Dependent Numerals in Bengali: A Case for Covert Adverbial D-operators ${ }^{1}$<br>Ishani GUHA - University of Delhi


#### Abstract

The semantic analysis of adnominal dependent numerals is much debated among two theoretical positions regarding the extent of distributivity involved. One school of analyses proposes that a covert Adverbial $D$ operator is key to the licensing of these numerals and another school claims that the $D$ operator is part of the lexical meaning of the numerals. In this work, I use lack of cumulative interpretations of nominals as a diagnostic for distributivity. This is implemented by probing the interpretation of nominal co-arguments of dependent numerals in ditransitive constructions and thereby finding that distributivity can be traced beyond the immediate bounds of the dependent numerals. Based on that evidence, I argue in favor of the covert Adverbial $D$ operator based analysis, against recent influential lexicalist proposals. Thus the work provides a crucial piece in this on-going debate about the semantics of these numerals.


Keywords: Dependent Numerals, Distributivity, Adverbial D operator, Ditransitives.

## 1. Introduction

Dependent numerals are morphologically augmented numerals that must convey distributive readings (Farkas, 1997). The dependent numerals in Bengali, as in (2) below, are formed by adding a suffix -kore 'do.Pfv' (or a reduplicating suffix) to cardinality phrases like (1).
(1) du-ṭo-bakšo 'two boxes'
(2) du-to-kore-bakšo 'two boxes each'

These numerals must be licensed by a c-commanding plural expression in the same clause (see Guha (2020) for details). In (3) Robi ar Rina is the licensor for du-to-kore-bakšo.
(3) Robi ar Rina du-ṭo-kore-bakšo boeche

Robi and Rina two-Cl.-do.Pfv-box carried
'Robi and Rina carried different sets of two boxes.'
The sentence requires there to be two boxes for Robi and a different set of two boxes for Rina. We describe this by saying that the dependent numeral 'covaries' with its antecedent. If Robi ar Rina is replaced with a singular antecedent, Rina, the result is infelicitous (4). Unlike Telugu, as discussed by (Balusu, 2006), dependent numerals in Bengali cannot lead to occasion distributive readings without an overt plural licensor.

> \#Rina du-to- kore-bakšo boeche
> Rina two-Cl.-do.Pfv-box carried
> \#'Rina carried different sets of two boxes on every occasion.'

The semantic analysis of adnominal dependent numerals is debated among two theoretical positions regarding the source of distributivity in the interpretation of the numeral. One school of analyses advocates that an operator like the covert Adverbial Distributivity operator $D$ (Link, 1987; Roberts, 1987: a.o.) is instrumental in licensing the covarying interpretation of the dependent numerals. Henderson (2014)'s analysis of Kaqchikel dependent indefinites, Cham-

[^0]pollion (2015) on distant distributive items in general or Oh (2005)'s analysis of these items in Korean adopt this position. Let me call this approach "Covert D-op." In Covert D-op the analysis of (3) can be represented as (5) below.
(5) Robi and Rina ${ }_{V P} \mathbf{D}\left[{ }_{V P}[\right.$ two-KORE boxes $]$ carried ] ]

The contrary school of analyses upholds the position that the source of distributivity resides within the dependent numeral itself. Kuhn (2017) strongly argued in favor of this position. Different implementations of this position has been presented in Balusu (2006)'s work on Telugu, Cable (2014)'s work on Tlingit among others. Let me call this approach "Lexical distributivity". Lexical distributivity will analyze (3) as (6) below.
(6) Robi and Rina [VP [two-KORE ${ }_{D}$ boxes] carried ]

In this paper, I will put forward arguments in favor of Covert D-op, primarily based on facts from dependent numerals in Bengali. A dynamic semantic analysis inspired by Henderson's and Kuhn's work will be presented, which accommodates these facts.

## 2. Previous Arguments against Covert D-op.

The arguments that have been presented against an analysis like Covert D-op are mainly based on predictions related to the scope of an adverbial $D$ operator. The covert adverbial $D$ operator being a Distributive universal quantifier induces certain effects in its scope. An indefinite occurring in the scope of a distributive quantifier can have a covarying interpretation. In (7) we can interpret the sentence to be true in a situation where every girl carried a different set of three boxes.
(7) Every girl carried three boxes.

Moreover, an indefinite occurring in the scope of a distributive quantifier cannot have a cumulative interpretation. That is to say, (7) cannot be true in a situation where Rina carried two boxes and Mira carried one box and together they carried a total of three boxes.

We expect $D$ to induce the same effects on the interpretation of indefinites in its scope: (i) $D$ will allow covariation of any indefinite in its scope, (ii) $D$ will block cumulative interpretation of indefinites in its scope. However, previous works have presented purported evidence showing that these effects are not attested in constructions involving a dependent numeral.

### 2.1. Covariation in conjunction of DPs

Kuhn (2017) argued that conjunction of object DPs provides a testing ground for the predictions regarding the scope of $D$. If a dependent numeral is conjoined with a plain numeral, the VP containing this conjunction of DPs will be in the scope of $D$ that licenses the dependent numeral as shown by (8). So the plain numeral would be expected to have a covarying interpretation or to not be interpreted cumulatively with respect to the subject.

$$
\begin{equation*}
\left[\text { Subject }\left[_{V P} \mathbf{D}\left[{ }_{V P}\left[\& P\left[{ }_{D P} \text { Plain Numeral }\right] \&\left[{ }_{D P} \text { DepNum }\right]\right] \mathrm{V}\right]\right]\right] \tag{8}
\end{equation*}
$$

However Kuhn showed that cross-linguistically the plain numeral in such a construction can receive cumulative interpretation. (9) below is an example from Bengali illustrating Kuhn's point. Here ' 3 biriyanis' in the first conjunct is interpreted cumulatively relative to the girls, but the dependent numeral in the second conjunct mandates that the firnis covary with the girls.
mee-ra [tin-ṭ-biriyani ar æk-ta-kore-firni] kheeche
girl-Pl. three-Cl.-biriyani and one-Cl.-do.Pfv.-firni ate
'The girls ate a total of three biriyanis and N firnis where N is the $\#$ of girls.'
If a conjunction of DPs is to be analyzed like (8), the cumulative interpretation of the plain numeral would indicate that $D$ does not scope over the VP containing the conjunction. Thus in effect it would be an evidence against an adverbial $D$ linked to the dependent numeral. However, I will show, in section 5.4, that using a conjunction reduction analysis can nullify this argument against Covert D-op.

### 2.2. The Extent of covariation in ditransitives

One might argue against Covert D-op by showing that a dependent numeral does not induce covariation on its neighboring DPs. For example, as an internal argument of a ditransitive, a dependent numeral does not force its coargument to have a covarying interpretation. ${ }^{2}$

Take a look at (10) from Bengali, where the direct object is a dependent numeral and the indirect object is a plain numeral. The sentence can describe a situation with more than two books but it cannot describe a situation with more than three boys. It must describe a situation with exactly three boys. The example shows that the indirect object cannot have a covarying interpretation just by virtue of the direct object being a dependent numeral.

Robi ar Rina [tin-te-chele-ke] [du-to-kore-boi] dieche
Robi and Rina three-Cl.-boy-Dat. two-Cl.-do.Pfv.-book gave
'Robi and Rina gave different sets of two books to three boys.'
Similarly, a sentence like (11) with a dependent numeral indirect object and a plain numeral direct object is incompatible with a situation with more than two books. This shows that the dependent numeral at the indirect object position does not induce covariation on the direct object. In fact, the sentence (11) itself is degraded.
??Robi ar Rina [tin-te-kore-chele-ke] [du-to-boi] dieche
Robi and Rina three-Cl.-do.Pfv.-boy-Dat. two-Cl.-book gave
'Robi and Rina gave different sets of three boys two books.'
These examples establish that covariation is strictly limited to the dependent numerals. If, as proposed by Covert D-op, there was a $D$ on the VP containing the internal arguments of the ditransitive predicate, the plain numerals should be able to express covarying readings. I will argue later on that the plain numerals are incapable of expressing covariation in these cases and hence these examples do not necessarily constitute an argument against an adverbial $D$.

