

Higher-order quantification and free choice in *how many*-questions¹

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Abstract. In Gentile and Schwarz (2018), we observed that certain *how many*-questions carry uniqueness presuppositions that can be understood as instances of Dayal’s (1996) *Maximal Informativity Presupposition*. We also observed, however, that uniqueness presuppositions can unexpectedly go missing in the presence of a possibility modal. In this paper, we offer an explanation for the obviation of such uniqueness presupposition by possibility modals. Building on a suggestion in Spector (2008), we propose that obviation is due to higher-order wh-quantification feeding free choice strengthening within the question nucleus.

Keywords: wh-questions, *how many*-questions, uniqueness presuppositions, Maximal Informativity Presupposition, modals, higher-order quantification, free choice strengthening.

1. Introduction

In Gentile and Schwarz (2018), we observed that *how many*-questions with non-distributive predicates, like *solved this problem together*, carry a “uniqueness-of-size” presupposition. The question in (1), for example, is intuited to presuppose that there is a unique cardinality such that a group of students of that cardinality jointly solved the problem. A response that is incompatible with this presupposition, such as *3 and 4*, is accordingly judged deviant.²

(1) How many students solved this problem together?

We also observed that when a non-distributive predicate in a *how many*-question appears under a possibility modal, like *allowed*, no corresponding uniqueness presupposition is attested. To illustrate, (2) clearly does not presuppose that there is a unique allowed cardinality for groups of students jointly solving the problem. This is confirmed by the observation that *3 or 4*, for example, can be felicitous as a response conveying that both 3 and 4 are permitted group sizes.

¹This paper is a sequel to Gentile and Schwarz (2018), offering a solution to a puzzle that we identified there without managing to resolve. We thank Luis Alonso-Ovalle and, in particular, Aron Hirsch for comments that steered us towards our proposal. We also thank Danny Fox for discussion that helped us more clearly see some theoretical choice points and their consequences. In fact, after working out our analysis, we learned that Danny (Fox, 2019, 2020) had independently developed a solution to our puzzle, a solution that resembles ours in its appeal to free choice strengthening, but differs significantly from ours in how this notion is deployed. For reasons of space, we will in this paper lay out our analysis as initially conceived and not provide a structured review of Fox’s analysis or comparison with our own. We will, however, include sporadic footnotes with pointers and commentary on Fox’s proposal. For comments and discussion, we also thank audiences at the workshop *Exhaustivity in Questions and Answers* at the University of Tübingen and *Sinn und Bedeutung 22* at the University of Osnabrück, as well as the members of the McGill Semantics Research Group. Of course, none of the people thanked here are responsible for any of our errors. Both authors gratefully acknowledge support from the Social Sciences and Humanities Research Council of Canada (Insight grants #435-2019-0143 and #435-2016-1448).

²In Gentile and Schwarz (2018), we note that (1) may carry a stronger presupposition, viz. that there is a *unique group* of students who solved the problem together. Like Fox (2019, 2020), we will focus here on the uniqueness-of-size presupposition, leaving open how it might be strengthened to a uniqueness-of-group presupposition.

- (2) How many students are allowed to solve this problem together?

In Gentile and Schwarz (2018), we noted that the uniqueness-of-size presupposition in non-modalized examples like (1) can be understood as an instance of the *Maximal Informativity Presupposition* (MIP) that Dayal (1996) posited in order to capture the uniqueness presupposition of singular *which*-questions. However, we did not provide a satisfactory explanation for the absence of uniqueness-of-size presuppositions in cases with possibility modals like (2).

In this follow-up, we will offer a way of filling this gap, building on a proposal motivated on independent grounds in Spector (2008). Following Spector, we suggest that *wh*-quantification in *wh*-questions can be higher-order, that is, can range over generalized quantifiers, and that such higher-order *wh*-quantification can feed so-called free choice strengthening within the question nucleus. For cases like (2), this will be shown to have the effect of closing the set of true Hamblin answers under conjunction, ensuring that the MIP in such cases amounts to a mere presupposition of existence, and thereby capturing the observed absence of a uniqueness-of-size presupposition.

Section 2 reviews the puzzle described in Gentile and Schwarz (2018). Section 3 lays out the proposed explanation for the obviation of a uniqueness-of-size presupposition in cases with possibility modals; building on Spector (2008), we attribute this obviation to higher-order *wh*-quantification feeding free choice strengthening. Section 4 points out that, to prevent over-obviation of uniqueness presuppositions, higher-order *wh*-quantification must be constrained, and we propose a constraint that has the intended effect for the examples we consider. Section 5 offers concluding remarks, regarding the analysis of free choice strengthening.

2. The uniqueness puzzle

This section reviews the puzzle described in Gentile and Schwarz (2018). We will lay out assumptions about the syntax and semantics of *how many*-questions that support an account of the uniqueness-of-size presuppositions as an instance of Dayal's Maximal Informativity Presupposition (MIP). We then point out that this account fails to capture the absence of uniqueness presupposition in cases with possibility modals.

2.1. A semantics for *how many*-questions

We begin by laying out a proposal about the syntax and semantics of *how many*-questions, a proposal that adopts central assumptions from Beck and Rullmann (1999), whose analysis instantiates the so-called Hamblin-Karttunen semantics for questions (Hamblin, 1973; Karttunen, 1977). On this analysis, example (3) is taken to denote the function in (4), where n ranges over cardinalities. This function characterizes the Hamblin set in (5).

- (3) How many students smiled?

$$(4) \quad \lambda p_{st}. \exists n[p = \lambda w. \exists x[\text{students}(x)(w) \wedge |x|=n \wedge \text{smile}(x)(w)]]$$

$$(5) \quad \left\{ \begin{array}{l} \lambda w. \exists x[\text{students}(x)(w) \wedge |x|=1 \wedge \text{smile}(x)(w)], \\ \lambda w. \exists x[\text{students}(x)(w) \wedge |x|=2 \wedge \text{smile}(x)(w)], \\ \lambda w. \exists x[\text{students}(x)(w) \wedge |x|=3 \wedge \text{smile}(x)(w)], \\ \dots \end{array} \right\}$$

In a modern rendition of Karttunen (1977) (spelled out in, e.g., Fox, 2013), this denotation for (3) can be attributed to the logical form in (6), by assigning to the relevant functional elements the denotations given in (7) and (8). (Here we write n for the type of cardinalities.)

$$(6) \quad \lambda_7[\text{CP how } \lambda_1[\text{C}' [\text{C} ? \text{t}_{7,\text{st}}] [\text{TP} [\text{DP } \exists [[\text{t}_{1,n} \text{ many}] \text{ students}]] \text{ smiled}]]]$$

$$(7) \quad \begin{array}{l} \text{a. } \llbracket \text{many} \rrbracket = \lambda n_n. \lambda x_e. |x|=n \\ \text{b. } \llbracket \exists \rrbracket = \lambda f_{\text{et}}. \lambda g_{\text{et}}. \exists x[f(x) \wedge g(x)] \end{array}$$

$$(8) \quad \begin{array}{l} \text{a. } \llbracket \text{how} \rrbracket = \lambda h_{\text{nt}}. \exists n[h(n)] \\ \text{b. } \llbracket ? \rrbracket = \lambda p_{\text{st}}. \lambda q_{\text{st}}. p = q \end{array}$$

So overtly pied-piped *many* is taken to reconstruct into the question nucleus, composing with the cardinality-denoting trace of *how* into a predicate of individuals. This predicate is taken to combine with the nominal predicate it modifies to form the restrictor of a silent existential determiner \exists . The operator $?$ in the C position combines with the question nucleus, the TP, to form the equivalent of Karttunen's (1977) Proto-Question. With *how* in the specifier of C analyzed as an existential quantifier over cardinalities, this derives for (6) the denotation in (4).

