# Why a Few? And Why Not *A Many? * 

Stephanie D. Solt, The CUNY Graduate Center<br>ssolt@gc.cuny.edu


#### Abstract

The expressions few and $a$ few are typically considered to be separate quantifiers. I challenge this assumption, showing that with the appropriate definition of few, a few can be derived compositionally as $a+$ few. The core of the analysis is a proposal that few has a denotation as a one-place predicate which incorporates a negation operator. From this, argument interpretations can be derived for expressions such as few students and a few students, differing only in the scope of negation. I show that this approach adequately captures the interpretive differences between few and $a$ few. I further show that other such pairs are blocked by a constraint against the vacuous application of $a$.


## 1 Introduction

The starting point for the present paper is the often-overlooked contrast exemplified below:
(1) a. Few students came to the party.
b. A few students came to the party.
(2) a. Many students came to the party.
b. *A many students came to the party.

The expressions few and many have long been recognized as problematic for treatments of quantification, on account of their vagueness and context dependence (or even ambiguity), and their resistance to classification on the standard dimension of strong versus weak (Milsark 1974; Barwise \& Cooper 1981; Westerståhl 1985; Keenan \& Stavi 1986; Lappin 1988, 2000; Partee 1989; Herburger 1997).
But one idiosyncrasy of few that has received little serious attention (though see Kayne 2005) is that it forms a pair with the superficially similar expression a few, the only such pair in the English count noun quantifier system. In particular, while few and many otherwise exhibit very similar properties, there is no *a many in parallel to a few.
My goal in this paper is to present some interesting facts and contrasts relating to the semantics of few and a few, to show that, despite their differences, a few can be derived from few, and finally to address why $a$ few does not have a counterpart in *a many. I also discuss some broader implications for the semantics of few and many, and of the indefinite article.

### 1.1 Does $a$ few $=a+f e w$ ?

It is not immediately clear that a few should receive a compositional treatment at all. And in particular, it is not obvious that $a$ few is composed of the $a$ in $a$ student plus the few in few students. Within basic accounts of generalized quantifiers (e.g., Keenan \& Stavi 1986) as

[^0]well as introductory semantics texts (e.g., Gamut 1990), the standard if unspoken assumption would seem to be that $a \mathrm{few}$ is an idiom, that is, a fixed, unanalyzable unit.

But on closer examination, it is clear that $a$ few does not always function as a unit: $a$ and few may be separated by an adverb (as in (3)) or, more interestingly, by an adjective modifying the head noun (as in (4)):
(3) a. A very few students got perfect scores on the test.
b. An incredibly few collectors have the good fortune to own one.
a. A lucky few students will get fellowships.
b. We spent a happy few days at John's house in the country.

The conclusion must be that $a$ few is composed of an independent $a$ and few which combine in the syntax; in light of this, a compositional semantic treatment is desirable as well.

### 1.2 Outline of the paper

The organization of the paper is the following. I begin in Section 2 by presenting some facts in the semantics of few and a few that must be captured by a compositional account. In Section 3, I introduce two further properties of few that will prove crucial to the present analysis. Section 4 is the core of the paper, where I present a proposal regarding the semantics of few and the derivation of a few. In Section 5, I address the obvious question that arises: why a few does not have a counterpart in *a many. I summarize in Section 6 with some conclusions and questions for further study.

## 2 The Interpretation of Few and A Few

### 2.1 Basic facts

Considering again examples (1a) and (1b), it can be observed that these sentences have overlapping truth conditions: Both are true if some small but unspecified number of students attended the party. But from there, the interpretations of few and a few diverge.
Specifically, diagnostics such as those proposed by Horn $(1989,2003)$ show that few is defined by its upper bound. That is, few means at most some maximum value. Thus for example "few students came to the party" can be followed felicitously by "in fact, hardly any did" and so forth, but not by "in fact, many did" or the like, evidence that the former but not the latter are encompassed within the possible interpretations of few:
(5) a. Few students came to the party; in fact, hardly any/almost none/only one did.
b. Few students came to the party; in fact, *many/*lots/*dozens did.

Likewise, (6) can only mean that I'm surprised that more students did not come to the party:
(6) I'm surprised that few students came to the party.

Furthermore, although speakers' intuitions differ with regards to this point, similar diagnostics show that few can even be none. Thus suppose I make you the bet in (7). If it later turns out that no students come to the party in question, it would seem that I have won the bet.
(7) I'll bet you that few students will come to the party.

Few is therefore monotone decreasing in its right argument, as seen by the validity of the entailment in (8a), and thus licenses negative polarity items, as in (8b):
(8) a. Few students in the class own cars. $\Rightarrow$ Few students in the class own red cars.
b. Few students in the class have ever owned a car.