## 3. Arguing in favor of Covert D-op: Cumulativity in ditransitives

Usually plural numerals receive cumulative interpretations easily as internal arguments (12).
(12) Robi ar Rina [pãc-ṭa-ṭebil-e] [chota-phul-dani] rekhechilo

Robi and Rina five-Cl.-table-Loc six-Cl.-flower-holder placed
'Robi and Rina placed a total of six vases on a total of five tables.'
If a ditransitive predicate has a dependent numeral as one of its internal arguments, Covert D-

[^1]op would postulate a $D$ operator on the $v \mathrm{P}$ containing the internal arguments. Notably, the $D$ operator blocks the cumulative interpretation of nominals occurring in its scope. So in effect Covert D-op would predict (13), that when a ditransitive has a dependent numeral internal argument, the other internal argument would resist a cumulative interpretation.
a. Subject $\mathbf{D}\left[{ }_{v P} \mathrm{t}_{\text {Sub }}\left[v P\right.\right.$ Indirect Object \#Cumulative [ ${ }^{2} P$ DepNum V ] $]$ ]
b. Subject D [ ${ }_{\nu P} \mathrm{t}_{\text {Sub }}\left[{ }_{\nu P}\right.$ DepNum [ ${ }_{V P}$ Direct Object\#cumulative V $\left.]\right]$ ]

In this section, I will present novel data from Bengali ditransitives showing that plain numeral coarguments of dependent numerals indeed can't get cumulative readings.

### 3.1. Cumulativity of indirect object

Let us first consider a double object construction in Bengali with a plain numeral at the indirect object position and a dependent numeral at the direct object position. The plain numeral can be interpreted cumulatively with respect to a plural subject, if the dependent numeral covaries with the plurality at the indirect object position. Crucially however, the plain numeral plurality cannot be cumulative in relation to the subject when the dependent numeral covaries with the subject. Compare Figures 1 and 2 below schematically representing the two cases.


Figure 1: Cumulative indirect object allowed


Figure 2: Cumulative indirect object blocked
Consider a 'Cumulative situation'. The sentence (14) from Bengali is false in this situation.
Cumulative situation: In a room there were two tables and Robi placed three flower vases on them. In another room there were three tables and Rina placed three flower vases on them. Cumulatively Robi and Rina placed six flower vases on a total of five tables.
(14) Robi ar Rina [pãc-ta-tebil-e] [tin-ṭe-kore-phul-dani] rekhechilo

Robi and Rina five-Cl.-table-Loc three-Cl.-do.Pfv-flower-holder placed
\#'Robi and Rina placed three flower vases each (per person) on a total of five tables.'
$\checkmark$ 'Robi and Rina placed three flower vases each (per table) on a total of five tables.'
The restriction is parallel to Figure 2 and matches the prediction (13a) above. However, (14) is true in the following situation where the indirect object licenses the dependent numeral: There were a total of five tables. Robi placed on two tables three flower vases each. Rina placed on three tables, three flower vases each. Cumulatively, Robi and Rina placed fifteen flower vases on a total of five tables.

This is parallel to Figure 1 and that (14) is true in this situation, is not in conflict with the prediction (13a). The same can be observed for other double object constructions from Bengali.
(15) tin-ṭ-company [pəncaš-jon-kormocari-ke] [æk-lokkho-taka-kore -maine] dieche three-Cl.-company fifty-Cl.-worker-Dat one-lakh-rupee-do.Pfv-salary gave \#‘ 3 companies paid 1 lakh rupees each (per company) to a total of 50 workers.' \' 3 companies paid 1 lakh rupees each (per worker) to a total of 50 workers.'

Cumulative situation: Company A paid its twenty employees one lakh ${ }^{3}$ rupees; company B paid its twenty employees one lakh rupees; company C paid its ten employees one lakh rupees. Cumulatively A, B and C paid a total of fifty workers three lakhs.
(15) is false in this cumulative situation, where the salary amount covaries with the companies. But it is true when the salary covaries with the workers ( 50 workers got paid 50 lakhs in total).

$$
\begin{align*}
& \text { du-jon-jubok [tin-te-company-te] [car-te- kore -cakrir-dorkhasto] }  \tag{16}\\
& \text { two-cl-young.man three-cl-company-Loc four-cl-do.Pfv-job.Gen-application } \\
& \text { pathiechilen } \\
& \text { sent } \\
& \text { \#‘Two young men sent } 4 \text { applications each (per man) to a total of } 3 \text { companies.' } \\
& \checkmark \text { 'Two young men sent } 4 \text { applications each (per company) to a total of } 3 \text { companies.' }
\end{align*}
$$

Cumulative situation: Robi sent four applications to a certain company. Abir sent four applications to two other companies. Cumulatively the two men sent eight applications to a total of three companies.
(16) is false in this cumulative situation, but true when applications covary with companies, such that Robi and Abir cumulatively sent twelve applications to three companies.

### 3.1.1. Interim conclusions against Lexical distributivity

These inference patterns indicate that the availability of a cumulative interpretation of a plain numeral indirect object is dependent on two factors: (a) the presence of the dependent numeral as direct object and (b) which plural the dependent numeral covaries with. The pattern exactly looks like what Covert D-op predicted.

Lexical distributivity however, would have the distributivity operator located inside the dependent numeral direct object as in (17). $D$ localized at the direct object position cannot block the cumulative interpretation of a plain numeral in the indirect object position. So the lexicalist analysis cannot explain why the cumulative interpretation is absent in these (Figure 2) cases. Thus the inference patterns constitute an argument against Lexical distributivity.

## (17) $\quad$ Subject $\left[{ }_{v P} \mathrm{t}_{\text {Sub }}\left[{ }_{v P}\right.\right.$ Indirect Object \#Cumulative $\left[{ }_{V P}\right.$ DepNum $_{\mathbf{D}}$ V ]] $]$

To reinforce the argument against Lexical distributivity, it must be noted that in Bengali double object constructions, the indirect object always scopes over the direct object in the canonical order. As Bhattacharya and Simpson (2011) observed, when the indirect object precedes the direct object in linear order, the relative scope between the quantificational DPs in these posi-

[^2]tions must match the linear sequence. Example (18) (Bhattacharya and Simpson's ex.43) has the surface scope reading but not the inverse scope.
(18) Hori [kono-æk-jon-šikkhok-ke] [prottek-ta-boi] dilo

Hori some-one-Cl.-teacher-Dat every-Cl.-book gave
$\checkmark$ 'Hori gave every book to some particular professor.' \#'Hori gave every book to a different professor.'

Maintaining the same pattern of matching between the relative scope and linear order, a dependent numeral indirect object cannot be licensed by a direct object that linearly follows it, as (19) illustrates.
\#Robi [æk-jon-kore-šikkhok-ke] [prottek-ta-boi] dilo
Robi one-Cl.-do.Pfv-teacher-Dat every-Cl.-book gave
'Robi gave every book to a different professor.'
We can safely conclude that the dependent numeral direct object in the ditransitive examples (14)-(16) discussed above, scopes below the linearly preceding plain numeral indirect object. So the dependent numeral on its own is not in a position to influence the interpretation of the indirect object. That makes the explanation of the systemic absence of the cumulative reading by an adverbial scope of $D$ more plausible.

### 3.2. Cumulativity of direct object

If indeed, the covert adverbial $D$ operator is behind the lack of cumulative interpretation of the indirect object in cases like (14)-(16), then we also expect to see the same effect when the positions of the dependent numeral and the plain numeral are switched. Thus Covert Dop predicts that a plain numeral direct object cannot be cumulative with respect to the subject when the indirect object is a dependent numeral (13b).


