Vagueness in Degree Constructions

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Abstract

This paper presents a novel semantic analysis of unit names and gradable adjectives, inspired by measurement theory (Krantz et al 1971). Based on measurement theory's typology of measures, I claim that different predicates are associated with different types of measures whose special characteristics, together with features of the relations denoted by unit names, explain the puzzling limited distribution of measure phrases.

1 Introduction: Measurement theory in grammar

1.1 The aims and structure of this paper

Measures can be described as mappings of individuals to degrees along dimensions (height, width, loudness, etc.; cf., Krantz et al, 1971). My main claim in this paper is that the grammar of natural language is sensitive to the distinctions of measurement theory's taxonomy of measure types. This four level taxonomy goes back to Stevens (1946), who dubbed the by now widely used names for the four measure types – *ratio-scale* measures (also known as *extensive* measures), *interval-scale*, *ordinal-scale* and *nominal-scale* measures. Measurement theory was not only found useful in the analysis of the correct use of measurement in natural sciences such as physics. Its taxonomy is extensively used in statistics and its application for research methods in the social sciences (Babbie, 2004). It is also an important source of influence in the field of psychophysics, where it is found useful in describing the way subjects perceive and represent scalar properties of stimuli, ranging from properties such as sound, color, and weight to scales of, for instance, pain, well-being and even grammaticality judgments (Featherstone, to appear).

In order to demonstrate that grammar is sensitive to measurement theory's distinctions, in this paper I address problems pertaining to the interpretation and distribution of unit names, like *pound* and *meter*, that form measure phrases (as in *two pounds*). Measure phrases occur in constructions like *two meters tall* ('numerical

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degree predicates'), as well as *two pounds of cheese* ('classifier constructions'). I focus on the former structure, whose distribution is highly restricted.

Numerical degree phrases are not used with most predicates (*happy, beautiful, intelligent*, etc.) or nominalizations (*happiness, beauty, intelligence*, etc.), including adjectives for which conventional measuring systems exist (as the infelicity of *#two degrees warm, #two dollars expensive* and *#two kilos heavy* illustrates;Kennedy 2001; Schwarzschild, 2005). While in some languages, numerical degree modifiers are allowed with a restricted set of positive predicates (e.g., English), in other languages (like Hebrew), numerical degree modifiers are allowed with a restricted set of nominalizations (as in *gova* (*shel*) *shney meter* 'height (of) two meters'). In addition, the set of positive predicates, which indeed allows this modification, varies considerably between languages (Schwarzschild, 2005). Despite this limited distribution, in many languages, measure phrases do occur freely in the comparative form of *all* predicates (as in *two meters taller/shorter; two degrees warmer; two kilos heavier*, etc.), and speakers often creatively produce new numerical degree constructions, such as *two heads taller*, *two fingers wide(r)* and *two aspirins sick(er)* (I thank Louise McNally for this last example).

The licensing of ratio modifiers like *twice* is related in intricate ways to that of measure phrases. Ratio modifiers are most acceptable and most often used with positive adjectives that license measure phrases, as in, for instance, *Dan is twice as tall as Sam*. Negative adjectives in the positive form, combine neither with measure phrases (cf. *#Dan is two meters short*), nor with ratio modifiers (cf. *#Dan is twice as short as Sam)*. However, many positive adjectives (like *happy*), which resemble negative ones in not licensing measure phrases, *are* acceptable with *twice* (e.g., *I am twice as happy as I used to be* is not as bad as, e.g., *#Dan is twice as short as Sam*). In fact, speakers use *twice* more often with *happy* than with *short*. In a study of Google-search results, the proportion of ratio constructions ('*twice as Adj. as*') out of the total amount of equative constructions ('*as Adj. as*') was more than five times greater with *happy* (15%) than with *short* (~3%; Sassoon 2008, Table 1).

In part 1, I present the relevant taxonomy of measurements. In part 2, I present a new analysis of unit names and measure phrases, which is directly inspired by measurement theory (cf. Krantz et al, 1971; Klein, 1991). In part 3, I explore the consequences of this analysis. I show that different gradable predicates are associated with different types of measures, whose special characteristics, together with features of the relations denoted by unit names, explain the limited distribution of measure phrases.

1.2 Measurement types in measurement theory

I propose that in addressing grammatical facts pertaining to measurment, gradability and comparison, it is useful to consider the following classification of scalar properties or degree functions (assignments of numbers to objects along a dimension; Stevens 1946; Krantz et al 1971; see also Wikipedia, the free Encyclopedia under 'Level of measurement').

