'ganz'*. *'ganz'* is usually translated into English as 'whole', but, unlike its cognate, can associate not only with singular count nouns (1a) but also with mass nouns (1b) and plural count nouns (1c). The ambiguity arises in precisely these cases.[2]

(1) a. *Beth baute das ganze Auto.*
Beth built the.SG *ganz*.SG car.SG
'Beth built the whole car.'

b. *Jolene aß das ganze Brot.*
Jolene ate the.SG *ganz*.SG bread.SG
(i) 'Jolene ate all the bread.'
(ii) 'Jolene ate the whole slice/loaf... of bread.'

c. *Jolene aß die ganzen Brote.*
Jolene ate the.PL *ganz*.PL bread.PL
(i) 'Jolene ate all the bread.'
(ii) 'Jolene ate every whole slice/loaf... of bread.'

I take the parallel ambiguities in (1b) and (1c) as evidence that the underlying mechanism is the same. Both sentences are analyzed as a scope ambiguity between *'ganz'* and the plural operator * (Link, 1983). The issue is explained in more detail in Section 2, where I also reject the null hypothesis that the phenomenon (particularly (1c)) can be analyzed as a simple lexical ambiguity. A basic framework for analyzing plurality is presented in Section 3 and used to develop a preliminary lexical entry for *'ganz'* in Section 4. This lexical entry is then used to offer a structural analysis able to derive the ambiguity. Some of the predictions made by this analysis are tested and confirmed in Section 5, which offers further support to the analysis. In Section 6, I elaborate on the idea of 'wholeness' in natural language, which enables me to formulate the lexical entry for *'ganz'* more precisely. Section 7 concludes the paper.

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[2] Glosses throughout the paper contain morphological information only with regards to number marking (SG for singular, PL for plural), and only where they further understanding.

2. The Issue

Reconsider the example in (1b). In contexts where there is only one unit of bread, e.g. a single loaf or slice, the ambiguity disappears, as 'all the bread' and 'the whole unit of bread' are synonymous in these contexts. Let us therefore examine a context where (1b) is truly ambiguous, cf. Figure 1.

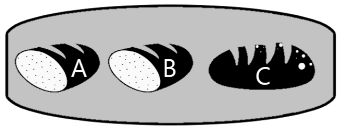


Figure 1: C_{1w}-context

Contexts like this one, containing exactly one 'whole object' but more than one object in total, will be referred to as *one whole contexts* (C_{1w}). Here, (1) can have two different interpretations: Jolene either ate A, B, and C (all the bread); or she only ate C (the 'whole unit' of bread). I will refer to the readings in (1b-ii)/(1c-ii), where 'ganz' makes reference to a form of 'wholeness', as *integrity readings* and (1b-i)/ (1c-i), where it seems cognate with English 'all', as *universal readings* (cf. Wągiel, 2018: Ch. 3.5 for a cross-linguistic perspective on 'whole'-adjectives). Sentences with overt plural morphology (1c) will be referred to as PM^+-sentences, sentences without (1b) as PM^--sentences.

2.1. The Null Hypothesis: Mass-Count-Ambiguity

A straightforward solution is to situate the ambiguity at a lexical level in the word *'Brot'*. Like its English cognate *'bread'*, *'Brot'* appears to be, at base level, a mass noun[3] . When appearing with a definite article, a maximal interpretation is derived. For example, (2) is true if Beth baked all the bread available in the context. This holds for English (2a) as well as the German word-for-word translation (2b).

(2) a. *Beth baked the bread.*
b. *Beth buk das Brot.*

However, this is as far as the English *'bread'* will go. Further attempts to use it as a count noun require an explicit unit or measure phrase to be made salient:

(3) a. **Beth baked two breads.*
b. *Beth baked two loaves of bread.*

'Brot', on the other hand, allows for a unit, but works just as well without:

(4) a. *Beth buk zwei Brote.*
Beth baked two bread.pl
b. *Beth buk zwei Brotlaibe.*
Beth baked two loaf-of-bread.pl

The fact that *'Brot'* can exhibit plural morphology, and be directly counted, could point towards a true lexical ambiguity, with two possible lexical entries for *'Brot'* sketched in (5).

(5) a. $[\![\text{Brot}_{\text{mass}}]\!] = \lambda x_e.x$ is a quantity of bread
b. $[\![\text{Brot}_{\text{count}}]\!] = \lambda x_e.x$ is a countable unit of bread

This assumption straightforwardly resolves the ambiguity in (1b):

[3]For an in-depth discussion of the mass-count distinction, cf. e.g. Krifka (1989); Chierchia (1998); Rothstein (2008, 2010).

(6) a. ⟦das ganze $\text{Brot}_{\text{count}}$⟧ = 'the unique maximal quantity of bread' $\triangleq$ A, B and C
b. ⟦das ganze $\text{Brot}_{\text{count}}$⟧ = 'the unique maximal countable unit of bread' $\triangleq$ C

I reject this approach for two reasons. The first is dogmatic: lexical ambiguity is not a very elegant solution. Situating the ambiguity in the lexicon does not provide much insight beyond the word itself. Even under the untested assumption that most or all German mass nouns are ambiguous in this way, the scope of the issue would be narrowly focused on the German lexicon. While there are certainly interesting observations to be made within this realm, applications in other languages or disciplines would be hard to find and even harder to justify.

The second reason for rejecting the mass-count ambiguity as a source of the ambiguity in (1b) lies in a parallel ambiguity found in PM^{+}-sentences in contexts like the one in Fig. 2 where more than one 'whole' countable unit exists. I will refer to this context as a *multiple-whole-context* (C_{nw}).

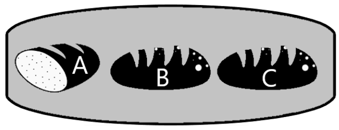


Figure 2: C_{nw}-Context

(7) *Jolene aß die ganzen Brote.*
Jolene ate the *ganz* bread.pl
a. 'Jolene ate all the bread.'
b. 'Jolene ate every whole (intact) countable unit of bread.'

Moltmann (1997) noted that, when combining with plural count nouns or mass nouns, *'ganz'* receives a maximizing interpretation, corresponding to the universal reading in (7a). However, the availability of the integrity reading (7b) shows that the picture is not quite that simple.

