A constraint on co-occurrence of partitive quantifiers and gradable predicates[1]

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Abstract. Quantifiers like *most of* are compatible with some predicates but not others, e.g., *Most of the road is* {*narrow / #long*}. In the context of collective predicates, this is known as the *gather/numerous* distinction. This paper focuses on gradable predicates—both singular and collective—and argues that only non-monotonic gradable predicates are compatible with quantifiers like *most of*. I show that if a monotonic predicate co-occurs with such a quantifier, the sentence does not have well-defined truth conditions, which makes it unacceptable.

Keywords: *gather/numerous*, proportional quantifiers, partitives, gradable predicates, monotonicity, intensive/extensive, mereology, partition.

1. Introduction

Singular and collective predicates can be classified into two categories based on whether or not they are compatible with quantifiers such as *all of* and *most of*. In the context of collective predicates, this classification is known as the *gather/numerous* distinction: *gather*-type collective predicates are compatible with these quantifiers (1), while *numerous*-type collective predicates cannot co-occur with them (2) (Kroch, 1979; Dowty, 1987; Champollion, 2010).

(1) a. All of the kids gathered outside.
    b. Most of the kids get along.

(2) a. #All of the kids were numerous.
    b. #Most of the kids elected Mary as class president.

The so-called *gather/numerous* distinction is not restricted to collective predicates but also found in singular ones (Löbner, 2000; Corblin, 2008). For instance, *be soft* and *be in the briefcase* behave like the *gather* type (3), while *be heavy* and *kill the gangster* pattern with the *numerous* type (4).[2]

(3) a. Most of the sofa is soft.
    b. All of the money is in the briefcase.

(4) a. #Most of the sofa is heavy.

[1]This work has greatly benefited from conversations with Bar Avineri, Noa Bassel, Ido Benbaji, Nora Boneh, Cleo Condoravdi, Ashwini Deo, Omri Doron, Berit Gehrke, Chris Kennedy, Malka Rappaport Hovav, Jessica Rett, Todd Snider, and the audience at the LLCC Research Brunch.

[2]Plural distributive predicates always pattern with the *gather* type. See Section 3.

© 2021, Omri Amiraz. In: P. G. Grosz, L. Martí, H. Pearson, Y. Sudo, and S. Zobel (eds.) *Proceedings of Sinn und Bedeutung 25*, pp. 55–72. University College London and Queen Mary University of London.

b. #All of the bullet killed the gangster.

The *gather/numerous* problem consists of two questions:

- Question 1: What semantic property distinguishes between *gather*-type and *numerous*-type predicates?
- Question 2: How does this property explain why *gather*-type predicates are compatible with quantifiers like *most of* but *numerous*-type predicates are not?

Several proposals have been made regarding the semantic basis of the *gather/ numerous* distinction in collective predicates (see Section 2). The scope of this paper is both broader and narrower than most previous studies: on the one hand, it covers both collective and singular predicates; on the other hand, the analysis is restricted to gradable predicates such as *be numerous*, *be heavy*, and *be soft*—i.e., I set aside non-gradable predicates such as *gather*, *be in the briefcase*, and *elect Mary as class president*, claiming that they require a different account.

Consider the following contrast: *be high* (5a) and *be wide* (5b) behave like *gather*-type predicates, while *be long* (5c) behaves like a *numerous*-type predicate. I propose the constraint in (6).

(5) a. Most of the Great Wall of China is 6-7 meters high.
b. Most of the Great Wall of China is 4-5 meters wide.
c. #Most of the Great Wall of China is 11,000 km long.

(6) *The non-monotonicity constraint* (to be revised)
A gradable predicate can co-occur with a quantifier like *most of* if and only if the dimension of the predicate is non-monotonic with respect to the dimension of the quantifier.

A dimension is monotonic on the part-whole structure of an entity if and only if the measure of every proper part of the entity is less than that of the entity as a whole (Schwarzschild, 2006: 73).[3] For instance, weight is monotonic since every proper part of a pizza weighs less than the pizza as a whole. In contrast, temperature is non-monotonic since the temperature of a slice of pizza is not necessarily less than the temperature of the pizza as a whole. Schwarzschild (2006: 89) argues that the dimension of quantifiers such as *much* and *most of* has to be monotonic on the part-whole structure of the entity denoted by the noun. For

[3]The notion of monotonicity is similar to the distinction between extensive and intensive measure functions, which is discussed in Krifka (1989) and later works.

example, *most of the pizza* in (7) can be interpreted as most of the pizza in terms of area or volume (monotonic) but not temperature or softness (non-monotonic).

