In: Emar Maier, Corien Bary & Janneke Huitink, eds. (2005) "Proceedings of SuB9"
www.ru.nl/ncs/sub9

TRUTH, BELIEF AND CERTAINTY IN EPISTEMIC LOGIC

Daniel Vanderveken
Université du Québec, Trois-Rivières
daniel@vanderveken.org

Abstract

Standard logic tends to reduce propositions to their truth conditions. However propositions with the same truth conditions are not the contents of the same thoughts just as they are not the senses of synonymous sentences. I will first define a much finer criterion of propositional identity that takes into account predications that we make in expressing propositions. In my view, propositions have a structure of constituents. We ignore in which possible circumstances most propositions are true because we ignore real denotations of their attributes and concepts. In understanding them we just know that their truth in each circumstance is compatible with certain possible denotation assignments to their constituents and incompatible with others. So propositions have possible in addition to real truth conditions. I will explain why strictly equivalent propositions can have a different cognitive value. I will define the notion of truth according to an agent and a strong propositional implication that is known *a priori*. I will also formulate a logic of belief that is compatible with philosophy of mind. Human agents are minimally rather than perfectly rational in my logic. Epistemic paradoxes are solved.

1 Propositional identity and truth according to predication

In philosophy, propositions are both *senses of sentences* with truth conditions and *contents of conceptual thoughts* like attitudes and illocutions. In order to take into account their double nature, I will proceed to a finer analysis in terms of predication of their logical form. Here are the basic principles of my predicative approach. [1]

1.1 A finite structure of constituents

In expressing propositions we *predicate* in a certain order a finite positive number of *attributes* (properties or relations) *of* objects to which we *refer*.[2] Understanding a proposition consists mainly of understanding which *attributes* objects of reference must possess in order that this proposition be true. We do not directly have in mind *individuals* like material bodies and persons. We rather have in mind *concepts* of individuals and we *indirectly* refer to them through these concepts. So our thoughts are directed towards *individuals under a concept* rather than pure *individuals*. Concepts can be deprived of denotation. By recognizing the indispensable role of concepts in reference, logic can account for thoughts directed at inexistent objects. It can also

[1] See *Formal Ontology, Propositional Identity and Truth According to Predication* [2003], “Propositional Identity, Truth According to Predication and Strong Implication” [2005] and “Universal Grammar and Speech Act Theory” [2001] for a more general presentation of the theory.
[2] Predication as it is conceived here is independent on force and psychological mood. We make the same predication when we express a belief and a doubt that something is the case.

account for the predication of *intensional properties* that objects only possess whenever they are subsumed under certain concepts. Frege's idea that propositional constituents are senses clearly explains the difference in cognitive value between the two propositions that Cicero is Cicero and that Cicero is Tullus. We *a priori* know by virtue of linguistic competence the truth of the first proposition while we have to learn the truth of the second. Frege's idea moreover preserves the *minimal rationality* of speakers.[3] We can wrongly ignore that Cicero has another proper name and believe that Cicero is not Tullus. But we cannot believe that Cicero is not Cicero. We could not be that irrational. So epistemic logic has to reject *direct reference*[4] and *externalism*. Like Frege, Church and Strawson I advocate that any *object of reference* is subsumed *under a concept*. Proper names are often introduced into language by an initial declaration. A speaker first gives such names to objects with which he is acquainted. Next other speakers of the linguistic community adopt these names and keep using them to refer to the same objects. Later speakers who do not know much of named objects can always refer to them under the concept of being the object *called by* that name. Possible interpretations of language must then consider in their domain two sets of senses: the set *Concepts* of individual concepts and the set *Attributes* of attributes of individuals in addition to the set *Individuals* of individual objects which are pure denotations.

1.2 A relation of correspondence between senses and denotations

To propositional constituents *correspond real denotations* of certain types in *possible circumstances*. Thus to each individual concept c_e corresponds in any circumstance the single individual which falls under that concept in that circumstance whenever there is such an object. Otherwise that concept is deprived of denotation in that circumstance. And to each attribute R_n of degree n of individuals corresponds the set of sequences of n objects under concepts which possess that attribute in that circumstance.[5] A *possible circumstance* is here a complete state of the real world at a moment m in a possible course of history h. As in the logic of ramified time, the set *Circumstances* of possible circumstances contains pairs of the form m/h where m is a moment belonging to the history h. Individual things change during their existence. So different denotations can correspond to the same concept or attribute at different moments. However individuals have their *essential attributes* in all circumstances where they exist. For example each human being keeps the same genetic code.

Our knowledge of the world is incomplete. We do not know real denotations of most propositional constituents. We ignore also many essential properties of objects. So we often *refer* to an object under a concept without knowing that object. The police officer who is pursuing the murderer of Smith can just refer to whoever is that murderer. The concept gives *identity criteria* for the object of reference (e.g. to be Smith's murderer). But few identity criteria enable us to *identify* that object. Moreover the *object* to which we refer is sometimes different from the *denotation* of our concept.[6] Presumed murderers are often innocent. It can also happen that no object satisfies our identity criteria. Smith could have died of a heart attack. Whoever *conceives* propositional

[3] The notion of *minimal rationality* comes from C. Cherniak [1986]

[4] The theory of *direct reference* is advocated by D. Kaplan in "On the Logic of Demonstratives" [1979]

[5] A. Church introduced the relation of correspondence in intensional logic. See "A Formulation of the Logic of Sense and Denotation" [1951]

[6] See S. Kripke "Speaker Reference and Semantic Reference" [1977]

constituents can always assign to them possible denotations of appropriate type. A chief of police who ignores the identity of the murderer can at least think of different persons who could have committed the crime. From a logical cognitive point of view, there are a lot of possible denotation assignments to senses. They are functions of the set ((*Concepts* x *Circumstances*) $\rightarrow$ *Individuals*) $\cup$ ((*Attributes* x *Circumstances*) $\rightarrow$ $\bigcup_{1 \leq n}$ Π(*Concepts*^{n})). However, from a cognitive point of view, only certain entities could be according to us possible denotations of attributes and concepts that we conceive in circumstances that we consider. Whoever refers to an object considers that it could have certain properties but not others. So not all possible denotation assignments are compatible with the beliefs of agents. Suppose that the chief of police believes at the beginning of his investigation that Smith was murdered by his wife. Only possible denotation assignments according to which the same person falls under the two concepts are then compatible with his beliefs.

Let us attribute to Smith's murderer the property of being mad. Clearly we do not *a priori* know actual denotations of the concept and property of that predication. But we can consider denotations that they could have. According to a first possible denotation assignment, a suspect Paul would be Smith's murderer and that suspect would be mad. According to a second, Smith's wife Julie would be the murderer but she would not be mad. According to a third Smith would not have been killed. Sure we need to make an empiric investigation in order to verify the predication in question. However we all know by virtue of competence that it is satisfied in a circumstance if and only if the individual that falls under its concept possesses its property in that circumstance. So we *a priori* know that the elementary proposition that Smith's murderer is mad is true according to the first possible denotation assignment considered above and false according to the two others.