Assuming a lexical ambiguity between mass and count noun interpretations of *'Brot'* is not sufficient to resolve the ambiguity in the PM^{+}-case since *'Brot'* in (7) has plural morphology. Mass nouns can generally not be pluralized (cf. e.g. Rothstein, 2008)[4] but the universal reading is still available. As the universal reading was assumed to correspond to the mass noun case in the PM^{-}-sentence, the null hypothesis predicts this reading to disappear in in the PM^{+}-case. This is not borne out.

Shortcomings of the null hypothesis can be illustrated further by studying the sentences without *'ganz'*:[5]

[4]There are some exceptions to this: pluralized mass nouns in Greek, for example, receive an abundance reading (Tsoulas, 2009). Similar constructions exist in German, e.g. *'die Wasser des Rheins'* (the waters of the Rhine), and receive a similar interpretation, but are beyond the scope of this paper.

[5] We assume the same truth conditions for sentences without *ganz* as for the universal reading with *ganz*. This raises questions about the word's actual semantic contribution. The most salient option seems to be that of imposing a 'good fit'-requirement (Morzycki, 2002; Brisson, 2003) on the cover functioning as the restrictor of *, similar to English 'all'. In prose, the cover in plural sentences can be structured in such a way that it allows for exceptions (ia), but not if the plural is modified by 'all' (ib).

(i) a. 'The chess players sat down.'
$\rightarrow$Exceptions possible (a few chess players are still standing)
b. 'All the chess players sat down'
$\rightarrow$No exceptions

However, as (ii) (Ex. 3:58 from Wągiel (2018)) shows, 'ganz' does not impose such a requirement:

(ii) Context: My neighbours have 10 children and I don't like them.
a. *Dann sind die Nachbarn mit ihren ganzen Kindern gekommen. Zwei waren krank zuhause, und*
then are the neighbours with their *ganz* children come two were ill at-home and

(8) C_{1w}-context:

a. *Jolene aß das Brot.*
Jolene ate the bread
(i) Available: Jolene ate the maximal quantity of bread available in the context
('she ate A,B and C' $\triangleq$ universal reading)
(ii) Not available: Jolene ate the unique countable unit of bread
('she ate C' $\triangleq$ integrity reading)

(9) C_{nw}-context:

a. *Jolene aß die Brote.*
Jolene ate the bread.pl
(i) Available: Jolene ate the maximal quantity of bread available in the context
('she ate A,B and C' $\triangleq$ universal reading)
(ii) Not available: Jolene ate the countable units of bread
('she ate B and C' $\triangleq$ integrity reading)

Non-maximal interpretations notwithstanding, both sentences are unambiguous in their respective contexts, deriving only the universal interpretation. If *'Brot'* were lexically ambiguous between mass and count noun interpretations, this ambiguity should be available even without *'ganz'*, which is not borne out. Instead, the analysis I suggest in Section 4 is structural. I locate the ambiguity in the scope relationship between German *'ganz'* and the plural operator * (Link, 1983), which solves both issues with the lexical ambiguity approach: Firstly, the (blocked) interpretation of *'Brot'* as a mass noun is no longer required to explain the ambiguity in (7). Secondly, if the scope relationship between *'ganz'* and * is the source of the ambiguity, it makes sense that the ambiguity disappears in sentences without *'ganz'*. In the next section, I present the framework used for this analysis.

3. Framework

3.1. Mereology

The standard framework for discussing parthood in natural language is referred to as *mereology*. Since it was first formalized by Link (1983), much work has been done on the formal implementation (cf. Champollion (2010) and the discussion therein). The version I will use follows Champollion and Krifka (2016), a handbook article on the topic, in which parthood is defined by imposing a structure on the domain of entities D_e. Formalizing this structure requires two ingredients, a join operation $\oplus$ and a parthood relation $\leq$.

(10) **Rules for $\leq$:**

a. *Reflexivity* (Every entity is a part of itself):
$\forall x \in D_e : x \leq x$

b. *Antisymmetry* (No entity is a part of one of its proper parts):

es waren immer noch acht!
it were still eight
'Then the neighbours came with their children. Two of them were ill and stayed home, but there were still eight.'

As the continuation shows, *'ganz'* in (ii) still allows for exceptions. The effect created by *'ganz'* instead seems to be one of abundance. (ii) would not be felicitous if, say, the neighbours only have two children, regardless of how many actually came. The contrast between sentences with and without *'ganz'* warrants further study, but is beyond the scope of this paper.

$\forall x,y \in D_e : x \leq y \& x \neq y \rightarrow y \nleq x$

c. *Transitivity* (Any part of a part of an entity is itself a part of that entity)[6]:
$\forall x,y,z \in D_e : x \leq y \& y \leq z \rightarrow x \leq z$

(11) **Proper Part:**

a. $x < y :\Leftrightarrow x \leq y \& x \neq y$

(12) **Rules for $\oplus$:**

a. $\forall x,y \in D_e[x \leq x \oplus y \& y \leq x \oplus y]$
Any 'member' of a plural individual is a part of that individual

b. $\forall x,y, \in D_e[x \oplus y \in D_e]$
D_e is closed w.r.t. $\oplus$: for any two objects in D_e, their join is also in D_e.

3.2. Plural Predication

For a predicate to be applicable to a plural individual, it has to be modified with the *-operator (Link, 1983; Krifka, 1986)[7].

(13) *: $D_{\langle e,t \rangle} \rightarrow D_{\langle e,t \rangle}$ is the function s.t. $\forall P_{\langle e,t \rangle}, x_e$:
$[^*P](x) = 1 \iff [P](x) = 1$ **or** $\exists x_1, x_2 \in D_e$ s.t. $x = x_1 \oplus x_2 \& [^*P](x_1) = [^*P](x_2) = 1$

If P is a predicate, $[^*P]$ contains all elements of P as well as all plural individuals made up of those elements:

(14) Context: The chess players are Beth, Harry and Vasily.

a. $[\![\text{chess player}]\!] = \{B; H; V\}$

b. $[\![^*\text{chess player}]\!] = \{B; H; V; B \oplus H; B \oplus V; H \oplus V; B \oplus H \oplus V\}$

Note: it is assumed that the use of * is licensed by plural morphology (e.g. English plural *-s*), but the locality of this licensing mechanism is controversial (Sauerland, 1998; Beck and Sauerland, 2000; Kratzer, 2007). This will be discussed further in the analysis.