Figure 3: Cumulative direct object not allowed
Examples (20), (21) and (22) show that the prediction is met. These sentences cannot be used to express that the direct object is cumulative relative to the subject. In fact the sentences are quite odd constructions to begin with.

Consider the following situation: In a room Robi placed three flower vases on two tables. In another room Rina placed two flower vases on a different set of two tables. Cumulatively Robi and Rina placed a total of five flower vases. (20) cannot describe this situation.
??Robi ar Rina [du-ṭo-kore-tebile] [pãc-ta-phuldani] rekheche
Robi and Rina two-Cl.-do.Pfv-table.Loc five-Cl.-flower.vase placed
\#'Robi and Rina placed on two tables each (per person) a total of five flower vases.'
Similarly (21) cannot describe a situation where three companies paid fifteen workers a total of four lakh rupees, neither can (22) be used for describing a situation where two men cumulatively sent a total of seven job applications.
??tin-te-company [pãc-jon-kore-kormocari-ke] [car-lokkho-taka-maine] dieche three-Cl.-company five-Cl.-do.Pfv-employee-Dat one-lakh-rupee-salary gave \#'Three companies paid 5 employees each (per company) a total of 4 lakh rupees.'

```
??du-jon-jubok [tin-te- kore -company-te] [šat-ta-cakrir-dorkhasto]
    two-Cl.-young.man three-Cl.-do.Pfv-company-Loc seven-Cl.-job.Gen-application
    pathhiechilen
    sent
    #`Two young men sent to 3 companies each (per man) a total of 7 job applications.'
```

Lexical distributivity does not predict the $D$ operator inside the indirect object to have any direct influence over the direct object and block its cumulative reading. Thus the lack of cumulative interpretation of the coarguments of the dependent numeral supports a Covert D-op analysis.

## 4. Universal quantifiers as licensors of dependent numerals

An additional argument (following Champollion (2015)) in favor of Covert D-op can be made based on the fact that Universal Quantifiers can license Bengali dependent numerals.

> prottek-ṭi-mee du-ṭo-kore-bakšo boeche
each.one-Cl.-girl two-Cl.-do.Pfv-box carried
'Every girl carried a different set of two boxes.'
In Covert D-op, dependent numerals do not introduce a distributivity operator themselves. They can be licensed by any c-commanding distributive quantifier. But in Lexical distributivity, dependent numerals introduce a distributivity operator themselves, as in (24).
(24) Every girl [ $V P$ [two-KORE $\mathbf{D}_{\text {b }}$ boxes] carried ]

Consequently, under a universal quantifier, a dependent numeral in this account could be predicted to generate additional distributive readings (i.e., more than two boxes per girl in (23)). But in sentences like (23) we do not get any such additional distributive readings.

## 5. A Unified Analysis of the dependent numerals

The data from the ditransitives showed that the effects of a distributivity operator can be traced in the entire VP containing the dependent numeral. The evidence seems to fit Covert D-op neatly while being at odds with the predictions of Lexical distributivity.

A very important element of Kuhn's analysis is the formal encoding of the dependency between a dependent numeral and its licensor with the use of indices. As Kuhn shows, this is necessary because when provided with multiple potential licensors, a dependent numeral can covary with any of them and in fact in ASL the relation between the numeral and its particular licensor is marked by overt morphology. He further shows that identical anaphoric marking appears on adjectives same and different and their sentence internal licensors. The marker 'b' in (25) indicates that the books covary with the girls and not the boys. Similarly in (26) 'a' marks anaphoric linking between the adjectives and their licensor.

ALL-a BOY GAVE ALL-b GIRL ONE-arc-b BOOK
(ASL) 'All the boys gave all the girls one book per girl.'
a. ALL-a BOY READ SAME-arc-a BOOK
'All the boys read the same book.'
b. ALL-a BOY READ DIFFERENT-redup-a BOOK
'All the boys read a different book.'
In section 3.1, we saw that the interpretation possibilities of the indirect object plural changed depending upon whether the dependent numeral at direct object position covaried with the indirect object or with the subject. In order to formally capture this difference I will need to use Kuhn's anaphoric linking of the dependent numeral and its licensor. Without this mechanism for 'relativizing' covariation, a Covert D-op analysis could not distinguish between the two relevant readings. The analysis I propose in this paper blends Henderson's (or Champollion (2015)'s) stand on the source of distributivity in dependent numerals with Kuhn's position about the need for anaphoric linking between the numerals and their licensors.

### 5.1. Analysis of the ditransitives

In ditransitives, syntactically there are two potential verbal projections that can be the scope of the $D$ operator: the lower $V P$ containing just the direct object and the higher $v \mathrm{P}^{4}$ containing both the Indirect and the direct object. The location of the $D$ operator depends upon the licensor of the dependent numeral. When the dependent numeral is licensed by the indirect object, $D$ scopes over a lower $V \mathrm{P}$ containing only the direct object. But when the licensor is the subject, $D$ scopes over higher $v \mathrm{P}$ containing both of the internal arguments. (27) and (28) below show the two different scope possibilities of $D$ and their effects.


Analysis (27) allows the indirect object to be cumulative with respect to the subject, as it is not in the scope of $D$. By contrast, analysis (28) does not allow the indirect object to be cumulative with respect to the subject, as it is in the scope of $D$.

```
Subject [D 斿[Ind-Object }\mp@subsup{\lambda}{2}{}[\mathrm{ [DepNum }\mp@subsup{\lambda}{1}{}[\mp@subsup{}{vP}{}\mp@subsup{\textrm{t}}{3}{}[\mp@subsup{}{vP}{}\mp@subsup{\textrm{t}}{2}{}[vPP\mp@subsup{\textrm{t}}{1}{}\textrm{V}]]]]]
```

$D$ in this analysis takes as argument the lambda abstract resulting from the Quantifier raising of the plural licensor. So the scope of $D$ is not affected by any further quantifier raising out of its argument. For example in (28), Quantifier raising the indirect object a second time will not let it escape the scope of $D .{ }^{5}$

### 5.2. The Non-Cumulative Interpretation and its Analysis

The crucial piece of the argument in favor of Covert D-op and against Lexical distributivity comes from the absence of the cumulative interpretation of the plain numeral. Naturally the question arises, what is the interpretation available when cumulativity is blocked. Sentence (29) below ((14) rep.) is true when there are exactly five tables and false if there are more than five tables. In other words, the plain numeral in this case does not allow a covarying interpretation.

[^3]> Robi ar Rina $\left[{ }_{v P} \mathbf{D}\left[{ }_{v P}\right.\right.$ [pãc-ta-tebil-e] [tin-te- kore-phul-dani]
> Robi and Rina ${ }_{{ }_{v P} \mathrm{D}} \mathrm{D}{ }_{v P}$ five-Cl.-table-LOC three-Cl.-do.Pfv-flower-holder rekhechilo]]
> placed]]
> \#'Robi and Rina placed three flower vases each (per person) on a total of five tables.'
> $\checkmark$ 'Robi and Rina placed different sets of 3 flower vases (per person) on the same 5 tables.'

The crucial reading of (29) that we are after here says there are five tables on which both Robi and Rina placed three flower vases. So Robi and Rina have access to the same five tables, involving a collective interpretation of the tables. Let me name this the 'same' situation. Thus in the 'same' situation, the flower vases covary with Robi and Rina but the tables don't! ${ }^{6}$

Recall, (29) is false in the 'cumulative' situation (where Robi placed three flower vases on two tables and Rina placed three flower vases on three tables). Therefore, we need to be able to grammatically differentiate between the 'same' situation and the 'cumulative' situation, even though both are situations where there are exactly five tables. I propose that it is the $D$ operator licensing the dependent numeral that rules out the 'cumulative' situation as one of the possible interpretations of (29). But I still need to be able to allow the interpretation 'exactly five tables' for the plain numeral under the $D$ operator, which licenses the dependent numeral. To achieve this I will propose to use the device of postsuppositions (Brasoveanu, 2013) that can yield the 'exactly five' interpretation of the plain numeral.