A few, by comparison, has essentially mirror image semantics relative to few. A few is defined by its lower bound. It is existential (in that it must be non-zero), and marginally allows an "at least" reading, similar to the cardinal numbers. Thus for example "a few students came to the party" can be continued with "in fact, many did," but not with "in fact, none did" or "in fact, one did":
(9) a. A few students came to the party; in fact, many/lots/dozens/over twenty did.
b. A few students came to the party; in fact, *none/*one/(?) two did.

Similarly, (10) seems to mean that I am surprised that any students at all came to the party (or perhaps that I am surprised that some particular students attended, a point that I will not address here).
(10) I'm surprised that a few students came to the party.

On its "at least" reading, a few is therefore monotone increasing (as seen in (11a)), and thus does not license negative polarity items (as in (11b)):
(11) a. A few students in the class own red cars. $\Rightarrow$ A few students in the class own cars.
b. *A few students in the class have ever owned a car.

Finally, for completeness, I consider also many, which will be relevant below. As seen through the contrasts in (12), many is lower bounded like a few, but of course specifies a larger number of individuals:
(12) a. Many students came to the party; in fact, dozens/hundreds did.
b. Many students came to the party; in fact, *none/*one/*a few did.

Within a generalized quantifier framework (Barwise \& Cooper 1981), the above facts might as a first approximation be summed up by the expressions in (13) as the denotations of few, $a$ few and many.
a. $\llbracket$ few $\rrbracket=\lambda \mathrm{P} \lambda \mathrm{Q}(|\mathrm{P} \cap \mathrm{Q}| \leq \mathrm{n}$, where n is some small number $)$
b. $\quad$ a few】 $=\lambda \mathrm{P} \lambda \mathrm{Q}(|\mathrm{P} \cap \mathrm{Q}| \geq \mathrm{m}$, where m is some small number $\geq 2)$
c. $\llbracket m a n y \rrbracket=\quad \lambda \mathrm{P} \lambda \mathrm{Q}(|\mathrm{P} \cap \mathrm{Q}| \geq \mathrm{p}$, where p is some large number $)$

But this approach does not provide an account of the relationship of a few to few. Nor is it apparent why a few does not have a counterpart in * a many.

### 2.2 Some additional complexities

Beyond these issues, there are some further subtleties that the expressions in (13) do not adequately capture. As is now well known, the semantics of few is notoriously difficult to specify precisely (Partee 1989). In some contexts, few would appear to have a proportional interpretation. For example, the intuition seems to be that few Americans in (14a) could refer to a larger number of individuals than few senators in (14b), which in turn could be a larger number than few students in my class in (14c) (assuming a class of ten students or so).
(14) a. Few Americans voted for Ralph Nader in 2004.
b. Few senators supported the bill.
c. Few students in my class solved the problem.

In fact, (14a) is clearly true - and perfectly felicitous - in a situation where one hundred thousand Americans (out of millions) voted for Nader in 2004. These facts could be readily be captured by giving few proportional semantics, so that few $N$ is interpreted as "a small proportion of the Ns."

But the situation is not as simple as this: In other contexts, few has a purely cardinal interpretation, where few $N$ could be paraphrased as "a small number of Ns." On this reading, few $N$ could even be all of the Ns. Thus for example (15) could best be paraphrased as "a small number of truly qualified candidates applied," rather than "a small proportion of all qualified candidates applied."
(15) Few truly qualified candidates applied for the position.

In fact, (15) could be judged true if there were only a small number of really qualified candidates (perhaps because the job requirements were particularly onerous), and all of them applied.

Likewise, (16), an example from Partee (1989), could be true if there were only a small number of faculty children in 1980, and all of them were at the picnic.
(16) There were few faculty children at the 1980 picnic.

The possibility of a cardinal reading for few is particularly clear when it appears in object position. Thus (17) means that my reasons are small in number, not that of all such reasons I subscribe to only a small proportion.
(17) I have few reasons to trust John.

Along with its difficult-to-specify interpretation, few also exhibits inconsistent formal properties. On the most simple test, namely allowability in there-insertion contexts (Milsark 1974), few can be classified as weak, patterning with other weak determiners such as some or no:
(18) There are few cars in the parking lot.