The definite determiner, when applied to a plural (starred) predicate, picks out the maximal element (Sharvy, 1980; Link, 1983). This version is modeled on Schwarz (2013):

(15) a. Sum Formation - The maximizing function σ (Link, 1983) returns the maximal element of a given set. $\sigma[P]$ is called the *sum* of P.
$\sigma = \lambda P_{\langle e,t \rangle}.\lambda x_e.P(x) \& \forall y : P(y) = 1 \rightarrow y \leq x$

b. Definite determiner presupposes existence of unique maximum and returns it:
$[\![\text{the}]\!] = \lambda P_{\langle e,t \rangle} : \exists! x_e[\sigma(P)(x) = 1].\iota x[\sigma(P)(x) = 1]$

c. For example, in (15):
$[\![\textit{chess player}]\!] = \{\textit{Beth; Harry; Vasily; } B \oplus H; B \oplus V; H \oplus V; B \oplus H \oplus V\}$

d. $[\textit{the chess players}]$
$= [\textit{the}\,[^*\textit{chess player}]]$
$= \iota x[\sigma([^*\textit{chess player}])(x)]$
$= \iota x[[^*\textit{chess player}](x) \& \forall y[^*\textit{chess player}](y) \rightarrow y \leq x]]$

[6]Transitivity is not uncontroversial for parthood in natural language. We will return to this in Section 6.

[7]The *-operator usually requires a restrictor set (type $\langle$e,t$\rangle$) known as a cover, for example for cases of non-maximality. It is essential, but left out of the current analysis for purposes of readability. For more on this, see Schwarzschild (1994); Brisson (2003); Schwarz (2013), among others.

$= B \oplus H \oplus V$
$\rightarrow$ *'the chess players'* refers to the plural individual consisting of all three players.

As an example, consider (16). It is true either if the chess players won \$1000 as a group (**collective interpretation**), or if each won \$1000 individually (**distributive interpretation**)[8].

(16) *The chess players won 1000 dollars.*
a. $[$ *The* $[{}^{*}$*chess player*$]][{}^{*}$*won \$1000*$] = 1$ iff
$[{}^{*}$*won \$1000*$](B \oplus H \oplus V) = 1$ iff
$B \oplus H \oplus V$ won \$ 1000 **or** B won \$1000, H won \$1000, V won \$1000

3.3. Number Agreement in German
The framework discussed works for both English and German, with one notable addition for German. German verbs and articles are marked for agreement with regards to number:

(17) a. ***Das*** *Schachbrett ist aus Glas.*
the.SG chess.board.SG be.SG from glass
'The chess board is made from glass.'
b. ***Die*** *Schachbretter sind aufgebaut.*
the.PL chess.board.PL be.PL set.up
'The chess boards are set up.'

The singular feature [SG] is analyzed as an identity function expressing the presupposition "my sister [node] denotes an atom or mass" (Sauerland, 2003). The distribution of [PL] is governed by Heim (1991)'s Maximize Presupposition principle: if the PSP of [SG] is met, [SG] should be used. [PL] thus carries the anti-presupposition that the PSP of [SG] is not met, meaning its sister denotes something which is neither atom nor mass (e.g. a plural noun). Agreement is tangential to the analysis, but important for testing predictions in Section 5. I will include morphological information in glosses only where relevant. Armed with these basic tools, the next section will be used to formulate a preliminary lexical entry for *ganz* and provide an analysis of (1) that derives both readings[9].

4. Analysis

4.1. A lexical entry
Any lexical entry for *'ganz'* has to somehow encode the concept of 'wholeness'. Like parthood, this concept seems deeply ingrained in natural language. However, it is far from trivial to define the concept within a framework. We will return to this in Section 6; for our current purposes, it is enough to point out that 'wholeness' is usually tethered to a predicate (Moltmann (1997, 2005), a.o.). For example, *'Beth's head'* can be classified as a *'whole head'*, but not a *'whole person'*. A given group of musicians may make up a *'whole trio'*, but not a *'whole orchestra.'* I will use the shorthand $[whole_P](\mathrm{x})$ to mean 'x is a *whole* P', and revisit this part of the analysis

[8] A third potential option is *partial cumulation*, e.g. Beth won \$1000 on her own, while Harry and Vasily won \$1000 together. The analysis as presented returns true in these scenarios. This is by no means uncontroversial, but outside the scope of this paper. For a discussion on cumulation and distributivity, see e.g. Winter (2000), Beck and Sauerland (2000), Kratzer (2007).

[9] For further background on plurality cf. Lasersohn (1989); Schein (1986); Beck and Sauerland (2000), for part-whole-modification cf. Brisson (2003); Morzycki (2002). For another approach based on set theory, cf. Schwarzschild (1994, 1996).

in more detail in Section 6. For now, we make do with the preliminary lexical entry in (18).

(18) **'ganz'** (preliminary version)
$[\![\text{ganz}]\!] = \lambda P_{\langle e,t\rangle} \lambda x_e\, [P(x) \,\&\, [whole_P](x)]$[10]

4.2. Tackling the Issue
4.2.1. PM^--Sentences

(19) *Beth isst das ganze Brot.*
Beth eat the.3sg whole bread
a. 'Beth eats all the bread.' (*universal*)
b. 'Beth eats the loaf/slice/... of bread which is whole.' (*integrity*)

