Partitives, multipliers and subatomic quantification[1]

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Abstract. In standard lattice-theoretic approaches to natural language (e.g., Link, 1983, Landman, 2000, Champollion, 2017) singularities and pluralities are presumed to involve two distinct mereological structures and it is commonly supposed that quantificational expressions do not access subatomic part-whole relations. In this paper, I argue that i) certain quantificational expressions are sensitive to subatomic part-whole structures, ii) quantification over parts is subject to identical restrictions as quantification over wholes and iii) counting presupposes certain topological relations. I present new evidence in favor of a mereotopological approach to natural language (cf. Grimm, 2012) as well as novel data concerning the interaction between quantification and subatomic part-whole relations.

Keywords: mereology, mereotopology, quantification, partitivity, countability.

1. Introduction

There is an ontological intuition dating back at least to Pre-Socratics that entities are often made up of smaller entities, i.e., parts, related to each other in a particular manner (see Varzi, 2016 for an historical overview). This is in accord with a cognitive fact that humans conceive entities as being made up of smaller entities related to each other in a particular manner (e.g., Elkind et al., 1964). The vital question this article will attempt to address is to what extent this fact is relevant for natural language semantics.

In this paper, I will provide novel evidence that natural language semantics is sensitive to subatomic part-whole structures and argue that SUBATOMIC QUANTIFICATION, i.e., quantification over parts, is subject to identical restrictions as quantification over wholes. Specifically, I will postulate that counting presupposes particular topological relations that cannot be captured in frameworks grounded in standard mereology but rather call for a more fine-grained mereotopological account (see Casati and Varzi, 1999 and Grimm, 2012). The evidence comes from the cross-linguistic distribution of proportional quantifiers, different types of partitives in Italian and Polish as well as multipliers such as English *double*. Though the focus of the article is limited to concrete nouns only, I believe that the novel perspective presented here will eventually allow to develop also a proper approach to the semantics of abstract nouns.

The paper is structured as follows. In Section 2, I briefly revise several standard assumptions in mereology that faced criticism due to their inadequacy with respect to modelling wholes. In Section 3, I present a conceptual framework consisting of a meretopological approach, general counting principles and constraints on subatomic quantification. Through this lens, I will dis-

[1]I would like to sincerely thank Viola Schmitt and Nina Haslinger for insightful comments and inspiring discussions. Furthermore, I am grateful to Muriel Assmann, Jonathan Bobaljik, Daniel Büring, Pavel Caha, Lucas Champollion, Mojmír Dočekal, Jakub Dotlačil, Hana Filip, Enrico Flor, Katie Fraser, Izabela Jordanoska, Ora Matushansky, Erlinde Meertens, Martin Prinzhorn, Max Prüller, Magdalena Roszkowski, Yasu Sudo, Peter Sutton and Guy Tabachnick as well as the audience at SuB23 for their helpful questions, comments, judgments and discussion of the data. All errors are, of course, my own. The paper is based on certain issues discussed in my Ph.D. thesis on subatomic quantification (Wągiel, 2018). I gratefully acknowledge that the research was supported by a Czech Science Foundation (GAČR) grant to the Department of Linguistics and Baltic Languages at the Masaryk University in Brno (GA17-16111S).

Proceedings of Sinn und Bedeutung 23, vol. 2, pp. 443–460. Universitat Autònoma de Barcelona, Bellaterra (Cerdanyola del Vallès).

cuss novel evidence for a mereotopological account for nominal semantics presented in Section 4. In particular, I will examine the parallelism between partitive constructions involving singulars and plurals, partitives with Italian irregular plurals, Polish topology-sensitive proportional quantifiers and, finally, multipliers. In Section 5, I will propose a semantics for a significant subset of the discussed expressions which will be based on mereotopology. Finally, Section 6 concludes the article.

2. Mereology and it limits

2.1. Standard assumptions in lattice-theoretical approaches to natural language

Since Link (1983) the main stream of the research on pluralities is grounded in standard mereology.[2] It is commonly assumed that there is only one primitive notion, i.e., the PARTHOOD relation ($\sqsubseteq$), which is accompanied with the derived notion of SUM ($\sqcup$).[3] A consequence of such a move is that in models employing mereology entities are essentially equivalent to sums of their parts neglecting the manner in which those parts are arranged.[4]

Another standard assumption that dates back to Link is that of SORTED DOMAINS distinguishing between the domain of individuals and the domain of portions of matter over which different parthood relations are defined, i.e., the individual parthood relation $\sqsubseteq_i$ as opposed to the material parthood relation $\sqsubseteq_m$, respectively. Those domains are related via mapping between the two.[5] Though $\sqsubseteq_i$ and $\sqsubseteq_m$ are analogous in a sense, formally they are different relations.[6]

The final standard assumption in theories of plurality to be discussed here concerns ATOMICITY. Though particular theories differ significantly, the mass/count distinction is usually accounted for by postulating atoms, i.e., minimal building blocks, in the denotations of count nouns and the lack thereof in the denotations of mass nouns.[7] Atoms are typically defined as mereological entities that have no proper parts.

The combination of the assumptions described above results in that there is virtually no relationship between $\sqsubseteq$ and the intuitive notion of parthood as expressed by the use of the English phrase *part of*. Though certain semanticists argue that this is a flaw of mereology (e.g., Moltmann, 1997), the mainstream response is that formal notions should not be constrained by the meaning of natural-language expressions they might be inspired by (e.g., Pianesi, 2002 and Champollion, 2017). In principle, I agree with such an argument. However, it does not rule out a possibility that the way how an expression such as *part of* is understood can give us certain hints with respect to how to model parthood so that it fits well natural language. As I will argue in the following sections, I believe we can gain a lot by exploring such a possibility.

