Remarks on two approaches to NPI licensing[1]

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Abstract. In relation to the notion of informativity, two types of approaches to DPs headed by *any* (*any*-DPs) have been distinguished. At a first approximation, one approach takes an *any*-DP to be accompanied by a requirement that a clause containing it be more informative than all its relevant alternatives (cf. Kadmon & Landman 1993), while the other approach requires a clause containing it to be more informative than all its relevant alternatives that are true (cf. Chierchia 2013). The goal of this paper is to compare these approaches with respect to their predictions about the distribution of plural *any*-DPs in modal environments.

Keywords: maximal informativity, exhaustification, negative polarity items, plurality.

1. Two approaches to *any*-DPs

In relation to the notion of informativity and to how it may be utilized, two approaches to *any*-DPs, whose limited distribution is exemplified in (1), can be distinguished. We introduce the approaches first in their most rudimentary form, which we then stepwise update.

(1) a. #Tal read any book(s).
b. Tal didn't read any book(s).

The Max approach. Abstracting aways from various details, the first type of approach takes the distribution of *any*-DPs to be governed by the mechanism described in (2): given a selected set of alternatives to a sentence with an *any*-DP, Max induces the inference that the sentence is more informative than all the selected alternatives (cf. Kadmon & Landman 1993; Lahiri 1998; Crnič 2014, among others). If this inference is consistent, the *any*-DP is acceptable; if it is not, it is unacceptable. (The Max operator can be seen to fill in for a more involved *even* operator, e.g., Lee & Horn 1994; Krifka 1995; Lahiri 1998; Crnič 2014.)

(2) $[\![\text{Max S}]\!]^{g,w} = [\![\text{S}]\!]^{g,w} \wedge \forall \text{S}' \in \textsc{SelAlt}(\text{S})\text{: } \lambda \text{w}.[\![\text{S}]\!]^{g,w} \subset \lambda \text{w}.[\![\text{S}']\!]^{g,w}$
$\big(\approx$ 'sentence S is more informative than its selected alternatives'$\big)$

The Exh approach. The second type of approach takes the distribution of *any*-DPs to be governed by the mechanism described in (3) (to be revised below): given a selected set of alternatives to a sentence with an *any*-DP, Exh induces the inference that the sentence is more informative than all the selected true alternatives (cf. Krifka 1995; Chierchia 2013; Dayal 2013). If this inference is consistent, the *any*-DP is acceptable; if it is not, it is unacceptable. (The Exh operator is an exhaustification operator and will be revised below.)

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(3) $[\![\text{Exh S}]\!]^{g,w} = [\![\text{S}]\!]^{g,w} \wedge \forall \text{S}' \in \textsc{SelAlt}(\text{S}): [\![\text{S}']\!]^{g,w} \rightarrow \lambda\text{w}.[\![\text{S}]\!]^{g,w} \subset \lambda\text{w}.[\![\text{S}']\!]^{g,w}$
$\big(\approx$ 'sentence S is more informative than its selected true alternatives'$\big)$

Given the above formulations of the two types of inferences, the latter approach is more permissive than the former (compare the domains of the universal quantifiers over selected alternatives). Accordingly, there may be areas in which the two can be distinguished.

Re: (1). If we assume that the selected alternatives to a sentence with *any* are its counterparts in which the *any*-DP (an existential quantifier) is replaced by a stronger expression, as in (4), the approaches are indistinguishable when it comes to data like (1). (Notational convention: S[any NP] is a clause containing at least one occurrence of *any NP*, and S[any NP/DP] is identical to it up to the replacement of all the occurrences of *any NP* with *DP*.)

(4) Assumption about selected alternatives (to be revised):
$\textsc{SelAlt}(\text{S[any NP]}) = \{\text{S[any NP/DP]} \mid \text{for all g,w: } [\![\text{DP}]\!]^{g,w} \subseteq [\![\text{any NP}]\!]^{g,w},$
$\text{for some g,w: } [\![\text{DP}]\!]^{g,w} \subset [\![\text{any NP}]\!]^{g,w} \}$

Since in (1a), the *any*-DP occurs in a sentence that is upward-entailing with respect to it, all its selected alternatives will entail it. On the one hand, this contradicts the informativity requirement on the Max approach, as illustrated in (5). On the other hand, negation of all the stronger alternatives contradicts the sentence on the Exh approach, as in (6) (but see Chierchia 2013 on quantified sentences). The NPI is correctly predicted to be unacceptable.