But as is well known, few does not possess the properties characteristic of prototypical weak determiners (Barwise \& Cooper 1981; Lappin 1988, 2000; Partee 1989). One such property is symmetry. As an example of symmetry, the two sentences in (19a) are logically equivalent. But it is not as clear that the equivalence in (19b) holds, and it is obvious that the one in (19c) does not:
(19) a. Some students are anarchists. $\Leftrightarrow$ Some anarchists are students.
b. ?Few students are anarchists. $\Leftrightarrow$ Few anarchists are students.
c. Few women are great-grandmothers. $\Leftrightarrow$ Few great-grandmothers are women.

Similar issues arise with other characteristic properties of weak determiners, such as intersection and persistence/antipersistence (upward/downward monotonicity in a determiner's left argument).

Finally, few does not even appear to possess the property of conservativity, long argued to be a universal characteristic of natural language determiners (Barwise \& Cooper 1981). Thus consider (20), based on a well-known example from Westerståhl (1985).

Few Americans have won the Nobel Prize in Physics.
The number of Americans who have won the Nobel Physics prize - and certainly the proportion - is without doubt small. Nevertheless, on one reading, (20) could be judged false if Americans make up a large proportion of the winners. But if the cardinality of the predicate is factored into the truth conditions of a sentence such as this, conservativity does not obtain.

Importantly, the interpretation of $a$ few is largely free of these complexities. To start with, $a$ few is purely cardinal. Regardless of the context or the nominal expression with which it combines, a few specifies a small number of individuals in an absolute sense. Thus (21a-c) could all be judged true if a handful of individuals within the domain (Americans, senators or students in my class) satisfied the predicate:
(21) a. A few Americans voted for Ralph Nader in 2004.
b. A few senators supported the bill.
c. A few students in my class solved the problem.

Furthermore, in a situation in which one hundred thousand Americans voted for Nader in 2004, (21a) is pragmatically odd if not actually untrue, evidence that a few does not exhibit the proportionality that I have shown is characteristic of few.
A few, like few, can be classified as weak, as seen by the acceptability of (22a). But unlike few, it displays the characteristic properties of this class, such as symmetry, as seen by the equivalence in (22b):
(22) a. There are a few cars in the parking lot.
b. A few senators are anarchists. $\Leftrightarrow$ A few anarchists are senators.

Finally, a few is clearly conservative; for example, the truth or falsity of (23) cannot depend on the total number of prize winners.
(23) A few Americans have won the Nobel Prize in Physics.

In short, a few is altogether a better-behaved expression than few. Any attempt to establish a compositional relationship between the two must capture this fact.

## 3 Two Crucial Properties

In this section, I introduce two further properties of few (and in parallel, many) that will serve as the starting point for the analysis to follow.

### 3.1 Few and many are adjectives

Within a standard generalized quantifier framework (Barwise \& Cooper 1981), all noun phrases are uniformly represented as objects of semantic type $\langle\langle e, t\rangle, t\rangle$, such that "quantificational determiners" - including few and many - must have the semantic type $\langle\langle e, t\rangle,\langle\langle e, t\rangle, t\rangle\rangle$. But this uniform approach has been challenged in other frameworks which distinguish indefinites from truly quantificational expressions, holding that the former are not inherently quantificational (Kamp 1981; Heim 1982; Landman 2004).
While the status of few and many with regards to this dichotomy is not completely clear (an issue which itself merits further investigation), in one respect there is clear evidence that these terms do not always have the semantics of determiners which produce generalized quantifiers: In many respects, few and many exhibit the morphological properties and syntactic distribution of adjectives rather than determiners (Hoeksema 1983; Partee 1989; Kayne 2005).

To begin with the most basic facts, both few and many pattern with adjectives in having comparative and superlative forms:
fewer, fewest; more, most (cf. taller, tallest)

Both may combine with degree modifiers:
(25) so few/many; too few/many; very few/many (cf. so/too/very tall)

Both may appear in predicative position:
(26) His good qualities are few/many (cf. numerous/evident/remarkable)

Both may be sequenced after determiners other than $a$ :
(27) a. The few/many advantages of his theory (cf. the important advantages)
b. His few/many friends (cf. his close friends)
c. Those few/many students who understood the problem (cf. those smart students...)

Finally, perhaps the most convincing evidence, both may be conjoined with other adjectives:
a. Study shows few - and small - inheritances for baby boomers.
b. Precious and few are the moments we two can share.
c. ... the many and complex processes involved in the development of an organism...

Since few and many exhibit the morphosyntactic behavior of adjectives, it is also desirable to represent them semantically as adjectives (i.e., noun modifiers), rather than as determiners. (For a related proposal, see Partee 1989, where few and many in their cardinal interpretations are associated with adjectival semantics.) Such an approach aligns these expressions within the broader treatment of indefinites as not inherently quantificational. In particular, this view of few and many finds a parallel in recent semantic analyses of cardinal numbers as noun modifiers lacking in quantificational force (Link 1983; Krifka 1999; Ionin \& Matushansky 2004; Landman 2004).