[2]But see, e.g., Schwarzschild (1996) for a system based on set theory.
[3]See also Champollion and Krifka (2016) for an overview of systems taking $\sqcup$ to be the primitive and deriving $\sqsubseteq$.
[4]Opposing views have been expressed, e.g., by Grimm (2012) and Sutton and Filip (2017).
[5]The distinction can be also postulated in the domain of eventualities where it distinguishes between events and processes, as proposed by Bach (1986).
[6]Again, dissenting views have been expressed in the literature, e.g., by Moltmann (1997) and Landman (2016).
[7]Notable exceptions include, e.g., Krifka (1989), Grimm (2012) and Landman (2016).

2.2. Criticism of mereology

As pointed out by many authors, one factor that motivates extending standard mereology with auxiliary notions is its inadequacy in modeling objects in the real world. In particular, mereology is committed to unrestricted sum formation and as such it is insufficient to capture what it means to be a whole. This results in diametrical discrepancies between intuitions regarding the nature of entities in the world and objects mereology actually delivers. To use Casati and Varzi (1999)'s example, imagine a cup and broken glass. Intuitively, the first constitutes an object, i.e., an individuated whole, something that counts as one, whereas the other is just a collection of shards. However, this distinction cannot be captured by describing entities purely in terms of parthood. This is because in mereology for every whole there is a set of parts and for every arbitrary collection of parts there is their sum, i.e., a complete whole. As a result, a cup and broken glass have the very same mereological status. In other words, the allowance of scattered entities makes it impossible to differentiate between them and individuals constituting integrated wholes. Consequently, mereology has faced criticism that in principle it fails as a theory of individuals.

Before I move on to the discussion of novel evidence indicating that standard mereology is insufficient for a proper account for nominal semantics in natural language and instead a more subtle mereotopological perspective is required, I will introduce the key elements of the conceptual background I assume here.

3. Conceptual background

3.1. Mereotopological notions in natural language

It is trivial to state that topological notions play an important role in natural language. The existence of locative prepositions such as *inside*, *near* and *between* shows that relations concerning how we conceptualize position and space are deeply rooted in grammar. Though the research on locatives is well-established and contributed to our understanding what means language uses to encode information regarding location of objects with respect to each other (e.g., Zwarts and Winter, 1997 and Kracht, 2002), the question concerning spatial constitution of entities remains somewhat elusive in the study of meaning. One line of argumentation justifying why this kind of issues should remain unaddressed is that they simply stem from every-day world knowledge, and thus as extra-linguistic factors are not supposed to be incorporated into semantic theory (e.g., Schwarzschild, 1996).

At first sight, such an approach might seem plausible. However, there are a number of natural language expressions that are sensitive to topological properties of part-whole structures corresponding to their referents. For instance, recent research on different types of collective nouns and mass terms points to the relevance of topology in nominal semantics. In particular, Grimm (2012) proposes that a subset of mass nouns that denote aggregates of granular objects such as *rice* and *gravel* involve reference to clustered individuals, i.e., bundled entities transitively connected to each other. Similar, Henderson (2017) argues that swarm nouns such as *grove* and *horde* denote large pluralities whose constituents remain in proximity within a certain spatial configuration. Finally, as proposed by Grimm (2012) referents of concrete count nouns can be viewed as entities whose part-whole structure is constrained in such a way that it forms an integrated object in its own right.

The distinction between integrated objects and arbitrary sums can be captured by means of mereotopology, i.e., mereology augmented with topological notions.[8] The crucial topological notion for the purpose of this paper is CONNECTEDNESS (C). This relation is introduced in such a way that it interacts with other definitions and axioms of standard mereology. It is reflexive and symmetric (but not transitive) and it is implied by OVERLAP (O).

Given such an extension, a number of mereotopological properties can be defined in order to enable us to draw subtle distinctions between different spatial configurations that entities may be in. One of such notions is INTERNAL PART (IP), as defined in (1). For instance, an individual *a* is an internal part of an individual *b* if every entity that is connected to *a* overlaps with *b* or, in other words, *b* includes *a*.

(1) $\textsc{ip}(x,y) \stackrel{\text{def}}{=} x \sqsubseteq y \wedge \forall z[\textsc{c}(z,x) \rightarrow \textsc{o}(z,y)]$

This allows us to define the notion of INTERIOR (*i*), see (2), which is taken to be the sum of an individual's internal parts.

(2) $ix \stackrel{\text{def}}{=} \sqcup X$ where $X = \{y \ : \ \textsc{ip}(y,x) = \textsc{True}\}$

As a result, it is possible to derive even more complex properties in order to distinguish between entities which come in one piece as opposed to arbitrary sums which bear no topological commitments. For this purpose, it is essential to introduce the property of SELF-CONNECTED (SC), see (3). Unlike arbitrary sums, SC entities cannot be divided into separated parts since an entity is self-connected if and only if any two parts that form the whole of that entity are connected to each other.

(3) $\textsc{sc}(x) \stackrel{\text{def}}{=} \forall y \forall z[\forall w(\textsc{o}(w,x) \leftrightarrow (\textsc{o}(w,y) \vee \textsc{o}(w,z))) \rightarrow \textsc{c}(y,z)]$

Though self-connectedness is a great improvement, this notion is still insufficient to capture what it means to be an integrated whole since it allows for configurations involving only external connection. In order to rule out cases in which objects only touch each other, it is necessary to guarantee that not only boundaries of parts of a whole are connected to each other but also that their internal parts are shared. This can be achieved by the property of STRONGLY SELF-CONNECTED (SSC), as defined in (4).