(5) $[\![$ [Max [Tal read any book]] $]\!]^{g,w} \Rightarrow$
(λw. Tal read a book in w) $\subset$ (λw. Tal read every book in w) (inconsistent)

(6) $[\![$ [Exh [Tal read any book]] $]\!]^{g,w} \Rightarrow$
(Tal read a book in w) $\wedge$ (for all books x, $\neg$(Tal read x in w)) (inconsistent)

In (1b), the sentence with the *any*-DP entails all its selected alternatives, and thus Max and Exh generate tautologous inferences. The NPI is correctly predicted to be acceptable.

(7) $[\![$ [Max [Neg [Tal read any book]]] $]\!]^{g,w} \Rightarrow$
(λw. $\neg$Tal read a book in w) $\subset$ (λw. $\neg$Tal read every book/a short book/etc. in w)

(8) $[\![$ [Exh [Neg [Tal read any book]]] $]\!]^{g,w}$ = $\neg$(Tal read a book in w)

Plan. Unlike in the examples like (1), the two approaches can be distinguished when it comes to modal environments (see Crnič 2014 on distinguishing between them in non-monotone environments). We begin by reviewing in Section 2 how the two approaches can capture the acceptability

of singular *any*-DPs in existential modal sentences, exemplified in (9), as well as the unacceptability of *any*-DPs in universal modal sentences, exemplified in (10). In order to do this, the approaches must be updated. Our updates are conservative, and for simplicity partially distinct from the actual proposals in the literature.

(9) Tal is allowed to read any book.

(10) #Tal is required to read any book.

We turn in Section 3 to the unacceptability of plural *any*-DPs in existential modal sentences, exemplified in (11). The contrast in the acceptability between (9) and (11) follows straightforwardly only on the Max approach, thus providing new support for it.

(11) #Tal is allowed to read any books.

In the conclusion of the paper, we discuss a case of potential undergeneration of the two approaches: contrary to our above description, a specific class of occurrences of *any*-DPs – namely, *any*-DPs with numeral modifiers – is acceptable in universal modal sentences, as exemplified in (12) (Dayal, 2004). (A more comprehensive discussion of several of the issues attended to in this paper, as well as of many related issues, can be found in Crnič 2020.)

(12) Tal is required to read any three books.

2. Modal sentences

While the acceptability of *any*-DPs in existential modal sentences is unexpected on the characterizations of the Max and Exh approaches above (not least since the *any*-DP occurs only in constituents that are upward-entailing with respect to it), their behavior can be captured on an amendment of the selected alternatives and the formulation of Exh.

2.1. The Max approach

Revision. On the first approach, the acceptability of the *any*-DPs is accounted for on, first, the assumption that the selected alternatives for Max can be restricted to the so-called subdomain alternatives (cf. Kadmon & Landman 1993),

(13) Assumption about selected alternatives (revised):

$$\textsc{SelAlt}(\text{S}[\text{any}_D\ \text{NP}]) = \{\text{S}[\text{any}_D/\text{a}_{D'}] \mid \text{for all g,w: } [\![\text{a}_{D'}\ \text{NP}]\!]^{g,w} \subseteq [\![\text{any}_D\ \text{NP}]\!]^{g,w}, \text{ for some g,w: } [\![\text{a}_{D'}\ \text{NP}]\!]^{g,w} \subset [\![\text{any}_D\ \text{NP}]\!]^{g,w}\}$$

and, second, the assumption that the grammar furnishes a strengthening mechanism that can derive conjunctive/universal inferences for certain occurrences of disjunction/existential quantifiers. One

such mechanism is precisely the mechanism pointed to in (3) (esp., Fox 2007). However, a more sophisticated formulation of it is required than we provided.[2] One such characterization is in (14)-(15) (see Bar-Lev & Fox 2020 for a full definition): the set of alternatives that get negated is determined by 'innocent exclusion', (15a); in addition, some of the alternatives get asserted, as determined by 'innocent inclusion', (15b). (Exh also combines with a resource domain, which we do not represent. We treat only the universal quantifier alternatives as relevant for simplicity. See Crnič 2020, App. A, for discussion.)

(14) $[\![\text{Exh S}]\!]^{g,w} = \forall S' \in \text{Excl(S)}: \neg[\![\text{S}']\!]^{g,w} \wedge \forall S' \in \text{Incl(S)}: [\![\text{S}']\!]^{g,w}$.