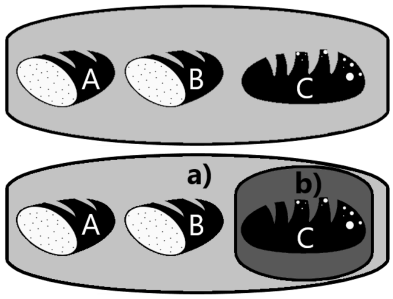


Figure 3

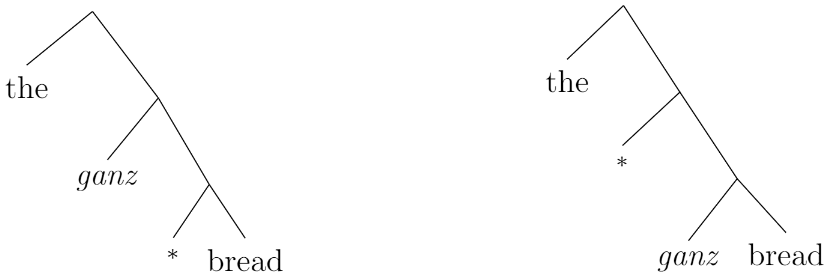
Let's return to our starting PM^- example repeated in (19) and have a closer look at the ambiguity in the C_{1w}-context repeated in Fig. 3, where *A* and *B* are halved loaves of bread while *C* is a complete loaf. Here, the universal reading (19a) is true if Beth eats *A*, *B* and *C*. The integrity reading (19b) is true if Beth eats *C*. As indicated by the picture, the universal entails the integrity reading. If Beth eats all the bread available, she also eats the complete loaf C. I point this out here, as we will make use of this fact in Section 5. Armed with the lexical entries for * and $[\![\text{ganz}]\!]$ repeated in (20), the two readings of (19) can be derived structurally. The analysis is facilitated by two important assumptions:

1. * and $[\![\text{ganz}]\!]$ share the semantic type $\langle\langle e,t\rangle,\langle e,t\rangle\rangle$
2. * is morphologically licensed, but unpronounced; its LF-position is independent of spell-out

(20) a. $[\![\text{ganz}]\!] = \lambda P_{\langle e,t\rangle} \lambda x_e.[P(x) \,\&\, [whole_P](x)]$[11]
b. $* = \lambda P_{\langle e,t\rangle} \lambda x_e.P(x) = 1 \text{ or } \exists x_1, x_2 \in D_e[x = x_1 \oplus x_2, [^*P](x_1) = [^*P](x_2) = 1]$

Taken together, these observations indicate that the scopal relationship between * and $[\![\text{ganz}]\!]$ is not overtly spelled out. It is possible for * to scope over *'ganz'* or vice versa, allowing for two distinct possible LFs for the object NP in (19)[12]:

[10]Like *, $[\![\text{ganz}]\!]$ is assumed to carry a contextual restriction, as part structures and the perception of 'wholeness' vary situationally (Moltmann, 1997; Brisson, 2003). I only explicity mention this restriction when it is relevant for the discussion at hand. Also see footnote 5.
[11]*'ganz'* can be said to carry a presupposition requiring the modified entity to have an accessible part structure (cf. Moltmann (1997)'s [ACC]). It formalizes the intuition that explicitly referencing 'wholeness' is strange in sentences where wholeness was never in question, but is not required for the present analysis.
[12]Marcin Wągiel (p.c.) pointed out that this LF structure can be further supported by data including numerals:
(i) a. *die zehn ganzen Brote*
the ten *ganz* bread
'the ten whole/complete loaves of bread'
b. *die ganzen zehn Brote*
the *ganz* ten bread
'all the ten loaves of bread'
These sentences seem to display a parallel structure, with the scope relationship between the numeral and 'ganz' reflected in the two readings.



(a) *ganz* scopes over * (b) * scopes over *ganz*

Figure 4: Possible LFs for (19)

Compositional analysis of these two structures derives two different readings as expected. We first examine the LF in 4a.

(21) $[\text{the}][\text{ganz}[{}^{*}\text{bread}]]$
$= [\text{the}][\lambda x.[{}^{*}\text{bread}](x)\,\&\,[\mathit{whole}_{[{}^{*}\text{bread}]}](x)]$
$=$ the unique sum of $[\lambda x.[{}^{*}\text{bread}](x)\,\&\,[\mathit{whole}_{[{}^{*}\text{bread}]}](x)]$
$\triangleq$ *'the unique x s.t. x is a quantity of bread, is whole as a quantity of bread, and contains all other whole quantities of bread'*
$\triangleq A{\oplus}B{\oplus}C$ (**universal reading**)

When $[\![\text{ganz}]\!]$ is allowed to scope over *, the requirement of 'wholeness' it imposes is in reference to the plural predicate $[\![{}^{*}\text{bread}]\!]$, which contains all individual pieces of bread as well as their respective joins. ($[\![{}^{*}\text{bread}]\!] = \{A,B,C,A{\oplus}B,A{\oplus}C,B{\oplus}C,A{\oplus}B{\oplus}C\}$). The sum of that set, picked out by the definite determiner as the object of the sentence, is the plural individual $A{\oplus}B{\oplus}C$. The universal reading is derived. Compare this to the inverse case in Fig. 4b:

(22) $[\text{the}][{}^{*}[\text{ganz}[\text{bread}]]]$
$= [\text{the}][{}^{*}[\lambda x.[\text{bread}](x)\,\&\,[\mathit{whole}_{[\text{bread}]}(x)]$
$=$ the unique sum of $[\lambda x.[{}^{*}[[\text{bread}](x)\,\&\,[\mathit{whole}_{[\text{bread}]}(x)]]$
$= \iota x[\forall y_e[{}^{*}[[\text{bread}](y)\,\&\,[\mathit{whole}_{[\text{bread}]}(y)]]] \rightarrow y \leq x]$
'the unique x s.t. x is a plurality of [whole bread], and any other such plurality is contained in x'
$\triangleq C$ (**integrity reading**)

In this case, $[\![\text{ganz}]\!]$ takes the unstarred predicate $[\![\text{bread}]\!]$ as its argument, creating the property of being a '*ganz* bread' – a singular object which can be considered 'whole' as bread. The only object in the context which satisfies this criterion is C: [*ganz* bread] $= \{C\}$ This set is then pluralized, and the sum of the pluralized set is created. On a singleton set, these operations trivially return its only member C. The integrity reading is derived.

4.2.2. The Plural Case

As stated in Section 2, the PM^{-}-examples could be explained by a mass-count ambiguity in the lexicon. However, this explanation fails to capture the ambiguity exhibited by the PM^{+}-example repeated in (23) in the C_{nw}-context in Fig. 5.