2.2. Uniqueness from a Maximal Informativity Presupposition

Singular *which*-questions carry a presupposition of uniqueness that is not attested in their plural counterparts (e.g., Higginbotham and May, 1981; Dayal, 1996). To illustrate, (9a) is judged to presuppose that only one student smiled, a presupposition that is not attested in (9b).

- (9) a. Which student smiled?
 b. Which students smiled?

To capture this contrast, Dayal (1996) proposed that any question carries a so-called Maximal Informativity Presupposition about its Hamblin set: the presupposition that the Hamblin set's true members includes one that entails all the other true members. In symbols, for a given Hamblin set Q , the Maximal Informativity Presupposition is the proposition $\text{MIP}(Q)$ in (10).³

$$(10) \quad \text{MIP}(Q) = \lambda w. \exists p \in Q[w \in p \wedge \forall q[w \in q \rightarrow p \subseteq q]]$$

This proposal can be shown to capture the contrast between (9a) and (9b) under the assumption that *student* is true only of atomic individuals that are students, while *students* also applies to pluralities of students (Link (1983)). To spell this out in the Karttunen-Hamblin semantics, the

³Dayal (1996) takes the MIP to be triggered by a syntactically represented answer operator *Ans*, an operator that is intended to characterize the notion of a question's complete answer. Since we are not directly concerned with answerhood in this paper, we will continue to suppress reference to *Ans*.

questions in (9) have the logical forms in (11). Assuming that *which* is an existential determiner with the denotation in (12), the resulting sentence denotations, in a given possible world w , are as shown in (13). Illustrating for a world w with only two (atomic) students, a and b , the Hamblin set for (9a) and (9b) is then (14a) and (14b), respectively.

- (11) a. $\lambda_{7[CP} [\text{which student}] \lambda_1[CP' [C ? t_{7,st}] [t_{1,e} \text{ smiled}]]]$
 b. $\lambda_{7[CP} [\text{which students}] \lambda_1[CP' [C ? t_{7,st}] [t_{1,e} \text{ smiled}]]]$

(12) $\llbracket \text{which} \rrbracket = \lambda f_{et}. \lambda g_{et}. \exists x[f(x) \wedge g(x)]$

- (13) a. $\lambda p_{st}. \exists x[\text{student}(w) \wedge p = \lambda w. \text{smile}(x)(w)]]$
 b. $\lambda p_{st}. \exists x[\text{students}(w) \wedge p = \lambda w. \text{smile}(x)(w)]]$

- (14) a. $\{\lambda w. \text{smile}(a)(w), \lambda w. \text{smile}(b)(w)\}$
 b. $\{\lambda w. \text{smile}(a)(w), \lambda w. \text{smile}(b)(w), \lambda w. \text{smile}(a \oplus b)(w)\}$

The members of (14a) are logically independent, hence not related by entailment. In contrast, *smile* being distributive, $\text{smile}(a \oplus b)(w)$ is equivalent to $\text{smile}(a)(w) \wedge \text{smile}(b)(w)$, hence (14b) is closed under conjunction. For a Hamblin set whose members are not related by entailment, the MIP amounts to the requirement that only one of those members be true. For (14a), this yields the intended presupposition that *only one* of the students a and b smiled. In contrast, when a Hamblin set is closed under conjunction, the MIP amounts to a mere presupposition of existence. As long as the set of true Hamblin answers is non-empty, it will contain a member that entails all the others, viz. the conjunction of all the true Hamblin answers. As intended, therefore, the MIP for (14b) merely requires that *at least one* of the students a and b smiled.

We now apply Dayal's proposal to *how many*-questions. Attending first to (3) above, note that due to the distributivity of *smile* and *students*, the Hamblin set in (5) is totally ordered by entailment, with larger cardinalities giving rise to stronger Hamblin answers than smaller cardinalities: for $n < m$, the existence of a plurality of m smiling students guarantees the existence of a plurality of n smiling students. But a set that is totally ordered by entailment is thereby also closed under conjunction. The MIP, then, in this case amounts to a mere presupposition of existence. So example (3) is predicted to merely presuppose that there were smiling students.

We now return to example (1), repeated below as (15), which features the non-distributive predicate *solve the problem together* instead of distributive *smile*, and which we said is judged to presuppose that there is only one cardinality such that a group of students of that cardinality solved the problem together.

- (15) How many students solved this problem together?

Under present assumptions, (15) has the logical form (16), which determines the denotation in (17) and the Hamblin set (18). (We write *spt* to abbreviate *solved this problem together*.)

- (16) $\lambda_{7[CP} \text{how} \lambda_1[CP' [C ? t_{7,st}] [TP [DP \exists [[t_{1,n} \text{ many}] \text{ students}]] \text{spt}]]]$

$$(17) \quad \lambda p_{st}. \exists n[p = \lambda w. \exists x[\text{students}(x)(w) \wedge |x|=n \wedge \text{spt}(x)(w)]]$$

$$(18) \quad \left\{ \begin{array}{l} \lambda w. \exists x[\text{students}(x)(w) \wedge |x|=1 \wedge \text{spt}(x)(w)], \\ \lambda w. \exists x[\text{students}(x)(w) \wedge |x|=2 \wedge \text{spt}(x)(w)], \\ \lambda w. \exists x[\text{students}(x)(w) \wedge |x|=3 \wedge \text{spt}(x)(w)], \\ \dots \end{array} \right\}$$

The members of the set in (18) are logically independent. For different cardinalities m and n , the existence of a plurality of m students who solved the problem together is logically independent of the existence of a plurality of n students who solved it together. The members of (18) are therefore not related by entailment, and so the MIP requires that only one of them be true. This requirement amounts to the attested uniqueness-of-size presupposition, viz. that there is *only one* cardinality such that a group of students of that cardinality jointly solved the problem.

How many-questions, then, appear to provide striking support for the view that uniqueness presuppositions in questions arise from their Hamblin sets' logical makeup. While *how many*-questions and singular *which*-questions differ greatly in their functional skeleton and compositional structure, the resulting Hamblin sets in both cases are equally unordered by entailment, so that in both cases the MIP correctly derives the attested presupposition of uniqueness.⁴

2.3. But: obviation of uniqueness

The *how many*-question in (19), which repeats (2), differs from (15) in the presence of the possibility modal *allowed* in the question nucleus.⁵ Under present assumptions, this question is assigned the logical form in (20), which will be assigned the denotation in (21), thereby determining the Hamblin set in (22).