(7) Most of the pizza is still frozen.

Similarly, in (5), the dimension of *most of* is length—i.e., most of the Great Wall of China along its length axis. Height and width are non-monotonic with respect to length since a proper part of the wall is not necessarily lower or narrower than the wall as a whole. In contrast, length is monotonic with respect to itself since every proper part of the wall along its length axis is necessarily shorter than the wall as a whole. Therefore, (5a) and (5b) obey the non-monotonicity constraint (6), while (5c) violates it.

As for Question 2, I show that sentences violating the non-monotonicity constraint do not have well-defined truth conditions. On a relational view of quantification, a quantifier like *most of* is a binary quantifier $Q(A,B)$, which denotes a binary relation between two sets of objects. Equivalently, the quantifier denotes a set of partitions of the objects in A into those that are in B and those that are not. For example, *Most cats like music* signifies that given a partition of the set of cats into music lovers and non-music lovers, the subset of music lovers contains more than half of the cats. I argue that sentences like (5c) are unacceptable because the entity does not have a well-defined partition with respect to the relevant property, and so the truth conditions of the quantified proposition are not well-defined.

This paper is structured as follows: Section 2 discusses previous analyses of the *gather/numerous* distinction. Section 3 argues that in the domain of gradable predicates, the *gather/numerous* distinction is due to the non-monotonicity constraint. Section 4 shows that sentences violating this constraint do not have well-defined truth conditions, which makes them unacceptable. Section 5 concludes.

2. Previous analyses

Several proposals have been made regarding the semantic basis of the *gather/numerous* distinction in collective predicates. A common claim is that *gather*-type collective predicates are similar in a way to plural distributive predicates—e.g., by having distributive subentailments (Dowty, 1987); by ranging over sets (Winter, 2002); or by having stratified reference (Champollion, 2015; Kuhn, 2020). Another line of analysis relates the *gather/numerous* distinction to lexical aspect: *gather*-type predicates denote activities and accomplishments, while *numerous*-type predicates denote states and achievements (Taub, 1989; Brisson, 2003).

Unlike most studies on the *gather/numerous* distinction, Löbner (2000) and Corblin (2008) discuss not only collective predicates but also singular ones. According to Löbner (2000), singular predicates that are compatible with quantifiers like *most of* are summative, i.e., they are true of an argument if and only if they are true of every part in it; on the other hand, singular predicates that are incompatible with these quantifiers are integrative, i.e., they are true of an argument as an integral whole (cf. Corblin (2008) for a refinement of this taxonomy and Amiraz (2020) for a discussion of how it is related to the properties of non-maximality and homogeneity).[4]

More recently, Champollion (2015) and Kuhn (2020) proposed analyses that are similar in spirit to Löbner (2000) but avoid the pitfalls associated with the requirement that the predicate hold of *all* subparts, which is also known as having divisive reference (see Kuhn (2020: 229-230) for discussion). Discussing only collective predicates, Champollion (2015) and Kuhn (2020) argue that *gather*-type collective predicates have stratified reference, which is a weakening of divisive reference due to Champollion (2010). Informally, a predicate P has stratified reference if and only if whenever P holds of an entity x, there is a way to divide x into small parts that are also in P. For instance, a mass term like *water* has stratified reference since a body of water can be divided into small parts that are also water. On the other hand, a singular count term like *chair* does not have stratified reference because parts of a chair are not chairs themselves. Similarly, in (8a), the gathering event can be decomposed into small gathering subevents involving overlapping groups of kids. In (8b), however, the event cannot be decomposed into small subevents of being numerous since the participants of these subevents would not be numerous.

(8) a. The kids gathered outside.
  b. The kids were numerous.

As Kuhn (2020: 235) notes, this analysis overgenerates the class of *gather*-type predicates in the case of certain gradable predicates. In (9a), the predicate has stratified reference since if a plurality of kids is few in number, it follows that any subplurality would also be few in number. Therefore, it is expected to be a *gather*-type predicate, contra to fact (9b).[5]

(9) a. The kids were few in number.

[4] According to Löbner (2000), all collective predicates are integrative, but *gather*-type collective predicates involve summative macro-predication.
[5] See Kuhn (2020: 236-239) for a possible solution for this potential counterexample.

b. #All of the kids were few in number.

I observe that extending the stratified reference analysis to singular predicates also raises an undergeneration problem. Consider (10): the predicate behaves like a *gather*-type predicate even though a singular count term like *island* lacks stratified reference since no proper part of an island is itself an island. Therefore, the stratified reference analysis wrongly predicts this predicate to belong to the *numerous* type. I propose solutions for both of these problems in Section 4.