We respect *meaning postulates*. We associate denotations of appropriate types to each individual concept c_e and attribute R_n in possible circumstances. Thus $val\ (c_e,\ m/h) \in$ *Individuals* when according to *val* the concept c_e has a denotation in circumstance *m/h*.[7] And $val\ (R_n,\ m/h) \in \Pi(Concepts^n)$). Moreover our denotation assignments respect *internal relations* that exist between constituents because of their logical form. We *a priori* know that individuals subsumed under two concepts are *identical* when these two concepts have the same denotation. So for any possible assignment val, $<c_e^1, c_e^2> \in val$ $(=, m/h)$ when $val\ (c_e^1,\ m/h) = val\ (c_e^2,\ m/h)$. As one can expect, the set *Val* of all possible denotation assignments contains a special *real assignment* (in symbol *val**) that associates with each concept and attribute their actual denotation in every possible circumstance. We ignore *real denotations* of many concepts and attributes. However we cannot have them in mind without *eo ipso* believing that they could have certain denotations and not others in given circumstances. So to each agent *a* and moment *m* there corresponds a unique set *Val*(*a*,*m*) containing all possible denotation assignments which are compatible with the beliefs of that agent at that moment. Suppose that the agent *a* believes at a moment *m* that an individual under concept c_e has (or just could have) the property R_1 in a certain circumstance *m/h*. Then for every (or for at least one) denotation assignment $val \in Val(a,m)$, $c_e \in val\ (R_1,\ m/h)$. By nature, we, human agents,

[7] Otherwise either the assignment *val* is undefined for the concept c_e or that concept has an arbitrary denotation like the empty set according to that assignment. See Carnap *Meaning and Necessity* [1956]

are minimally consistent. We could not believe that the same individual under concept has and does not have a given property in the same circumstance. So the set *Val*(*a*,*m*) is a proper subset of the whole set *Val* when the agent *a* is conscious at the moment *m*.

1.3 Possible truth conditions

By definition, a predication of the form ($R_n\ c_e^1,\dots,c_e^n$) whose attribute R_n is applied to n individuals under concepts $c_e^1,\dots,c_e^n$ in that order is satisfied in a circumstance *m/h* according to a denotation assignment *val* when $<c_e^1,\dots,c_e^n> \in val\ (R_n, m/h)$. So any complete possible assignment *val* associates certain *possible truth conditions* with each elementary proposition. For that proposition *would be true* in all and only the possible circumstances where its predication is satisfied according to that assignment *val* if it *were real*. There are few *analytically true* elementary propositions that predicate of objects attributes that we *a priori* know that they possess. So we ignore in which possible circumstances most elementary propositions are true. However we know in apprehending their logical form that elementary propositions are true in a circumstance according to all possible denotation assignments that satisfy their predication in that circumstance and false according to others. So in my approach propositions have above all *possible truth conditions*. They *could be true* in different sets of possible circumstances given the possible denotations that their constituents could have in reality. If one considers a number n of possible circumstances, one can distinguish as many as 2^n possible truth conditions. Formally each *possible truth condition* of an elementary proposition corresponds to a unique set of possible circumstances where that proposition is true according to a particular possible denotation assignment. Of course, from a cognitive point of view, not all such possible truth conditions are compatible with our beliefs. In order that a proposition *could be true according to an agent* at a moment, that proposition must be true according to at least one possible assignment *val* $\in$ *Val*(*a*,*m*) compatible with the beliefs of that agent at that moment. So, for example, according to the chief of police at the beginning of his investigation Smith's wife but not Paul could be Smith's murderer.

Among all possible truth conditions of a proposition there are of course its *actual characteristic truth conditions* that correspond to the set of possible circumstances where it is true. Carnap did not consider other possible truth conditions By definition, the *real denotation assignment val** associates with each elementary proposition its actual truth conditions.

1.4 A recursive definition of propositions

In my analysis, propositions have *a structure of constituents*: they serve to make a finite positive number of predications. In order to make a predication of the form ($R_n\ c_e^1,\dots,$ c_e^n) one must have in mind its attribute and objects under concepts. One must also apply that attribute to these objects in the right order. One makes two different predications in thinking that Mary loves John and that John loves Mary. The two elementary propositions have the same constituents but different truth conditions. However the order of predication only matters whenever it affects truth conditions. Whenever the predicated binary relation is symmetric it does not matter at all. The propositions that Cicero is Tullus and that Tullus is Cicero contain the same predication. For that reason, a predication of the form ($R_n\ c_e^1,\dots,c_e^n$) cannot be identified with the corresponding sequence $<R_n\, c_e^1,\ldots, c_e^n>$. From a logical point of view, such a predication is rather an ordered pair whose first element is the set of its propositional constituents $\{R_n, c_e^1,\ldots, c_e^n\}$ and whose second element is the set of possible circumstances where it is satisfied according to the real assignment *val**. Such an account identifies predications whose different order determines the same truth conditions. So the set *Predications* of all predications is a subset of $\Pi(\mathit{Concepts} \cup \mathit{Attributes}) \times (\Pi \mathit{Circumstances})$. In addition to a structure of constituents propositions also have *possible truth conditions*. Their truth in each possible circumstance is compatible with a certain number of possible denotation assignments to their constituents and incompatible with the others.

Elementary propositions are the simplest propositions. They serve to make a single predication and their truth in each possible circumstance is only compatible with possible denotation assignments according to which their predication is satisfied. Other *more complex propositions* are obtained by applying truth functional, modal and other operations. Complex propositions are in general composed from several elementary propositions. When they contain a single one, they are true according to different possible denotation assignments.

Truth functions do not change the structure of constituents. They only make the predications of their arguments. Thus the *negation* $\neg$P has the structure of constituents of P. The *conjunction* (P $\wedge$ Q) and the *disjunction* (P $\vee$ Q) of two propositions P and Q are composed from elementary propositions of both. Unlike truth functions, modal and epistemic propositions serve to make new predications of *modal and epistemic attributes*. In thinking that it is impossible that God makes mistakes we do more than predicate of God the property of not making mistakes. We also predicate of Him the modal property of infallibility, namely that He does not make a mistake in any possible circumstance. Infallibility is the necessitation of the property of not making mistakes. Similarly when we think that the pope believes that God exists we do more than predicate of God the property of existence, we also predicate of Him the property of being existent according to the pope. The property of being existent according to an agent is an *epistemic property* different from that of being existent. Agents can wrongly believe that an object exists. Moreover they can ignore the existence of many objects.