(19) How many students are allowed to solve this problem together?

$$(20) \quad \lambda \gamma_{[CP \text{ how } \lambda_1[C' [C ? t_{7,st}] [TP \text{ allowed } [DP \exists [[t_{1,n} \text{ many}] \text{ students}]] \text{ spt}]]]$$

$$(21) \quad \lambda p_{st}. \exists n[p = \lambda w. \diamond_w[\exists x[\text{students}(x) \wedge |x|=n \wedge \text{spt}(x)]]]$$

$$(22) \quad \left\{ \begin{array}{l} \lambda w. \diamond_w[\exists x[\text{students}(x) \wedge |x|=1 \wedge \text{spt}(x)]], \\ \lambda w. \diamond_w[\exists x[\text{students}(x) \wedge |x|=2 \wedge \text{spt}(x)]], \\ \lambda w. \diamond_w[\exists x[\text{students}(x) \wedge |x|=3 \wedge \text{spt}(x)]], \\ \dots \end{array} \right\}$$

The central observation about this analysis is that the addition of the modal operator preserves

⁴However, we observe in Gentile and Schwarz (2018) that, surprisingly, uniqueness-of-size presuppositions are found in certain cases where the Hamblin sets' logical makeup does not lead us to expect them. For example, *How many students have the same name?* intuitively presupposes that there is only one cardinality n such that n students have the same name. This is surprising, since *have the same name* is distributive (down to groups of two), so that the question's Hamblin set should be ordered by entailment. Here we will set this problem aside.

⁵Examples of this sort were first discussed in Beck and Rullmann (1999). Beck and Rullmann did, however, not observe the uniqueness-of-size presupposition in cases like (15), and accordingly did not discuss the obviation effect observed in modalized examples like (19).

the logical relations between Hamblin answers. Accordingly, the Hamblin set (22) is no more ordered by entailment than the one in (18). For different cardinalities m and n , the existence of a permissible world where m students solve the problem together is logically independent of the existence of a permissible world where n students do. As a consequence, the MIP will once again derive a presupposition of uniqueness, here that there is *only one* cardinality such that it is allowed for a group of students of that cardinality to solve the problem together.

However, as we reported above, (19) is *not* judged to carry a uniqueness presupposition comparable to the one attested for (15). That is, the use of the modalized question is intuited to be compatible with the assumption that there are two or more permitted group sizes. Accordingly, there is nothing deviant about responses like *3 or 4, between 5 and 15, or any even number*, interpreted as conveying that both 2 and 4, all numbers between 5 and 15, or all even numbers, are permitted group sizes.

What explains this obviation effect? Why does the presence of the possibility modal obviate a uniqueness presupposition of the sort attested in its absence? Left open in Gentile and Schwarz (2018), we will now offer an answer to this question.

3. An account of uniqueness obviation

An account of the obviation effect must either reject the MIP as stated in Dayal (1996) or revise current assumptions about the membership of the participating Hamblin sets. Here we will pursue the latter avenue.⁶ Maintaining the MIP unchanged, we propose that the relevant Hamblin answers need not be those assumed above. Specifically, we propose that a modalized *how many*-question with a non-distributive predicate lacks a uniqueness-of-size presupposition in virtue of the set of its true Hamblin answers being closed under conjunction.

We will first explain why such closure under conjunction would have the intended effect (Section 3.1). We then propose that closure under conjunction can result under a proposal that Spector (2008) motivated on independent grounds: *wh*-quantification can be higher-order, in the sense of ranging over quantifiers, and such higher-order *wh*-quantification can feed free choice strengthening in the question nucleus (Section 3.2).

3.1. Obviation of uniqueness from closure under conjunction

The obviation effect observed in the interpretation of (19) would be captured if this question's Hamblin set, instead of (22), could be the closure under conjunction of (22). This would have the intended effect of weakening the MIP into a mere existence presupposition. That is, the MIP would merely derive the presupposition that there is a cardinality such that it is allowed for a group of students of that cardinality to solve the problem together. Not entailing uniqueness, this presupposition is weak enough to be compatible with intuitions about (19). Closure under conjunction, then, would capture the observed obviation of uniqueness.

For a more concrete illustration of this line of attack, suppose that tacit domain restriction had reduced the set of cardinalities in *how*'s domain to $\{3, 4\}$. The classic Hamblin set for (19)

⁶In contrast, Fox (2019, 2020) revises Hamblin sets' membership and *in addition* updates Dayal's MIP.

corresponding to (22) would then be the two-membered set in (23a); closure under conjunction would add the conjunction of the two classic members, yielding (23b).

$$(23) \quad \begin{array}{l} \text{a.} \quad \left\{ \begin{array}{l} \lambda w. \diamond_w \exists x [\text{students}(x) \wedge |x|=3 \wedge \text{spt}(x)], \\ \lambda w. \diamond_w \exists x [\text{students}(x) \wedge |x|=4 \wedge \text{spt}(x)] \end{array} \right\} \\ \\ \text{b.} \quad \left\{ \begin{array}{l} \lambda w. \diamond_w \exists x [\text{students}(x) \wedge |x|=3 \wedge \text{spt}(x)], \\ \lambda w. \diamond_w \exists x [\text{students}(x) \wedge |x|=4 \wedge \text{spt}(x)], \\ \lambda w. \diamond_w \exists x [\text{students}(x) \wedge |x|=3 \wedge \text{spt}(x)] \wedge \\ \quad \diamond_w \exists x [\text{students}(x) \wedge |x|=4 \wedge \text{spt}(x)] \end{array} \right\} \end{array}$$

The pair of Hamblin sets in (23) is parallel in its logical profile to the pair in (14) above, which we presented to illustrate Dayal's (1996) assumptions about singular and plural *which*-questions. The effects of the MIP are parallel as well. That is, while for (23a) the MIP delivers the unattested uniqueness-of-size presupposition that *only one* of the two cardinalities 3 and 4 is a permitted group size, for (23b) the MIP merely yields the weaker presupposition that *at least one* of those two cardinalities is permitted, capturing the observed obviation of uniqueness.

The question that remains, of course, is how the Hamblin sets for the relevant *how many*-questions might come to be closed under conjunction. This is the question we turn to next.

3.2. Closure under conjunction from free choice strengthening

Based on observations about how *which*-questions can be answered, Spector (2008) proposed that wh-questions have readings where the wh-phrase quantifies over quantifiers, and that such a construal can lead to free choice strengthening of the resulting Hamblin answers. We will review Spector's proposal (Section 3.2.1), and we will then suggest that free choice strengthening is also a plausible source of a Hamblin set's closure under conjunction in cases of *how many*-questions where obviation of uniqueness is observed (Section 3.2.2). In support of the proposed analysis, we will also present independent evidence for the assumption that *how many*-questions participate in higher-order wh-quantification (Section 3.2.3).

3.2.1. Free choice strengthening in *which*-questions

Spector (2008) reports that, as a response to the *which*-question in (24a), the disjunctive sentence fragment (24b) permits two different types of interpretation.