In the account proposed here, the dependent numerals and the plain numerals have the same at-issue meaning. But they differ in terms of their not-at-issue meaning content, namely their 'post'-suppositions. Postsuppositions (Brasoveanu, 2013) are not-at-issue elements of meaning that are evaluated on the output of the scope of certain operators, particularly $D$ for us.

Brasoveanu (2013) used cardinality postsuppositions to account for the cumulative readings of modified numerals like exactly three or exactly five in sentences like (30). In his analysis, a cardinality postsupposition on a modified numeral ensured that its interpretation remained 'exactly n' particularly under distributive quantifiers, ruling out 'more than n' readings.

Exactly three boys saw exactly five movies.
The purpose of using postsuppositions for Bengali dependent numerals is to allow a dependent numeral and a plain numeral yield 'more than n' and 'exactly n' readings respectively, when placed in the scope of $D$. The postsupposition on the dependent numeral, which I have named DIFFERENT, ensures that there are different sets of entities satisfying the properties of the indefinite, evaluating the output of $D$. Unlike previous works, I have also introduced a post-

[^4]supposition called SAME on a plain numeral to block covariation, which ensures that there is only one set of entities picked out by the plain numeral.

The postsupposition DIFFERENT requires the presence of $D$ to be licensed. It also needs $D$ to distribute over a plurality. Thus the need for covariation in a dependent numeral arises from the postsupposition. The postsupposition SAME on the other hand, does not require $D$ or a plurality to be licensed. The analysis thus construed can be schematically represented as follows:
(a) A mono-transitive case with the subject licensing an object dependent numeral:

$$
\begin{equation*}
\left.\left.\operatorname{Subject}^{I}\left[\mathbf{D}^{i} \lambda_{2}\left[\operatorname{DepNum}_{\operatorname{DIFF}_{i}^{j}}^{j} \lambda_{1}\left[v P \mathrm{t}_{2}\left[V P \mathrm{t}_{1} \mathrm{~V}\right]\right]\right]\right]\right]\right] \tag{31}
\end{equation*}
$$

(b) ditransitive cases with the subject or indirect object as licensors

$$
\begin{align*}
& \text { Subject }^{k} \lambda_{3}\left[\text { Ind-Object }^{I}\left[\mathbf{D}^{i} \lambda_{2}\left[\operatorname{DepNum}_{\text {DIFF }_{i}^{j}}^{j} \lambda_{1}\left[{ }_{v P} \mathrm{t}_{3}\left[{ }_{v P} \mathrm{t}_{2}\left[{ }_{V P} \mathrm{t}_{1} \mathrm{~V}\right]\right]\right]\right]\right]\right]  \tag{32}\\
& \text { Subject }^{I}\left[\mathbf{D}^{i} \lambda_{3}\left[\text { Ind-Object }_{\mathrm{SAME}_{j}^{j}}^{j} \lambda_{2}\left[\operatorname{DepNum}_{\text {DIFF }_{i}^{k}}^{k} \lambda_{1}\left[{ }_{v P} \mathrm{t}_{3}\left[v P \mathrm{t}_{2}\left[v P \mathrm{t}_{1} \mathrm{~V}\right]\right]\right]\right]\right]\right] \tag{33}
\end{align*}
$$

The formal details will be laid out in sections 6, 7 and 8 .

### 5.3. Covarying indirect and direct objects

If, as proposed in the analysis in (33), both of the internal arguments are in the scope of $D$, we predict that, apart from the dependent numeral direct object, the indirect object too potentially can host an indefinite covarying with the subject. I have already discussed that a plain numeral is barred from expressing covariation in this case (10). However, instead of a plain numeral, we could use a dependent numeral indirect object. Example (34) below is a case where both of the internal arguments are dependent numerals covarying with the subject.