The new attributes of modal and epistemic propositions remain of the first order. Modal attributes of individuals are obtained from simpler attributes of individuals by quantifying universally or existentially over possible circumstances. They are *necessitations* and *possibilizations* of simpler attributes. Thus an object under concept c_e possesses the *necessitation* $\blacksquare R_1$ of a property R_1 when it possesses that property in all possible circumstances.[8]

Epistemic attributes of the form aR_n are also of the first order. They are satisfied by sequences of objects under concepts which satisfy according to agent *a* the simpler attribute R_n. One can analyze them thanks to a *relation of compatibility* Belief_m^a between possible denotation assignments that takes into consideration beliefs that agent *a* could

---

[8] More generally, the necessitation $\blacksquare R_n$ of an attribute R_n satisfies the meaning postulate: $<c_e^1,\ldots,c_e^n> \in val(\blacksquare R_n, m/h)$ when, for every m'/h', $<c_e^1,\ldots,c_e^n> \in val\,(R_n, m'/h')$. See G. Bealer *Quality and Concept* [1982] for the intensional logic of attributes.

[8] A belief with undetermined truth conditions would be a belief without real content. It would not be a belief at all.

have at each moment *m*. First of all, whoever has a particular belief is able to determine under which conditions that belief is true.[9] He or she has then in mind attributes and concepts of that belief. In order to believe that Descartes is not a janissary one must understand the property of being a janissary. According to any possible denotation assignment *val* each agent a has in mind a certain set *val(a,m)* of propositional constituents at each moment *m* and the agent has then beliefs about the denotations of these constituents in certain (generally not all) possible circumstances. The relation $Belief^a_m$ serves to determine the exact nature of these beliefs. Suppose that according to the denotation assignment *val* the agent *a* believes at the moment *m* that certain concepts and attributes have such and such denotations in such and such possible circumstances. A possible denotation assignment *val'* is *compatible with what the agent a then believes according to the assignment val* (in symbols val*'* $\in Belief^a_m(val)$) when according to val*'* the same concepts and attributes have the same possible denotations in the same possible circumstances. So if according to assignment *val* the agent *a* believes at moment *m* that an individual object u falls under a certain concept c_e in the circumstance *m/h* then according to any compatible assignment val*'* $\in Belief^a_m(val)$, u = val*'*(c_e,*m/h*). The concept c_e however could have according to a compatible assignment val*'* a different possible denotation val*'*(c_e,*m'/h'*) $\neq$ val(c_e,*m'/h'*) in another possible circumstance *m'/h*' that agent *a* does not consider.

By definition, the relation of epistemic compatibility corresponding to $Belief^a_w$ is reflexive and transitive: $val \in Belief^a_m(val)$ and if $val' \in Belief^a_m(val)$ and $val'' \in Belief^a_m(val')$ then $val'' \in Belief^a_m(val)$. But it is not symmetric.[10] Moreover, the set $Belief^a_m(val^*)$ that serves to determine the real beliefs of agent *a* at moment *m* is the set *Val(a,m)* already defined. As one can expect, an object under concept c_e possesses according to the agent *a* the property R_1 in a circumstance *m/h* (in symbols $c_e \in$ val*(aR_1,*m/h*) when according to all assignments $val' \in Val(a,m)$ that object has that property in that circumstance.[11] Of course, the agent *a* has no beliefs at all at the moment *m* according to *val* when the set $Belief^a_m(val)$ is the whole set *Val*. In that case, the set *val(a,m)* is empty. He or she does not then have in mind anything.

What are the *possible truth conditions* of *complex propositions*? We determine them by respecting obvious meaning postulates. A truth functional negation $\neg$P is true in a possible circumstance according to a possible denotation assignment to its constituents when the proposition P is not true in that circumstance according to that assignment. A conjunction (P $\wedge$ Q) is true in a circumstance according to a denotation assignment when both conjuncts P and Q are true in that circumstance according to that assignment.

The modal proposition $\blacksquare$P that it is logically necessary that P is true according to a denotation assignment in a possible circumstance when proposition P is true according to that assignment in all possible circumstances. Finally, the proposition *Bel*aP that

---

[10] For according to a compatible assignment $val' \in Belief^a_w(val)$, agent *a* at moment *m* can believe that certain propositional constituents of *val(a,m)* have possible denotations in other circumstances. He or she can also have in mind other constituents. The assignment *val* could not respect these new beliefs that agent *a* has according to assignment *val*' at moment *m*.

[11] More generally, epistemic attributes of the form aR_n satisfy the meaning postulate: $<c^1_e,\ldots,c^n_e> \in val(aR_n,m/h)$ when according to all assignments $val' \in Belief^a_m(val)$. $<c^1_e,\ldots,c^n_e> \in val(aR_n,m/h)$.

agent *a* believes that P is true in a circumstance *m/h* according to a denotation assignment *val* when according to that assignment the agent *a* has in mind all the constituents of P at moment *m* and the proposition P is true in that circumstance according to all assignments $val' \in Belief^{a}_{m}(val)$ which are compatible with what that agent believes at that moment according to assignment *val*.

There are two borderline cases of truth conditions. Sometimes the proposition is true according to every possible denotation assignment to its constituents. It is a pure *tautology*. Sometimes it is true according to none. It is a pure *contradiction*. In my analysis, *tautologies* only have the *universal truth condition* that corresponds to the set of all possible circumstances and *contradictions* the *empty truth condition* that corresponds to the empty set. So tautologies (and contradictions) are special cases of *necessarily true* (and *necessarily false*) propositions. Tautologies are also *a priori* and *analytically true* (and contradictions *a priori* and *analytically false*).[12]

1.5 The new criterion of propositional identity

Identical propositions have the same structure of constituents and they are true in the same possible circumstances according to the same possible denotation assignments to their constituents. Propositions which are *true according to the same possible denotation assignments* have the *same possible truth conditions*. So the set of propositions U_p is a subset of $\Pi Predications$ x ($Circumstances \rightarrow \Pi Val$). Each proposition is an ordered pair containing first the finite non empty set of its predications and second a function associating with each possible circumstance the set of all possible denotation assignments according to which it is true in that circumstance.

My criterion of propositional identity is much finer than that of modal, temporal, intensional and relevance logics. My logic distinguishes strictly equivalent propositions with a different structure of constituents. We do not make the same predications in expressing them. So there are a lot of different necessarily true and necessarily false propositions and not only two as classical logic wrongly claims. Predicative logic moreover distinguishes strictly equivalent propositions which are not true in the same circumstances according to the same possible denotation assignments. They do not have the same possible truth conditions. So we do not understand in the same way under which conditions they are true. Consider the elementary proposition that the biggest whale is a fish and the conjunction that the biggest whale is and is not a fish. Both are composed from a single elementary proposition predicating of the biggest whale the property of being a fish. And both are necessarily false. In all possible circumstances where they exist, whales are mammals. They all have in common that *essential property*. However the two propositions have a different cognitive value. We can believe the first but not the second. Unlike Parry[13] I distinguish such strictly equivalent propositions with the same structure of constituents. The first is true according to many possible denotation assignments but the second according to none. It is a contradiction.