- (24) a. Which books is Jack allowed to read?
 b. The French or the Russian novels.

It can be read as conveying that Jack is allowed to read the Russian novels *or* is allowed to read the French novels, thereby suggesting that the speaker is unsure which. But it can also be taken to make the stronger statement that Jack is allowed to read the Russian novels *and* is allowed to read the French novels. Spector refers to these readings as *high* and *low*, respectively.

Spector proposes that the high-low ambiguity is due to the question itself having a corresponding ambiguity. His assumptions about the syntax of wh-questions are somewhat different from

those we have introduced above, so we will adjust his proposal slightly to fit the present setting. So adjusted, the analysis takes (24a) to permit logical forms of the sort sketched in (25).

- (25) a. $\lambda_7[{}_{CP} [\text{which books}] \lambda_1[{}_{C'} [C ? t_{7,st}] [{}_{TP} \text{ allowed } [Jack \text{ read } t_{1,e}]]]]]$
 b. $\lambda_7[{}_{CP} [\text{which books}] \lambda_2[{}_{C'} [C ? t_{7,st}] [{}_{TP} \text{ allowed } [t_{2,(et)t} \lambda_1[Jack \text{ read } t_{1,e}]]]]]]]$

These logical forms differ in terms of the semantic type of the highest wh-trace in the question nucleus. In (25a), this trace (index 1) denotes in type e, whereas the corresponding trace in (25b) (index 2) denotes in type (et)t. Spector takes the logical type of the highest wh-trace to fix the fragment's scope relative to the modal operator. If this trace denotes in type e, as in (25a), the disjunction in (24b) is taken to be interpreted as scoping over *allowed*, yielding the high reading. If it denotes in type (et)t, as in (25b), the disjunction is taken to be interpreted as scoping below the modal. Specifically, a fragment response on its low reading is analyzed as the proposition obtained by interpreting the question nucleus relative to a variable assignment that maps the index on the type (et)t trace to the fragment's type (et)t denotation.⁷ So, (24b) in its low reading is taken to be given by the question nucleus (i.e., the TP) when interpreted relative to the variable assignment $2 \rightarrow \llbracket \text{the Russian or the French novels} \rrbracket$.

As Spector notes, however, this will not actually account for the low reading without further assumptions. Assuming that *allowed* is an existential operator, the modal and disjunction should be scopally commutative. Such scopal commutativity should neutralize the high-low contrast, making (24b) convey unambiguously that Jack is allowed to read the Russian novels *or* is allowed to read the French novels. However, as Spector also points out, the stronger low reading actually attested can plausibly be analyzed as instantiating the familiar free choice effect, sketched in (26), whereby a statement with a disjunction in the scope of a possibility modal is strengthened to a conjunction of possibility statements about the individual disjuncts.

- (26) $\diamond(p_1 \vee \dots \vee p_n) \rightsquigarrow \diamond p_1 \wedge \dots \wedge \diamond p_n$

Spector does not actually propose an analysis of the free choice effect. We will likewise leave it unanalyzed here. Our point is merely that, if free choice strengthening, however analyzed, applies with the effect in (26) within the question nucleus in (25b), it will capture the intended low reading of (24b), conveying that Jack is allowed to read the Russian novels *and* is allowed to read the French novels.⁸

Returning now to the two logical forms in (25), Spector assumes that, corresponding to the dif-

⁷On this analysis, then, fragment responses are not, as Merchant (2005) has argued, clauses reduced by ellipsis. Instead, they acquire propositional content by composing directly with (a constituent given by) the question, as proposed in Jacobson (2016) and Xiang (2016). Spector actually takes the question nucleus to denote a property, and proposes that the fragment answer comes to express a proposition in virtue of composing with that property. In the case of low readings, the question nucleus in Spector's view denotes a property of generalized quantifiers. He takes the fragment response to express the proposition obtained by applying the property denoted by the nucleus to the fragment's denotation.

⁸What this commits us to, of course, is that free choice strengthening applies in the semantics. That higher-order wh-quantification can feed free choice strengthening in the question nucleus is also proposed in Xiang (2016), which moreover spells out an analysis of the effect in terms of a syntactically represented exhaustification operator. See Section 5, as well as Fox (2020), for brief discussion of potential problems for this approach.

ference in the nucleus, they also differ in the interpretation of the wh-phrase itself. While the occurrence of *which books* in (25a) is taken to quantify over individuals — books and pluralities of books — the one in (25b) is taken to quantify over quantifiers — quantifiers over books and pluralities of books. The Hamblin set determined by (25b), then, will consist of propositions based on such quantifiers. To illustrate, suppose that tacit domain restriction reduces the domain of wh-quantification in (25a) to the set of pluralities {the Russian novels, the French novels}. The Hamblin set determined by (25a) will then be the set of classic Hamblin answers (27a). Turning to (25b), suppose there the domain of wh-quantification is the set of quantifiers $\{\lambda f_{et}.f(\text{the Russian novels}), \lambda f_{et}.f(\text{the French novels}), \lambda f_{et}.f(\text{the Russian novels}) \vee f(\text{the Russian novels})\}$, which in addition to two quantifiers corresponding to individuals, includes the disjunctive quantifier denoted by the fragment response (24b). Assuming free choice strengthening within the question nucleus, this domain will for (25b) yield the Hamblin set in (27b), which in addition to the two classic Hamblin answers, includes the conjunctive proposition that expresses the low reading of (24b).⁹

$$(27) \quad \begin{array}{l} \text{a.} \\ \text{b.} \end{array} \left\{ \begin{array}{l} \lambda w. \diamond_w \text{read}(\text{the Russian novels})(\text{Jack}), \\ \lambda w. \diamond_w \text{read}(\text{the French novels})(\text{Jack}) \end{array} \right\}$$

$$\left\{ \begin{array}{l} \lambda w. \diamond_w \text{read}(\text{the Russian novels})(\text{Jack}), \\ \lambda w. \diamond_w \text{read}(\text{the French novels})(\text{Jack}), \\ \lambda w. \diamond_w \text{read}(\text{the Russian novels})(\text{Jack}) \wedge \\ \quad \diamond_w \text{read}(\text{the French novels})(\text{Jack}) \end{array} \right\}$$

Having reviewed Spector’s analysis of low readings of *which*-questions with possibility modals, we will show in the next subsection that this analysis naturally extends into an account of the obviation of uniqueness in *how many*-questions.