(10) Mombasa is partly an island.[6]

To summarize, several previous analyses argue that the property that distinguishes between *gather*-type and *numerous*-type predicates is whether or not it is the case that if the predicate holds of an argument, it also holds of its parts. In contrast, this paper argues that the relevant property is whether or not it is possible to partition an argument into parts of which the predicate holds and parts of which it does not hold. This property is shown to explain the non-monotonicity constraint (6) on gradable predicates, which are the focus of this paper.

3. Empirical generalization: The non-monotonicity constraint

I argue that the distinction between *gather*-type and *numerous*-type gradable predicates is due to the non-monotonicity constraint (6). Henceforth, the dimension of the quantifier will be marked as Q, and the dimension of the gradable predicate will be marked as P; monotonic dimensions are in red, and non-monotonic dimensions are in blue. For example, (12a) means that most of the dish in terms of volume has a low temperature.

(11) Monotonic gradable predicates
  a. #Most of the sofa is heavy. (Q-volume; P-weight)
  b. #All of the kitchen is spacious. (Q-area; P-area)

(12) Non-monotonic gradable predicates
  a. Most of the dish is cold. (Q-volume; P-temperature)
  b. All of the sofa is soft. (Q-area; P-softness)

The class of quantificational expressions that are subject to the non-monotonicity constraint consists of proportional quantifiers (13) and non-proportional partitive constructions (14). I will refer to this class of expressions as partitive quantifiers since proportional quantifiers involve a parthood relation just like partitives.

[6]Mombasa, the capital of Kenya, is centered around Mombasa Island but extends to the mainland.

(13) a. #{All of / Most of / Part of / 80% of} the kitchen is spacious.
b. #{All of / Most of / Some of / None of} the kids were numerous.

(14) a. #One square meter of the kitchen is spacious. (Q-area; P-area)
b. #Forty of the kids were numerous. (Q-cardinality; P-cardinality)

In contrast, non-partitive numerals (15) and pseudopartitives (16) are not subject to the non-monotonicity constraint, i.e., they allow monotonic predicates.

(15) Six bottles cost 10$ (altogether). (Q-cardinality; P-cost)

(16) One liter of mercury weighs 13.53 kg. (Q-volume; P-weight)

From a syntactic perspective, the non-monotonicity constraint applies both to determiners (17) and to adverbial quantifiers (18).

(17) a. All of the kitchen is {new / #spacious}.
b. Most of the kitchen is {new / #spacious}.
c. Part of the kitchen is {new / #spacious}.

(18) a. The kitchen is entirely {new / #spacious}.
b. The kitchen is mostly {new / #spacious}.
c. The kitchen is partly {new / #spacious}.

The non-monotonicity constraint is revised as follows:

(19) *The non-monotonicity constraint* (revised)
A gradable predicate can co-occur with a partitive quantifier if and only if the dimension of the predicate is non-monotonic with respect to the dimension of the partitive quantifier.

In some cases, it depends on the context whether or not a sentence violates the non-monotonicity constraint. First, the dimension of the partitive quantifier is contextually determined (Schwarzschild, 2006: 90). For example, the relevant dimension in (20) can be length, e.g., in the case of a 100-meter-long wall where just ten (horizontal) meters of the wall are visible; alternatively, the relevant dimension can be height, e.g., in the case of a 10-meter-high wall where just one (vertical) meter of the wall sticks above the ground.

(20) Most of the wall is buried under the sand.

Second, the dimension of the predicate is not necessarily inherently monotonic or non-monotonic. For example, spatial dimensions such as length, height, width, and depth can be interpreted as monotonic or non-monotonic, depending on the context (Schwarzschild, 2006: 74-75). When a predicate refers to the maximal measure of an entity, it is monotonic. When a predicate refers to the measures of different parts of an entity, it is non-monotonic. For example, in (21a), the predicate is monotonic because it refers to the maximal height of the tower; in (21b), the predicate is non-monotonic since it refers to the heights of different parts of the wall along its length axis.

(21) a. Shanghai Tower is 632 m high. (height)
b. The Great Wall of China is 5-14 m high. (height)

In light of these facts, the prediction is that predicates that are associated with spatial dimensions such as length can co-occur with a partitive quantifier in some cases but not others—depending on whether or not the dimension of the predicate is monotonic with respect to the contextually determined dimension of the quantifier. This prediction is borne out, as demonstrated by the contrast between (22c) and (23). Given the shape of the Great Wall of China, the most salient dimension in *most of the wall* is length. Examples (22a) and (22b) obey the non-monotonicity constraint since height and width are non-monotonic with respect to length, while (22c) violates the constraint since every proper part of the wall along its length axis is necessarily shorter than the wall as a whole. In contrast, the predicate *be long* is acceptable in (23) since hair length is non-monotonic with respect to the dimension of the quantifier, which in this case is scalp area.