1.6 Truth definition

In the philosophical tradition, from Aristotle to Tarski, *truth* is based on *correspondence*

[12] The necessary truth of tautologies is then metaphysical, logical and epistemic.
[13] Parry elaborated a logic of *analytic implication*. See “Comparison of Entailment Theories” [1972]

with reality. True propositions represent how objects are in the actual world. Objects of reference have properties and stand in relations in possible circumstances. However they could have many other properties and stand in many other relations in these circumstances. In addition to ways in which things are, there are possible ways in which they could be. We consider a lot of possible truth conditions in expressing and understanding propositional contents. The truth of propositions is compatible with many possible ways in which objects could be. However in order that a proposition be true in a given circumstance, things must be in that circumstance as that proposition represents them. Otherwise, there would be no correspondence. Along these lines, a proposition *is true in a possible circumstance* when it is true according to any denotation assignment associating with its constituents their real denotation in every circumstance. Many possible circumstances are not actual: their moment just belong a possible inactual course of history of this world True propositions correspond to existing facts. So they are true at a moment in the actual course of history of this world. Classical laws of truth theory follow from my concise definition.

1.7 Cognitive aspects in the theory of truth

As I said earlier, to each agent a and moment m there corresponds a unique set $Val(a,m)$ containing all the possible denotation assignments to senses *compatible with what that agent believes at that moment*. Whenever the agent a is provided with consciousness, the set $Val(a,m)$ is restricted; $Val(a,m) \neq Val$. Thanks to my conceptual apparatus logic can now define the subjective notion of *truth according to an agent*: *a proposition is true in a circumstance* according to *an agent a at a moment m* when that agent a has in mind at the moment m all its constituents and that proposition is true in that circumstance according to all possible assignments $val \in Val(a,m)$ that are compatible with his or her beliefs at that moment. As one can expect, tautological propositions are true and contradictory propositions are false according to all agents who have them in mind. But impossible propositions which are not contradictory can be true and necessary propositions which are not tautological can be false according to agents at some moments. For they have other possible truth conditions than the empty and the universal truth condition respectively.

So the logic of language imposes different limits on reality and thought. Necessarily false propositions represent impossible facts that could not exist in reality and that we could not experience. In my view, there is need to postulate impossible circumstances where such impossible facts would exist. Impossible facts are objectively impossible. In any possible circumstance where there are whales they are mammals and not fishes. So many ways in which we can think of objects do not represent possible ways in which these objects could be. Certain objectively impossible facts e.g. that whales are fishes are subjectively possible. We can wrongly believe that they exist. Their existence is compatible with certain possible denotation assignments to senses that do not respect essential properties. But corresponding possible truth conditions are *subjective* rather than *objective possibilities*.

1.8 The notion of strong implication

We, human beings are not perfectly rational. Not only we make mistakes and are sometimes inconsistent. But moreover we do not draw all valid inferences. We believe (and assert) propositions without believing (and asserting) all their logical consequences. However we are not completely irrational. On the contrary, we manifest a *minimal rationality* in thinking and speaking. First we *a priori* know that certain propositions are necessarily false (for example, contradictions). So we cannot believe them nor attempt to do things that we know impossible.[14] Moreover, we always draw certain valid inferences. We *a priori* know that certain propositions cannot be true unless others are also true, since we *a priori* know the truth of tautologies with a conditional propositional content. In that case we cannot believe (or assert) the first propositions without believing (or asserting) the others. There is an important *relation of strict implication* between propositions due to C.I. Lewis that Hintikka[15] and others have used to explain to which beliefs agents are committed. A proposition *strictly implies* another whenever that other proposition is true in every possible circumstance where it is true. According to Hintikka, whoever believes a proposition *eo ipso* believes all others that it strictly implies. However we ignore which propositions are related by strict implication, just as we ignore which are necessarily true.

So we need a propositional implication much finer than strict implication in epistemic logic. Predicative logic can rigorously define that finer propositional implication that I call ***strong implication.*** A propositio*n strongly implies another* proposition when firstly, it has the same or a richer structure of constituents and secondly, it *tautologically implies* that other proposition in the following sense: whenever it is true in a possible circumstance according to a possible denotation assignment the other is also true in that circumstance according to that same assignment. Unlike strict implication, *strong implication is a priori known*. Whenever a proposition P strongly implies another Q, we cannot express that proposition without *a priori* knowing that it strictly implies the other. For in expressing P, we have by hypothesis in mind all elementary propositions of Q. We make all the corresponding acts of reference and predication. Furthermore, in understanding the truth conditions of proposition P, we distinguish *eo ipso* all possible denotation assignments to its propositional constituents which are compatible with its truth in any circumstance. These are by hypothesis compatible with the truth of proposition Q in the same circumstance. Thus, in expressing P, we know for certain that Q follows from P. Belief and knowledge are then closed under strong rather than strict implication in epistemic logic.

2 Formal semantics for a minimal epistemic logic

2.1 The ideographical object – language **L** of that epistemic logic

Its lexicon contains a series of *individual constants* naming agents and a series of *propositional constants* expressing propositions.

The *syncategorematic expressions* are: $\neg$, $\blacksquare$, *Tautological*, *Bel*, $\wedge$, $\geq$, (and)
Here are the **rules of formation.** Any propositional constant is a *propositional formula* of **L**. If A_p and B_p are propositional formulas of **L** so are $\neg A_p$, $\blacksquare A_p$, *Tautological*A_p, $(A_p \geq B_p)$, $(A_p \wedge B_p)$ and *Bel*aA_p , for any individual constant a. $\neg A_p$ expresses the *negation* of the proposition expressed by A_p . $\blacksquare A_p$ expresses the *modal proposition* that

[14] See my contribution "Attempt, Success and Action Generation: A Logical Study of Intentional Action" [2005]
[15] See J. Hintikka ***Knowledge and Belief*** [1962]

it is logically necessary that A_p and *Tautological*A_p the proposition that it is tautological that A_p. *Bel*aA_p expresses the *proposition* that the agent named by a believes that A_p. $(A_p \wedge B_p)$ expresses the *conjunction* of the two propositions expressed by A_p and B_p . Finally, $(A_p \geq B_p)$ means that the proposition that A_p has the same or a richer structure of constituents than the proposition that B_p.