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Robi ar Rina \(\left[{ }_{\nu P} \mathbf{D}\left[{ }_{\nu P}\right.\right.\) [pãc-ta- kore -tebil-e] [tin-te- kore -phul-dani]
Robi and Rina \({ }^{[ }{ }_{\nu} \mathrm{D}\left[{ }_{v}\right.\) five-Cl.-do.Pfv-table-LOC three-Cl.-do.Pfv-flower-holder rekhechilo]]
placed
'Robi and Rina placed different sets of 3 flower vases (per person) on different sets 5 tables (per person).'
```


### 5.4. Re-analysis of the Conjunction of object DPs

Any argument in favor of Covert D-op would have to account for Kuhn's argument based on object DP conjunction. I will propose that Kuhn's evidence is not conclusive and what he presented as a conjunction of DPs is in fact a conjunction of $v \mathrm{Ps}$. Hirsch (to appear) has extensively argued in favor of a Conjunction Reduction analysis for conjunction of DPs. Following Hirsch's lead I will reanalyze (35) ((9) rep.) as in (36).
mee-ra [tin-ṭ-biriyani ar æk-ta-kore-firni] kheeche girl-Pl. three-Cl.-biriyani and one-Cl.-do.Pfv.-firni ate
'The girls ate three biriyanis in total and N firnis where N is the \# of students.'
[mee-ra $\lambda_{2}\left[{ }_{\&} P\left[{ }_{\nu P} \mathrm{t}_{2}\right.\right.$ tin-ṭe-biriyani $\left.\mathrm{t}_{3}\right]$ ar $\left[{ }_{\nu P} \mathrm{t}_{2} \mathbf{D} \boldsymbol{\lambda}_{1}\left[{ }_{\nu P} \mathrm{t}_{1} æ \mathrm{k}\right.\right.$-ta-kore-firni $\left.\left.\left.\left.\mathrm{t}_{3}\right]\right]\right]\right] \lambda_{3}$ kheeche.

The analysis in (36) involves raising of the subject out of the two $v P$ conjuncts and rightnode raising of the verb out of each conjunct. $D$ has scope over the second conjunct and is instrumental in the working of the covarying interpretation of the dependent numeral. Thus $D$ has no influence over the first conjunct letting the object DP in the first conjunct be open to any kind of interpretation, cumulative or non-cumulative.

## 6. Formal details in Dynamic Plural Logic

The analysis of the dependent and the plain numerals sketched above will be implemented in van den Berg's DPlL (Dynamic Plural Logic) as extended in Brasoveanu $(2011,2013)$ and Henderson (2014). In DPIL formulas are relations between pairs of sets of variable assignments $\langle G, H\rangle$. A set of assignments $G$ is a plural information state, represented as a matrix (Figure 4). The rows of the matrix represent individual assignments (with $\mathrm{g}, \mathrm{h}$ as variables over the individual assignment functions) $\mathrm{g}_{1}, \mathrm{~g}_{2}, \mathrm{~g}_{3}$, etc. in the set of assignments $G$. The columns represent the discourse referents (drefs) $x, y$, etc. An object in a cell of the matrix is the value assigned to a dref by an assignment: $\operatorname{girl}_{1}=\mathrm{g}_{1}(x), \operatorname{girl}_{2} \oplus \operatorname{girl}_{3}=\mathrm{g}_{2}(x)$, $\operatorname{book}_{1}=\mathrm{g}_{1}(y)$, book $_{2} \oplus$ book $_{3}=\mathrm{g}_{2}(y)$, etc. The matrices also express that there is some relation between $\operatorname{girl}_{1}$ and $b o o k_{1}$ in a row and there are relations between columns as well.

| $G$ | $\ldots$ | $x$ | $y$ | $\ldots$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{~g}_{1}$ | $\ldots$ | girl $_{1}$ | book $_{1}$ | $\ldots$ |
| $\mathrm{~g}_{2}$ | $\ldots$ | girl $_{2} \oplus$ girl $_{3}$ | book $_{2} \oplus$ book $_{3}$ | $\ldots$ |
| $\mathrm{~g}_{3}$ | $\ldots$ | girl $_{1}$ | book $_{4}$ | $\ldots$ |
| $\mathrm{~g}_{4}$ | $\ldots$ | girl $_{2} \oplus$ girl $_{3}$ | book $_{4} \oplus$ book $_{5}$ | $\ldots$ |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |

Figure 4: The information state $G$

- The values of a dref $x$ stored in a column of the matrix $G$ is $G(x)$.
(37) $\quad G(x):=\{g(x): g \in G\} \ldots G(x)$ is the set of values assigned to the dref $x$ in $G$.
- $\left.G\right|_{x=d}$ is the substate containing the set of all assignments in $G$ where $x$ has been assigned the value $d$.

$$
\begin{equation*}
\left.G\right|_{x=d}:=\{g: g \in G \& g(x)=d\} \tag{38}
\end{equation*}
$$

- $\left.G\right|_{x=d}(y)$ is defined as the set of values of $y$, relative to the value $d$ of $x$ in a substate of $G$.
(39) $\left.G\right|_{x=d}(y):=\{g(y): g \in G \& g(x)=d\}$

Dependency is defined as follows:
(40) In an information state $G, y$ is dependent on $x$ iff

$$
\exists d,\left.e \in G(x) \cdot G\right|_{x=d}(y) \neq\left. G\right|_{x=e}(y)
$$

- Atomic formulas for lexical relations are tests. They are interpreted distributively

$$
\begin{equation*}
R\left(x_{1}, \ldots, x_{n}\right)^{\langle G, H\rangle}=\mathbb{T} \text { iff } G=H \text { and } \forall \mathrm{h} \in H,\left\langle\mathrm{~h}\left(x_{1}\right), \ldots, \mathrm{h}\left(x_{n}\right)\right\rangle \in \Im(R) \tag{41}
\end{equation*}
$$

- New discourse referents are introduced by random assignment indicated by the notation [x]

Random assignment: $[x]^{|G, H\rangle}=\mathbb{T}$ iff $G[x] H$, where
a. $\quad G[x] H:=\forall g \in G[\exists h \in H(g[x] h)] \wedge \forall h \in H[\exists g \in G(g[x] h)]$
b. $\quad \mathrm{g}[x] \mathrm{h}:=\forall i[i \neq x \rightarrow \mathrm{~g}(i)=\mathrm{h}(i)]$

- Dynamic conjunction is defined as follows:

$$
\begin{equation*}
\phi \wedge \psi^{\langle G, H\rangle}=\mathbb{T} \text { iff there is a } K \text { such that } \phi^{\langle G, K\rangle}=\mathbb{T} \text { and } \psi^{\langle K, H\rangle}=\mathbb{T} \tag{43}
\end{equation*}
$$

- (44) Truth: A formula $\phi$ is true relative to an input set of assignments $G$ iff there is an output set of assignments $H$ such that $\phi^{\langle G, H\rangle}=\mathbb{T}$.
- Basic lexical relations and the theta-roles are cumulatively closed by default.

A universal quantifier is defined as a combination of max and $\delta$ operators (Brasoveanu, 2013).
$\max (i)(\phi)^{\langle G, H\rangle}=\mathbb{T}$ iff
a. $\quad[i] \wedge \phi^{\langle G, H\rangle}=\mathbb{T}$
b. there is no $H^{\prime}$ such that $[i] \wedge \phi^{\left\langle G, H^{\prime}\right\rangle}=\mathbb{T}$ and $H(i)<H^{\prime}(i)$

The max $(i)$ operator introduces a new dref $i$ and stores in $H$ the maximal set of individuals satisfying the formula $\phi$ it scopes over.
$\delta(i)(\phi)^{\langle G, H\rangle}=\mathbb{T}$ iff
a. $\quad G(i)=H(i)$
b. for any $a \in G(i), \phi^{\left\langle G_{i=a}, H_{i=a}\right\rangle}$

The distributivity operator (Brasoveanu, 2013: section 4.3) $\delta$ takes the output of maximization and distributively updates over the variable $i$ the set of assignments $G$ with the nuclear scope formula $\phi$.

The postsuppositions introduced in the scope of the distributivity operator must be evaluated relative to the operator's output set of assignments. ( $\zeta, \zeta^{\prime}$ are sets of postsuppositional tests)

$$
\begin{equation*}
\delta(i)(\phi)^{\left\langle G[\zeta], H\left[\zeta^{\prime}\right]\right\rangle}=\mathbb{T} \text { iff } \zeta=\zeta^{\prime} \quad \& \tag{47}
\end{equation*}
$$

a. $\quad G(i)=H(i)$
b. There is a possibly empty set of tests $\left\{\psi_{1}, \ldots, \psi_{n}\right\}$ such that for all $a \in G(i), \phi^{\left\langle G_{i=a}[\zeta], H_{i=a}\left[\zeta \cup\left\{\psi_{1}, \ldots, \psi_{n}\right\}\right]\right\rangle}=\mathbb{T}$ and $\left\{\psi_{1}, \ldots, \psi_{n}\right\}^{<H[\zeta], H[\zeta]\rangle}=\mathbb{T}$

Truth and satisfaction is defined relative to sets of assignments indexed with $\zeta$ and $\zeta^{\prime}$. (48) shows that postsuppositions (marked with an overline) do not update input sets of assignments. They just get added to the input set of tests $\zeta$ to yield $\zeta^{\prime}$, which is passed along into the output. (48) interacts with the definition of truth in (49) to ensure that postsuppositions are evaluated after the at-issue content.