(23) Jolene aß die ganzen Brote.
Jolene ate the whole bread.
a. 'Jolene ate all the bread.' $\triangleq$ she ate A, B and C
b. 'Jolene ate every intact countable unit of bread.' $\triangleq$ she ate B and C, but not A

The structural analysis handles this case just fine. Note that the only difference to the singular example is the overt plural morphology present in (23). In our framework, plural morphology serves to license *. Since we already assume * to be licensed by the mass noun 'Brot' in the PM^{-} case, LFs and truth-conditional analysis for the PM^{+}-case are identical to those calculated in (21) and (22). In fact, I assume that the LFs in Fig. 4, as well as the calculations in (21) and (22), can derive both readings also for the PM^{+}-sentences. Let us re-examine the final lines of the calculations, repeated in (24), in a C_{nw}-context.

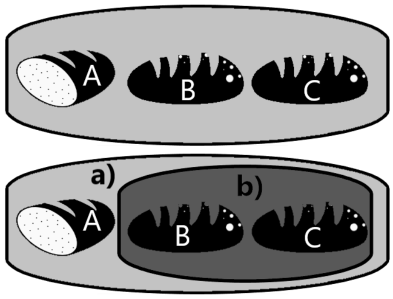


Figure 5

(24) a. **Universal** (LF Fig 4a):
[die ganzen Brote] = *the unique sum of* $\lambda x.[{}^{*}\text{bread}](x)\,\&\,[whole_{[{}^{*}\text{bread}]}](x)$
b. **Integrity** (LF Fig 4b):
[die ganzen Brote] = *the unique sum of* $\lambda x.{}^{*}[[\text{bread}](x)\,\&\,[whole_{[\text{bread}]}(x)]]$

In (24a), we derive the unique x such that x is a quantity of bread, is whole as a quantity of bread, and contains all other quantities of bread. This only applies to the plural individual $A \oplus B \oplus C$. The universal reading is derived. In (24b), the calculation returns the individual x such that x is a plurality of *whole* loaves of bread, and contains all *whole* loaves available in the context. This applies to $B \oplus C$ only. The integrity reading is derived. Due to the parallel calculations, this analysis makes fairly strong predictions regarding cross-contextual uses of the respective examples. We will examine this more closely in the next section.

5. Predictions

In this section, I will examine and confirm two sets of predictions made by the analysis.

5.1. Predictions regarding Cross-Contextual Use

In Section 4, we assumed parallel structures and identical calculations for PM^{+} and PM^{-} sentences. * having scope over *'ganz'* yields the integrity reading, while the inverse scope relationship yields the universal reading. This is true for both PM^{+} and PM^{-} sentences. This close relationship suggests that the PM^{+} sentence might also work in a C_{1w}-context and vice versa. I will refer to these examples as *cross-contextual uses*. The analysis makes some interesting predictions in this regard.

Since structure and lexical entries are identical, the analysis also predicts identical truth conditions for PM^{+} and PM^{-}. However, there is a difference in the PSP of the definite determiner, which is marked for number. The analysis makes predictions regarding the availability of both readings in cross-contextual uses, depending on the status of this PSP.

To facilitate further calulations, we first calculate the denotation we would expect from both LF structures in the respective contexts (repeated in Figure 6), if they were defined.

(a) C_{1w}-context (b) C_{nw}-context

Figure 6

(25) Preliminary calculations - **If defined**, we predict these identities:
a. [das [ganz [*Brot]]]
 = [the][$whole_{[^*bread]}$](C)
 = the unique sum of $\{A \oplus B \oplus C\} = A \oplus B \oplus C$ for $C \in \{C_{1w}, C_{nw}\}$
b. [das [*[ganz Brot]]]
 = [the]$\{x : x$ is $[whole_{[\text{bread}]}](C)$ or x is a plurality of $[whole_{[\text{bread}]}](C)\}$
 (i) = the unique sum of $\{B, C, B \oplus C\} = B \oplus C$ for $C = C_{nw}$
 (ii) = the unique sum of $\{C\} = C$ for $C = C_{1w}$

Let us now have a look at the respective universal readings.

(26) a. Context: C_{nw}
 (i) Beth isst das ganze Brot. (PM$^-$)
 (ii) [das [ganz [*Brot]]]
b. Context: C_{1w}
 (i) Beth isst die ganzen Brote. (PM$^+$)
 (ii) [die [ganz [*Brot]]]

The LFs are identical, and in both cases the uniqueness-PSP of the definite determiner is easily met, since all that is required is a maximal amount of bread. Barring contexts that do not contain any bread[13], a maximal quantity of bread can always be assumed to exist. The analysis thus predicts that, when trying to evoke the universal reading, PM$^+$ and PM$^-$ sentences can be used interchangeably, regardless of the context.

Next, I turn to the integrity readings. In these examples, singular [SG]- and plural [SG]-features have been marked, as agreement (cf. Section 3.3) plays a role.

(27) a. Context: C_{nw}
 (i) Beth isst das.SG ganze.SG Brot.SG. (PM$^-$)
 (ii) [das [* [ganz Brot]]]
b. Context: C_{1w}
 (i) Beth isst die.PL ganzen.PL Brote.PL. (PM$^+$)
 (ii) [die [*[ganz Brot]]]

At first glance, the PSP of (27a) is met. As we calculated in (25b-i), [*[ganz Brot]] has a unique sum in C_{nw}, namely $B \oplus C$. However, this is where agreement saves the day. As mentioned above, the definite determiner as used in (27a) carries a singular feature, expressing the presupposition "my sister denotes an atom or mass" (Sauerland, 2003). Under the universal reading, this poses no issue, as a maximal 'mass' of bread was targeted. Under the integrity reading however, the definite determiner is supposed to target $B \oplus C$, which is neither atom nor

[13]Which would be interesting, but my intuition is that neither sentence is felicitous in those contexts.

mass. The agreement PSP fails. I thus predict the integrity reading to be unavailable for the PM^{-}-sentence in C_{nw}.