(15) a. $\text{Excl(S)} = \bigcap\{\text{M} \mid \text{M is a maximal subset of ALT(S)}$
such that $\{\neg[\![S']\!] \mid S' \in \text{M}\} \cup \{[\![\text{S}]\!]\}$ is consistent$\}$

b. $\text{Incl(S)} = \bigcap\{\text{M} \mid \text{M is a maximal subset of ALT(S)}$
such that $\{[\![S']\!] \mid S' \in \text{M}\} \cup \{\neg[\![S']\!] \mid S' \in \text{Excl(S)}\}$ is consistent$\}$

Existential modal sentences. With these amendments in hand, we can now derive the acceptability of (9) (see Crnič 2017, 2019, 2020 for further details). The output of the exhaustification of the modal sentence is provided in (17) (for simplicity, we treat *any*-DPs as de re throughout). The inclusion inferences are called 'free choice inferences'.

(16) [Exh [$\Diamond$ [Tal read $\underline{\text{any}_D \text{ book}}$]]]

(17) $\forall \text{D}' \subseteq \text{D}(\text{card}(\text{D}' \cap \text{book}) \geq 1 \rightarrow \Diamond(\text{Tal read a book in D}')) \wedge$ (inclusion)
$\forall \text{D}' \subseteq \text{D}(\text{card}(\text{D}' \cap \text{book}) > 1 \rightarrow \neg\Diamond(\text{Tal read every book in D}')) \wedge$ (exclusion)
$\forall \text{D}' \subseteq \text{D}(\neg\Box(\text{Tal read a book in D}'))$

The full parse of the sentence in (9) is provided in (18), where Exh is intercalated between Max and the *any*-DP. The inference induced by Max, computed in (19), is trivially true, that is, the sentence is downward-entailing with respect to the domain of the *any*-DP. Accordingly, the *any*-DP is correctly predicted to be acceptable.

(18) [Max [Exh [$\Diamond$ [Tal read any_D book]]]]

(19) For all $\text{D}^* \subset \text{D}$ such that $\text{D}^* \cap \text{book} \subset \text{D} \cap \text{book}$,
λw. $\forall \text{D}' \subseteq \text{D}(\text{card}(\text{D}' \cap \text{book}) \geq 1 \rightarrow \Diamond_w(\text{Tal read a book in D}')) \wedge$
$\forall \text{D}' \subseteq \text{D}(\text{card}(\text{D}' \cap \text{book}) > 1 \rightarrow \neg\Diamond_w(\text{Tal read every book in D}')) \wedge$

[2] Exhaustification of a simple disjunctive sentence results in a contradiction given (3) and (13). This is avoided by adopting 'innocent exclusion' in (14); in contrast, Chierchia 2013 relies on the pruning of alternatives. See Spector 2016 for a review. For brevity, we skirt the issues of pruning in the main text.

$\forall D' \subseteq D(\neg \Box_w(\text{Tal read a book in } D'))$

$\subset$ $\lambda w.\ \forall D' \subseteq D^*(\text{card}(D' \cap \text{book}) \geq 1 \rightarrow \Diamond_w(\text{Tal read a book in } D')) \wedge$
$\forall D' \subseteq D^*(\text{card}(D' \cap \text{book}) > 1 \rightarrow \neg \Diamond_w(\text{Tal read every book in } D')) \wedge$
$\forall D' \subseteq D^*(\neg \Box_w(\text{Tal read a book in } D'))$

It is worth noting that calling on Exh in (9) does not present an amendment of the Max approach. Exh is an independent mechanism in grammar that can be employed to rescue an otherwise illicit sentence. Exh is thus not a constitutive ingredient of the approach – recall from our discussion of (1) that Exh need not be used in negated sentences like (1b) (and perhaps cannot be used there, cf. Fox & Spector 2018). The approach is summarized in (20).

(20) The Max approach to *any*-DPs is modular:
(i) Max is a constitutive ingredient
(ii) Exh is called on as a rescue mechanism

Universal modal sentences. Before proceeding to the Exh approach, an immediate payoff of assuming Max should be pointed out: it rules out occurrences of *any*-DPs in universal modal sentences like (10), repeated below. The sentence can be assigned the structure in (21); the meaning of the sister of Max is in (22). Although this is not important at this point, note that the sentence does not entail the negation of the alternatives based on the universal modal. (In order to obtain these otherwise missing inferences, Bar-Lev & Fox 2020, Sect. 5.5, introduce a special assumption about alternatives derived by a substition of modal elements, which we do not adopt here, primarily for ease of the presentation below.)

(10) #Tal is required to read any book.