2.2 Rules of abbreviation

I will use the usual rules of abbreviation for the elimination of parentheses and the connectives $\vee$ of *disjunction*, $\Rightarrow$ of m*aterial implication*, $\Leftrightarrow$ of m*aterial equivalence*, $\blacklozenge$ of *logical possibility* and $-\!\!\!-\!\!\in$ of s*trict implication*. Here are rules of abbreviation for new notions:
Analytic implication: $A_p \rightarrow B_p =_{df} (A_p \geq B_p) \wedge (A_p -\!\!\!-\!\!\in B_p)$
Strong implication: $A_p \mapsto B_p =_{df} (A_p \geq B_p) \wedge$ *Tautological* $(A_p \Rightarrow B_p)$
Propositional identity: $A_p = B_p =_{df} A_p \mapsto B_p \wedge B_p \mapsto A_p$
Same structure of constituents: $A_p \equiv B_p =_{df} (A_p \geq B_p) \wedge (B_p \geq A_p)$
Strong psychological commitment: *Bel*a$A_p \blacktriangleright$*Bel*a$B_p =_{df}$ *Bel*a$A_p \mapsto$ *Bel*aB_p
Weak psychological commitment: *Bel*a$A_p \triangleright$ *Bel*a$B_p =_{df}$ *Bel*a$A_p \mapsto \neg$*Bel*a$\neg B_p$
Certainty: *Certain*a$A_p =_{df}$ *Bel*a$A_p \wedge$ *Tautological*A_p[16]

2.3 Definition of a model structure

A *standard model* M for **L** is a structure < *Moments*, *Individuals*, *Agents*, *Concepts*, *Attributes*, *Val*, *Predications*, *Belief*, * , ⊗, ≡≡>, where *Moments*, *Individuals*, *Agents*, *Concepts*, *Attributes*, *Val* and *Predications* are non empty sets and Belief, * , ⊗ and ≡≡ are functions which satisfy the following clauses:

- The set Moments is a set of moments of time. It is partially ordered by a temporal relation $\leq$ as in ramified temporal logic. m1 < m2 means that moment m1 is anterior to moment m2. By definition, < is subject to historical connection and no downward branching. Any two distinct moments have a common historical ancestor. Moreover, the past is unique: if m1 < m and m2 < m then either m1 = m2 or m1 < m2 or m2 < m1. A maximal chain h of moments is called a history. It represents a possible course of history of the world. The set Circumstances of all possible circumstances contains all pairs m/h where m is a moment belonging to the history h.

- The set Individuals is a set of possible individual objects. For each moment m, Individualsm is the set of individual objects existing at that moment. Agents is a non empty subset of Individuals containing persons.

- Concepts is the set of individual concepts and Attributes is the set of attributes of individuals considered in model . For each natural number n, Attributes(n) is the subset of Attributes containing all attributes of degree n .

- The set Val is a proper subset of ((Concepts x Circumstances) $\rightarrow$ (Individuals $\cup$ {∅})) $\cup \bigcup_n$ ((Attributes(n) x Circumstances) $\rightarrow$ (Conceptsn)). Val contains all possible denotation assignments of the model M. Such assignments are also called possible

[16] I can only deal here with certainties whose propositional content is necessarily true.

valuations of constituents. For any possible circumstance m/h, val (ce,m/h) $\in$ Individuals when individual concept ce has a denotation in the circumstance m/h according to assignment val. Otherwise val (ce,m/h) = $\emptyset$. For any attribute Rn of degree n, val (Rn, m/h) $\in$ (Conceptsn). The set Val contains a real valuation val which assigns to concepts and attributes their actual denotation in each possible circumstance according to the model . Moreover, there corresponds to each agent a, moment m and assignment val a particular set val(a,m) containing all propositional constituents that the agent a has in mind at that moment according to that assignment.

- Belief is a function from Agents $\times$ Moments $\times$ Val into (Val) that associates with any agent a, moment m and valuation val, the set Belief^a_m(val) $\subseteq$ Val of all possible denotation assignments to which are compatible with the beliefs that agent a has at the moment m according to that valuation. The relation of epistemic compatibility corresponding to Belief^a_m is reflexive and transitive. Moreover, val(a,m) $\subseteq$ val'(a,m) when val' $\in$ Belief^a_m(val). As one can expect, Belief^a_m(val) = Val when a $\notin$ Individualsm.17

- The set Predications is a subset of (Attributes $\cup$ Concepts) x Circumstances that contain all predications that can be made in the language L. Each member of that set is an ordered pair of the form (Rn c^1_e,…, c^n_e) whose first element is the set of propositional constituents {Rn,c^1_e,…, c^n_e} and whose second element is the set of all possible circumstances m/h such that <c^1_e,…,c^n_e> $\in$ val (Rn,m/h)}. The power set Predications is closed under union $\cup$, a modal unary operation * and, for each agent a, a unary epistemic operation $\otimes$a of the following form: For any Γ, Γ1 and Γ2 $\in$ Predications, Γ $\subseteq$ *Γ and *Γ $\subseteq$ $\otimes$a Γ. Moreover, *(Γ1 $\cup$ Γ2) = *Γ1 $\cup$ *Γ2 and **Γ =*Γ. Similarly, $\otimes$a (Γ1 $\cup$ Γ2) = $\otimes$a Γ1 $\cup$ $\otimes$a Γ2 and $\otimes$a $\otimes$a Γ = $\otimes$a Γ. By definition, when Belief^a_m(val) $\neq$ Val and Γ $\subseteq$ val(a,m), Γ $\subseteq$ $\otimes$a val(a,m).18

- ≡≡ is an interpreting function which associates with each individual constant a an agent ≡a≡$\in$ Agents and with each propositional formula Ap the proposition ≡Ap≡ that is expressed by that formula according to the model . In my analysis, each proposition has two essential features: the set of all its predications and the set of possible denotation assignments according to which it is true. So the set Up of all expressible propositions is the smallest subset of Predications $\times$ (Circumstances $\rightarrow$ Π*Val*) that is defined recursively as follows:

- U_p contains all *elementary propositions P* whose first element id_1P is a singleton of the form {(R_n c^1_e,…, c^n_e)} and whose second element id_2P is the function that associates with each circumstance *m/h* the set {*val* / <c^1_e,…,c^n_e> $\in$ *val*(R_n ,*m/h*)}.

The set U_p is closed under operations corresponding to our logical connectives:

- id_1≡¬B_p≡ = id_1≡B_p≡) and id_2≡¬B_p≡(*m/h*) = *Val* – id_2(≡B_p≡(*m/h*) .

---

[17] Only existing agents can have beliefs.
[18] As one can expect, each agent who has beliefs has beliefs about himself. In particular given the reflexivity and transitivity of *Belief*^a_m, whoever has a belief also believes that he or she has that belief.

- $id_1 \equiv Tautological B_p \equiv = id_1 \equiv B_p \equiv$ and $id_2 \equiv Tautological B_p \equiv (m/h) = Val$ when $\equiv B_p \equiv = Val$. Otherwise, $id_2 \equiv Tautological B_p \equiv (m/h) = \emptyset$.