In the reverse case, the PSP of the definite determiner is also straightforwardly met. If $[^{*}$[ganz Brot]] is a singleton set, it trivially has a unique sum. Agreement does not pose a direct hurdle here, as [PL] itself does not carry a presupposition (Sauerland, 2003). Instead, its distribution is governed by the principle "Maximize Presupposition" (max-PSP) Heim (1991). In order to be as informative as possible, speakers should presuppose as much as possible. Knowingly employing the [PL]-feature where the [SG]-alternative would be felicitous is a violation of this principle. This directly applies to the analysis. The PSPs of (27a) and (27b) are identical, except that (27a) carries an [SG]-feature:

(28) PSPs of Integrity readings
  a. $PM^{-} : \exists! x[x = \sigma[^{*}\textit{ganz Brot}] \& \textsc{sg}(x)]$
  b. $PM^{+} : \exists! x[x = \sigma[^{*}\textit{ganz Brot}]]$

The analysis thus predicts that uttering (27b) in the provided context should be possible. However, due to the entailment relation between the PSPs, knowingly uttering (27b) when (27a) would also be felicitous again violates max-PSP. In these cases, I predict the integrity reading to be possible, but degraded. Predictions for cross-contextual use are summed up in Table 1.

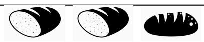
	 C_{1w}	 C_{nw}
PM^{-} 'das ganze Brot'	(Universal (a) ✓) (Integrity (b) ✓)	Universal ✓ Integrity X
PM^{+} 'die ganzen Brote'	Universal ✓ Integrity ?	(Universal ✓) (Integrity ✓)

Table 1: Predictions regarding cross-contextual use

This picture is empirically adequate: Uttering 'das ganze Brot' in a C_{nw}-context only evokes the universal reading. The integrity reading is unavailable.

(29) Context: At a bakery at the end of the day, all but 2 full loaves and one half loaf of bread have been sold (C_{nw}). The baker tells her apprentice:
  a. *Räum das ganze Brot weg, bitte!*
     put the *ganz* bread away please
     (i) Available: 'Put away all the bread, please.'
     (ii) Unavailable: 'Put away the whole loaf/loaves of bread, please.'

In contrast, uttering 'die ganzen Brote' in a C_{1w} context can evoke the integrity reading under certain conditions. Assume the same situation as in (29), but with only one complete loaf and two halves remaining (C_{1w}):

(30) *Räum die ganzen Brote weg, bitte!*
  put the *ganz* bread away please
  a. 'Put away all the bread, please.'
     (i) Available
  b. 'Put away the whole loaves of bread, please.'

(i) Available only if the baker does not know the exact number of complete loaves remaining.
(ii) Unavailable otherwise

The integrity reading in (30) is perfectly attainable. Evoking the integrity reading in a PM$^+$-sentence is possible even in cross-contextual use. However, ignorance is key here. If the baker and her apprentice are standing together, with a clear view of the remaining inventory, the integrity reading of (30) is blocked.

Thus, the predictions the analysis makes for the cross-contextual use are confirmed. We can now turn to another prediction, which concerns more complex plural operators.

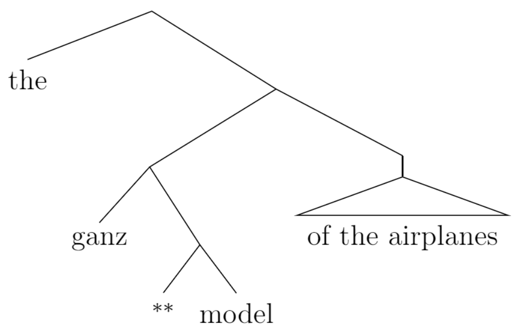
5.2. Higher Order Plural Operators

Another prediction concerns plural operators of a higher type (Krifka, 1986; Sternefeld, 1998). These operators are necessary when pluralizing predicates which take two or more arguments as in (31).

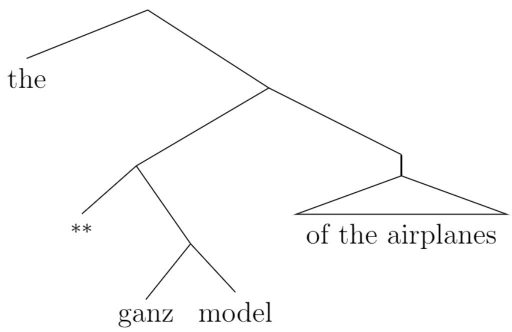
(31) *die ganzen Modelle von den Flugzeugen*
the *ganz* model.pl of the airplanes
a. 'all the models of the airplanes'
b. 'the complete(d) models of the airplanes'

The version in (32a) for ⟨e,⟨e,t⟩⟩-predicates, comes from Beck (2000). Under the assumption that *'ganz'* is typeshifted to ⟨⟨e,⟨e,t⟩⟩,⟨e,⟨e,t⟩⟩⟩ as in (32b), two LFs are again possible (Fig. 7).

(32) a. $^{**} : D_{\langle e,et\rangle} \to D_{\langle e,et\rangle}$ is the function s.t. $\forall P_{\langle e,et\rangle} : [^{**}P](x)(y) = 1 \iff P(x)(y)=1$ **or** $\exists x_1x_2y_1y_2[x=x_1\oplus x_2, y=y_1\oplus y_2, [^{**}P](x_1)(y_1)=[^{**}P](x_2)(y_2)=1]$
b. $[\![\text{ganz}]\!] = \lambda P_{\langle e,\langle e,t\rangle\rangle}.\lambda x_e.\lambda y_e.P(x)(y) = 1\&[whole_{P(x)}](y)$



(a) *ganz* scopes over **



(b) ** scopes over *ganz*

Figure 7: LFs for (31)

Without delving into the compositional analysis too deeply, the existence of the two LFs indicates that the sentences with ****** exhibit the same ambiguity as the ***** ones.

(33) a. the [*ganz* [[**model][of the airplanes]]]
$\triangleq$ the unique member of [λx. x is a 'whole' totality of airplane models]
$\triangleq$ all the models $\to$ **universal reading**

b. the [** [*ganz*[model][of the airplanes]]]
$\triangleq$ the unique sum of [$^{**}\lambda x$ x is a 'whole' model (of the airplanes)]
$\triangleq$ the totality of 'whole' models $\rightarrow$ **integrity reading**

Again we see that mass-count-ambiguity is not a sufficient explanation for these cases, as 'Modell' is unambiguously a count noun. There is no option of a mass count interpretation to derive the universal interpretation. The structural analysis, on the other hand, allows for the calculation of two sets of truth conditions correspondent to the two readings, predicting the ambiguity for higher-order plural operators such as ******.