(4) $\textsc{ssc}(x) \stackrel{\text{def}}{=} \textsc{sc}(x) \wedge \textsc{sc}(ix)$

Finally, a proper treatment of integrated wholes should accommodate both topological integrity and mereological maximality. Also, integrity should be evaluated relative to a property. Hence, the notion of being MAXIMALLY STRONGLY SELF-CONNECTED (MSSC), see (5), provides the final mereotopological definition of what a whole is. In prose, if an entity satisfies MSSC, then it is the largest strongly self-connected entity satisfying that property.

(5) $\textsc{mssc}(P)(x) \stackrel{\text{def}}{=} P(x) \wedge \textsc{ssc}(x) \wedge \forall y[P(y) \wedge \textsc{ssc}(y) \wedge \textsc{o}(y,x) \leftrightarrow y \sqsubseteq x]$

With this tool in hand, we can distinguish between integrated wholes and other types of entities.

[8] See Casati and Varzi (1999) for an excellent survey and Grimm (2012) for the first application to natural language.

3.2. General counting principles

Usually in the literature on the mass/count distinction, at some point when it comes to defining what counting is something like ‘counting is quantification over what counts as one’ is stated. Typically, ‘what counts as one’ is understood in terms of atomicity, i.e., counting is often implicitly assumed to be simply establishing one-to-one correspondence between numbers and atoms. However, such an assumption ignores certain properties of the cognitive mechanism invovled in counting (see Dehaene, 1997 for an overview). Therefore, I will attempt to provide a more explicit characterization of what counts as one. I propose that counting is a quantificational operation that is governed by three general principles restricting what kind of object can be assigned a number. I will refer to those constraints as the principle of NON-OVERLAP, MAXIMALITY, and INTEGRITY.[9]

The principle of non-overlap ensures that things that count as one need not overlap, i.e., do not share a part (cf. Landman, 2011, 2016). Guaranteeing disjointness of units of counting is necessary to avoid a possibility of an entity being counted twice. For instance, assume portions of matter a, b, c and d arranged in such a way that c overlaps with both a and b, specifically $c = a \sqcup b$. Now, one could imagine an operation that would assign numbers to all a, b, c and d. Summing them up would yield number 4 but this result is incorrect if one wanted to count how many portions of matter there are. The reason is that c is not disjoint from a and b, and thus should not be associated with a number.

The second principle concerns maximality. It states that counting requires that what is associated with a number needs to be a maximal entity of which a certain property holds. In other words, objects need to be counted in their entirety, i.e., it is disallowed to leave some parts out. To illustrate this, let us consider three distinct entities a, b and c such that c consists of two parts d and e. Now, assume a quantificational operation that satisfies the non-overlap constraint but is not restricted by the principle of maximality. When applied to the set of entities in question, it might very well yield 4 since a, b, d and e are disjoint, whereas c shares a part with both d and e. However, such an operation would not be of great help if one wanted to know how many entities are there since it fails to differentiate between wholes and their parts.[10]

Finally, the principle of integrity requires what counts as one to be conceptualized as an integrated whole, i.e., parts of an entity need to be viewed as being connected. This means that scattered entities such as substances and arbitrary sums of individuals normally are not assumed to count as one. With this respect counting differs from measuring which is not sensitive to integrity (cf. Rothstein, 2017). To illustrate this, let us consider a scenario in which someone has spilled some liquid on an empty table in such a way that there are two separate blobs a and b whose volume is 1.5 milliliters each. Though the statement in (6a) is true despite the fact that one of the 3 milliliters must be split between a portion of a and a portion of b since each of the blobs consists of 1.5 milliliters of liquid, (6b) is simply false.

[9] In accordance with the focus of this paper the proposed principles are intended to account for quantification over concrete physical objects though ultimately they would need to be more abstract in order to account for phrases such as *two ideas*. However, I will not pursue this issue here.

[10] Notice also that this constraint accounts for how we count homogeneous entities such as twigs and rocks. Given a particular counting situation, what counts as one is always the maximal entity irrespective how its part-whole structure is construed in that situation.

(6) a. There are three milliliters of liquid on the table.
 b. #There are three objects on the table.

The contrast discussed above shows that units of measurement such as milliliters have different properties than objects. In other words, measuring, unlike counting, does not care about individuation in terms of integrity and it indeed appears to be a distinct operation.

3.3. Subatomic quantification

There is yet another reason to believe that defining counting as quantification over atoms, i.e., entities that have no proper parts, is not on the right track since natural language involves expressions dedicated to subatomic quantification, i.e., quantification over parts of entities. For instance, certain types of partitives and multipliers like English *double* to be discussed in Section 4 trigger this kind of quantification. I argue that the proposed set of counting principles constitutes a universal mechanism that can be applied not only to whole individuals but also when counting parts of objects. In other words, I posit that subatomic quantification is subject to the very same constraints as quantification over wholes. In particular, counting at the subatomic level presumes non-overlap, mereological maximality and topological integrity of entities subject to quantification.

Typically, counting is sensitive to the fact that some parts are cognitively more salient within the part-whole structure of an object than others. This seems to correspond to what Champollion and Krifka (2016) call structured parthood as well as to the distinction between specific and arbitrary subdivisions of a whole into parts (e.g., Markosian, 1998). Though what counts as cognitively salient parts may vary with respect to a particular context, what such parts have in common is that given a particular context they need to be disjoint. Thus, only those divisions of a whole into parts that involve non-overlapping parts can be enumerated.