### 3.2 Few is negative

A second crucial fact about few is that it is negative. This is in one sense an obvious point, and certainly not a new one. As early as Barwise \& Cooper (1981) we find the proposal that few can be defined as "not many":

Semantic Postulate: $\llbracket$ few】 $=\neg \llbracket$ many】
More recently, McNally (1998) proposes that few is equivalent to a variant of many which has the morphosyntactic licensing condition that it appear within the scope of clausal negation.

But not all accounts have treated few as explicitly negative. An alternate approach is to represent few and many as opposites, related as $\leq$ is related to $>$. Thus for example Partee (1989) proposes the following as a first approximation of the semantics of few and many in their cardinal interpretations:
(30) $\llbracket$ few $N \rrbracket=\{X:|X \cap N| \leq n\}$, where n is some small number
$\llbracket$ many $N \rrbracket=\{X:|X \cap N|>n\}$, where $n$ is some large number
Lappin $(1988,2000)$ similarly remarks that the denotation of few can be obtained from that of many by replacing > with $\leq$ in the relevant formula.

Now, it is not immediately apparent that the distinction between Partee's and Lappin's approach (few and many as opposites) and that of Barwise \& Cooper and McNally (few as the negation of many) is an important one. After all, there is an obvious equivalence between the two, stemming from the equivalence of a formula of the form $|\mathrm{X} \cap \mathrm{N}| \leq \mathrm{n}$ to one of the form $\neg|\mathrm{X} \cap \mathrm{N}|>\mathrm{n}$. Thus we can of course move transparently from one type of definition to the other. But on another level, the difference between these two approaches is a more fundamental one. In the expressions in (30), few and many are of equal status; either one can be viewed as the opposite of the other. But with semantics such as Barwise \& Cooper's (29), many is the primary term, while few is derived from it. Or to put this differently, the denotation of few includes an additional element that is not present in that of many, namely a negation operator. This is a basic asymmetry between the two expressions, which we might predict would have syntactic or semantic consequences. Thus it seems to matter which of these two approaches to few we choose.

I would like to argue that there is ample evidence that few is in fact negative, and should be represented as such. As a first point of support, the syntactic distribution of few parallels that of explicitly negative expressions. On standard tests for negativity (e.g., Klima 1964), few patterns with overtly negative quantifiers such as $n o$, rather than positive quantifiers such as some or many. For example, few, like no, takes either rather than too tags:
a. Some men like Brussels sprouts, and some women do, $\checkmark$ too/*either. (POS)
b. Many men like Brussels sprouts, and many women do, $\checkmark$ too/*either. (POS)
c. No men like Brussels sprouts, and no women do, *too $/ \checkmark$ either. (NEG)
d. Few men like Brussels sprouts, and few women do, $*$ too $/ \checkmark$ either. (NEG)

Few is also similar to no and other negative expressions in being somewhat awkward in object position, at least in colloquial speech. In either case, the most natural way to express the same proposition would be by means of an explicit negator higher in the clause:
a. ?He has no books. $\quad \rightarrow \quad$ He doesn't have any books.
b. ?He has few books. $\quad \rightarrow \quad$ He doesn't have many books.

Perhaps the most compelling evidence that the representation of few contains a negative component is provided by the existence of so-called "split scope" readings (Jacobs 1980) when it appears in the scope of an intensional verb or modal operator. For example, the most natural reading of (33a) is roughly that given by the paraphrase in (33b), where negation is interpreted outside the scope of the verb need, while many reasons is interpreted as within its scope. This is distinct from the narrow scope or de dicto reading in (33c), where both negation and many reasons are within the scope of need, and which could be paraphrased as "to fire you, they need it to be the case that they have not many (i.e. a small number of) reasons." It is also distinct from the true wide scope or de re interpretation in (33d), where both negation and many reasons scope outside of need, and which could be paraphrased as "to fire you, there are not many (specific) reasons such that they need them."
a. They need few reasons to fire you.
b. "to fire you, it is not the case that they need many reasons"
$\neg>$ need $>$ many reasons
c. "to fire you, they need there to be not many reasons" need > $\neg>$ many reasons
d. "to fire you, there are not many (specific) reasons such that they need them" $\neg>$ many reasons > need

Similarly, (34a) could be best paraphrased by (34b), where negation outscopes the modal operator, which in turn outscopes many reasons:
a. You can have few reasons to doubt my story.
b. "it is not possible that you have many reasons...." $\neg>\diamond>$ many reasons
In light of these facts, as well as the previously discussed distributional patterns, I propose that at the level of logical form, few must be decomposed into a negation operator and a positive term.