In this section, we have investigated two phenomena, cross-contextual use and higher-order plural operators. I reject the null hypothesis of a lexical ambiguity, since it fails in at least one of these cases. Conversely, the structural analysis correctly predicts the available readings. However, there is still one key ingredient missing in the lexical entry of *'ganz'*, which we turn to in the next section.

6. What it Means to be Whole

In order to fully understand *'ganz'*, it is necessary to talk about the concept of 'wholeness' as encoded in the $[whole]$ within the lexical entry (39). Before we can tackle this, we have to discuss a more specific kind of 'wholeness', namely integrated wholes.

6.1. Integrated Wholes

In plural semantics, the term 'integrated whole' (Moltmann (1997), Wągiel (2018: Ch. 6)) refers to the form of 'wholeness' most readily available in natural language. It is the type of 'wholeness' that sentences like those in (34) make reference to.

(34) a. *The apple is whole.*
b. *Beth took a whole bottle of pills.*

Basic mereology falls short when trying to define integrated wholes, since the parthood relation it employs is unrestrictive.

(35) a. $[\![\text{Beth's arm}]\!] \leq [\![\text{Beth}]\!]$
b. $[\![\text{Beth}]\!] \leq [\![\text{Beth} \oplus \text{Harry}]\!]$

Intuitively, $[\![\text{Beth}]\!]$ constitutes an integrated whole, while $[\![\text{Beth} \oplus \text{Harry}]\!]$ does not. However, there is no way to distinguish between the two on the basis of mereological parthood alone, as it applies to parts of integrated wholes and parts of less well-organized objects equally. Instead, the more complex notion of *mereotopology* is required to explain the underlying structure of part-whole modification.

Mereotopology refers to the merging of mereology with topology, the study of those properties of objects which remain invariant under certain spatial deformations. A combination of these fields has repeatedly been advocated for and attempted on a philosophical basis (e.g. Whitehead, 1920; Clarke, 1981). A more formal approach to mereotopology (Varzi, 1996; Casati et al., 1999) was transposed for the realm of natural language semantics by Grimm (2012a, b). The version sketched here follows Wągiel (2018: Ch. 6), a concise version can be found in Wągiel (2019).

One of the most basic topological notions employed in mereotopological approaches is a connectivity relation $\odot$ on D_e. $\odot$ is reflexive (everything is connected to itself) and symmetric (if x is connected to y, then y is also connected to x). Unlike the parthood relation, it is not transitive: The head bone is connected to the back bone, and the back bone is connected to the hip bone, but the head bone is not connected to the hip bone. For (relative) brevity's sake, I repeat here only those mereotopological definitions required to define an integrated whole.

(36) Mereotopological definitions (Wągiel, 2018)[14]:

a. Internal Parts

(i) Overlap $\mathcal{O}$:
$x\mathcal{O}y :\Longleftrightarrow \exists z[z \leq x \& z \leq y]$

(ii) Internal Part IP
$\textsc{ip}(x,y) :\Longleftrightarrow x \leq y \& \forall z[z \odot x \rightarrow z\mathcal{O}y]$

(iii) Interior x_{int}
$x_{int} := \bigcup\{y : \textsc{ip}(y,x)\}$

b. States of self-connectedness

(i) Self-connected:
$sc(x) :\Leftrightarrow \forall y \forall z[\forall w[x\mathcal{O}w \leftrightarrow (w\mathcal{O}y \vee w\mathcal{O}z) \rightarrow y \odot z]]$
An entity is self-connected if and only if any two parts that form the whole of that entity are connected to each other.

(ii) Strongly self-connected:
$ssc(x) :\Leftrightarrow sc(x) \& sc(x_{int})$
An entity is strongly self-connected if it is self-connected and its interior is self-connected.

(iii) Maximally-strongly-self-connected ($\triangleq$ **x is integrated whole**):
$IW(x) :\Leftrightarrow mssc(x) :\Longleftrightarrow ssc(x) \& \forall y[ssc(y) \& y\mathcal{O}x \rightarrow y \leq x]$
An entity is maximally strongly self-connected if it is strongly self-connected, and anything which overlaps with it and is strongly self connected is a part of it (maximality).

(iv) Maximally-strongly-self-connected relative to a property P
(**$\triangleq$ x is integrated whole w.r.t. P:)**
$IW_P(x) :\Leftrightarrow mssc(P)(x) :\Longleftrightarrow P(x) \& ssc(x) \& \forall y[P(y) \& ssc(y) \& y\mathcal{O}x \rightarrow y \leq x]$
x is maximally strongly self-connected relative to P if it is strongly self-connected and any other P-entity y that overlaps x is a part of x

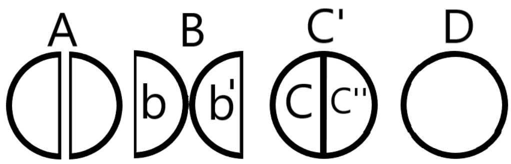


Figure 8: States of Self-Connectedness

An entity is an integrated P-whole if it is maximally strongly self-connected relative to P. For example, consider the objects in Fig. 8 relative to P := $\{x$: x is a circle or half-circle$\}$. A has two unconnected parts, and is therefore not self-connected at all. B is self-connected ($b \odot b'$), but not

[14]With minor changes suggested by Marcin Wągiel (p.c.).

strongly ($b_{int} \neg \odot b'_{int}$). C is strongly self-connected, but part of a larger strongly self-connected entity C', therefore C is not maximal. D is maximally strongly self-connected relative to P and therefore constitutes an integrated P-whole: it is a P-entity, strongly self-connected, and maximal.

I now make use of Wągiel's definition of integrated wholes to define the integrated closure of a predicate P as the set containing the 'missing parts' of all the entities in P.

6.2. Integrated Closure

Integrated wholes are an important concept in natural language. Humans possess not only the ability to judge something as an integrated whole, but also have an intuitive knowledge of missing pieces: If you find a broken table with only three legs, the existence (at some point in time and space) of the fourth leg is a natural assumption. This allows for the definition of the *integrated closure* of a given subset P of D_e.