Another issue concerns the principle of maximality. What counts as part of an object can also consist of smaller parts. Notice that such parts of parts also satisfy the property of being part of that object. However, the general counting principles require that only entities in their mereological entirety can can be associated with a number. This means that once a particular division of an individual into parts has been executed in a given counting context, those parts are immutable and treated as objects in their own right. Consequently, the principle of maximality applies as it would in a situation when one counts whole individuals. In other words, given a partition, non-overlapping parts are assumed to be maximal with respect to how the whole has been divided.

Finally, countability is also assumed to be governed by the principle of integrity. Note that extensions of expressions referring to parts of objects do not necessarily involve topological commitments, i.e., parts need not be continuous. For instance, there is definitely a sense in which two or more separated portions of matter within an object are part of that object. Nevertheless, similar to any arbitrary sum such an entity is not countable since associating it with a number would clearly violate the principle of integrity. In other words, a random collection of fragments of an object is not A PART of that object (cf. Acquaviva, 2008: 90–93, Champollion and Krifka, 2016). Therefore, only sets including parts that are mereologically maximal integrated entities that do not overlap can be enumerated, i.e., only a subset of possible divisions of an object is fit for counting.

Given the conceptual framework sketched here, let us turn to the question whether there is linguistic evidence that natural language is sensitive to the notions and constraints introduced. In the next section, I will argue that there is such evidence.

4. Novel evidence for a meretopological approach to natural language

4.1. Unified part-whole structure

Moltmann (1997) observes an analogy between partitives involving singular and plural terms. For instance, in German the same expression *Teil* 'part' can be used both in partitive constructions involving a singular and a plural DP in order to quantify over parts of atomic individuals and subsets of groups of individuals, respectively, see (7). In particular, in (7a) *Teil* quantifies over material parts of singular objects, i.e., functional parts or portions of a substance making up the whole individual, whereas in (7b) it quantifies over whole individuals. This fact indicates a unified parthood structure for both singular and plural individuals.

(7) a. Teil des Apfels
part of-the apple$_{\text{GEN}}$
'part of the apple'
b. Teil der Äpfel
part of-the apples
'some of the apples'

Though for some reason the analogy does not hold in the case of the English quantifier *part* (Schwarzschild, 1996: 165–166), the pattern is cross-linguistically widespread and can be observed at least in Slavic, Germanic, Romance, Celtic, Ugric, Semitic and Basque (see Wągiel, 2018 for the data). Moreover, it is systematic in that it holds also for other proportional quantifiers, see (8)–(9).

(8) a. most of the apple
b. half of the apple
c. two thirds of the apple

(9) a. most of the apples
b. half of the apples
c. two thirds of the apples

Furthermore, in many different languages partitives involving number-neutral expressions such as object mass nouns and pluralia tantum display an ambiguity between a singular and plural reading, see (10).

(10) a. část obuvi
part footwear$_{\text{GEN}}$
'part of the footwear/some of the footwear'
b. část nůžek
part scissors$_{\text{GEN}}$
'part of the scissors/some of the scissors'

A similar effect is reported to appear in some partitive constructions in languages with general number such as Japanese (Sauerland and Yatsushiro, 2004, Watanabe, 2013). For instance, despite the fact that the nominals in (11) are number-neutral, both sentences are ambiguous between a singular and plural interpretaton.

(11) a. Ringo-no ichibu-ga kusatteiru.
apple-GEN part-NOM is.rotten
'Part of the apple is rotten/Some of the apples are rotten.'
b. Ringo-no hotondo-ga kusatteiru.
apple-GEN most-NOM is.rotten
'Most of the apple(s) is/are rotten.'

The evidence discussed above indicates that in many languages proportional quantifiers are able to simultaneously access elements making up pluralities of individuals as well as subatomic part-whole structures. This suggests a unified part-whole structure for both plural and singular entities, contrary to what is typically assumed in mereological approaches to natural language. However, before drawing a conclusion let us look more closely at the discussed partitives.

4.2. Counterargument from the interaction with numerals

As observed by Schwarzschild (1996), nominal expressions such as *part* have different properties depending on whether they occur in partitive constructions with singular or plural DPs. Specifically, they are countable only if they refer to material parts of a singular object. For instance, the Italian phrase (12b) cannot be interpreted as referring to three subsets of the relevant set of the walls and can only have a meaning similar to (12a), i.e., denoting three material parts of the walls making up the plurality. This kind of behavior is cross-linguistically widespread.

(12) a. tre parti del muro
three parts of-the wall
'three parts of the wall'
b. #tre parti dei muri
three parts of-the walls
Intended: 'three subsets of the walls'

A possible consequence of the effect observed in (12) concerns the ontology encoded in the part of natural language semantics that deals with parthood. Specifically, what is implied by the evidence is that after all singularities and pluralities involve two distinct part-whole structures and it is just a coincidence that two different types of quantifiers happen to sound identically. Such an explanation would, of course, corroborate the recieved view in theories of parthood.

However, there is a good reason to believe that the phenomenon discussed in this section results from an independent factor. The extensions of regular plurals comprise arbitrary sums of individuals, i.e., scattered entities which bear no topological commitments with respect to the configuration of the parts of a plurality. It is well-known that numerals simply do not count pluralities and require the domain of quantification to consist only of singularities (e.g., Kratzer, 1989). Hence, (13a) is understood as referring to a plurality of three parts of the walls and not three pluralities of parts of the walls for the same reason why (13b) does not denote three pluralities of walls but rather a plurality of three walls. So, it turns out that partitives acctually pattern with regular nominals with respect to numeral modification.