## 4 The Semantics of Few and the Derivation of A Few (Or: Why A Few? )

In this section, I build on the conclusions of the previous discussion with a proposal for the formal semantics of few, which I show addresses many of the difficulties discussed above, and also allows a few to be derived in a compositional manner.

### 4.1 Few

I begin with the lattice theoretic framework of Link (1983), in which the domain of individuals is extended to include plural individuals formed as the sums over sets of atomic individuals. Within this framework, the cardinal numbers may be represented as follows (e.g. Landman 2004):

$$
\begin{equation*}
\llbracket \text { three } \rrbracket=\lambda x[|x|=3] \tag{35}
\end{equation*}
$$

Here, three is defined as a one-place cardinality predicate, that is, an expression of type $\langle\mathrm{e}, \mathrm{t}\rangle$.
I propose that a similar approach can be applied to few and many, the primary difference being that these terms require a contextual component to their interpretations. My proposal for the semantics of many and few is given in formal terms in (36):
a. $\llbracket$ many $\rrbracket=\lambda x\left[\operatorname{large}^{\mathrm{C}}(|\mathrm{x}|)\right]$
b. $\llbracket$ few】 $=\lambda x\left[\neg \operatorname{arge}^{\mathrm{C}}(|\mathrm{x}|)\right]$

Here large ${ }^{\mathrm{C}}$ is a contextually defined value that may reflect the size of the domain of quantification, contextual information, prior expectations, and perhaps other factors. To paraphrase (36) in less formal language, few and many thus denote sets of (plural) individuals of (contextually specified) small or large cardinality, respectively.
I further follow Link (1983) in introducing the pluralization operator *, defined as follows for any one-place predicate P :

$$
\begin{equation*}
* P=\{x \in D: \exists Z \subseteq P: x=\sqcup Z\} \text {, where } \sqcup Z \text { is the sum of the elements in } Z \tag{37}
\end{equation*}
$$

With this in place, few and many may combine with a plural noun such as students by intersective modification, giving the following for few:

$$
\begin{align*}
& \llbracket \text { few students }\langle(e, t\rangle  \tag{38}\\
&=\llbracket \text { few】 } \cap \llbracket \text { students } \rrbracket \\
&=\lambda x\left[\neg \operatorname{arge}{ }^{\mathrm{C}}(|\mathrm{x}|) \& * \operatorname{student}(\mathrm{x})\right]
\end{align*}
$$

The resulting expression is again of semantic type $\langle\mathrm{e}, \mathrm{t}\rangle$, a one-place predicate or set of plural individuals (cf. previous non-quantificational treatments of indefinites, e.g. McNally 1998; de Swart 2001; Landman 2004; among other). Beyond this, I assume that the plural morphology on the noun restricts the denotation of few students to proper plural (i.e., non-atomic) individuals; that morphological pluralization can have this effect is seen through the contrast in (39), where (39a) must refer to a single student, while (39b) must be two or more:
a. some student
b. some students

The advantages of this approach to the semantics of few and many are several. First and most obviously, the vagueness and context-sensitivity of their interpretations can be accounted for. In particular, both cardinal and proportional readings of few can be obtained with the appropriate choice of large ${ }^{\mathrm{C}}$, as can the "reverse" reading available for examples such as (20). Secondly, the non-determiner-like properties of few - notably lack of conservativity - receive an explanation: Few is not a determiner, and so it is not surprising that it does not behave like one.

It should be mentioned that there are two important questions that I am not addressing here, the first being precisely how large ${ }^{\mathrm{C}}$ receives its value within a particular context, and the second being whether the denotations of few and many should reference the same or different values. There is much of interest to pursue here, but the definitions in (36) are sufficient for the present purposes.
An issue that must be addressed in this sort of treatment is that, within a classical generalized quantifier framework, an expression of type $\langle e, t\rangle$ such as (38) is not the appropriate type to appear in argument position. Within "adjectival" theories of indefinites, the standard approach to resolving this issue is to invoke a shift to type $\langle\langle e, t\rangle, t\rangle$, an operation that has come to be known as existential closure (Partee 1986; de Swart 2001; Landman 2004). I follow this approach here, using the following definition of existential closure:
（40）Existential closure（EC）
For any one－place predicate $P$ ：
$\mathrm{EC}(\mathrm{P})=\lambda \mathrm{Q} \exists \mathrm{x}[\mathrm{P}(\mathrm{x}) \& \mathrm{Q}(\mathrm{x})]$
I further propose that under existential closure，the negation operator in the underlying semantic representation of few is able to detach and take higher scope，above the existential operator．The necessity of such an operation is separately motivated by the existence of split scope readings，discussed in Section 3.2 above，which provide evidence that the negative component of few is able to take separate scope from the remainder of the expression（though I should note that the precise mechanism by which this occurs requires further investigation）．