Before doing so, however, we would like to clarify that our analysis of uniqueness obviation will not depend on Spector’s analysis of fragment responses being correct. What we will make use of below is merely the assumption about wh-questions that underlies this analysis of fragment responses. In fact, on an alternative analysis that comes to mind, the fragment response (24a) is a clause reduced by ellipsis (Merchant, 2005), and permits a free choice reading simply in virtue of sharing a logical form with the non-reduced clause *Jack is allowed to read the Russian or the French novel* in its free choice reading. By the same token, we think that free choice readings of fragment responses do not by themselves provide a compelling evidence for higher-order wh-quantification. Moreover, conjunctive Hamblin answers of the sort posited by Spector could conceivably arise from distribution over pluralities such as *the Russian*

⁹Spector (2008) proposes that the domain of higher-order quantification by *which*-phrases can include any upward entailing quantifiers over the set of entities given by the wh-phrase’s overt restrictor. Under the higher-order parse of (24a), the domain of wh-quantification could then also include, for example, the conjunctive quantifier $\lambda f_{et}. f(\text{the Russian novels}) \wedge f(\text{the French novels})$. Spector’s assumption is compatible with the analysis of uniqueness obviation in modalized *how many*-questions that we are about to propose (as it does not interfere with Hamblin sets’ closure under conjunction). Fox (2018, 2020) in fact argues that conjunctive quantifiers are *obligatorily* included in the domain of higher-order wh-quantification. (Fox’s argument comes from constraints on low readings described by Spector but not reviewed here.) However, the inclusion of conjunctive quantifiers undermines the central premise that our proposal is based on, viz. the proposal, reviewed in Section 2, that the uniqueness-of-size presupposition in non-modal cases can be attributed to the MIP. We return to this important issue in Section 4.

novels ⊕ *the French novels* with scope over the possibility modal. So while Spector's analysis of free choice fragments sets up our analysis of uniqueness obviation, the existence of free choice fragment responses does not by themselves furnish compelling independent motivation for it. However, extending our report on Spector (2008), we will offer independent evidence for the account in Section 3.2.3.

3.2.2. Application to *how many*-questions

We return to the problematic *how many*-question in (19), here repeated again as (28a). We now note that the fragment response (28b) participates in the sort of high-low ambiguity identified in the last subsection.

- (28) a. How many students are allowed to solve this problem together?
 b. 3 or 4

The response has a high reading stating that either 3 *or* 4 is a permitted size of student group solving the problem together, suggesting that the speaker is unsure which. It also has a stronger, low, reading, conveying that both 3 *and* 4 are permitted group sizes. In fact, we already identified this low reading above, where we used it to confirm the absence of a uniqueness-of-size presupposition in the meaning of (28a).

In an obvious extension of Spector's (2008) analysis, the two readings arise from two different logical forms for (28a). The high reading of (28b) arises when (28a) has the logical form (20) above, the logical form that we saw is associated with an unwanted uniqueness-of-size presupposition. According to this logical form, *wh*-movement of *how* leaves a trace that denotes in type *n*, the type of cardinalities. In contrast, the low reading is attributed to the logical form (29), where *how* leaves a trace denoting in type (nt)t, the type of quantifiers over cardinalities.

- (29) $\lambda_7[\text{CP how } \lambda_2[\text{C' [C ? } t_{7,\text{st}}] [\text{TP allowed } [t_{2,(\text{nt})t} \lambda_1[[\text{DP } \exists [[t_{1,n} \text{ many}] \text{ students}]] \text{ spt}]]]]]]$

Correspondingly, the *wh*-phrase *how* in (29) is taken to quantify not over cardinalities, but over quantifiers over cardinalities, functions of type (nt)t. For illustration, suppose that tacit domain restriction has reduced *how*'s domain to the set $\{\lambda_{f_{\text{nt}}.f(3)}, \lambda_{f_{\text{nt}}.f(4)}, \lambda_{f_{\text{nt}}.f(3) \vee f(4)}\}$. Note that this set of quantifiers is closed under disjunction. Once again assuming free choice strengthening in the question nucleus, this domain will result in the Hamblin set (30).

$$(30) \left\{ \begin{array}{l} \lambda_{\text{w}}. \diamond_{\text{w}} \exists x [\text{students}(x) \wedge |x| = 3 \wedge \text{spt}(x)], \\ \lambda_{\text{w}}. \diamond_{\text{w}} \exists x [\text{students}(x) \wedge |x| = 4 \wedge \text{spt}(x)], \\ \lambda_{\text{w}}. \diamond_{\text{w}} \exists x [\text{students}(x) \wedge |x| = 3 \wedge \text{spt}(x)] \wedge \\ \quad \diamond_{\text{w}} \exists x [\text{students}(x) \wedge |x| = 4 \wedge \text{spt}(x)] \end{array} \right\}$$

We saw this Hamblin set before, in (23b) above. We observed that this Hamblin set is closed under conjunction and noted that, as a consequence, the MIP amounts to a mere presupposition of existence, capturing the attested obviation of uniqueness.

Extrapolating from this illustration, we arrive at a general analysis of uniqueness obviation

for *how many*-questions with non-distributive predicates. On this analysis, uniqueness can be obviated by a possibility modal in virtue of *how* leaving a type (nt)t trace in the modal's scope and quantifying over quantifiers over cardinalities. If the set of quantifiers that forms *how*'s domain is closed under disjunction, then free choice strengthening will lead to the resulting Hamblin set being closed under conjunction. Under those circumstances, the MIP amounts to a mere presupposition of existence, capturing the obviation effect.

In summary, we have shown that the observed obviation of the uniqueness-of-size is predicted under Spector's (2008) analysis of the high-low ambiguity of fragment answers to questions with possibility modals. If accepted, our analysis of the obviation effect therefore provides an argument for Spector's analysis of the high-low ambiguity. In the next subsection, we identify data that add independent motivation for a central ingredient of this analysis, viz. the availability of higher-order wh-quantification.

3.2.3. Support from other types of modalized questions

Spector (2008) reports that a high-low ambiguity is also found in *which*-questions with universal modals in the question nucleus. He notes that, as response to (31a), the fragment in (31b) can be read in two ways.

- (31) a. Which books must Jack read?
b. The French or the Russian novels.

On its high reading, it conveys that Jack is required to read the Russian novels or is required to read the French novels, suggesting that the speaker is unsure which. On its low reading, it states that Jack is required to either read the Russian novels or read the French novels, suggesting that either will do.

In the present rendition of Spector's analysis, these two readings of (31b) are associated with the two logical forms for (31a) shown in (32). Assuming the same domains of wh-quantification introduced for illustration above, these logical forms determine the Hamblin sets in (33).

- (32) a. $\lambda_7[\text{CP} [\text{which books}] \lambda_1[\text{C}' [\text{C} ? t_{7,\text{st}}] [\text{TP must} [\text{Jack read } t_{1,e}]]]]]$
b. $\lambda_7[\text{CP} [\text{which books}] \lambda_2[\text{C}' [\text{C} ? t_{7,\text{st}}] [\text{TP must} [t_{2,(et)} \lambda_1[\text{Jack read } t_{1,e}]]]]]]]$

- (33) a. $\left\{ \begin{array}{l} \lambda w. \Box_w \text{read}(\text{the Russian novels})(\text{Jack}), \\ \lambda w. \Box_w \text{read}(\text{the French novels})(\text{Jack}) \end{array} \right\}$
b. $\left\{ \begin{array}{l} \lambda w. \Box_w \text{read}(\text{the Russian novels})(\text{Jack}), \\ \lambda w. \Box_w \text{read}(\text{the French novels})(\text{Jack}), \\ \lambda w. \Box_w (\text{read}(\text{the Russian novels})(\text{Jack}) \vee \\ \text{read}(\text{the French novels})(\text{Jack})) \end{array} \right\}$

How many-questions, including *how many*-question with non-distributive predicates, participate in the very same sort of ambiguity. As a response to (34a), (34b) has a high reading stating that either a group of three is required to solve the problem or a group of four is required (sug-

gesting that the speaker is unsure which). It also has a low reading, which conveys that it is required for there to be a group of three or four solving the problem (suggesting that either would do).