(22) Most of the Great Wall of China is...
a. 6-7 meters high. (Q-length; P-height)
b. 4-5 meters wide. (Q-length; P-width)
c. #11,000 km long. (Q-length; P-length)

(23) Most of Mary's hair is 60 cm long, but she keeps it shorter on her left temple. (Q-area; P-length)

Singular gradable predicates can thus be monotonic or non-monotonic, sometimes depending on the context. On the other hand, it appears that collective gradable predicates are almost always monotonic, e.g., (24). I have only been able to come up with one example of a non-monotonic collective predicate, namely *be dense*. Unexpectedly, *be dense* is incompatible with partitive quantifiers as a collective predicate (25a), contra to the non-monotonicity constraint. In contrast, as a singular predicate, *be dense* can co-occur with a partitive quantifier (25b). I suggest that the problem with (25a) might be that density is undefined for pluralities of atoms without an additional dimension of area or volume, which makes the sentence uninterpretable. If this is correct, (25a) is bad for independent reasons.

(24) a. #Most of the kids were numerous. (Q-cardinality; P-cardinality)
b. #Most of the boxes weigh less than 100 kg. (Q-cardinality; P-weight)

(25) a. #Most of the trees in the park are dense.[7] (Q-cardinality; P-density)
b. Most of the fishing net is dense. (Q-area; P-density)

In contrast to singular and collective predicates, distributive plural predicates are always non-monotonic since the measure of an entity in a set does not depend on the size of the set. Consider (26a), which has both a distributive and a collective reading—only the distributive reading is compatible with partitive quantification (26b). The reason is that the weight of all the bottles taken together (collective reading) is monotonic with respect to the cardinality of bottles, but the weight of each bottle (distributive reading) is non-monotonic with respect to this dimension.

(26) a. The bottles are heavy. (✓distributive; ✓collective)
b. Most of the bottles are heavy. (✓distributive; ✗collective)

4. Explaining the non-monotonicity constraint

Before presenting the analysis, I will first rule out an obvious explanation that comes to mind. It might seem that sentences violating the non-monotonicity constraint are unacceptable due to general pragmatic principles. For instance, it is unclear what the literal meaning of (27) would even be. However, this is not always the case. Consider the minimal pair in (28): it is reasonable to assume that the literal meaning of (28a) is identical to the meaning of (28b). However, only the latter is acceptable, even if rather uninformative. This contrast is expected under the proposed analysis since the non-monotonicity constraint does not apply to part-whole denoting nouns and adjectives such as *part*, *totality*, *whole*, and *entire*. In (28a), *part of* functions as a determiner.[8] On the other hand, in (28b), *part* is used as a common noun, and the quantifier is *some*; the noun phrase *part of the Great Wall of China* denotes the set of parts of the wall, and the predication is distributive. Therefore, (28a) violates the non-monotonicity constraint, while (28b) obeys it. Given that the non-monotonicity constraint only applies to partitive quantifiers (see Section 3), an explanation of why this constraint holds has to refer to the semantics of these quantifiers.

[7] Compare (25a) with the acceptable *Most of the trees in the park are densely packed together*, where *dense* functions as an adverb modifying the predicate. I have no explanation for this contrast.

[8] The evidence for this is that English does not allow bare singular count nouns in argument position, so *part of* in (28a) can only be a determiner.

(27) #Most of the Great Wall of China is 11,000 km long.

(28) a. #Part of the Great Wall of China is 10 km long.
b. Some part of the Great Wall of China is 10 km long.

On a relational view of quantification, a binary quantifier $Q(A,B)$ denotes a binary relation between two sets of objects. A sentence like *All cats are carnivores* states that the set of cats is a subset of the set of carnivores. A partitive quantifier denotes the proportion or cardinality of objects in A that are also in B, where A is the restriction and B is the nuclear scope of the quantifier. Equivalently, a partitive quantifier denotes a (possibly trivial) partition or set of partitions of A into objects that are in B and objects that are not in B.[9]

A partition of a set is defined in Definition 1.