$$
\begin{equation*}
\left.\llbracket \text { few students }\langle\langle\mathrm{e}, \mathrm{t}\rangle, \mathrm{t}\rangle \pm=\lambda \mathrm{Q} \underset{\sim}{\neg \mathrm{x}[ } \operatorname{large}^{\mathrm{C}}(|\mathrm{x}|) \& * \operatorname{student}(\mathrm{x}) \& \mathrm{Q}(\mathrm{x})\right] \tag{41}
\end{equation*}
$$

To paraphrase（41），few students at the generalized quantifier level denotes the set of sets （properties）that do not contain an element of large cardinality composed of students，but that may contain a small plural individual composed of students，an atomic member of the set student，or no elements of the set student at all．This seems to capture the meaning of few as it was outlined above；it also correctly follows from（41）that few is monotone decreasing．

## 4．2 A few

With the analysis I have proposed above for few，the derivation of a few－the primary objective of this paper－is now straightforward．
As a first step，it is necessary to take a position on the semantics of the indefinite article $a$ ． While one standard approach would be to say that $a$ introduces existential quantification，here I will again follow recent theories of indefinites as non－quantificational（e．g．Heim 1982； Landman 2004），and propose that the existential force of an expression such as a student（or for that matter，a few students）originates externally，again via an operation of existential closure．As a first approximation（to be revised below），we could therefore view $a$ as a modifier（type $\langle\langle\mathrm{e}, \mathrm{t}\rangle,\langle\mathrm{e}, \mathrm{t}\rangle\rangle$ ）which is semantically vacuous．

Under this view，the semantics of an expression such as a few students at the set level（type $\langle\mathrm{e}, \mathrm{t}\rangle$ ）can now be derived in one of two ways．As the first option，few may first combine with students as above，with $a$ then applying to the resulting combination：

$$
\begin{align*}
& \text { 【students】 } \quad=\lambda x[* \operatorname{student}(\mathrm{x})]  \tag{42}\\
& \llbracket \text { few students }\left\langle_{\langle, \mathrm{e}\rangle} \rrbracket=\lambda \mathrm{x}\left[\neg \operatorname{arge}^{\mathrm{C}}(|\mathrm{x}|) \& * \text { student }(\mathrm{x})\right]\right. \\
& \llbracket \mathrm{a} \text { few students }\left\langle(\mathrm{e}, \mathrm{t}\rangle \pm=\lambda \mathrm{x}\left[\neg \operatorname{large}{ }^{\mathrm{C}}(|\mathrm{x}|) \& * \text { student }(\mathrm{x})\right]\right.
\end{align*}
$$

In this version of the derivation，a few is not a constituent．While this might initially seem counterintuitive，this option is necessary to account for the possibility of positioning a noun modifier between $a$ and few，as in a luckyfew students．

As the second option，a may first combine with few，with the resulting expression then combining with students：

$$
\begin{array}{ll}
\llbracket \text { few } \rrbracket & =\lambda x\left[\neg \operatorname{arge}{ }^{\mathrm{C}}\left(\left.\right|_{\mathrm{x}} \mid\right)\right]  \tag{43}\\
\llbracket \text { a few } \rrbracket & =\lambda \mathrm{x}\left[\neg \operatorname{arge}{ }^{\mathrm{C}}\left(\left.\right|_{\mathrm{x}} \mid\right)\right] \\
\llbracket \text { a few students }\langle(\mathrm{e}, \mathrm{t}\rangle \\
& =\lambda \mathrm{x}\left[\neg \operatorname{arge}^{\mathrm{C}}\left(\left.\right|_{\mathrm{x}} \mid\right) \& * \operatorname{student}(\mathrm{x})\right]
\end{array}
$$

Here the constituency of $a$ few has been restored，a welcome outcome from an intuitive point of view；this option will prove necessary below．

In either case, existential closure may apply to the resulting set expression to yield a generalized quantifier interpretation. Importantly, in this case, I propose that the presence of the indefinite article $a$ blocks the raising of the negator over the existential operator, as occurs in (41). As evidence that $a$ may have this effect, note that a similar pattern is seen when the overt negator not appears within the scope of $a$. For example, (44a) must mean that some students solved the problem; it cannot be true in the case where no students did so, as would be the case if the negator had scope over the existential operator. In this, (44a) contrasts directly with (44b), where negation has sentential scope, and which is clearly true in the case where there were no problem-solvers.
a. A not large number of students solved the problem.
b. It is not the case that a large number of students solved the problem.