- (34) a. How many students are required to solve this problem together?
b. 3 or 4

The two readings can be attributed to the logical forms in (35). For the domains of wh-quantification introduced above for illustration, those logical forms determine the Hamblin sets in (36).

- (35) a. $\lambda_7[\text{CP how } \lambda_1[\text{C' [C ? } t_{7,\text{st}}] [\text{TP required } [\text{DP } \exists [[t_{1,n} \text{ many}] \text{ students}]] \text{ spt}]]]$
b. $\lambda_7[\text{CP how } \lambda_2[\text{C' [C ? } t_{7,\text{st}}] [\text{TP required } t_{2,(\text{nt})\text{t}} \lambda_1[[\text{DP } \exists [[t_{1,n} \text{ many}] \text{ sts}]] \text{ spt}]]]]]$

- (36) a. $\left\{ \begin{array}{l} \lambda w. \Box_w \exists x [\text{students}(x) \wedge |x| = 3 \wedge \text{spt}(x)], \\ \lambda w. \Box_w \exists x [\text{students}(x) \wedge |x| = 4 \wedge \text{spt}(x)] \end{array} \right\}$
b. $\left\{ \begin{array}{l} \lambda w. \Box_w \exists x [\text{students}(x) \wedge |x| = 3 \wedge \text{spt}(x)], \\ \lambda w. \Box_w \exists x [\text{students}(x) \wedge |x| = 4 \wedge \text{spt}(x)], \\ \lambda w. \Box_w (\exists x [\text{students}(x) \wedge |x| = 3 \wedge \text{spt}(x)] \vee \\ \exists x [\text{students}(x) \wedge |x| = 4 \wedge \text{spt}(x)]) \end{array} \right\}$

On Spector's analysis of low readings with necessity modals, the proposition given by the response's low reading is a member of the Hamblin set. As Spector argues, the inclusion of this proposition in the Hamblin set is indeed called for on empirical grounds, to capture that fact that the response's low reading can constitute an intuitively complete answer to the question.

From our present perspective, Spector's analysis provides an account of the apparent obviation of the *existence* presupposition that would otherwise be entailed by the MIP. Applied to (32a), the MIP requires that there be a particular book or plurality of books that Jack must read. Similarly, the MIP for (45a) demands that there be a particular cardinality such that a group of students of that cardinality must solve the problem together. The intuition that (31b) and (34b) can be complete answers (despite not portraying any particular books or cardinalities as being required) indicates that those existence presuppositions can be absent. The logical forms (32b) and (35b), and the Hamblin sets they determine, can account for this observation. This is apparent from the fact the disjunctive, non-classic, Hamblin answers in (33b) and (36b) can be the only true propositions in those sets. In that case, those disjunctive Hamblin answers will be the strongest true members of their Hamblin sets, and so the MIP will be satisfied despite all the classic Hamblin answers being false.

Questions with necessity modals, then, provide independent evidence for the assumption that wh-questions, and *how many*-questions in particular, permit wh-quantification over quantifiers – evidence that does not rely on assumptions about the analysis of fragment answers. Further support of this sort comes from *how many*-questions with a possibility modal in the scope of negation, such as (37). Consider the logical forms in (38) and the Hamblin sets in (39).

- (37) How many students are not allowed to solve this problem together?
- (38) a. $\lambda_7[\text{how } \lambda_1[C' [C ? t_{7,\text{st}}] [\text{TP not [allowed [DP } \exists [[t_{1,n} \text{ many}] \text{ students}] \text{ spt}]]]]]$
 b. $\lambda_7[\text{how } \lambda_2[C' [C ? t_{7,\text{st}}] [\text{TP not [allowed } t_{2,(\text{nt})t} \lambda_1[[\text{DP } \exists [[t_{1,n} \text{ many}] \text{ sts}] \text{ spt}]]]]]]]$
- (39) a. $\left\{ \begin{array}{l} \lambda w. \neg \diamond_w \exists x[\text{students}(x) \wedge |x| = 3 \wedge \text{spt}(x)], \\ \lambda w. \neg \diamond_w \exists x[\text{students}(x) \wedge |x| = 4 \wedge \text{spt}(x)] \end{array} \right\}$
 b. $\left\{ \begin{array}{l} \lambda w. \neg \diamond_w \exists x[\text{students}(x) \wedge |x| = 3 \wedge \text{spt}(x)], \\ \lambda w. \neg \diamond_w \exists x[\text{students}(x) \wedge |x| = 4 \wedge \text{spt}(x)], \\ \lambda w. \neg \diamond_w (\exists x[\text{students}(x) \wedge |x| = 3 \wedge \text{spt}(x)] \vee \\ \exists x[\text{students}(x) \wedge |x| = 4 \wedge \text{spt}(x)]) \end{array} \right\}$

Note that the classic Hamblin answers in (39a) are logically independent, so the MIP requires that only one of them be true. So if only the logical form (38a) were available, (37) should presuppose that there is a unique cardinality such that it is not allowed for a group of that cardinality to solve the problem. Intuitions indicate, however, that (37) carries no such presupposition. This is shown by the observation that (37) can be answered felicitously with, say, *3 or 4* to convey that neither 3 nor 4 is a permitted group size.¹⁰ This judgment is captured by the logical form (38b) and its Hamblin set (39b), given that this set includes the proposition that neither 3 nor 4 is allowed. More generally, higher-order quantification over quantifiers allows for the Hamblin set for (37) to be closed under conjunction. (37) is therefore predicted to merely carry a weaker presupposition of existence, viz. that there is some cardinality such that groups of that cardinality are not allowed to solve the problem.

So *how many*-questions with a possibility modal under negation provide further independent evidence for the assumption that *how many*-questions permit higher-order wh-quantification. Notably, this evidence is, once again, independent of the analysis of fragment answers.

Summarizing again, Spector's analysis of the high-low ambiguity of fragment responses to *which*-questions permits a parsimonious extension that captures the obviation of the uniqueness-of-size presupposition in *how many*-questions. This is a satisfying result. But it also leads to an important remaining question. What remains to be investigated is whether the proposed analysis of obviation preserves the account of uniqueness-of-size presupposition in those cases where it *is* attested. This is the question that we will now turn to.

4. Reining in higher-order wh-quantification

To explore the consequences of our proposal for the analysis of non-modalized questions, let us return to example (15), repeated here once again in (40). In Section 2, we proposed to attribute the uniqueness-of-size presupposition carried by this example to the MIP.

- (40) How many students solved this problem together?