Definition 1 (Partition (set-theoretic))**.** *A family of sets Π is a partition of a set S if and only if:*

- $\forall A_i \in \Pi, A_i \neq \emptyset$
  (no subset is empty)
- $\bigcup_{A_i \in \Pi} A_i = S$
  (the union of the subsets is equal to S)
- $\forall A_i, A_j \in \Pi, A_i \neq A_j \Rightarrow A_i \cap A_j \neq \emptyset$
  (the subsets are pairwise disjoint)

For example, *Most cats like music* signifies that given a partition of cats into music lovers and non-music lovers, the subset of music lovers contains more than half of the cats; *Some cats like music* means that the partition of cats contains a subset of music lovers. The equivalence between the relational view and the partition view of quantification is illustrated in (29).[10]

Definition 2. *Let $A/\sim_B$ be the partition of A into the subset of elements in B, denoted by $A_{|B}$, and the subset of elements not in B, denoted by $A_{|B^c}$.*[11]

(29) a. SOME(A)(B) = 1 if and only if:
$A \cap B \neq \emptyset \Leftrightarrow A_{|B} \in A/\sim_B$

[9]The partition of B by membership in A is irrelevant to the truth conditions due to conservativity. In natural language quantification, the restriction of the quantifier limits the domain of quantification to objects in that set, so objects in B that are not in A are irrelevant (Keenan and Stavi, 1986: 260).

[10]For simplicity's sake, I am assuming that neither set is empty.

[11]This definition can be generalized to partitions with more than two equivalence classes, but such partitions are not relevant for this paper.

b. ALL(A)(B) = 1 if and only if:
$A \subseteq B \Leftrightarrow A_{|B} = A$

c. NO(A)(B) = 1 if and only if:
$A \cap B = \emptyset \Leftrightarrow A_{|B^c} = A$

d. MOST(A)(B) = 1 if and only if:
$|A \cap B| > \frac{|A|}{2} \Leftrightarrow |A_{|B}| > \frac{|A|}{2}$

Quantification over individuals may thus be represented in terms of a partition of a set, but what about quantification over parts of a non-plural entity? E.g., what set is partitioned in (30)? In this case, the entity is divided into subsets, and any point in the entity lies in exactly one of the subsets. I define a partition of an entity *x* in mereological terms in Definition 3 (cf. Ionin et al., 2006: 358), where $\bigoplus$ represents mereological sum and $\sqsubseteq$ represents proper parthood. For example, a partition of the Earth's landmass into a northern and a southern hemisphere consists of two subsets: one that contains the points in the Earth's landmass located in the Northern Hemisphere, and one containing the points located in the Southern Hemisphere.

(30) Most of the Earth's landmass is in the Northern Hemisphere.

Definition 3 (Partition (mereological))**.** *A family of sets Π is a partition of an entity x if and only if:*

- $\forall A_i \in \Pi, A_i \neq \emptyset$
  (no subset is empty)
- $\bigoplus_{A_i \in \Pi} A_i = x$
  (the sum of the subsets is equal to x)
- $\forall A_i, A_j \in \Pi, A_i \neq A_j \Rightarrow \neg\exists y, z \in A_i, A_j [\exists a, a \sqsubseteq y \wedge a \sqsubseteq z]$
  (the subsets do not contain shared parts, i.e., they do not overlap)

Note that while each point in *x* lies in exactly one subset of *x*, it is not always the case that the points themselves are defined for the property by which the entity is partitioned. For example, in (31), population density is not defined for points. Therefore, the process of partitioning the entity must make use of a set containing larger parts of the entity. I refer to the set that is used for the partition of the entity with respect to the relevant property as the partitioned set.

(31) Most of the Earth's landmass is sparsely populated.

Also note that the partitioned set cannot be the set of *all* parts of the entity. In the context of (30), consider the part covered by Ecuador, which straddles the equator. If the Earth's landmass is to be partitioned into a northern and a southern hemisphere, the part covered by Ecuador cannot be a member of the partitioned set because in that case, the subsets in the partition would overlap. In other words, not every part of x is in the partition of x since otherwise the third condition in Definition 3 can only be met by a trivial partition, where the partition contains just one subset.

Thus, the partitioned set is a subset of the set of parts of x, whose sum is x. I.e., a mereological partition of an entity x is a set-theoretic partition of a cover of x, as defined in Definition 4.