(13) a. three parts of the walls
b. three walls

Furthermore, there is an intriguing twist corroborating the objection formulated above. What if there were a language with a plural that is similar to, say, the German plural in that it denotes sums of individuals but in addition it asserts a specific spatial relation holding between individuals making up a plurality that guarantees that such a sum has object-like properties? If the claim relating countability with being an integrated entity is on the right track, one would expect that when a partitive involving a proportional quantifier and such an untypical plural expression is modified by a numeral, the quantificational behavior of the whole phrase should differ from what we observed with respect to regular plurals. It turns out that such a construction can be found in Italian.

4.3. Italian irregular plurals

There is a relatively small set of irregular nouns in Italian that display both morpho-syntactic and semantic peculiarities with respect to the singular/plural distinction. These nouns form an inflectional class whose defining characteristic is that it exhibits an idiosyncratic agreement pattern involving a gender shift in the plural. Such forms are known as irregular plurals in *-a* (see Acquaviva, 2008 and references therein). The class of Italian irregular nouns can be divided into two groups. The first group consists of nouns that take irregular plural forms exclusively. More interestingly, however, there are also a number of nouns with both regular and irregular plural counterparts, see (14).[11]

(14) a. muro ∼ muri ∼ mura
wall$_{\text{SG.M}}$ wall$_{\text{PL.M}}$ wall$_{\text{PL.F}}$
'wall ∼ walls ∼ walls (in a complex)'
b. osso ∼ ossi ∼ ossa
bone$_{\text{SG.M}}$ bone$_{\text{PL.M}}$ bone$_{\text{PL.F}}$
'bone ∼ bones ∼ bones (in a skeleton)'

In the cases of doublets, concurrent irregular *-a* plurals tend to differ from run-of-the-mill *-i* forms in meaning. As witnessed by the translations of irregular forms in (14), there is often a sense of collectivity or cohesion in addition to the ordinary plural interpretation. For instance, the irregular form *ossa* 'bones' evokes an interpretation that the bones belong together as in a skeleton, whereas regular *ossi* 'bones' simply refers to a plurality of unrelated bones. Similar, the only way to interpret *mura* 'walls' is by picturing a walled complex, e.g., a perimeter of city walls. In both cases, there is a strong intuition that the referents of the irregular plural forms are naturally related and due to this kind of semantic flavor Italian *-a* plurals have been analyzed as collectivizers of a particular sort (Ojeda, 1995) or expressions inherently encoding the cohesion of referents (Acquaviva, 2008).

Therefore, it is plausible to posit that when an Italian plural in *-a* alternates with a regular *-i* form, its extension does not simply comprise arbitrary sums of individuals but rather a more complex type of entities. Specifically, I propose that the topological notion of connectedness or stable proximity of parts making up a plurality denoted by the irregular plural is involved in the way how the referents of such an expression are conceptualized. At least in some cases there is a good reason to assume that the reported collective flavor is due to the fact that Italian

[11] Acquaviva (2008: pp. 124–129) introduces a more subtle classification. For the sake of clarity of the main argument, I will not discuss all the nuanced morphological and semantic intricacies here.

irregular plurals encode integrated plural individuals, i.e., pluralities that unlike referents of regular plurals involve particular spatial relations holding between individual parts, and thus have the potential to form clusters, cohesive aggregates and even individuated wholes.

Italian is similar to many other languages in that it displays the same analogy as discussed in Section 4.1 with respect to singulars and regular plurals. Intriguingly, however, everything changes when a cardinal numeral modifies a partitive involving an irregular plural form such as those in (14). When an irregular doublet is swapped for a regular one, suddenly new interpretations are possible. For instance, similar to (12b) the phrase in (15a) refers to a plurality of three material parts but also it can be interpreted as denoting three subsets of the walls as long as individual walls form continuous sections. Similar, (15b) can refer either to three parts of the bones or to three continuous pluralities such as femur + knee, ulna + radius and skull + neck.

(15) a. tre parti delle mura
three parts of-the walls$_{\text{COLL}}$
'three parts of the complex formed by the walls'
b. tre parti delle ossa
three parts of-the bones$_{\text{COLL}}$
'three parts of the skeleton formed by the bones'

The evidence discussed above indicates that counting pluralities is possible only if they are conceptualized as topologically contiguous entities similar to units of some sort, a fact that a purely mereological treatment fails to capture. In the next section, I will present further evidence for the relevance of the topological notion of connectedness in partitive constructions.

4.4. Polish topology-sensitive proportional quantifiers

Polish distinguishes lexically between three morphologically related proportional quantifiers expressing the meaning *half*. As presented in (16), *pół* 'half$_2$' consists only of a root and a null inflectional marker, whereas *połowa* 'half$_1$' and *połówka* 'half$_3$' involve also additional morphology, i.e., the morpheme *-ow-/-ów-* marking the stem as well as the diminutive suffix *-k-* in the latter case.

(16) pół-∅ ∼ poł-ow-a ∼ poł-ów-k-a
root-infl.marker root-stem-infl.marker root-stem-suffix-infl.marker
all 'half'

At first sight, the expressions in (16) seem to be mere synonyms making the same contribution to the interpretation of a sentence. However, closer investigation reveals interesting differences in their distribution and meaning. Specifically, as witnessed in (17) there are no constraints on what the proportional quantifier *połowa* can combine with. On the other hand *pół* and *połówka* have a more restricted distribution since they are incompatible with cumulative predicates such as plurals and mass nouns and are only felicitous with count singulars. Furthermore, *połówka* prototypically combines with nominals denoting solid objects one could easily cut or divide into separate parts such as food terms or building materials like bricks.