$$
\begin{equation*}
\bar{\phi}^{\left\langle G[\zeta], H\left[\zeta^{\prime}\right]\right\rangle}=\mathrm{T} \text { iff } \phi \text { is a test, } G=H \text { and } \zeta^{\prime}=\zeta \cup\{\phi\} . \tag{48}
\end{equation*}
$$

(49) Truth: $\phi$ is true relative to an input context $G[\emptyset]$ iff there is an output set of assignments $H$ and a (possibly empty) set of tests $\left\{\psi_{1} \wedge \ldots \wedge \psi_{m}\right\}$ such that $\phi^{<G[0], H\left[\left\{\psi_{1}, \ldots, \psi_{m}\right\}\right]>}=\mathrm{T}$ and $\psi_{1} \wedge \ldots \wedge \psi_{m}{ }^{<H[0], H[0]>}=\mathbb{T}$.

### 6.1. Meaning Components of the Numerals

In this analysis, numerals have two components of meaning (i) an at-issue Cardinality test and (ii) not-at-issue postsuppositions. In order to define the not-at-issue components of meaning we must introduce a partition in a column (a set of values) relative to individual values of another dref. Formally the partition is defined as in (50) (Kuhn, 2017: ex. 56).

$$
\begin{equation*}
\left\{S: \exists d\left[\left.d \in G(x) \& G\right|_{x=d}(y)=S\right]\right\} \tag{50}
\end{equation*}
$$

Thus for the information state $G$ in Figure 4, when the value $\operatorname{girl}_{1}$ is assigned to $x, y$ has been assigned the set of values $\left\{\right.$ book $_{1}$, book $\left._{4}\right\}$. Similarly, when the value girl $_{2} \oplus$ girl $_{3}$ is assigned to $x, y$ has been assigned the set of values $\left\{\right.$ book $_{2} \oplus$ book $_{3}$, book $_{4} \oplus$ book $\left._{5}\right\}$. The relevant partition of the set of values of $y$, with respect to the particular values of $x$ in $G$ would be (51).

$$
\begin{align*}
& \left\{S: \exists d\left[\left.d \in G(x) \& G\right|_{x=d}(y)=S\right]\right\}  \tag{51}\\
& =\left\{\left\{\text { book }_{1}, \text { book }_{4}\right\},\left\{\text { book }_{2} \oplus \text { book }_{3}, \text { book }_{4} \oplus \text { book }_{5}\right\}\right\}
\end{align*}
$$

The at-issue Cardinality test is defined as follows:

$$
\begin{equation*}
|j|=n^{\langle G, H\rangle}=\mathbb{T} \text { iff } G=H \text { and } \forall \mathrm{h} \in H:|\mathrm{h}(j)|=n, \ldots \text { where } \tag{52}
\end{equation*}
$$

a. $\quad \operatorname{atom}(a):=\forall b \leq a(b=a)$
b. $\quad|\mathrm{h}(j)|:=\mid\{b: b \leq h(j) \wedge$ atom $(b)\} \mid$

The Cardinality test counts the atoms that are part of a value assigned to a dref in a cell by an individual assignment. The atoms are recognized via the metalanguage predicate atom. ${ }^{7}$

Note that the at-issue cardinality test inside(j/i)=n in Kuhn (2017: ex.72), defined for dependent numerals is as follows:

$$
\begin{align*}
& \text { inside }(j / i)=n:=  \tag{53}\\
& \lambda G H . G=H \& \forall T \in S: \exists d\left(\left.d \in H(i) \& H\right|_{i=d}(j)=S\right) .|T|=n
\end{align*}
$$

inside $(j / i)=n$ requires every cell in the partition of $H(j)$ with respect to individual values $d$ of $H(i)$ has the cardinality $n$.
This at-issue test makes the dependent numerals in Kuhn (2017) inherently distributive. Compare it with the cardinality test in (52) above, which only checks cardinality of each row of a column and not the cardinality of each cell of a partition of a column. Thus, (52) does not make the distributive numerals inherently distributive.

I will encode Covariation in terms of the postsuppositional test DIFFERENT $(j)_{i}$ on a dependent numeral. It is defined as follows, where $i, j$ are metalanguage variables over drefs: ${ }^{8}$

$$
\begin{equation*}
\operatorname{DIFFERENT}(j)_{i}{ }^{\langle G, H\rangle}=\mathbb{T} \text { iff } G=H \text {, and } \exists a, b \in H(i)\left[\left.H\right|_{i=a}(j) \neq\left. H\right|_{i=b}(j)\right] \tag{54}
\end{equation*}
$$

DIFFERENT $(j)_{i}$ introduces a partition in $H(j)$ with respect to individual values in $H(i)$. It checks if at least two cells in the partition of $H(j)$ are distinct from each other. Crucially, DIFFERENT is not a feature on a plain numeral.

If we compare the postsuppositional cardinality test 'Evaluation plurality' from Henderson (2014: ex. 43), we will see that Evaluation plurality is not relativized to the individual values

[^5]of another dref. Hence it is not equipped to differentiate between two potential licensors as in the case of ditransitives or similar such constructions.
\[

$$
\begin{equation*}
x>1^{\langle G, H\rangle}=\mathbb{T} \text { iff } G=H \text { and }|H(x)|>1 \tag{55}
\end{equation*}
$$

\]

Similar to covariation, in the present analysis, Lack of covariation is encoded as the postsuppositional test $\operatorname{SAME}(j)_{i}$ on a dependent numeral.

$$
\begin{equation*}
\operatorname{SAME}(j)_{i}{ }^{\langle G, H\rangle}=\mathbb{T} \text { iff } G=H, \text { and } \forall a, b \in H(i)\left[\left.H\right|_{i=a}(j)=\left.H\right|_{i=b}(j)\right] \tag{56}
\end{equation*}
$$

$\operatorname{SAME}(j)_{i}$ introduces a partition in $H(j)$ with respect to individual values in $H(i)$ and then it checks if all the cells in the partition of $H(j)$ are identical. This postsupposition is needed to account for interpretations where a dependent numeral does not covary relative to a potential licensor.

The Lack of covariation in case of a plain numeral is encoded in terms of the postsuppositional test $\operatorname{SAME}(i)_{i}$ on the plain numeral.

$$
\begin{equation*}
\operatorname{SAME}(i)_{i}{ }^{\langle G, H\rangle}=\mathbb{T} \text { iff } G=H, \text { and } \forall a, b \in H(i)\left[\left.H\right|_{i=a}(i)=\left.H\right|_{i=b}(i)\right] \tag{57}
\end{equation*}
$$

SAME $(i)_{i}$ introduces a trivial partition in $H(i)$ and checks if the values assigned to a dref $i$ by the set of assignments $H$ are identical. SAME $(i)_{i}$ cannot be a feature on a dependent numeral, as it will conflict with DIFFERENT.
$\operatorname{SAME}(i)_{i}$ is different from Evaluation singularity in Henderson (2014: ex. 42). Evaluation singularity is not a postsupposition in Henderson (2014), it is part of the at-issue updates. Being an at-issue update, Evaluation singularity cannot account for the simultaneous absence of covariation and cumulativity on a plain numeral.

$$
\begin{equation*}
x=1^{\langle G, H\rangle}=\mathbb{T} \text { iff } G=H \text { and }|H(x)|=1 \tag{58}
\end{equation*}
$$

### 6.2. Translations

Translations for lexical items are as follows:
a. rekhechilo $\lambda e . \operatorname{PLACE}(e) \quad$ verbs are predicates of events
b. tebil $\quad \lambda i$.TABLE $(i) \quad$ nouns denote sets of individuals
c. robi ar rina $\quad \lambda P \cdot[x] \wedge \operatorname{robi} \oplus \operatorname{rina}(x) \wedge P(x)$
d. $\Theta \quad \lambda i . \lambda e . \Theta(e, i) \quad \quad$ thematic roles of type $\langle e,\langle v, e\rangle\rangle$
e. ExClo $\quad \lambda V_{v t} \cdot[e] \wedge \operatorname{SAME}(e)_{e} \wedge V(e) \quad$ Ex-Closure $\langle v t, t\rangle$ for events
f. $\mathrm{D}^{i} \quad \lambda P . \lambda j \cdot \max (i)($ atom $(i) \wedge i \leq j) \wedge \delta(i)(P(i)) \quad$ Adverbial Distributivity operator

The Plain numerals will be translated using the Cardinality test and the postsupposition SAME $(i)_{i}$. Here the postsuppositions are indicated with an overline. The plain numeral pãc-ta-tebil 'five tables' is translated as follows:

$$
\begin{equation*}
\text { pãc-ṭa-ṭebil }{ }_{i}^{i} \rightsquigarrow \lambda P \cdot[i] \wedge \operatorname{TABLE}(i) \wedge|i|=5 \wedge \overline{\operatorname{SAME}(i)_{i}} \wedge P(i) \tag{59}