I propose that a similar pattern obtains in the case of $a$ few. This gives (45) as the derivation of the generalized quantifier interpretation of $a$ few students:

$$
\begin{align*}
& \llbracket \mathrm{a} \text { few students }\langle\langle\mathrm{e}, \mathrm{t}, \mathrm{t}\rangle  \tag{45}\\
&=\mathrm{EC}\left(\llbracket \mathrm{a} \text { few } \operatorname{students}_{\langle e, t\rangle} \rrbracket\right) \\
&=\lambda \mathrm{Q} \exists \mathrm{x}\left[\neg \operatorname{arge}^{\mathrm{C}}(|\mathrm{x}|) \& * \operatorname{student}(\mathrm{x}) \& \mathrm{Q}(\mathrm{x})\right]
\end{align*}
$$

To express this less formally, a few students is interpreted as the set of sets (properties) that contain a plural individual of not-large cardinality made up of students.
Thus the $a$ of $a$ few does have a semantic contribution, namely to ensure wide scope for the existential operator (that is, to maintain the ordering $\exists \neg$ rather than $\neg \exists$ ).
Before proceeding, it should be noted that the expression in (45) accurately captures the semantics of $a$ few as discussed in Section 2 above.
First, the "at least" interpretation of a few falls out from the semantics of the existential operator: If there is some large plural student individual $y$ within the denotation of the predicate Q , there also must be a not-large plural student individual $y^{\prime}$ (an individual part of $y$ ) within its denotation. This in turn establishes that a few is monotone increasing, as demonstrated above.
Second, regardless of how large ${ }^{\mathrm{C}}$ is interpreted in a given context, the existential in (45) is only guaranteed to pick out the minimal element of the set few students, namely an element of cardinality two. This means that the proportionality or context dependence inherent to few is not passed along to $a$ few. Thus with this analysis we have captured the fact that $a$ few, unlike few, has a purely cardinal interpretation, and thus patterns consistently with weak determiners.
In short, the present analysis of few allows a compositional derivation of $a$ few, and provides a neat account for the interpretive differences between the two.

In turn, facts relating to a few provide further support for the proposal that the denotation of few must include a negation operator. To see this, consider the expressions in (46):
a. Not every student solved the problem.
b. Not many students solved the problem.
c. Not a student solved the problem.
d. Not five minutes later, the professor walked in.
e. Not a few students solved the problem.

We have here a puzzling contrast. In (46a-d), not + quantifier +N specifies a number of individuals smaller than would be specified by quantifier +N . Thus not every student is less than every student, not many students is less than many students, not five minutes later is less than five minutes later, and so forth. But oddly, in (46e) not a few students means more than a few students.

Under the present proposal，an explanation suggests itself：In not a few students，the negator in few is able to cancel with not．To capture this formally，I begin with the standard assumption that not is interpreted logically as the negation operator：

$$
\begin{equation*}
\llbracket \mathrm{not} \rrbracket=\neg \tag{47}
\end{equation*}
$$

Then the denotation of not a few students can be derived as follows：

$$
\begin{align*}
& \llbracket \mathrm{afew} \rrbracket=\lambda \mathrm{x}\left[\neg \operatorname{large}{ }^{\mathrm{C}}(|\mathrm{x}|)\right]  \tag{48}\\
& \text { 【not a few】 }=\lambda \mathrm{x} \neg\left[\neg \operatorname{arge}^{\mathrm{C}}(|\mathrm{x}|)\right] \\
& =\lambda x\left[\operatorname{large}^{\mathrm{C}}(|\mathrm{x}|)\right] \\
& \llbracket \text { not a few students }\langle(e, t\rangle) \rrbracket=\lambda x\left[\operatorname{large}^{\mathrm{C}}(|\mathrm{x}|) \& * \operatorname{student}(\mathrm{x})\right] \\
& \llbracket \text { not a few students }\left\langle\left\langle\langle, t, t, t\rangle=\lambda \mathrm{Q} \exists \mathrm{x}\left[\operatorname{large}^{\mathrm{C}}(|\mathrm{x}|) \& * \operatorname{student}(\mathrm{x}) \& \mathrm{Q}(\mathrm{x})\right]\right.\right.
\end{align*}
$$