Definition 4 (Cover (mereological))**.** *A family of sets C is a cover of an entity x if and only if:*

- $\forall A_i \in C, A_i \neq \emptyset$
  (no subset is empty)
- $\bigoplus_{A_i \in \Pi} A_i = x$
  (the sum of the subsets is equal to x)

Now, for every entity x there can multiple, and possibly infinitely many, covers of x—which one is used for the partition of x? One requirement is that the cover must be partitionable with respect to the relevant property. For example, a cover of the Earth's landmass that contains the part corresponding to Ecuador does not have a partition with respect to the property of being or not being in the Northern Hemisphere. Thus, a cover C of x is partitionable with respect to a binary property P if and only if for every $a \in C$, $P(a) \vee \neg P(a)$, i.e., a is in the domain of P.[12]

In some cases, there is also a contextual restriction on what covers are relevant. This is what I referred to as the dimension of the quantifier in Section 3. For example, in (32), the salient covers consist of parts of the Great Wall of China along its length axis (i.e., from the ground to the top and from side to side). This means that while every part in the cover can be further decomposed into vertical parts of lesser height, these parts are not members of any salient cover, and thus they are irrelevant for the truth conditions of (32).

(32) Most of the Great Wall of China is 6-7 meters high.

I argue that other than that, one need not assume any particular type of cover since

[12] I am abstracting away from the problem of vague predicates, which is orthogonal to the issue under discussion.

if the partition is well-defined, it is necessarily the case that all the partitionable covers have the exact same partition, i.e., each point is always contained in the same subset of the partition. For example, in a partition of the Earth's landmass into a northern and a southern hemisphere, there is a unique line dividing the subsets, i.e., the equator. Therefore, any cover that has a partition with respect to this property (i.e., any cover that does not contain parts straddling the equator) will necessarily produce the same partition of the Earth's landmass in the sense that the sum of the parts in each subset is identical across covers.

However, note that some properties are not associated with a unique line or set of lines that divide the subsets. Consider a partition of the Earth's landmass into parts that border with a sea and parts that do not. Given that every part that borders with a sea can be further divided into smaller parts such that some of which do not border with a sea, there is no unique line or set of lines that divide the subsets.

I propose that a partition of an entity x with respect to a property P is well-defined if an only if x has a unique partition with respect to P, as defined in Definition 5.

Definition 5 (Well-defined partition (mereological))**.** *A partition Π of an entity x with respect to a binary property P is well-defined if and only if:*
For all salient covers C_1 and C_2 of x, if $C_1/\sim_P$ and $C_2/\sim_P$ are defined, then $\bigoplus C_{1_{|P}} = \bigoplus C_{2_{|P}}$ *and* $\bigoplus C_{1_{|P^c}} = \bigoplus C_{2_{|P^c}}$.
All salient, partitionable covers of x have the same partition, i.e., the sum of each subset is identical across covers.

For example, a partition of the Earth's landmass into parts that border with a sea and parts that do not is not well-defined because the partition depends on which parts are in the cover: a cover of 1-square-km parts and a cover of 10-square-km parts will produce different partitions, and there is no principled way of choosing between them.

Given that a partitive quantifier Q denotes a certain partition of an entity x with respect to a property P, a proposition of the form $Q(x)P(x) = 1$ has well-defined truth conditions if and only if x has a well-defined partition with respect to P. I propose the constraint in (33), where a suitable argument means that there is a cover of x where all the elements in the cover are in the domain of P.

(33) *Constraint on partitive quantification* (to be revised)
A partitive quantifier Q can co-occur with a predicate P if and only if:
Any suitable nominal argument x of Q has a well-defined partition with

respect to P.

Note that the constraint in (33) is not relativized to a particular nominal argument but rather generalizes over possible arguments. The reason for formulating the constraint in this way is that a property which fails to produce a well-defined *non-trivial* partition can nonetheless produce a well-defined *trivial* partition according to Definition 5. For example, there is a unique partition of a landlocked country into parts that border with a sea and parts that do not—i.e., a trivial partition where all the parts are in the same subset.[13] According to (33), a predicate must allow well-defined, non-trivial partitions in order to be compatible with a partitive quantifier.

With these tools at hand, we can explain the non-monotonicity constraint (19). Monotonicity is defined in Definition 6.

Definition 6 (Monotonicity)**.** *A measure function μ is monotonic on the part-whole structure of an entity x if and only if for all $a, b \in C$ such that C is a salient cover of x, if $a \sqsubset b$, then $\mu(a) < \mu(b)$.*

If a predicate P is non-monotonic with respect to the dimension of the cover of an entity x, then x has a well-defined partition with respect to P. For example, in (34), a cover consisting of the parts of the Great Wall of China along its length axis can be partitioned into parts that are 6-7 meters high and parts that are not, and the boundaries between the subsets are well-defined. On the other hand, a partition of x is not well-defined with respect to a monotonic predicate P. The Great Wall of China cannot be partitioned into parts that are 11,000 km long and parts that are not since different covers will produce different partitions. For example, one can devise two distinct covers that consist of one part that is 11,000 km long and ten non-overlapping parts that are 1,000 km long—in one cover the long part is in the eastern end of the wall, and in the other cover it is in the western end. These two partitions do not agree on the classification of the parts into subsets, so they are not well-defined according to Definition 5.