(17) a. połowa / pół / połówka jabłka
half$_1$ half$_2$ half$_3$ apple$_{\text{GEN}}$
'half of the apple'

b. połowa / #pół / #połówka jabłek
half$_1$ half$_2$ half$_3$ apples$_{\text{GEN}}$
'half of the apples'

c. połowa / #pół / #połówka błota
half$_1$ half$_2$ half$_3$ mud$_{\text{GEN}}$
'half of the mud'

Based on the distributional differences presented above, it is plausible to assume distinct semantic properties of the discussed lexical items. In particular, the evidence suggests that *pół* and *połówka* require their arguments to denote integrated objects, and thus reject arbitrary sums and scattered entities.

Given that the only type of nominals that is compatible with all the proportional quantifiers in question is singular count nouns, let us explore in detail the meanings of the phrases in (17a). As already discussed in Section 3.3, parts need not be integrated. For instance, though the marked area in both Figure 1 and 2 constitute approximately 50% of the whole, only Figure 1 illustrates a continuous part. Intriguingly, it turns out that while *połowa jabłka* and *pół jabłka* are true of both Figure 1 and 2, *połówka jabłka* cannot denote an entity such as that depicted in Figure 2. Consequently, partitives with *połówka* refer only to integrated halves of an object denoted by the 'downstairs' DPs. Given the morphological complexity, I assume that the suffix *-k-* is responsible for introducing this constraint.

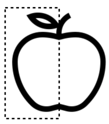

Figure 1: Continuous half

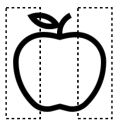

Figure 2: Discontinuous half

The novelty of the data introduced here is twofold. First of all, Polish proportional quantifiers provide strong evidence that natural language is sensitive to the topological arrangement of parts of an entity. Second, the evidence implies that individuation is also possible at the subatomic level, i.e., quantification over parts in natural language reflects the fact that some parts might be assigned the status of an individual in its own.

4.5. Multipliers

The final piece of evidence comes from an intriguing class of numerical expressions such as English *double* and *triple* which I will refer to as multipliers (following Quirk et al., 1985). Though such expressions display non-trivial semantic properties and constitute a crosslinguistically widespread category attested both within and outside the Indo-European, see (18)–(19), they have been surprisingly neglected in the formal study of quantification in natural language (but see Wągiel, to appear for an analysis of Slavic multipliers).

(18) a. double
b. doppio *Italian*
c. dvojnoj *Russian*
d. dvigubas *Lithuanian*

(19) a. dupla *Hungarian*
b. kaksinkertainen *Finnish*
c. kafúl *Hebrew*
d. shuāng *Mandarin*

Unlike cardinal numerals, multipliers do not count entities, but rather their particular parts.[12] For instance, consider examples such as those in (20).[13] Though object DPs in both (20a) and (20b) involve a quantifier, there is a crucial difference between the truth conditions of the two sentences. While (20a) is true if John ate a collection of two entities, (20b) does not require that John ate a plurality of objects. Rather, it is true if John ate one burger as along as it consisted of two particular parts, specifically two patties in a bun. In other words, while the cardinal simply counts entities denoted by the modified NP, the multiplier seems to quantify over elements within the inner structure of a denoted entity.

(20) a. John ate two burgers.
b. John ate a double burger.

The interpretative contrast discussed above indicates that *double* in (20b) restricts the denotation of the noun to only those burgers that have a particular complex form. Such an intuition seems to be further corroborated by the fact the fact that (21a) entails (21b). Thus, it is plausible to assume that multipliers trigger subatomic quantification or, in other words, are expressions dedicated to counting parts.

(21) a. The Burgenator is a double burger.
b. $\models$ The Burgenator consists of two parts.

Importantly, however, parts the multiplier quantifies over are not arbitrary. In fact, they appear to be the most salient parts of the whole entity. Intuitively, in the fast food context it is commonly assumed that in a way the essential element of a burger is a patty whereas other parts are considered to be merely a garnish. In extreme cases, such essential parts are what could be referred to as self-sufficient elements, i.e., parts that have a property very similar to the property of the whole entity. Many frequent collocates in the Corpus of Contemporary American English (COCA) fall into this category, e.g., *double bracket*, *double sink* and *double tomb*.

The fact that a certain part of an entity is perceived as significantly more salient than others corresponds to what is sometimes called structured parthood (Champollion and Krifka, 2016; see also Simons, 1987). On such an approach, certain parts might be viewed as more important than others or even essential for a particular individual. However, what counts as an essential part is somewhat vague and can be subject to different conceptualizations under different circumstances. To illustrate this claim, let us consider two more examples from the COCA collocate list for the English multiplier *double*, specifically *double arrow* and *double dagger*. One can stumble across the first in the context of typography where it can be used to designate the symbols $\Rightarrow$, $\leftrightarrow$ and $\gg$. Thus, what appears to be considered the most salient part, and thus multiplied, can be either the horizontal line or the arrowhead. Similar, in typography the phrase

[12] Though the distribution of multipliers is relatively broad, in this paper I will only focus on a subset of environments that are relevant with respect to the phenomena discussed here. For possible extensions, see Wągiel (2018, to appear).
[13] All burgers in the following examples are of course vegan.

double dagger refers to the symbol ‡. Its design indicates that the parts worth quantification over are the handles. However, in a military, martial arts, or gaming context the very same expression is much more likely to denote thrusting weapons with two blades.

Notice also that multipliers force individuation in such a way that the whole phrase is always countable. When a multiplier combines with a mass term, it triggers an obligatory shift into the count denotation, e.g., via the Universal Packager. For instance, similar to (22a) the phrase (22b) gets only a portion reading.

(22) a. two coffees
b. double coffee

Though many European languages borrowed their multipliers from Latin, in other languages multipliers and cardinals are formally related. For instance, Slavic multipliers are morphologically complex expressions derived from numeral roots by means of affixation which might be analyzed as encoding a classifier element. Similar patterns can be observed in Baltic and Finnic, as witnessed by the correspondences given in 1.