(21) [Max [Exh [Exh [$\Box$ [Tal read $\underline{\text{any}_D\ \text{book}}$]]]]]

(22) $\Box(\text{Tal read a book in D}) \wedge$ (inclusion)
$\forall D' \subseteq D(\text{card}(D' \cap \text{book}) \geq 1 \rightarrow \Diamond(\text{Tal read a book in } D')) \wedge$
$\forall D' \subseteq D(\text{card}(D' \cap \text{book}) > 1 \rightarrow \neg \Diamond(\text{Tal read every book in } D'))$ (exclusion)

While the meaning in (22) is consistent, it is not more informative than that of any of the selected alternatives, in violation of the Max requirement, as stated in (23) (Crnič, 2017, 2019). For example, due to the first conjunct, being required to read a $\underline{\text{book}}$ and allowed to read any book fails to entail being required to read a $\underline{\text{long book}}$ and allowed to read any long book. Consequently, the *any*-DP in (10) is correctly ruled out as unacceptable.

(23) For all $D^* \subset D$ such that $D^* \cap \text{book} \subset D \cap \text{book}$,
$\lambda w.\ \Box_w(\text{Tal read a book in D}) \wedge$

$\forall D' \subseteq D(\text{card}(D' \cap \text{book}) \geq 1 \rightarrow \Diamond_w(\text{Tal read a book in } D')) \wedge$
$\forall D' \subseteq D(\text{card}(D' \cap \text{book}) > 1 \rightarrow \neg\Diamond_w(\text{Tal read every book in } D'))$

$\not\subset$ $\lambda w.\ \Box_w(\text{Tal read a book in } D^*) \wedge$
$\forall D' \subseteq D^*(\text{card}(D' \cap \text{book}) \geq 1 \rightarrow \Diamond_w(\text{Tal read a book in } D')) \wedge$
$\forall D' \subseteq D^*(\text{card}(D' \cap \text{book}) > 1 \rightarrow \neg\Diamond_w(\text{Tal read every book in } D'))$

2.2. The Exh approach

The derivation of the acceptability of *any*-DPs in existential modal sentences is prima facie simpler on the second approach. Specifically, if one adopts the new definition of Exh in (14) in the approach, the inventory of mechanisms employed by it is a proper subset of the inventory employed on the Max approach. Moreover, (9) has the LF in (16), repeated below, which is a proper subconstituent of the LF on the Max approach. If we assume the same alternatives as we did in (16), this LF has a consistent interpretation, provided in (17), repeated below. Consequently, the *any*-DP is correctly predicted to be acceptable.

(16) [Exh [$\Diamond$ [Tal read $\underline{\text{any}_D \text{ book}}$]]]

(17) $\forall D' \subseteq D(\text{card}(D' \cap \text{book}) \geq 1 \rightarrow \Diamond(\text{Tal read a book in } D')) \wedge$ (inclusion)
$\forall D' \subseteq D(\text{card}(D' \cap \text{book}) > 1 \rightarrow \neg\Diamond(\text{Tal read every book in } D')) \wedge$ (exclusion)
$\forall D' \subseteq D(\neg\Box(\text{Tal read a book in } D'))$

The current setup of the Exh approach is thus as minimal as it gets. It is summarized in (24). Unlike the Max approach, cf. (20), it only has to assume a single covert operator, Exh.

(24) The Exh approach to *any*-DPs (to be revised):
(i) Exh is a constitutive ingredient

Avoiding overgeneration. Due to a switch to a new definition of Exh, however, the Exh approach now overgenerates in two areas pertinent to this paper: with respect to the unacceptability of *any*-DPs in episodic upward-entailing environments, exemplified in (1a), and to their unaccepatibility in universal modal sentences, exemplified in (10). (Overgeneration arises on all contradiction-free formulations of Exh, e.g., Fox 2007; Katzir 2014.)

Consider the unacceptable sentence *#Tal read any book* in (1a). On the new characterization of Exh, the sentence is admitted as acceptable: no contradiction is generated due to innocent exclusion/inclusion. Thus, the import of Exh is vacuous in both sentences in (1) – but only one of them should be admitted as acceptable. An amendment of the approach is thus called for (as acknowledged by Chierchia 2013, Ch. 3, App. I).

A stand-in for such an amendment is provided in (25), courtesy of Bar-Lev & Margulis (2014, Sect. 5.1): the exhaustified meaning of a sentence with an *any*-DP must entail every formal alternative to the sister of Exh or its negation (cf. Fox 2018; Bar-Lev & Fox 2020).