- $id_1 \equiv \blacksquare B_p \equiv = {}^* id_1 \equiv B_p \equiv)$ and $id_2 \equiv \blacksquare B_p \equiv (m/h) = \bigcap_{m'/h' \in Circumstances} id_2 \equiv B_p \equiv (m'/h'\}$

- $id_1(\equiv B_p \wedge C_p \equiv) = id_1(\equiv B_p \equiv) \cup id_1(\equiv C_p \equiv)$; $id_2 \equiv B_p \wedge C_p \equiv (m/h) = id_2 \equiv B_p \equiv (m/h) \cap id_2 \equiv C_p \equiv (m/h)$.

- $id_1(\equiv B_p > C_p \equiv) = id_1(\equiv B_p \equiv) \cup id_1(\equiv C_p \equiv)$ and $id_2 \equiv B_p > C_p \equiv (m/h) = Val$ when $id_1 \equiv B_p \equiv \subseteq id_1 \equiv C_p \equiv$. Otherwise, $id_2 \equiv B_p > C_p \equiv (m/h) = \emptyset$.

- Finally, $id_1 \equiv Bel a B_p \equiv = \otimes_a id_1 \equiv B_p \equiv)$ where $\equiv a \equiv = a$ and $id_2 \equiv Bel a B_p \equiv (m/h) = \{val \in Val$ / firstly, for all $(R_n\ c_e^1, \ldots, c_e^n) \in id_1 \equiv B_p \equiv$, $\{R_n, c_e^1, \ldots, c_e^n\} \subseteq val(\equiv a \equiv, m)$ and secondly, $Belief_m^a(val) \subseteq id_2 \equiv B_p \equiv (m/h)$. [19]

2.4 Definition of truth and validity

A propositional formula A_p of **L** is *true* in a possible circumstance *m/h* according to a standard model M if and only $\equiv A_p \equiv$ is true in *m/h* according to *val*M. The formula A_p is *valid* (in symbols: $\models A_p$) when it is true in all possible circumstances according to all standard models of **L**.

3 An axiomatic system

I conjecture that all and only valid formula are provable in the following axiomatic system:

3.1 Axioms

The axioms of my system are all the instances in the object- language L of classical axiom schemas of truth functional logic and S5 modal logic and instances of the following new schemas:

Axiom schemas for tautologies

(T1) $Tautological A_p \Rightarrow \blacksquare A_p$
(T2) $Tautological\ A_p \Rightarrow Tautological\ Tautological A_p$
(T3) $\neg Tautological\ A_p \Rightarrow Tautological \neg Tautological A_p$
(T4) $Tautological A_p \Rightarrow (Tautological\ (A_p \Rightarrow B_p) \Rightarrow Tautological B_p)$

Axiom schemas for propositional identity

(I1) $A_p = A_p$

[19]In other words, any valuation *val* compatible with the fact that an agent *a* believes a proposition $\equiv B_p \equiv$ in a circumstance w must satisfy two conditions. Firstly according to that valuation the agent *a* must have beliefs about all propositional constituents of the believed proposition $\equiv B_p \equiv$ in circumstance *w*. Secondly, all possible denotation assignments *val*‘ which are compatible with the beliefs of that agent in that circumstance according to that valuation must themselves be compatible with the truth of that believed proposition in circumstance *w*.

(I2) $(A_p = B_p) \Rightarrow (C \Rightarrow C^*)$ where C* and C are propositional formulas which differ at most by the fact that an occurrence of B_p in C* replaces an occurrence of A_p in C.
(I3) $(A_p = B_p) \Rightarrow$ *Tautological* $(A_p = B_p)$
(I4) $\neg(A_p = B_p) \Rightarrow$ *Tautological* $\neg(A_p = B_p)$

Axiom schemas for belief

(B1) $(\mathit{Bel}aA_p \wedge \mathit{Bel}aB_p) \Rightarrow \mathit{Bel}a(A_p \wedge B_p)$
(B2) $\mathit{Tautological}A_p \Rightarrow \neg\mathit{Bel}a\neg A_p$
(B3) $\mathit{Bel}aA_p \Rightarrow ((A_p \mapsto B_p) \Rightarrow (\mathit{Bel}aB_p))$
(B4) $\mathit{Bel}aA_p \Leftrightarrow (\mathit{Bel}a\mathit{Bel}aA_p)$
(B5) $\mathit{Bel}aA_p \Rightarrow \mathit{Bel}a\blacklozenge A_p$

Axiom schemas for propositional composition

(C1) $(A_p \geq B_p) \Rightarrow \mathit{Tautological}(A_p \geq B_p)$
(C2) $\neg(A_p \geq B_p) \Rightarrow \mathit{Tautological}\neg(A_p \geq B_p)$
(C3) $A_p \geq A_p$
(C4) $(A_p \geq B_p) \Rightarrow ((B_p \geq C_p) \Rightarrow (A_p \geq C_p))$
(C5) $(A_p \wedge B_p) \geq A_p$
(C6) $(A_p \wedge B_p) \geq B_p$
(C7) $((C_p \geq A_p) \wedge (C_p \geq B_p)) \Rightarrow C_p \geq (A_p \wedge B_p)$
(C8) $A_p \equiv \neg A_p$
(C9) $A_p \equiv \mathit{Tautological}A_p$
(C10) $(A_p \wedge B_p) \equiv (A_p \geq B_p)$
(C11) $\blacksquare A_p \geq A_p$
(C12) $\mathit{Bel}aA_p \geq \blacksquare A_p$
(C13) $\blacksquare\neg A_p \equiv \blacksquare A_p$ And similarly for *Bel*a. (C14)
(C15) $\blacksquare(A_p \wedge B_p) \equiv (\blacksquare A_p \wedge \blacksquare B_p)$ And similarly for *Bel*a. (C16)
(C17) $\blacksquare\blacksquare A_p \equiv \blacksquare A_p$ And similarly for *Bel*a. C18)

3.2 Rules of inference

The two rules of inference of my axiomatic system are:
The *rule of Modus Ponens*: (MP) From sentences of the form A and $(A \Rightarrow B)$ infer B .
The *tautologization rule*: (RT) From a theorem A infer *Tautological*A .

4 Important valid laws of epistemic logic

4.1 Laws about the structure of constituents

A proposition has all the elementary propositions of its arguments. $\models A_p \geq B_p$ when B_p occurs in A_p. However modal and epistemic propositions have in general more elementary propositions than their arguments. Thus $\not\models A_p \geq \blacksquare A_p$ and $\not\models \blacksquare A_p \geq \mathit{Bel}aA_p$.

4.2 Laws for tautologyhood

Tautologyhood is stronger than necessary truth and contradiction stronger than necessary falsehood. $\models$ (*Tautological*A_p) $\Rightarrow$ $\blacksquare A_p$. But $\not\models$ $\blacksquare A_p$ $\Rightarrow$ *Tautological*A_p There are modal and epistemic tautologies. Thus $\models$ *Tautological* ($\blacksquare A_p \Rightarrow A_p$).