\end{equation*}
$$

The dependent numerals will be translated using the cardinality test and the postsuppositions $\operatorname{different}(j)_{i}$ and $\operatorname{SamE}(j)_{i}$. Two different possibilities of translations for the dependent numeral tin-te-kore-phuldani 'two flower vases each' are listed below.
(60) a. tin-te-kore-phuldani ${ }_{i}^{j} \rightsquigarrow \lambda P \cdot[j] \wedge \operatorname{FLOWER}-\operatorname{VASE}(j) \wedge|j|=3 \wedge \overline{\operatorname{DIFFERENT}(j)_{i}} \wedge$ $P(j)$
b. $\quad \underline{\text { tin-te-kore-phuldani }}{ }_{i, k}^{j} \rightsquigarrow \lambda P \cdot[j] \wedge$ FLOWER- $\operatorname{VASE}(j) \wedge|j|=3 \wedge \overline{\operatorname{DIFFERENT}(j)_{i}} \wedge$ $\overline{\operatorname{SAME}(j)_{k}} \wedge P(j)$
A Universal quantifier like prottekti-mee 'each one of the girls' will be translated as follows:

In the derivations, all DPs must be QR-ed leaving a trace indexed with the index of the DP.

### 6.3. Illustration of Interpretation

A transitive sentence like (62) containing plain numerals will be translated as in (63).

$$
\begin{equation*}
\text { du-jon-mee } e_{x}^{x} \text { tin-tee-boi } y_{y}^{y} \quad \text { porechilo } \tag{62}
\end{equation*}
$$

two-Cl.-girl three-Cl.-book read
'(A total of) two girls read (a total of) three books.'

$$
\begin{align*}
& (62) \rightsquigarrow[\mathrm{x}] \wedge \overline{\operatorname{SAME}(x)_{x}} \wedge \operatorname{GIRL}(x) \wedge|x|=2 \wedge[y] \wedge \overline{\operatorname{sAME}(y)_{y}} \wedge \operatorname{BOOK}(y) \wedge|y|=3 \wedge  \tag{63}\\
& {[e] \wedge \operatorname{SAME}(e)_{e} \wedge \operatorname{READ}(e) \wedge \operatorname{AGENT}(e, x) \wedge \operatorname{THEME}(e, y)}
\end{align*}
$$

However, the postsuppositions, marked by overline, will be evaluated after the at-issue updates.

$$
\begin{align*}
& \left([x] \wedge \operatorname{GiRL}(x) \wedge|x|=2 \wedge[y] \wedge \operatorname{Book}(y) \wedge|y|=3 \wedge[e] \wedge \operatorname{SAME}(e)_{e} \wedge \operatorname{READ}(e) \wedge\right.  \tag{64}\\
& \operatorname{AGENT}(e, x) \wedge \operatorname{THEmE}(e, y)) \wedge \operatorname{sAME}(x)_{x} \wedge \operatorname{SAME}(y)_{y}
\end{align*}
$$

The output of a successful path updates by (64) might look like Figure 5, on which the postsuppositions $\operatorname{SAME}(x)_{x}$ and $\operatorname{SAME}(y)_{y}$ will be evaluated. Since all the values of $x$ are identical $\operatorname{SAME}(x)_{x}$ is satisfied Since all the values of $y$ are identical $\operatorname{SAME}(y)_{y}$ is satisfied.

\[

\]

Figure 5: Output of the at-issue updates in (64)

## 7. Dependent numerals in transitive sentences

In sentence (65), a dependent numeral is licensed by a plain numeral plurality. Sentence (65) will be translated as (66). The postsupposition of the dependent numeral DIFFERENT $(y)_{x^{\prime}}$ finds its antecedent in the dref introduced by the (max operator in the) distributivity operator $\mathrm{D}^{x^{\prime}}$.
du-jon-mee ${ }_{x}^{x} \mathrm{D}^{x^{\prime}}$ tin-te- kore-boi ${ }_{x^{\prime}}^{y} \quad$ porechilo
two-Cl.-girl D three-Cl.-do.Pfv-book read
'Two girls read different sets of three books.'

$$
\begin{align*}
& (65) \rightsquigarrow[x] \wedge \overline{\operatorname{SAME}(x)_{x}} \wedge|x|=2 \wedge \operatorname{GIRL}(x) \wedge \max \left(x^{\prime}\right)\left(\operatorname{atom}\left(x^{\prime}\right) \wedge x^{\prime} \leq x\right) \wedge \delta\left(x^{\prime}\right)([y] \wedge  \tag{66}\\
& |y|=3 \wedge \operatorname{BOOK}(y) \wedge \operatorname{DIFFERENT}(y)_{x^{\prime}} \wedge[e] \wedge \operatorname{SAME}(e)_{e} \wedge \operatorname{READ}(e) \wedge \operatorname{AGENT}\left(e, x^{\prime}\right) \wedge \\
& \operatorname{THEME}(e, y))
\end{align*}
$$

The postsupposition DIFFERENT $(y)_{x}$ is tested on the output set of assignments of the $\delta x^{\prime}$ operator.

$$
\begin{align*}
& \left([x] \wedge|x|=2 \wedge \operatorname{GIRL}(x) \wedge \max \left(x^{\prime}\right)\left(\operatorname{atom}\left(x^{\prime}\right) \wedge x^{\prime} \leq x\right) \wedge \delta\left(x^{\prime}\right)([y] \wedge|y|=3 \wedge \operatorname{BOOK}(y) \wedge\right.  \tag{67}\\
& \left.\left.[e] \wedge \operatorname{SAME}(e)_{e} \wedge \operatorname{READ}(e) \wedge \operatorname{AGENT}\left(e, x^{\prime}\right) \wedge \operatorname{THEME}(e, y)\right) \wedge \operatorname{DIFFERENT}(y)_{x^{\prime}}\right) \\
& \wedge \operatorname{SAME}(x)_{x}
\end{align*}
$$

The output of $\delta x^{\prime}$ operator would look like Figure 6, a substate of which satisfies the postsupposition DIFFERENT $(y)_{x^{\prime}}$ of the distributive numeral. As before, DIFFERENT $(y)_{x^{\prime}}$ would introduce an $x^{\prime}$-partition into column under $y$ in the set of assignments in Figure 6 and check if for the pair of individual values under $x^{\prime}$, the corresponding values in $y$ differ. Figure 7 represents the relevant substate with the partition.

| $x^{\prime}$ | $y$ | $e$ |
| :---: | :---: | :---: |
| girl $_{1}$ | book $_{1} \oplus$ book $_{2} \oplus$ book $_{3}$ | read $_{1}$ |
| girl $_{2}$ | book $_{1} \oplus$ book $_{2} \oplus$ book $_{4}$ | read $_{2}$ |

Figure 6: Output satisfying DIFFERENT $(y)_{x^{\prime}}$

| $H$ | $x^{\prime}$ | $y$ |
| :---: | :---: | :---: |
| $\mathrm{~h}_{1}$ | girl $_{1}$ | book $_{1} \oplus$ book $_{2} \oplus$ book $_{3}$ |
| $\mathrm{~h}_{2}$ | girl $_{2}$ | book $_{1} \oplus$ book $_{2} \oplus$ book $_{4}$ |

Figure 7: $x^{\prime}$-partition of $H(y)$
Without a distributivity operator, we would get an output set of assignments where the same two girls are related to one or many sets of three books via the same event of reading. The translation of (65) would be (68). It would lead to outputs that fail to satisfy the postsupposition DIFFERENT $(y)_{x}$, as there would be only one value in the $x$ column, whereas DIFFERENT $(y)_{x}$ needs at least two values in the $x$ column to introduce a non-trivial partition into $H(y)$.

$$
\begin{align*}
& {[x] \wedge \overline{\operatorname{SAME}(x)_{x}} \wedge \operatorname{GIRL}(x) \wedge|x|=2 \wedge[y] \wedge \overline{\operatorname{DIFFERENT}(y)_{x}} \wedge \operatorname{BOOK}(y) \wedge|y|=3 \wedge[e] \wedge}  \tag{68}\\
& \operatorname{SAME}(e)_{e} \wedge \operatorname{READ}(e) \wedge \operatorname{AGENT}(e, x) \wedge \operatorname{THEME}(e, y)
\end{align*}
$$

Having a singularity as a licensor of the dependent numeral would lead to an output where the postsupposition DIFFERENT $(y)_{x}$ cannot be met. The output would fail to provide two distinct values for $x^{\prime}$ for the postsupposition DIFFERENT $(y)_{x^{\prime}}$ to work on.

## 8. Dependent numerals in ditransitive sentences

In this section I will briefly go over the crucial ditransitive case discussed in the paper, in which a dependent numeral direct object covaries with the subject of the ditransitive and as a side effect ends up blocking the cumulative interpretation of the indirect object.

Robi ar Rina ${ }^{x} D^{x^{\prime}}$ pãc-ṭa-tebil-e ${ }_{y}^{y}$ tin-te- kore-phuldani ${\underset{x}{ }{ }^{z}}_{z}^{\text {rekhechilo }}$ Robi and Rina $D$ five-Cl.-table-Loc three-Cl.-do.Pfv-flower.holder placed \#'Robi and Rina placed three flower vases each (per person) on a total of five tables.’ $\checkmark$ 'Robi and Rina placed different sets of 3 flower vases (per person) on the same 5 tables.'

The translation (70) of (69) shows that the indirect object (Goal) cannot be cumulative relative to the subject as it is in the scope of $\delta$.