(25) A clause S[any NP] is acceptable only if for all $S' \in$ ALT(S[any NP]):
$([\![\text{Exh S[any NP]}]\!]^{g,w} \rightarrow [\![S']\!]^{g,w})$ or $([\![\text{Exh S[any NP]}]\!]^{g,w} \rightarrow \neg[\![S']\!]^{g,w})$

In the case of acceptable (1b), all alternatives induced by the *any*-DP are entailed by the original sentence (and thus its exhaustification). In the case of acceptable (9), some alternatives are entailed (the subdomain alternatives) and others have their negation entailed (the universal quantifier and the universal modal alternatives) by the exhaustification of the sentence. Accordingly, the behavior of the *any*-DPs in (1b) and (9) is correctly captured. In contrast, in the case of unacceptable (1a), no subdomain alternatives induced by the *any*-DP nor their negations are entailed by the exhaustification of the sentence. Accordingly, the behavior of the *any*-DP in (1a) is correctly captured as well.

The summary of the updated approach is provided in (26).[3]

(26) The Exh approach to *any*-DPs (revised):
(i) Exh is a constitutive ingredient
(ii) An additional condition admits specific applications of Exh

Several questions are raised by this rendition of the Exh approach. One is whether (25), or something akin to it, can be encoded in the meaning of Exh, thus eliminating (ii) in (26) as a separate condition. Another is in what relation condition (25) stands to the inferences induced by Max – that is, can the approaches still in principle be distinguished empirically?

About the first question: The condition in (25) cannot be encoded as a general constraint on Exh. This conclusion is supported by, for example, disjunction activating parallel alternatives to *any*-DPs but having a non-idiosyncratic, broader distribution of exhaustified meanings. Consider (27). With recursive exhaustification, we can derive its observed meaning in (28) (Bar-Lev & Fox 2020, Sect. 5.5). However, the sentence violates the condition in (25). Another example are plain disjunctive sentences that are acceptable and induce exclusive inferences, but would violate the generalization of condition (25), etc.

(27) Every student ate cake or soup.

[3]The Exh approach described here differs from those of Chierchia 2013 and Dayal 2013 in several respects. Chierchia assumes a contradiction-admitting Exh and two additional constraints in order to deal with the distribution of *any*-DPs in modal sentences: obligatory wide-scope of *any*-DPs in modal contexts (Wide-Scope Constraint) and the alternatives in the domain of Exh obligatorily differing in a specific way in the conversational backgrounds of the modals occurring in them (Modal Containment). Dayal 2013 adopts a Viability Constraint on exhaustified alternatives, requiring each of them to be true in some accessible worlds.

(28) Every student ate cake or soup ∧ (inclusion)
Some student ate cake ∧ Some student ate soup ∧
¬Every student ate cake and soup (exclusion)

More generally, and independently of the concrete condition suggested in (25), all the discrepancies in the distributions of *any*-DPs and disjunction make finding a significant condition that would correctly tease them apart extremely difficult.[4] The Exh approach must adopt either a characterization of Exh that admits contradictions, or a separate condition on when a contradiction-free exhaustification is admitted, a condition that cannot be encoded in Exh.

About the second question: The Max approach remains distinguishable the Exh approach also on its rendition with the condition in (25) – a sentence may be predicted to be unacceptable on the Max approach, but still be predicted to be acceptable on the Exh approach. In particular, consider a hypothetical sentence that satisfies the condition in (25) with its exhaustified meaning entailing the negation of at least some subdomain alternatives. Such a sentence would be admitted as acceptable on the Exh approach, but not on the Max approach. We turn to examples of such sentences in the following section.

Universal modals. Before proceeding to the next section, we review a positive side effect of the combination of the assumption of the condition in (25) and the characterization of Exh in (14). As computed above, the recursively exhaustified meaning of (10) is the one provided in (22), repeated below. Crucially, neither the subdomain alternatives based on the universal modal nor their negations (that is, that Tal is (not) required to read a book in D′, where D′ is a subset of D) are entailed by (22), in violation of the condition in (25). Thus, the unacceptability of *any*-DPs in universal modal sentences is correctly predicted. (If we followed Bar-Lev & Fox 2020 in restricting the alternatives to the universal modal, hinted at above, the condition in (25) would be satisfied and yet another condition would have to be stipulated to capture the behavior of *any*-DPs. See footnote 3 for some options.)