LANGUAGE	NUMBER	CARDINAL	MULTIPLIER
Russian	2	dva	dvojnoj
Lithuanian	2	du	dvigubas
Finnish	2	kaksi	kaksinkertainen

Table 1: Multipliers across languages

The morphological complexity of multipliers attested in some languages suggests that such expressions are semantically decomposable. Therefore, a proper treatment of multipliers not only should account for the discussed quantificational behavior but also it should involve a unified compositional mechanism allowing for derivation of both cardinals and multipliers.

4.6. Data summary

In Section 4.1 and 4.2, I have argued that the cross-linguistic distribution of proportional quantifiers in partitives indicates that singulars and plurals share a unified part-whole structure. The differences between the two types of expressions seem to stem from distinct ways in which topological notions play a role in their semantics. While singular count nouns can be viewed as encoding that their referents are integrated wholes, plurals denote collections of such entities but provide no information with respect to the topological relation holding between particular individuals. Crucially, numerals do not count such arbitrary sums and force the domain of quantification to consist of integrated wholes.

In Section 4.3, I discussed the evidence provided by Italian irregular plurals that indicates the existence of natural language expressions involving yet another type of structure. In particular, such nominals designate entities that are similar to plurals in that they consist of multiple integrated objects but at the same time the sum thereof is arranged in a particular way, i.e., it constitutes a cluster. The fact that when partitives involving such expressions are modified by numerals the resulting phrase can refer to a plurality of subsets as long as their members form a continuous configuration supports the claims presented in Section 3.

Moreover, in Section 4.4 I have presented novel evidence further corroborating the relevance of the notion of integrity in natural language. Specifically, certain Polish proportional quantifiers encode morphologically topological sensitivity. As a result, they require their arguments to denote integrated individuals or even constrain parts they yield to form continuous portions of matter.

Finally, in Section 4.5 I have discussed the relevance of the class of multipliers. I have argued that unlike cardinals, which quantify over wholes, multipliers are numerical expressions devised to count essential parts of wholes. Importantly, in both cases identical constraints on counting are imposed.

In the next section, I will demonstrate how at least some of the phenomena discussed here can be accurately captured by means of the mereotopological notions and assumptions concerning quantification introduced in Section 3.

5. Analysis

5.1. Singulars and plurals

In order to account for the data discussed in Section 4, I develop a mereotopological account on which singular count nouns are modeled as predicates of MSSC individuals, i.e., expressions encoding that the elements of their denotations are only MSSC entities, see (23). This allows us to capture the intuition that such expressions denote integrated wholes without employing the notion of atomicity.

(23) $[\![\text{apple}]\!] = \lambda x[\text{MSSC}(\text{APPLE})(x)]$

On the other hand, in order to capture the meaning of regular plurals I propose that pluralization is modeled in terms of algebraic closure (Link, 1983) but also involves a special presupposition which requires the pluralized noun to be a predicate of MSSC entities, see (24).[14] In other words, parts of referents of regular plurals are required to be MSSC individuals but no additional topological constraints are imposed on pluralities thereof.

(24) a. $[\![\text{PL}]\!] = \lambda P \,.\, P_{\text{MSSC}}[{}^{*}P]$
b. $[\![\text{apples}]\!] = [\![\text{PL}]\!]([\![\text{apple}]\!]) = \lambda x\big[{}^{*}\big(\lambda y[\text{MSSC}(\text{APPLE})(y)]\big)(x)\big]$

With the basic distinction between singulars and plurals, let us now move to the meaning of the two numerical expressions discussed in this paper.

5.2. Cardinals and multipliers

I propose that numerical expressions including cardinals and multipliers are complex semantic expressions derived from names of number concepts via different overt or covert classifiers specialized in counting distinct types of objects, e.g., MSSC entities and cognitively salient parts thereof, respectively.

I assume that cardinals are predicate modifiers (e.g., Ionin and Matushansky, 2006, Chierchia, 2010) derived by shifting numeral roots which I assume to be names of number concepts, i.e.,

[14]This formulation does not allow us to explain plural mass nouns. However, I will leave this issue for future research.

abstract entities of a primitive type *n*, by a classifier element $CL_{\#}$ which introduces an additive measure function $\#(P)$ which is standardized by the requirement in (25) (cf. Krifka, 1989). In essence, it maps MSSC individuals onto numbers.

(25) $\forall P \forall x[\#(P)(x) = 1 \text{ iff } \textsc{mssc}(P)(x)]$

Cardinal numerals are modeled as expressions that take predicates of MSSC individuals and yield sets of pluralities consisting of a particular number of MSSC entities. That number corresponds to the meaning of a numeral root, see, e.g., (26). Notice also that I assume that cardinals trigger pluralization and I take plural forms of modified nominals in languages such as English to be due to semantic agreement.

(26) $[\![\text{two}]\!] = [\![\text{CL}_{\#}]\!]([\![\sqrt{\text{tw}}]\!]) = \lambda P.\ P_{\text{MSSC}}\ \lambda x[{}^{*}P(x) \wedge \#(P)(x) = 2]$

I assume that multipliers, similar to cardinals, are semantically complex expressions. As discussed in Section 4.5, in some languages their decomposability is indicated by their morphology, i.e., they are derived from numeral roots. I propose that multipliers are also predicate modifiers but unlike cardinals they do not involve a classifier dedicated to counting integrated wholes but rather an element $CL_{\boxplus}$ which is specialized in counting essential parts of objects. This is ensured by the measure function $\boxplus(P)$ which satisfies the requirement specified in (27).