4.3 Agents are minimally rather than perfectly rational.

They do not believe all necessary truths and they can believe necessarily false propositions.

$\not\models$ $\blacksquare A_p \Rightarrow$ *Bel*aA_p and $\not\models$ $\neg\blacklozenge A_p \Rightarrow$ *Bel*a $\neg A_p$. However they are minimally consistent: they cannot believe that a tautology is false or that a contradiction is true. $\models$*Tautological*$A_p \Rightarrow \neg$*Bel*a$\neg A_p$ and $\models$*Tautological*$\neg A_p \Rightarrow \neg$*Bel*a$A_p$ Now in order to believe a proposition an agent must have in mind its attributes and concepts. Unlike God, human agents do not have in mind all propositional constituents. Consequently they do not know or even believe all tautologies. $\not\models$*Tautological*A_p $\Rightarrow$ *Bel*aA_p. The limits of their language imposes limits to their thoughts. However whenever they express a tautology and a contradiction, they know just by apprehending their logical form that the first is necessarily true and the second necessarily false.

So $\models$*Tautological*$A_p \Rightarrow$ (*Bel*a$A_p \Rightarrow$ (*Certain*aA_p)

4.4 Laws for tautological implication

Tautological implication is much finer than strict implication. $\models$ *Tautological* ($A_p \Rightarrow B_p$) $\Rightarrow$ ($A_p -\!\!\in B_p$) But $\not\models$ ($A_p -\!\!\in B_p$) $\Rightarrow$ *Tautological* ($A_p \Rightarrow B_p$). Necessarily true propositions are strictly implied by others. $\models$ $\blacksquare A_p \Rightarrow$ ($B_p -\!\!\in A_p$). But only tautologies can tautologically imply other tautologies. $\models$ ((*Tautological*B_p) $\wedge$ *Tautological* ($A_p \Rightarrow B_p$)) $\Rightarrow$ *Tautological* A_p. So $\not\models$ $\blacksquare A_p$ $\Rightarrow$ *Tautological*($B_p \Rightarrow A_p$). Similarly necessarily false propositions strictly imply all other propositions. $\models$ $\blacksquare\neg A_p \Rightarrow$ ($A_p -\!\!\in B_p$). But only contradictions can tautologically imply contradictions. So $\not\models$ $\blacksquare\neg A_p$ $\Rightarrow$ *Tautological*($A_p \Rightarrow B_p$).

Beliefs are not closed under tautological implication. $\not\models$ (*Tautological* ($A_p \Rightarrow B_p$)) $\Rightarrow$ (*Bel*aA_p $\Rightarrow$ *Bel*aB_p)) Because $\not\models$ (*Tautological* ($A_p \Rightarrow B_p$)) $\Rightarrow$ ($A_p \geq B_p$)). However whoever believes a proposition cannot believe the negation of a proposition that the first tautologically implies. For the conjunction of both is a contradiction. This is why tautological implication generates *weak psychological and illocutionary commitment*. Any assertion that P *weakly commits* the agent to asserting any proposition Q that P tautologically imply according to illocutionary logic.[20] Similarly, $\models$ *Tautological* ($A_p \Rightarrow B_p$) $\Rightarrow$ (*Bel*a$A_p -\!\!\in \neg$*Bel*a$\neg B_p$) in epistemic logic

[20] See "Success, Satisfaction and Truth in the Logic of Speech Acts and Formal Semantics" [2004]

4.5 Laws for strong implication

Strong implication is a stronger kind of propositional implication than strict, tautological and analytic implications. It requires the same or a richer structure of constituents in addition to tautological implication. There are two reasons why a proposition can fail to strongly imply another. Firstly, the second proposition requires new predications. $\models \neg(A_p \geq B_p) \Rightarrow \neg(A_p \mapsto B_p)$. In that case, one can think the first proposition without thinking the second. Secondly, the first proposition does not tautologically imply the other. In that case even if the first implies the second, one can ignore that implication.

So strong implication is finer than analytic implication which does not require tautological implication. $\not\models (A_p \rightarrow B_p) \Rightarrow (A_p \mapsto B_p)$ So $\not\models (A_p \rightarrow B_p) \Rightarrow \mathit{Bel}\mathrm{a}A_p \Rightarrow \mathit{Bel}\mathrm{a}\ B_p$.

Unlike strict and tautological implications, strong implication is anti-symmetric. Consequently, $\models A_p \mapsto B_p \Leftrightarrow ((A_p \wedge B_p) = A_p)$

Strong implication is *decidable*. For $\models A_p \geq B_p$ when all propositional constants which occur in B_p also occur in A_p. And $\models$ *Tautological* $(A_p \Rightarrow B_p)$ when any semantic tableau of S5 modal logic for $(A_p \Rightarrow B_p)$ closes.

Moreover, strong implication is *finite*: every proposition only strongly implies a finite number of others. For it contains a finite number of elementary propositions. In particular, a proposition only strongly implies the tautologies having its elementary propositions. $\models$ *Tautological* $B_p \Rightarrow (A_p \mapsto B_p \Leftrightarrow A_p \geq B_p)$. Similarly a contradiction only strongly the propositions having its elementary propositions. $\models$ *Tautological* $\neg A_p \Rightarrow (A_p \mapsto B_p \Leftrightarrow A_p \geq B_p)$

For all these reasons, strong implication is *a priori known*. $\models (A_p \mapsto B_p) \Rightarrow (\mathit{Bel}\mathrm{a}A_p \Rightarrow \mathit{Certain}\mathrm{a}(A_p \Rightarrow B_p))$. However $\mapsto$ does not obey the rule of *Modus Tollens*. $\not\models (A_p \mapsto B_p) \Rightarrow (\neg B_p \mapsto \neg A_p)$. For $\not\models (A_p \mapsto B_p) \Rightarrow (B_p \geq A_p)$. So $\not\models (A_p \mapsto B_p) \Rightarrow (\mathit{Bel}\mathrm{a}\neg B_p \Rightarrow \mathit{Bel}\mathrm{a}\neg A_p)$

4.6 Natural deduction

Valid laws of inference of natural deduction generate strong implication when their premises contain all propositional constants of their conclusion. Here are some laws:

The *law of introduction of belief* : $\models A_p \mapsto B_p \Rightarrow \mathit{Bel}\mathrm{a}A_p \mapsto \mathit{Bel}\mathrm{a}B_p$

The law of elimination of conjunction : $\models (A_p \wedge B_p) \mapsto A_p$ and $\models (A_p \wedge B_p) \mapsto B_p$

The law of elimination of disjunction : $\models ((A_p \mapsto C_p) \wedge (B_p \mapsto C_p)) \Rightarrow (A_p \vee B_p) \mapsto C_p$

Failure of the law of introduction of disjunction: $\not\models A_p \mapsto (A_p \vee B_p)$.
So strong implication is stronger than *entailment* which obeys the law of introduction of disjunction. Clearly $\not\models A_p \mapsto \mathit{Bel}\mathrm{a}\ (A_p \vee B_p)$.