(22) $\Box$(Tal read a book in D) ∧
$\forall D' \subseteq D(\text{card}(D' \cap \text{book}) \geq 1 \rightarrow \Diamond(\text{Tal read a book in } D')) \wedge$
$\forall D' \subseteq D(\text{card}(D' \cap \text{book}) > 1 \rightarrow \neg\Diamond(\text{Tal read every book in } D'))$

Given what we have seen so far, the two approaches to *any*-DPs are comparable: they have the same empirical coverage and they are of similar theoretical complexity. But, importantly, they are also, at least in principle, empirically distinguishable.

[4] An additional challenge comes from recent exhaustification-based approaches to so-called positive polarity items (esp., Spector 2014; Nicolae 2017), where the unacceptability of positive polarity items in downward-entailing environments is pinned on the illicit vacuity of exhaustification that resembles that in (1b).

3. The observation and its derivation

We saw that not all occurrences of *any*-DPs are acceptable in existential modal sentences: in particular, plural *any*-DPs are unacceptable, as exemplified in (11), repeated below (but see Crnič 2020 for some qualifications). This state of affairs is expected on the Max approach.

(11) #Tal is allowed to read any books.

3.1. The Max approach

Let us assume for concreteness that plural *any*-DPs quantify over non-atoms (e.g., Chierchia 1998). Accordingly, the output of the exhaustified meaning of (28) is the one provided in (30): all sub-domain alternatives whose domain of *any* contains at least one plurality consisting of exactly two books are included; all universal quantifier alternatives over at least three books, and all alternatives built on the universal modal, are excluded.

(29) [Exh [$\Diamond$ [Tal read any_D books]]]

(30) $\forall D' \subseteq D(\exists x(x \in D' \wedge x \text{ books} \wedge |x|=2) \rightarrow \Diamond(\text{Tal read some books in } D')) \wedge$ (inclusion)
$\forall D' \subseteq D(\text{card}(D' \cap \text{books}) > 2 \rightarrow \neg\Diamond(\text{Tal read all books in } D')) \wedge$ (exclusion) $\forall D' \subseteq D(\neg\Box(\text{Tal read some books in } D'))$

While the meaning in (30) is consistent, (31) entails an inconsistent Max requirement, provided in (32). In order to appreciate this, it suffices to identify one substitution of the domain of *any* on which the pertinent entailment fails to go through. One such example is a domain that contains only pluralities consisting of 3 books. The meaning that we obtain on this substitution is not entailed by the initial sentence – in fact, its negation is entailed. Due to the inconsistent Max inference, the *any*-DP in (11) is predicted to be unacceptable.

(31) [Max [Exh [Tal read any_D books]]]

(32) For any $D^* \subset D$ such that $D^* \cap \text{book} \subset D \cap \text{book}$,

$\lambda w.\ \forall D' \subseteq D(\exists x(x \in D' \wedge x \text{ books} \wedge |x|=2) \rightarrow \Diamond_w(\text{Tal read some books in } D')) \wedge$
$\forall D' \subseteq D(\text{card}(D' \cap \text{books}) > 2 \rightarrow \neg\Diamond_w(\text{Tal read all books in } D')) \wedge$
$\forall D' \subseteq D(\neg\Box_w(\text{Tal read some books in } D'))$

$\subset$ $\lambda w.\ \forall D' \subseteq D^*(\exists x(x \in D' \wedge x \text{ books} \wedge x \text{ has no proper parts in } D^*)$
$\rightarrow \Diamond_w(\text{Tal read some books in } D')) \wedge$
$\forall D' \subseteq D^*(\text{card}(D' \cap \text{books}) > 2 \rightarrow \neg\Diamond_w(\text{Tal read all books in } D')) \wedge$
$\forall D' \subseteq D^*(\neg\Box_w(\text{Tal read some books in } D'))$

Accordingly, the Max approach correctly captures the asymmetry in the acceptability of singular vs. pluraly *any*-DPs in existential modal sentences. The approach also gives rise to several correct predictions about factors that may ameliorate the acceptability of plural *any*-DPs in existential modal sentences. These are discussed in Crnič (2020).

3.2. The Exh approach

The sentence in (11) satisfies the condition in (25): all subdomain alternatives whose domain contains at least one plurality consisting of two books are entailed by the exhaustified meaning of the sentence (they are included); all other alternatives are negated by exhaustification and so, obviously, their negations are entailed by the exhaustified meaning of the sentence. Thus, the approach must look for the source of the unacceptability of (11) elsewhere. A natural candidate is intervention – that is, the plural morphology may act as an intervener.