(27) $\forall P \forall x[\boxplus(P)(x) = 1 \text{ iff } \textsc{mssc}(P)(x) \wedge \exists y[y \sqsubseteq x \wedge \textsc{essential}(P)(y) \wedge \#(y) = 1]]$

Similar to cardinals, multipliers take as their input a set of MSSC entities and yield its subset such that it consists of those MSSC individuals that involve a particular number of essential parts, see (28) for the Russian multiplier *dvojnoj* 'double'. Notice that multipliers do not involve pluralization, and thus a resulting phrase can be modified by a cardinal numeral.

(28) $[\![\text{dvojnoj}]\!] = [\![\text{CL}_{\boxplus}]\!]([\![\sqrt{\text{dv}}]\!]) = \lambda P.\ P_{\text{MSSC}}\ \lambda x[P(x) \wedge \boxplus(P)(x) = 2]$

The proposed mechanism provides a unified algorithm to derive different types of numerical expressions by combining names of number concepts with different kinds of classifiers. Let us now turn to the semantics of various partitives.

5.3. Partitives

Following Barker (1998), I model partitivity in terms of proper parthood. However, the main difference is that in the discussed partitive constructions I locate the source of partitivity not in a preposition such English *of* but rather in proportional quantifiers themselves. In general, I assume the meaning of a partitive expression such as *part* as in (29a). For quantifiers such English *half*, I postulate a generalized context-dependent measure function μ similar to *more* in the system by Bale and Barner (2009). Given a proper ordering of measure functions, the mechanism of contextual conditioning ensures that μ gives different measures for different DPs, specifically either volume for singulars or number for plurals. The denotation of proportional quantifiers such as *half* is provided in (29b).

(29) a. $[\![\text{PART}]\!] = \lambda y \lambda x[x \sqsubset y]$
b. $[\![\text{HALF}]\!] = \lambda y \lambda x[x \sqsubset y \wedge \mu(x) \approx \mu(y) \times 0.5]$

In order to account for topology-sensitive proportional quantifiers, I posit that two ingredients are required, namely non-overlap and integrity. Non-overlap can be ensured by the partitioning function π which when applied to a set guarantees that given a particular context all members of the resulting subset are disjoint, see (30).

(30) For any P and any x and y in $\pi(P)$: $\neg\exists z[z \sqsubseteq x \wedge z \sqsubseteq y]$ relative to a particular context.

However, since non-overlap is insufficient, disjoint parts need to be individuated by imposing the integrity condition. I propose that this is achieved by the individuating element IND which gets rid off those non-overlapping parts that are discontinuous by introducing the MSSC condition, see (31). Though IND is often null, it can have a formal exponent, as indicated by the data discussed in Section 4.4.

(31) $[\![\text{IND}]\!] = \lambda P \lambda x[\text{MSSC}\big(\pi(P)\big)(x)]$

With these tools, I propose the semantics for different types of proportional quantifiers. For instance, German topology-neutral partitive expression *Teil* 'part' simply gets the interpretation in (32a). The semantics of Polish topology-neutral *połowa* 'half' differs only in that it specifies that the resulting part constitutes approximately 50% of the whole, see (32b). On the other hand, Polish topology-sensitive *pół* 'half' involves a special presupposition ensuring that it only combines with a DP denoting an MSSC individual, see (32c). This explains why it is incompatible with plurals and mass terms. Furthermore, I posit that the suffix *-k-* in Polish proportional quantifiers is an exponent of IND, see (32d) which allows to derive the individuating proportional quantifier *połówka* 'half', see (32e). Thus, *połówka* can only combine with a DP denoting an MSSC entity and always yields a continuous half of such an object.

(32) a. $[\![\text{Teil}]\!] = \lambda y \lambda x[x \sqsubset y]$
b. $[\![\text{połowa}]\!] = \lambda y \lambda x[x \sqsubset y \wedge \mu(x) \approx \mu(y) \times 0.5]$
c. $[\![\text{pół}]\!] = \lambda y \,.\, y_{\text{MSSC}}\ \lambda x[x \sqsubset y \wedge \mu(x) \approx \mu(y) \times 0.5]$
d. $[\![\text{-k-}]\!] = [\![\text{IND}]\!] = \lambda P \lambda x[\text{MSSC}\big(\pi(P)\big)(x)]$
e. $[\![\text{połówka}]\!] = [\![\text{-k-}]\!]([\![\text{pół DP}]\!])$

The proposed denotations allow us to derive the meanings of different types of partitives via standard function application. Extending the repertoire of formal tools with the mereotopological notion of MSSC accompanied with several additional assumptions provides means to explain the so-far neglected issues in subatomic quantification.

6. Conclusion

The main aim of this paper was to contribute to our understanding of quantification in natural language by exploring the so far neglected domain of subatomic quantification, i.e., quantification over parts of building blocks of the denotations of singular count nouns. I have provided compelling evidence for the relevance of this phenomenon for natural language semantics. In particular, I have explored various aspects of meaning of a broad range of linguistic expressions such as different types of partitive constructions including partitives with Italian irregular plurals as well as multipliers such English *double* from a cross-linguistic perspective. Properties of different types of investigated expressions suggest that there is one unified parthood relation for various types of entities and different part-whole structures result from distinct topological relations holding between particular elements such as integrity or lack thereof. Furthermore, the existence of topology-sensitive proportional quantifiers in Polish demonstrates the relevance of the notion of integrity in quantification in natural language. In order to account for the data, I proposed a universal mechanism which allows for counting of both entire objects and their parts. Its formal implementation is based on mereotopology, i.e., a theory which extends standard mereological parthood with topological notions such as connectedness. Different aspects of meanings can arise as a result of the interaction between topology, partitivity and numerical quantification.

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