¹⁰Fox (2010) reports the corresponding judgment about *which*-questions, noting that as a response to *Which books are we not allowed to read?*, the fragment response *The French books or the Russian books* can convey that we are not allowed to read either set of books.

This proposal was based on the assumption that (40) is assigned the logical form (41a), which gives rise to a classic Hamblin set of propositions about particular cardinalities. However, parallel to the analysis of the modalized questions, we must now also consider the logical form (41b), where *wh*-movement has left a trace denoting in type (nt)t, and where *how*'s domain is accordingly taken to consist of quantifiers over cardinalities.

- (41) a. $\lambda_7[\text{CP how } \lambda_1[\text{C}' [\text{C ? } t_{7,\text{st}}] [\text{TP } [\text{DP } \exists [[t_{1,n} \text{ many}] \text{ students}]] \text{ spt}]]]$
 b. $\lambda_7[\text{CP how } \lambda_2[\text{C}' [\text{C ? } t_{7,\text{st}}] [\text{TP } t_{2,(\text{nt})t} \lambda_1[\text{DP } \exists [[t_{1,n} \text{ many}] \text{ students}]] \text{ spt}]]]$

In our discussion of *wh*-quantification over cardinalities, we have so far considered domains that include quantifiers that correspond to basic entities, such as $\lambda_{f_{\text{nt}}.f(3)}$, as well as the disjunction of such quantifiers, such as $\lambda_{f_{\text{nt}}.f(3) \vee f(4)}$. However, Spector (2008) actually proposed that the domain of higher-order *wh*-quantification can comprise any upward monotone quantifiers, including conjunctive quantifiers like $\lambda_{f_{\text{nt}}.f(3) \wedge f(4)}$.

In the context of our analysis, the inclusion of conjunctive quantifiers in the domain of higher-order *wh*-quantification is problematic. It conflicts with the starting point of our paper, the assumption that the uniqueness-of-size presupposition is an instance of the MIP. To illustrate, suppose the domain of *how* is the set of quantifiers $\{\lambda_{f_{\text{nt}}.f(3)}, \lambda_{f_{\text{nt}}.f(4)}, \lambda_{f_{\text{nt}}.f(3) \wedge f(4)}\}$. The logical form (41b) will yield the Hamblin set in (42).

$$(42) \quad \left\{ \begin{array}{l} \lambda w. \exists x[\text{students}(x)(w) \wedge |x| = 3 \wedge \text{spt}(x)(w)], \\ \lambda w. \exists x[\text{students}(x)(w) \wedge |x| = 4 \wedge \text{spt}(x)(w)], \\ \lambda w. \exists x[\text{students}(x)(w) \wedge |x| = 3 \wedge \text{spt}(x)(w)] \wedge \\ \quad \exists x[\text{students}(x)(w) \wedge |x| = 4 \wedge \text{spt}(x)(w)] \end{array} \right\}$$

This Hamblin set is closed under conjunction, and as a consequence, the MIP will amount to a mere presupposition of existence. It will be true as long as *at least one* of the cardinalities 3 and 4 is the size of a student group who solved the problem together. More generally, permitting conjunctive quantifiers in *how*'s domain amounts to permitting higher order domains that are closed under conjunction. In the absence of an additional operator, this would yield Hamblin sets that are closed under conjunction, predicting unwanted obviation of uniqueness even in non-modal cases, where the uniqueness-of-size presupposition is invariably attested.

We are led to conclude that, if logical forms like (41b) are available, then the domain of higher-order *wh*-quantification must be taken to exclude conjunctive quantifiers. The data presented in Spector (2008) indeed appear compatible with this hypothesis. To illustrate with an example of our own, note that (43a) permits (43b) as a felicitous response in a low reading, according to which Jack's reading requirements are met if he reads either *Buddenbrooks* and *The tin drum* or *Buddenbrooks* and *The magic mountain*.

- (43) a. Which books must Jack read?
 b. *Buddenbrooks* and either *The tin drum* or *The magic mountain*.

The Hamblin answer corresponding to this low reading could enter the question's Hamblin set in virtue of the *wh*-quantification's domain including the conjunctive quantifier $\lambda_{f_{\text{et}}. f(\textit{The$

Buddenbrooks) \wedge (f(*The tin drum*) \vee f(*The magic mountain*)). However, given that *read* is distributive in its object position, the relevant Hamblin answer can just as well be credited to the disjunctive quantifier $\lambda f_{\text{et.f}}(\text{Buddenbrooks} \oplus \text{The tin drum}) \vee \text{Buddenbrooks} \oplus \text{The magic mountain}$).

Given the availability of distribution over pluralities, then, *which*-questions with necessity modals do not seem to furnish evidence against (or for) our hypothesis that the domain of higher-order wh-quantification excludes conjunctive quantifiers.¹¹ This invites us to consider wh-questions where distribution over pluralities in the wh-phrase's domain is not available.

How many-questions are a case in point. The very fact that (40) carries a uniqueness-of-size presupposition indicates that distribution over pluralities of cardinalities is not available there. Such distribution too would result in Hamblin sets like (42), again losing the proposed account of the uniqueness-of-size presupposition.¹² Given this, consider again (34a), repeated below as (44), together with the logical forms in (45). With conjunctive quantifiers excluded from *how*'s domain, and in the absence of distribution over plural cardinalities, we are led to maintain that (44) has Hamblin sets like those in (46), repeated from (35) and (36).

(44) How many students are required to solve this problem together?

- (45) a. $\lambda_7[\text{CP how } \lambda_1[\text{C' [C ? } t_{7,\text{st}}] [\text{TP required } [\text{DP } \exists [[t_{1,n} \text{ many}] \text{ students}]] \text{ spt}]]]$
 b. $\lambda_7[\text{CP how } \lambda_2[\text{C' [C ? } t_{7,\text{st}}] [\text{TP required } t_{2,(\text{nt})t} \lambda_1[[\text{DP } \exists [[t_{1,n} \text{ many}] \text{ sts}]] \text{ spt}]]]]]$

- (46) a. $\left\{ \begin{array}{l} \lambda w. \Box_w \exists x [\text{students}(x) \wedge |x| = 3 \wedge \text{spt}(x)], \\ \lambda w. \Box_w \exists x [\text{students}(x) \wedge |x| = 4 \wedge \text{spt}(x)] \end{array} \right\}$
 b. $\left\{ \begin{array}{l} \lambda w. \Box_w \exists x [\text{students}(x) \wedge |x| = 3 \wedge \text{spt}(x)], \\ \lambda w. \Box_w \exists x [\text{students}(x) \wedge |x| = 4 \wedge \text{spt}(x)], \\ \lambda w. \Box_w (\exists x [\text{students}(x) \wedge |x| = 3 \wedge \text{spt}(x)] \vee \\ \exists x [\text{students}(x) \wedge |x| = 4 \wedge \text{spt}(x)]) \end{array} \right\}$

Note now that the classic Hamblin answers that are shared by the two Hamblin sets in (46) are logically independent. Since neither set includes the conjunction of those classic Hamblin answers, the MIP entails that only one of them is true, i.e. that only one of the cardinalities 3 and 4 is such that there must be a group of that size solving the problem. More generally, excluding conjunctions from the domain of higher-order wh-quantification predicts that, like its non-modal counterpart (40), (44) carries a uniqueness-of-size presupposition; here the presupposition that there is only one cardinality such that a group of that size must solve the problem.