Chierchia (2013) develops a theory of intervention that capitalizes on the assumption that the set of alternatives that Exh operates on may be greater than what we assumed above. Since it successfully accounts for various well-known empirical generalizations pertaining to intervention, the hope may be that it can also capture the contrasts between (9) and (11).

In relation to plural *any*-DPs in existential modal sentences, the set of their alternatives could be expanded with the alternatives built on the singular *any*-DP. It holds, however, that the alternatives based on singular *any*-DPs are all includable (vacuously so because they are entailed by their plural counterparts that are includable). Accordingly, their addition does not have obvious detrimental effects on exhaustification and the satisfaction of the condition in (25), and thus should not affect the acceptability of plural *any*-DPs. While a more sophisticated theory of number and intervention may be devised that would distinguish between plural *any*-DPs in modal and non-modal environments, we cannot attempt this here.

4. Plural *any*-DPs with numerals

The final issue that we attend to challenges not only the approaches to *any*-DPs presented above, but any approach designed to rule out *any*-DPs in universal modal sentences. The issue is exemplified in (33): although bare *any*-DPs are marked in universal modal sentences, as discussed in Sect. 2, their variants that are modified by numerals are acceptable (see, esp., Dayal 2004, 2013). This is puzzling since the presence of the numeral cannot by itself rescue the consistency of the Max inference or help with the satisfaction of the condition in (25).

(33) Tal is required to read any #(three) books.

We outline one strategy of how the acceptability of (33) can be captured on the Max approach (see Chierchia 2013; Dayal 2013 for two resolutions on an Exh approach). In order to do this, we need to shift from the notion of entailment in our definition of Max to a weaker notion of

Strawson entailment: a sentence S Strawson entails another sentence S′ iff S together with the presuppositions of S′ entail S′ (von Fintel, 1999). (Note that this relation may need to be weakened further, see Crnič 2014, 2019 for discussion.)

(34) $[\![\text{Max S}]\!]^{g,w} = [\![\text{S}]\!]^{g,w} \wedge \forall \text{S}' \in \textsc{SelAlt}(\text{S})\text{: } (\lambda \text{w}.[\![\text{S}]\!]^{g,w} \cap \lambda \text{w}.[\![\text{S}']\!]^{g,w} \vee \neg [\![\text{S}']\!]^{g,w}) \subset \lambda \text{w}.[\![\text{S}']\!]^{g,w}$

With this in hand, we suggest that the acceptability of (33) is aided by a focus presupposition of the sentence, that is, the sentence is Strawson downward-entailing with respect to the domain of *any two books*. We embed our proposal in a theory that takes discourses to be structured by questions that can be extrapolated from the focus structure of their sentences (esp., Roberts 2012; Beaver & Clark 2009) – this is crucial for avoiding overgeneration.

Existence presupposition. We follow a tradition of work on focus that assumes that focus may give rise to presuppositions (e.g., Geurts & van der Sandt 2004; Abusch 2010, among others). In particular, we put forward that the sentence in (33) is acceptable because it can trigger the presupposition that Tal is required to read three books. It does this by inducing an implicit question under discussion *What three books is Tal required to read?* (see, e.g., Dayal 1996 on existence presuppositions of questions and their derivation). Consequently, sentence (33) is Strawson stronger than all its selected alternatives, as required by Max:

(35) For any D′⊂D such that D′∩three books ⊂ D∩three books,

λw. Tal is required to read any three books in D ∩

λw. Tal is required to read three books in D′

⊂ λw. Tal is required to read any three books in D′

Avoiding overgeneration. An immediate reaction to this suggestion may be that it is bound to overgenerate. It might well – but not in the simple cases. For example, neither (36) nor (37) are admitted if we adopt the existence presupposition described above: (36) is ruled out on this proposal because its meaning is Strawson equivalent with that of all selected alternatives, that is, it is not stronger than them, as required by Max; (37) is ruled out for reasons similar to those that ruled out plural *any*-DPs in existential modal sentences.

(36) #Tal read any book.

(37) #Tal must read any books.

However, the state of affairs is more involved when it comes to examples like (38), which are also unacceptable, but which may seem to be admitted on the simpleminded statement of the presupposition above. In order to see how overgeneration is avoided in examples like this – as a consequence of an independent property of questions that structure discourse – we need to spell out how the described presuppositions are brought about in greater detail.

(38) #Tal must read any book.