(37) **Integrated Closure** of $P \subseteq D_e$ w. r. t. a predicate $Q \subseteq D_e$:
$IC_Q(P) := P \cup \{x \in D_e : \exists y \in P[(IW_Q(x) \& y \leq x) \vee IW_Q(x \oplus y)]\}$

The integrated closure of a set of entities P w.r.t. a predicate Q contains P, every integrated Q-whole that has a part in P, and every part of those integrated Q-wholes. As an example, we calculate the integrated closure of the set $P := \{C, D\}$ w.r.t. $Q := \{x : \text{x is circle or half-circle}\}$ in Fig. 8. To simplify a little, we assume D_e to be restricted to the items pictured.

(38) $IC_Q(\{C, D\})$
$= \{C, D\} \cup \{x \in D_e : \exists y \in \{C, D\}[IW_Q(x) \& y \leq x] \vee IW_Q(x \oplus y)\}$ From (37)
a. $\{x \in D_e : \exists y \in P[IW_Q(x) \& y \leq x)]\}$
Adds every integrated Q-whole with a part in P
$= \{D, C, C'\}$ Due to $IW_Q(C'), C \leq C', C' \leq C'; IW_Q(D), D \leq D$
b. $\{x \in D_e : \exists y \in P[IW_Q(x \oplus y)]\}$
Adds every Q-part of integrated Q-wholes in P
$= \{C''\}$ Due to $IW_Q(C \oplus C'')$
$\Rightarrow IC_Q(\{C, D\}) = \{C, C', C'', D\}$

In prose, the integrated closure of the set $P = \{C, D\}$ contains the set itself, as well as every integrated Q-whole with a part in P (in this case C') and every missing part of those Q-wholes (C'').

6.3. Back to *'ganz'*

The definitions of integrated wholes and integrated closure offer a solution to the original problem. Integrity readings can be assumed to make reference to integrated wholes in some way. Integrated closure is helpful because plural individuals are not integrated wholes. This allows me to define a more vague notion of 'wholeness' which can encapsulate plurals, but still check whether the singular objects in its argument set are integrated wholes.

Recall the lexical entry for *'ganz'* from Section 4[15]:

(39) $[\![\text{ganz}]\!] = \lambda C_{\langle e,t \rangle} \lambda P_{\langle e,t \rangle} \lambda x_e . [P(x) \& [whole_P](C)(x)]$

[15] The contextual restriction becomes relevant here, and is thus made explicit.

We now have the tools to provide a definition of $[whole_P]$:

(40) $[whole_P] = \lambda C_{\langle e,t\rangle} \lambda x_e . P(x) \,\&\, \neg\exists y \in IC_P(C) s.t.[x < y]$

Under this analysis, an entity x is considered a 'whole P' in a context C if there are no missing parts - e.g. there is no larger P-entity y such that x is a proper part of y and y is in the integrated closure of C with regards to P. This definition, in combination with the definition of integrated closure in (37) ensures that, if P is an unstarred predicate, the set $[whole_P](C)$ only contains integrated wholes. This is due to the fact that singular objects are generally strongly self-connected, meaning any P-entity that is not itself an integrated P-whole is a proper part of an integrated P-whole.

On the other hand, applying $[whole]$ to a starred predicate $[{}^*P]$ does not result in the same constraint. Since plural individuals do not constitute integrated wholes, the integrated closure is trivial:

(41) For any $P, Q \in D_{\langle e,t\rangle} : IC_Q([{}^*P]) = [{}^*P]$

Let us apply this to our original PM$^-$-example, repeated in (42) with two possible LFs for the object NP *'das ganze Brot'*, assuming the C_{1w}-context (Fig. 3, Section 4).

(42) *Jolene aß das ganze Brot.*
Jolene ate the whole bread
a. [the [*ganz* [*bread]]]
b. [the [**ganz* [bread]]]

In (42a), *'ganz'* (and therefore $[whole]$) are applied to the starred predicate [*bread].

(43) $[ganz(C_{1w})[{}^*\text{bread}]](x)$
$= [[{}^*\text{bread}](x) \& [whole_{[{}^*\text{bread}]}](C_{1w})(x)]$ (39)
$= [[{}^*\text{bread}](x) \& \neg\exists y \in IC_{[{}^*\text{bread}]}(C_{1w}) s.t.[x < y]]$ (40)
$= [[{}^*\text{bread}](x) \neg\exists y \in [{}^*\text{bread}] s.t. x < y]$ (41)
$= \{A \oplus B \oplus C\}$ All other members of $[{}^*bread]$ are proper parts
$\rightarrow$ [the $ganz(C_{1w})$ [*bread]] = $A \oplus B \oplus C$
$\triangleq$ **Universal Reading**

In (42b), $[whole]$ is applied to an unstarred predicate, demonstrating how a combination of the analysis developed in Section 4 with the lexical entry in (40) derives the two readings.

(44) Let A', B' be the respective missing halves of A and B.
$\rightarrow A \oplus A'$ and $B \oplus B'$ are integrated [bread]-wholes
$\Leftrightarrow A \oplus A' \in IC_{[\text{bread}]}(C_{1w}) \,\&\, B \oplus B' \in IC_{[\text{bread}]}(C_{1w})$ (37)
$\rightarrow [whole_{[\text{bread}]}](C_{1w})(A) = [whole_{[\text{bread}]}](C_{1w})(B) = 0$ (40)
$\rightarrow$ [the [$^*ganz(C_{1w})$[bread]] = C
$\triangleq$ **Integrity Reading**

This section functions as a refinement of the analysis. Wągiel (2018)'s definition of an integrated whole was used as a starting point to describe the 'wholeness' that *'ganz'* makes reference to, enabling me to give a more detailed version of the preliminary lexical entry developed in Section 4.

7. Conclusion

This paper presents an analysis of the ambiguities in (1b) and (1c), with the two readings being derived depending on the scope relationship between * and *'ganz'*. This solution is more attractive than the assumption of a lexical ambiguity for several reasons: it explains the disappearance of the ambiguity in sentences without 'ganz', it can be applied to both PM^- and PM^+-sentences equally, and it correctly predicts the availability of a similar ambiguity with higher-order plural operators such as **. Further support for the analysis comes from predictions regarding the availability of cross-contextual readings matching the empirical pattern. Additionally, I use Marcin Wągiel's definition of an integrated whole (2018) in developing the concept of *integrated closure* to be able to capture the different types of 'wholeness' encoded when *'ganz'* interacts with starred vs. unstarred predicates.

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