This can be paraphrased as the set of sets（properties）that contain a plural individual of large cardinality composed of students．We can compare this back to the denotation of a few students，which references＂a plural individual of not－large cardinality，＂to see that this gets the facts right，giving us an interpretation of not a few that is more than a few．Importantly，if we had not derived a few from few，as proposed，and if we had not specified that few incorporates a negation operator，it is not clear how we could approach capturing the facts in （46）．

## 5 Constraints on the Distribution of $\boldsymbol{A}$（Or：Why Not＊A Many？）

An obvious question arises from the preceding discussion，which can be simply stated as follows：＂Why is there no a many？＂If the indefinite article $a$ is able to combine with a set of plural individuals such as few or few students，we would predict that this process would be more widespread．But of course examples such as the following are bad：
a．＊An every student came to the party．
b．＊A most students came to the party．
c．＊A many students came to the party．
d．＊A three students came to the party．
Now，there is a relatively simple explanation for the ungrammaticality of（49a－b）．Every student and most students are presumably interpretable only at the generalized quantifier level （type $\langle\langle\mathrm{e}, \mathrm{t}\rangle, \mathrm{t}\rangle$ ），not the appropriate type to combine with $a$ ．
But（ $49 \mathrm{c}-\mathrm{d}$ ）are more problematic for the present account．Under the theory proposed here， expressions such as many students and three students－like few students－have interpretations at the level of sets（type $\langle e, t\rangle$ ）．But this implies that they should be able to combine with $a$ ，which in fact they do not．
In addressing this issue，note first that from the set many students，either existential closure alone or the application of $a$ followed by existential closure would produce the same generalized quantifier．This is illustrated in（50）：


Thus in the case of a nominal expression containing many, a does not make a semantic contribution beyond that which obtains through a non-lexical operation of existential closure alone. And the same point could be made for expressions involving the cardinal numbers, such as three students.

This contrasts directly with the case of few. The diagram in (51) recaps the material presented in the previous section. As is seen here, from the set few students, two different generalized quantifiers may be formed: a monotone decreasing expression derived via existential closure (namely few students), and a monotone increasing expression derived via the application of $a$ followed by existential closure (namely a few students).


In light of these observations, I propose the following generalization: The distribution of $a$ is limited by a requirement that $a$, when present, make a semantic contribution. This constraint effectively blocks the derivation of *a many students or *a three students, since in these cases $a$ would not do any semantic "work" for us. However, it is not invoked in the case of few, since the generalized quantifiers few students and a few students have different semantics.
Thus here we see the source of the uniqueness of the pair few/a few: Few is the only lexically simple quantifying expression of the appropriate semantic type whose interpretation is such that the application of $a$ is not vacuous; this follows from the presence of the negation operator, which allows for two different scope relationships between existential operator and negator.

## 6 Conclusions and Further Questions

In this paper, I have proposed an analysis of few as a one-place predicate that incorporates a negation operator. I have shown that this approach allows the compositional derivation of $a$ $f e w$ as $a+f e w$, and accurately captures the differences in interpretation and formal properties between expressions such as few students and a few students. I have further shown that parallel expressions such as *a many and *a five can be blocked by a constraint against the vacuous application of $a$.

In concluding, I will mention several further questions that arise from this analysis. The first relates to an apparent exception to the above-described restriction on the distribution of $a$ : While $a$ cannot directly precede many or the cardinal numbers, this is possible if a modifier intervenes (Ionin \& Matushansky 2004; Kayne 2005):
*(A) great many students came to the party.
a. *(A) lucky five students will win fellowships.
b. It cost me * (a) whole ten dollars.
c. *(An) incredible ten thousand soldiers died in the battle.

What is particularly interesting about these cases is that $a$ is not just allowed, it is required. For example, a lucky five students is fine, but lucky five students is not allowed. One possible explanation is that $a$ is required here for some independent (e.g., syntactic) reason, in which case the existence of these constructions would be further evidence that $a$ may combine with a plural expression. A second possibility is based on the observation that, in their requirement for an overt indefinite article, expressions such as great many students or lucky five students
show precisely the behavior of singular count nouns such as student, raising the question of whether they could in some respect be singular.
Finally, this paper began with a particular contrast between few and many. There are several other puzzling contrasts of this nature that also would benefit from further investigation. For example (Kayne 2005):
a. He visits every few/*many days.
b. Another few/*many students won fellowships.
c. The same few/*many students always get the best scores.

One approach would be to explore whether the present account of a few versus *a many could be extended to capture these facts as well. However, there is one fact that suggests a different analysis will be required: With respect to combination with $a$, the cardinal numbers pattern with many rather than few, but in the constructions in (54), they pattern with few (e.g., such that every five days is entirely acceptable). I must leave this question as a topic for future research.

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