Focus presuppositions are induced by sentences invoking (implicit) questions under discussion (cf. Beaver & Clark 2009). Consider (39). The sentence may invoke the question in (40), which has (39) as a complete answer. And this question entails that someone read a book.

(39) Tal_F read a book.

(40) Who read a book? $\big(\Rightarrow$ Someone read a book$\big)$

For concreteness, we assume that the invoking of an implicit question is mediated by a focus-sensitive operator OP, as represented in (41). The simplified meaning of the operator is provided in (42), which relies on the notion of maximal informativity (cf. Dayal 1996; Fox 2018; see Gentile & Schwarz 2018; Hirsch & Schwarz 2020 for discussion). The requirement for there to be a maximally informative alternative in the set of alternatives entails the existence presupposition described above (see, esp., Dayal 1996). Furthermore, if the sister of OP is not a complete answer to the question (once exhaustified), its use is pathological.

(41) [OP [Tal_F read a book]]

(42) $[\![\text{OP S}]\!]^{g,w}$ is defined only if F(S) contains a maximally informative alternative relative to g, w $\big($where S is a maximally informative alternative in M relative to g, w iff $[\![\text{S}]\!]^{g,w} \wedge \forall \text{S}' \in \text{M}$: $[\![\text{S}']\!]^{g,w} \rightarrow \lambda\text{w}.[\![\text{S}]\!]^{g,w} \subseteq \lambda\text{w}.[\![\text{S}']\!]^{g,w}\big)$. If defined, $[\![\text{OP S}]\!]^{g,w} = [\![\text{S}]\!]^{g,w}$.

Derivation. The sentence in (33) has the structure in (43). We stated above that the question needed for Max to yield a consistent interpretation is *What three books is Tal required to read?* More precisely, what is required is a higher-order construal of that question (cf. Spector 2008), which we assume is obtained by the domain of the *any*-DP inducing subdomain alternatives, as provided in (44). Given this set of alternatives, OP effectively triggers the presupposition that Tal is required to read three books in D. (The *any*-DP bears a specific stress pattern that requires further study; our hope is that the observed focal marking is compatible with the focus being assigned to the domain of the *any*-DP.)

(43) [Max [Exh [Exh [OP [$\Box$ [Tal read any_{D_F} three books]]]]]]

(44) $\{[\ \Box\ [\ \text{Tal read any}_{D'}\ \text{three books}\ ]\ ] \mid [\![\text{D}']\!]^{g,w} \subseteq [\![\text{D}]\!]^{g,w}\}$

If one would opt for a different construal, say, a simple question construal, with the alternatives in (45), the sentence in the scope of OP would not provide a complete answer to the question invoked (once exhaustified), resulting in its infelicity.

(45) $\{[\ \Box\ [\ \text{Tal read X}\ ]\ ] \mid [\![\text{X}]\!]^{g,w} \in \text{D}_e\}$

But how does the need for the resolution in (44) ensure that (38) is not admitted? It does so due to an independent, ill-understood property of questions. They may have a higher-order interpretation only if the alternatives-inducing DP (in the usual case, the *wh*-phrase; in our case, the *any*-DP) is plural (Elliott et al. 2018; Fox 2018, but see Xiang 2021). Accordingly, the *any*-DP in (11) may be acceptable (since its host sentence can induce the question under discussion that guarantees the required Strawson downward-entailingess), while the *any*-DP in (38) cannot be (since its host sentence cannot induce the required question under discussion). The contrast between singular and plural numeral *any*-DPs is thus captured.

The presented outline of how the unexpected distribution of plural *any*-DPs with numeral modifiers may be explained is merely a proof of concept. For it to qualify as a full-fledged account, it must be developed and explored further. The distribution of *any*-DPs in other environments that appear to be at odds with the expected inferences of Max must be studied as well (see Crnič 2019 for a review of some of these environments).

5. Conclusion and outlook

We described two approaches to *any*-DPs and explored some predictions that they make with respect to the distribution of *any*-DPs in modal sentences. On both approaches a special mechanism is introduced that is responsible for the constrained distribution of *any*-DPs (Max and condition (25)). While both mechanisms correctly capture the acceptability of singular *any*-DPs in existential and their unacceptability in universal modal sentences, only the former approach readily captures the unacceptability of plural *any*-DPs in existential modal sentences (see Crnič 2020 for a more comprehensive study of their distribution).

Many issues spring from the discussion in this paper, including issues pertaining to polarity items other than *any*-DPs. These issues, as well as a more comprehensive comparison of the two approaches, in a broader range of environments, will have to be pursued elsewhere.

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