The *law of introduction of negation*: $\models A_p \mapsto O_t \Rightarrow (A_p \mapsto \neg A_p)$ where O_t is any contradiction.

Failure of the law of elimination of negation: $\not\models (A_p \wedge \neg A_p) \mapsto B_p$

Agents can have relatively inconsistent beliefs. $\not\models (A_p \multimap\!\!\in \neg B_p) \Rightarrow \neg\blacklozenge \textit{Bel}a\ (A_p \wedge B_p)$

They are *paraconsistent*. $\not\models (A_p \multimap\!\!\in \neg B_p) \Rightarrow (\textit{Bel}a\ (A_p \wedge B_p) \Rightarrow \textit{Bel}a\ C_p)$

But they always respect the principle of non contradiction. $\models \neg\blacklozenge \textit{Bel}a(A_p \wedge \neg A_p)$

The *law of elimination of material implication*: $\models (A_p \wedge (A_p \Rightarrow B_p)) \mapsto B_p$

The *law of elimination of necessity* : $\models \blacksquare A_p \mapsto A_p$

The *law of elimination of possibility* : $\models \blacklozenge A_p \mapsto B_p \Rightarrow A_p \mapsto B_p$

4.7 Laws of propositional identity

All the classical *Boolean laws of idempotence, commutativity, associativity* and *distributivity* are valid laws of propositional identity: So $\models \textit{Bel}aA_p = \textit{Bel}a(A_p \wedge A_p)$; $\models \textit{Bel}a\ (A_p \wedge B_p) = \textit{Bel}a(B_p \wedge A_p)$; $\models \textit{Bel}a\neg(A_p \vee B_p) = \textit{Bel}a\ (\neg A_p \wedge \neg B_p)$; $\models \textit{Bel}a\ (A_p \wedge (B_p \vee C_p)) = \textit{Bel}a((A_p \wedge B_p) \vee (A_p \wedge C_p))$ and $\models \textit{Bel}a\ \blacksquare(A_p \wedge B_p) = \textit{Bel}a\ (\blacksquare A_p \wedge \blacksquare B_p)$.

The classical laws of *reduction* are also valid: $\models \neg\neg A_p = A_p$ and $\models \textit{Bel}a\textit{Bel}aA_p = \textit{Bel}aA_p$ Unlike hyperintensional logic, predicative logic does not require that identical propositions be *intensionally isomorphic*.[21] First of all, as I said earlier, the order of predication does not always affect truth conditions. Similarly, the order and number of applications of propositional operations does not always affect the logical form. Clearly, $\models \textit{Bel}a(A_p \Leftrightarrow B_p) = \textit{Bel}a(B_p \Leftrightarrow A_p)$ Intensional isomorphism is too strong a criterion of propositional identity.

However, propositional identity requires more than the *co-entailment* advocated in the logic of relevance. As M. Dunn points out, it is unfortunate that A_p and $(A_p \wedge (A_p \vee B_p)$ co-entail each other.[22] For most formulas of such forms are not synonymous. Co-entailment is not sufficient for synonymy because it allows for the introduction of new sense. $\not\models A_p \mapsto (A_p \wedge (A_p \vee B_p)$. $\not\models \textit{Bel}a\ A_p \mapsto \textit{Bel}a\ (A_p \wedge (A_p \vee B_p)$ in epistemic logic.[23]

References

Anderson A. R., N. Belnap & J. M. Dunn: 1992, *Entailment The Logic of Relevance and Necessity*, Princeton University

Bealer B.: 1982, G. *Quality and Concept* Oxford University Press

Carnap R.: 1956, *Meaning and Necessity*, University of Chicago Press

Cherniak C: 1986, *Minimal Rationality* Bradford Books, MIT Press

[21] See Max J. Cresswell, "Hyperintensional Logic". *Studia Logica* [1975].
[22] See his philosophical rumifications in Anderson *et al* [1992].
[23] A general predicative logic of propositions dealing with generalization, logical and historic modalities, ramified time and action is fully developed in my next book *Logic Truth & Thought*.

Church A.: 1951, “A Formulation of the Logic of Sense and denotation” in P. Henle & *al* (eds) *Structure, Method and Meaning*, Liberal Arts Press

Cresswell M. J.: 1975, "Hyperintensional Logic", *Studia Logica*, 34,: 25-38.

Fine K.: "Analytic Implication", *Notre Dame Journal of Formal Logic*, 27, 2, 1986

Frege G.: 1970, "On Sense and reference" in P. Geach and M. Black (eds), Translations from the philosophical Writings of Gottlob Frege, Blackwell

Hintikka J.: *Knowledge and Belief*, Cornell University Press, 1962

Kaplan D.: 1979, « On the Logic of Demonstratives», *Journal of Philosophical Logic*, 8, 716-29

Kripke S.: 1977, “Speaker Reference and Semantic Reference” in French P.A. *et al* (eds) *Contemporary Perspectives in the Philosophy of Language*, University of Minnesota Press

Barcan Marcus R.: *Modalities* , Oxford University Press, 1993

Lewis C. I.: *A Survey of Symbolic Logic*, University of California Press, 1918

Parry W.T.: "Comparison of Entailment Theories", *The Journal of Symbolic Logic*, 37, 1972

Strawson P.F.: *Subject and Predicate in Logic and Grammar*, Methuen, 1974

Vanderveken D.: *Meaning and Speech Acts*, 1990, Volume 1: *Principles of Language Use* and 1991, Volume 2: *Formal Semantics of Success and Satisfaction*, Cambridge University Press

Vanderveken D.: 2001, “Universal Grammar and Speech Act Theory” in D. Vanderveken & S. Kubo (eds), *Essays in Speech Act Theory*, Benjamins, P&b ns 77, 25-62

Vanderveken D.: 2003, *Formal Ontology, Propositional Identity and Truth According to Predication With an Application of the Theory of Types to Modal and Temporal Propositions Cahiers d’Épistémologie*, Cahier n° 2003-03, 294, Université du Québec à Montréal, http://www.philo.uqam.ca

Vanderveken D.: 2004, “Success, Satisfaction and Truth in the Logic of Speech Acts and Formal Semantics” in S. Davis & B. Gillan (eds) *Semantics A Reader*, Oxford University Press, 710-734

Vanderveken D.: 2005, “Propositional Identity Truth According to Predication and Strong Implication With a Predicative Formulation of Modal Logic” in D. Vanderveken (ed) *Logic, Thought & Action,* Springer,185-216

Vanderveken D.: 2005, "Attempt, Success and Action Generation: A Logical Study of Intentional Action", in D. Vanderveken (dir) *Logic, Thought & Action op. cit.*, 316-342

Vanderveken D.: Forthcoming, *Propositions, Truth & Thought*