(34) Most of the Great Wall of China is...
  a. 6-7 meters high. (Q-length; P-height)
  b. 4-5 meters wide. (Q-length; P-width)
  c. #11,000 km long. (Q-length; P-length)

[13] Note that trivial partitions should be allowed in some cases since they are equivalent to universal quantification (see examples in (29)).

I now prove that no monotonic predicate can satisfy the constraint on partitive quantification (33).

Theorem. *Let a gradable predicate P whose dimension is μ be monotonic on the part-whole structure of an entity x. If exist $y, z \in C$ such that C is a salient cover of x satisfying $P(y) \wedge \neg P(z)$, then x does not have a well-defined partition with respect to P.*

The proof of the theorem follows directly from the combination of the following two lemmas. Lemma 1 shows that if there are two arbitrary parts in x which belong to different subsets of a well-defined partition of x with respect to P, then the sum of the parts cannot be in the domain of P.

Lemma 1. *Assume that exist $y, z \in C$ such that C is a salient cover of x, and $P(y) \wedge \neg P(z)$. If $y \oplus z$ is in the domain of P, then x does not have a well-defined partition with respect P.*

Proof. Assume, for the sake of contradiction, that exist $y, z \in C_1$ such that C_1 is a salient cover of x, and $P(y) \wedge \neg P(z)$. Then, $y \sqsubseteq \bigoplus C_{1|P}$ and $z \sqsubseteq \bigoplus C_{1|P^c}$. Now, assume that $y \oplus z \in C_2$ such that C_2 is a potentially distinct salient cover of x, and $y \oplus z$ is in the domain of P. Given that $P(y \oplus z)$ or $\neg P(y \oplus z)$, then $y \oplus z \sqsubseteq \bigoplus C_{2|P}$ or $y \oplus z \sqsubseteq \bigoplus C_{2|P^c}$. It follows that $y \sqsubseteq \bigoplus C_{2|P^c}$ or $z \sqsubseteq \bigoplus C_{2|P}$ (contra to the partition of C_1). Therefore, $\bigoplus C_{1|P} \neq \bigoplus C_{2|P}$, which proves that the partition is not well-defined according to Definition 5. □

Lemma 2 shows that the assumption of Lemma 1 holds for any monotonic measure function, which concludes the proof of the theorem.

Lemma 2. *Let a gradable predicate P whose dimension is μ be monotonic on the part-whole structure of an entity x. For all disjoint $y, z \in C$ such that C is a salient cover of x, if $y \in dom(P)$ and $z \in dom(P)$, then $y \oplus z \in dom(P)$.*

Proof. I assume that a monotonic measure function is a measure in the sense of measure theory. By definition, a measure on a set is countably additive, from which it follows that a union of two measurable sets is measurable. Now, assume that exist disjoint $y, z \in C$ such that C is a cover of x and $y \in dom(P)$ and $z \in dom(P)$. Since the dimension of P is μ, then $y \in dom(\mu)$ and $z \in dom(\mu)$. Given that a union of two measurable sets is measurable, $y \oplus z \in dom(\mu)$. Since P partitions a scale S whose dimensions is μ into two subsets, then $P(y \oplus z)$ or $\neg P(y \oplus z)$. Therefore, $y \oplus z \in dom(P)$, which proves the lemma. □

Another problem that was raised in Section 2 is singular count nouns which behave like *gather*-type predicates, e.g., (10), repeated here as (35). This type of case requires further assumptions. If a cover contains Mombasa Island and does not contain its parts, the cover is partitionable, and (35) is true. On the other hand, suppose that another cover consists of Mombasa neighborhoods. Since Mombasa Island consists of several neighborhoods, no part in the cover is an island, and (35) is false. Therefore, (35) seems not to obey the constraint in (33) and is wrongly predicted to be unacceptable.

(35) Mombasa is partly an island.