```
(69) \(\rightsquigarrow[x] \wedge\) robi \(\oplus \operatorname{rina}(x) \wedge \max \left(x^{\prime}\right)\left(\operatorname{atom}\left(x^{\prime}\right) \wedge x^{\prime} \leq x\right) \wedge \delta\left(x^{\prime}\right)\left([y] \wedge \overline{\operatorname{SAME}(y)_{y}} \wedge\right.\)
\(\operatorname{TABLE}(y) \wedge|y|=5 \wedge[z] \wedge \overline{\operatorname{DIFFERENT}(z)_{x^{\prime}}} \wedge \operatorname{FLOWER}-\operatorname{VASE}(z) \wedge|z|=3 \wedge[e] \wedge \operatorname{SAME}(e)_{e}\)
\(\left.\wedge \operatorname{PLACE}(e) \wedge \operatorname{AGENT}\left(e, x^{\prime}\right) \wedge \operatorname{GOAL}(e, y) \wedge \operatorname{THEME}(e, z)\right)\)
```

$[x] \wedge$ robi $\oplus$ rina $(x) \wedge \max \left(x^{\prime}\right)\left(\operatorname{atom}\left(x^{\prime}\right) \wedge x^{\prime} \leq x\right) \wedge \delta\left(x^{\prime}\right)([y] \wedge \operatorname{TABLE}(y) \wedge|y|=5 \wedge[z] \wedge$ $\operatorname{FLOWER}-\operatorname{VASE}(z) \wedge|z|=3 \wedge[e] \wedge \operatorname{SAME}(e)_{e} \wedge \operatorname{PLACE}(e) \wedge \operatorname{AGENT}\left(e, x^{\prime}\right) \wedge \operatorname{GOAL}(e, y) \wedge$ $\operatorname{THEME}(e, z)) \wedge \operatorname{SAME}(y)_{y} \wedge \operatorname{DIFFERENT}(z)_{x^{\prime}}$

A successful update would yield an output like in Figure 8.

| $x$ | $x^{\prime}$ | $z$ |  |  |
| :---: | :---: | :---: | :---: | :---: |
| robi $\oplus$ rina | robi | table $_{1} \oplus$ table $_{2} \oplus$ table $_{3} \oplus$ table $_{4} \oplus$ table $_{5}$ | vase $_{1} \oplus$ vase $_{2} \oplus$ vase $_{3}$ | place $_{1}$ |
| robi $\oplus$ rina | rina | table $_{1} \oplus$ table $_{2} \oplus$ table $_{3} \oplus$ table $_{4} \oplus$ table $_{5}$ | vase $_{4} \oplus$ vase $_{5} \oplus$ vase $_{6}$ | place $_{2}$ |

Figure 8: Output for subject licensing situation
The postsupposition $\operatorname{SAME}(y)_{y}$ will be evaluated by introducing the trivial partition in $H(y)$, which looks like (72a). Since the cells in the partition are identical to each other, the postsupposition $\operatorname{SAME}(y)_{y}$ will be satisfied. The postsupposition DIFFERENT $(z)_{x^{\prime}}$ will be evaluated by introducing the $x^{\prime}$-partition in $H(z)$, which looks like (72b). Since the cells are distinct in this case, the postsupposition will be satisfied.
(72) a. $\quad\left\{\left\{\right.\right.$ table $_{1} \oplus$ table $_{2} \oplus$ table $_{3} \oplus$ table $_{4} \oplus$ table $\left._{5}\right\},\left\{\right.$ tabl $_{1} \oplus$ table $_{2} \oplus$ table $_{3} \oplus$ table $_{4} \oplus$ table $\left.\left._{5}\right\}\right\}$
b. $\quad\left\{\left\{\right.\right.$ vase $_{1} \oplus$ vase $_{2} \oplus$ vase $\left._{3}\right\},\left\{\right.$ vase $_{4} \oplus$ vase $_{5} \oplus$ vase $\left.\left._{6}\right\}\right\}$

## 9. Summary and Discussion

The analysis developed in the last three sections incorporates the proposal that dependent numerals do not lexicalize distributivity operators. Sections 7 and 8 show in detail: (i) how the postsupposition DIFFERENT $(j)_{i}$ brings about the covarying reading, (ii) why plain numerals cannot covary because of the postsupposition $\operatorname{SAME}(i)_{i}$ and (iii) how the covert adverbial distributivity operator interacts with nominals in its scope.

Finally, I would like to briefly discuss the cross-linguistic aspect of my argumentation in favor of an adverbial $D$ operator. The data from the Bengali ditransitives is not unique to Bengali and as far as I have investigated can be replicated for double object constructions at least in Hindi and Turkish, both languages with dependent numerals. I would expect double object constructions in English to show similar inference patterns regarding the lack of the cumulative interpretation ${ }^{9}$, but that needs to be investigated carefully in future work.

The argumentation about reducing object DP conjunction to conjunction reduction too has a clear cross-linguistic prediction. If a language has a conjunction that is only meant for conjoining DPs and is not reducible to conjunction of propositions, we expect Kuhn's prediction (73) to hold in those cases of object DP conjunction. That is to say, the cumulative reading should indeed be absent for the plain numeral in the first conjunct with such a conjunction.

$$
\begin{equation*}
\left[\text { Subject }{ }_{V P} \mathbf{D}\left[{ }_{V P}\left[\& P\left[{ }_{D P} \text { Plain Numeral }\right] \&[D P \text { DepNum }]\right] \text { V }\right]\right] \tag{73}
\end{equation*}
$$

[^6]It will be very interesting to see if this precise prediction of adverbial $D$ is borne out or not.

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[^0]:    ${ }^{1}$ I would like to thank Roger Schwarzschild for his detailed comments. I also benefited much from the comments of the reviewers and the participants of SuB25.

[^1]:    ${ }^{2}$ Similar arguments can be found in LaTerza (2014) on adnominal each or in Dotlačil (2013) on each other.

[^2]:    ${ }^{3} 1$ lakh=100000

[^3]:    ${ }^{4}$ or a projection of an Applicative head
    ${ }^{5}$ Thanks to Luisa Marti for asking me to explore this option.

[^4]:    ${ }^{6}$ Similar observations can me made for other ditransitive examples (i), (ii).
    (i) tin-te-company ${ }_{{ }_{\nu} P} \mathbf{D}$ [ ${ }_{\nu P}$ [poncaš-jon-kormocari-ke] [æk-lokkho-taka- kore -maine] dieche]] three-Cl.-company ${ }_{[\nu p} \mathrm{D}\left[{ }_{\nu p}\right.$ fifty-Cl.-worker-Dat one-lakh-rupee-do.Pfv-salary gave $]$ \#'3 companies paid 1 lakh rupees each (per company) to a total of 50 workers.' $\checkmark$ ' 3 companies paid 1 lakh rupees each (per company) to the same 50 workers.'
    (ii) du-jon-jubok ${ }{ }_{\nu P} \mathbf{D} \mathbf{D}\left[{ }_{v P}[\right.$ tin-te-company-te] [car-te- kore-cakrir-dorkhasto] pathiechilen]] two-cl-young.man [ ${ }_{\nu} \mathrm{D}$ D ${ }_{\nu p}$ three-cl-company-Loc four-cl-do.Pfv-job.Gen-application sent]] \#‘'Two young men sent 4 applications each (per man) to a total of 3 companies.' $\checkmark$ 'Two young men sent 4 applications each (per man) to the same 3 companies.'

[^5]:    ${ }^{7}$ It is similar to domain level cardinality constraints in Henderson (2012).
    ${ }^{8}$ The definition is identical to dependency (40).

[^6]:    ${ }^{9}$ Beck and Sauerland (2000) however discusses data showing the opposite