¹¹Xiang (2016) argues for higher-order wh-quantification over conjunctive quantifiers in *which*-questions with non-distributive predicates, like *Which students solved the problem together?*. This proposal allows for the Hamblin sets for such questions to be closed under conjunction, despite being built on non-distributive predicates. This, in turn, provides an account for the absence of an uniqueness presupposition in such cases, which Xiang (2016) was the first to draw attention to. However, Fox (2019, 2020) argues for an alternate route to closure under conjunction in such cases, viz. distribution over higher-order pluralities (Landman, 1989a, b). Assuming that Fox's proposal is viable, *which*-questions with non-distributive predicates do not after all shed light on the makeup of the domain of higher-order wh-quantification.

¹²Comments by Danny Fox (personal communication; also Fox, 2020) have helped us see this issue more clearly.

There are observations that conform with this prediction. As a responses to (44), conjunctive fragments like *3 and 4* strike us as a deviant, as do fragments like *3 and either 4 or 7*. What might account for the contrast between such responses and fully felicitous, purely disjunctive, responses like *3 or 4*? Our hypothesis that conjunctive quantifiers are excluded from the domain of wh-quantification, in conjunction with the assumption that distribution over pluralities of cardinalities in the question nucleus is unavailable, provides a straightforward answer.

Our negation example (37), repeated below as (47), can be used to make a related point. The relevant observation is that it seems quite impossible to interpret a conjunctive response to (47) like *3 and 4* in a low reading.

(47) How many students are not allowed to solve this problem together?

That is, this response cannot be understood as denying the existence of permissible worlds where the set of cardinalities of student groups who solve the problem includes both 3 and 4. If the question (47) permitted a Hamblin answer that expresses such a denial, the unavailability of the low reading for *3 and 4* would be surprising. In particular, it is hard to see what would account for the contrast between *3 and 4* and its disjunctive counterpart *3 or 4*, which we observed in Section 3 naturally lends itself to a low interpretation.¹³

We conclude that observations about *how many*-questions with necessity modals and possibility modals under negation furnish independent support for the assumption that the domain of *how* cannot include conjunctive quantifiers.¹⁴

5. Concluding remarks

Building directly on Spector (2008), we have proposed that obviation of the uniqueness-of-size presupposition in (28a), repeated once more in (48), is due to higher-order wh-quantification that feeds free choice strengthening of Hamblin answers. However, like Spector, we have remained silent on the workings of free choice strengthening, assuming merely that it applies in the question nucleus and takes the form sketched in (49), repeated from (26). Relying in this characterization of free choice strengthening, we illustrated its intended effect on (48) by crediting the Hamblin set (30), repeated here as (50), to the set $\{\lambda f_{nt}.f(3), \lambda f_{nt}.f(4), \lambda f_{nt}.f(3) \vee f(4)\}$ as the domain of wh-quantification.

(48) How many students are allowed to solve this problem together?

(49) $\diamond(p_1 \vee \dots \vee p_n) \rightsquigarrow \diamond p_1 \wedge \dots \wedge \diamond p_n$

¹³Under the assumption that the domain of *how* excludes conjunctive quantifiers, the unavailability of a low reading for *3 and 4* could be attributed to the observation that it would not actually address the question. As the reader is invited to confirm, under the assumption that conjunctive quantifiers are excluded from the domain of the wh-quantification, the low reading fails to discriminate between cells of the partition that the question determines.

¹⁴However, Fox (2020) reports judgments that are in conflict with those we reported above. For example, Fox reports that *How many students should solve this problem together?* can be answered felicitously with *Between 5 and 7 (the small group) and between 8 and 10 (the large group)*. We suspect that this response reflects that (47) marginally allows for a pair-list reading that pairs cardinalities with groups, but we are at present unable to substantiate this suspicion.

$$(50) \quad \left\{ \begin{array}{l} \lambda w. \diamond_w \exists x [\text{students}(x) \wedge |x| = 3 \wedge \text{spt}(x)], \\ \lambda w. \diamond_w \exists x [\text{students}(x) \wedge |x| = 4 \wedge \text{spt}(x)], \\ \lambda w. \diamond_w \exists x [\text{students}(x) \wedge |x| = 3 \wedge \text{spt}(x)] \wedge \\ \quad \diamond_w \exists x [\text{students}(x) \wedge |x| = 4 \wedge \text{spt}(x)] \end{array} \right\}$$

In an approach pioneered by Fox (2007), free choice strengthening can arise from exhaustification, the strengthening process that also gives rise to scalar implicatures, and exhaustification is credited to syntactically represented operator exhaustification, Exh. A central benefit of this approach in the present context is that it permits strengthening of individual Hamblin answers, by positing that Exh can appear in the question nucleus. However, exhaustification may not actually deliver (50). Exhaustification is relative to a set of alternative propositions. If this alternative set is equated with the Hamblin set that would obtain in the absence of exhaustification, then free choice strengthening should result in (51) instead of (50).

$$(51) \quad \left\{ \begin{array}{l} \lambda w. \quad \diamond_w \exists x [\text{students}(x) \wedge |x| = 3 \wedge \text{spt}(x)] \wedge \\ \quad \neg \diamond_w \exists x [\text{students}(x) \wedge |x| = 4 \wedge \text{spt}(x)], \\ \lambda w. \neg \diamond_w \exists x [\text{students}(x) \wedge |x| = 3 \wedge \text{spt}(x)] \wedge \\ \quad \diamond_w \exists x [\text{students}(x) \wedge |x| = 4 \wedge \text{spt}(x)], \\ \lambda w. \quad \diamond_w \exists x [\text{students}(x) \wedge |x| = 3 \wedge \text{spt}(x)] \wedge \\ \quad \diamond_w \exists x [\text{students}(x) \wedge |x| = 4 \wedge \text{spt}(x)] \end{array} \right\}$$

So the Hamblin answers resulting from free choice strengthening would be mutually incompatible propositions. This can be shown to be inconsequential for the content of the presupposition delivered by the MIP. For a set of exhaustified, mutually incompatible, Hamblin answers, the MIP is met just in case one of the exhaustified answers is true, which is equivalent to saying that the set of true classic, non-exhaustified, Hamblin answers contains one that entails all the others. In particular, the MIP maps (51) to the same proposition as (50), viz. the proposition that at least one of the cardinalities 3 and 4 is such that a student group of that cardinality is allowed to solve the problem together.

However, if free choice strengthening of Hamblin answers always rendered Hamblin answers mutually incompatible, then the obviation of uniqueness in questions embedded under responsive predicates (in the sense of Lahiri, 2002) should force strongly exhaustive readings in the sense of Groenendijk and Stokhof (1984). Exploring this prediction, and its consequences for the analysis of uniqueness obviation, is a task that we leave for future work.

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