In a sense, a partition of Mombasa into island parts and non-island parts is indeed not well-defined since proper parts of Mombasa Island are not islands by themselves, so the subsets of the partition overlap (in violation of the third condition in Definition 3). To account for the acceptability of (35), one needs to introduce atoms into the structure, i.e., objects that have no proper parts. I argue that in (35), Mombasa Island is conceived of as an atom, and so its parts are not in any salient cover. Thus, the subsets do not overlap.

This analysis raises the following question: what prevents us from introducing atoms in cases like (34c), which would make such sentences acceptable? Why not say that there is an atom in the structure which is 11,000 km long? I argue that the crucial difference between the two cases is that the predicate in (35) is a count noun. Assuming that count nouns make a set of atoms accessible (Chierchia, 1998), the cover may contain atoms. Otherwise, a singular entity does not contain accessible atoms.

Another problematic case is predicates such as *be inhabited*, which are true of a whole whenever they are true of some part in it. For example, in (36), if a is an inhabited area of the island and b is an uninhabited area, then $a \oplus b$ is inhabited (more generally, the island as a whole is inhabited). Therefore, the subsets necessarily overlap, and the partition is not well-defined.

(36) Most of the island is inhabited.

While a partition of an island into inhabited and uninhabited areas is indeed not well-defined, intuitively, one can nonetheless produce a partition of this sort, even if the boundaries between the subsets are somewhat arbitrary. To account for such partitions, I introduce the notion of a valid partition, as defined in Definition 7. According to Definition 7, there are multiple valid partitions of a given entity, which differ by the precise locations of the boundaries between the subsets of x but agree on the classification of the vast majority of points in x.

Definition 7 (Valid partition (mereological))**.** *A partition Π of an entity x with respect to a binary property P is valid if and only if:*
For all salient covers C_1 and C_2 of x, if $C_1/\sim_P$ and $C_2/\sim_P$ are defined, then $\bigoplus C_{1_{|P}} \approx \bigoplus C_{2_{|P}}$ *and* $\bigoplus C_{1_{|P^c}} \approx \bigoplus C_{2_{|P^c}}$.
All salient, partitionable covers of x have approximately the same partition, i.e., the sum of each subset is approximately identical across covers.

I revise the constraint on partitive quantification as follows:

(37) *Constraint on partitive quantification* (revised)
A partitive quantifier Q can co-occur with a predicate P if and only if:
Any suitable nominal argument x of Q has a well-defined or valid partition with respect to P.

The set of salient covers is contextually determined. It seems intuitive to assume that normally the salient covers are formed by grids that divide the entity into contiguous cells of a certain size. The covers differ from one another by the size of the cells (i.e., by their "resolution"), and covers where the cells are too small or too large for the partition to be meaningful are not in this set. In other words, the salient covers do not contain non-contiguous parts or very large parts that can be further divided into parts belonging to different subsets. For example, in the case of (36), a part consisting of a village and an uninhabited forest is not contained in any salient cover. Thus, the partitions of the salient covers cannot be significantly different from each other.

We saw that an entity x cannot have a well-defined, non-trivial partition with respect to a monotonic predicate P, but can it have a valid partition? I argue that it cannot because changing the size of the cells of the cover necessarily changes the measures of the parts in each cell due to monotonicity. Given that P partitions a scale of measurement, the measure of a part determines to which subset it belongs. Thus, the classification of each point in x into one subset or the other only depends on the size of the part in the cover that contains it. As a result, partitions of different covers do not agree on the classification of the points in x, and so the partition of x is not valid.

5. Conclusion

The starting point of this paper was the observation that some gradable predicates are compatible with partitive quantifiers like *most of*, while others are not. I argued that this distinction is due to the non-monotonicity constraint (19)—i.e., only non-monotonic gradable predicates can co-occur with partitive quantifiers. Sentences violating this constraint—e.g., *Most of the Great Wall of China is 11,000 km long*—are unacceptable because they do not have well-defined truth conditions. A partitive quantifier like *most of* is a binary quantifier $Q(A)(B)$ which denotes a set of partitions of A by membership in B. When a monotonic predicate occurs in the nuclear scope of the quantifier (set B), the restriction (set A) does not have a well-defined partition with respect to this property, and so the truth conditions of the quantified proposition are not well-defined.

Extending the proposed analysis to non-gradable predicates is left for future research. The constraint on partitive quantifiers (37) may turn out to be applicable to non-gradable predicates as well. For example, in a sentence like #*Most of the bullet killed the gangster*, it can be argued that it is impossible to partition the bullet into parts that killed the gangster and parts that did not. However, one would want to relate this intuition to the semantics of causative predicates in a similar fashion to the way this paper ascribed our intuitions about gradable predicates to the property of monotonicity.

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