The first level in this classification is called *nominal*. The only significance of nominal degree functions lies in the fact that entities are assigned the same or different values. If the values are numerals, the choice of numerals is irrelevant and the only comparisons to be made between variable values are equality and inequality. There are no "less than" or "greater than" relations among the values, nor operations such as addition or subtraction. Examples are the set of eye colors (brown, blue, green, etc.) and the set of truth values {0,1}.

The second level is called *ordinal*. Here, the numbers assigned to objects represent their rank order (1st, 2nd, 3rd etc.). Comparisons of "greater than" and "less than" can be made, in addition to equality and inequality. However, operations such as addition and subtraction are still meaningless. Examples include the results of a horse race or a swimming competition, which state only which competitors arrived first, second, third, etc. but do not state time intervals.

The third level is called *interval*, where, in addition to the features of an ordinal level, equal differences between values represent equivalent intervals. Thus, operations such as subtraction are meaningful. But the zero point on the scale is arbitrary and negative values can be used. Thus, neither sums of nor ratios between numbers on the scale are meaningful, and operations such as multiplication, division and addition cannot be carried out directly (only ratios of differences between pairs of values can be expressed; one difference can be twice the other, etc., as demonstrated in Section 3.3) Examples are the year date in many calendars and temperature in the Celsius or Fahrenheit scale. The fact that the water freezing point is mapped to the zero Celsius degree is arbitrary (as arbitrary as the fact that the water boiling point is mapped to the 100 Celsius degree). The freezing point does not correspond to non-existence of temperature, in fact it corresponds to 273 Kelvin degrees. Accordingly, it is meaningless to say that 20 degrees Celsius is twice as hot as 10 degrees Celsius, in the sense that 20 degrees Celsius does not represent a double amount of heat (for a more complete discussion of the Celsius scale see Section 3.4).

The forth level is called *ratio*. Ratio functions have all the features of interval functions, in addition to meaningful ratios between values. Operations such as multiplication, division and addition are therefore meaningful. The zero value on a ratio scale is non-arbitrary. Most physical quantities, such as mass, length or energy are measured in ratio scales; so is temperature measured in Kelvin, that is, relative to absolute zero. Other examples include age, length of residence in a given place, etc.

Having presented measurement theory's levels of measurements, I now give my account of the semantics and distribution of unit names and measure phrases.

2 My Proposal

2.1 Vagueness pertaining to degrees

My implementation of ideas from measurement theory in the semantics of adjectives is unique in that it crucially relies on observations regarding the information that different adjectival degree functions do or do not encode, i.e., the idea of vagueness as pertaining to degree constructions is central to the present analysis.

Let us call the linguistic and world knowledge of a given community of speakers *an actual context*. In standard vagueness models (van Fraassen 1969; Kamp 1975; Veltman 1984; etc.), expressions are assigned interpretation relative to information states (*contexts*) c, rather than relative to worlds w, so the interpretation of a statement in a context c may be *true*, *false* or *undetermined*. Only in (and in all) contexts of total information (*supervaluations*; van Fraassen 1969) is the truth value of all statements determined (it is either true or false). Let M_C be a vagueness model for a domain D and a set C of contexts. For any context $c \in C$, let $T_c \subseteq C$ be the set of contexts of total information extending c (c's *completions*). Let the set of statements that are *true in c* consist of the statements that are true in every completion $t \in T_c$ of c, and the set of statements that are *false in c* consist of the statements that are false in every completion t of c. For example, the truth of a statement like *it rains* is considered common knowledge in a given context c iff it holds true in every completion t in T_c; the falsity of a statement is considered common knowledge in c iff it is false (e.g., it does not rain) in every completion t in T_c. The truth value of a statement is *undetermined* in c iff T_c includes both a completion in which it is true (e.g., it rains) and a completion in which it is false (it does not rain). Generally, for any statement φ:

(1) a. $[\![\varphi]\!]_c = 1$ iff $\forall t \in T_c$, $[\![\varphi]\!]_t = 1$
b. $[\![\varphi]\!]_c = 0$ iff $\forall t \in T_c$, $[\![\varphi]\!]_t = 0$
c. Otherwise, $[\![\varphi]\!]_c$ is *undetermined*

Let T be the set of total contexts in C. In this paper, I associate gradable adjectives with the following semantics:

(2) For any $t \in T$, and any gradable adjective P:
a. Let $f_{P,t} \in \Re^D$ be *the degree function of P in t* (where $\Re$ is the set of real numbers).
b. Let $c_{P,t} \in \{0,1\}^D$ be *the characteristic function of P in t* (where 1 and 0 stand for truth values).
c. P denotes either $f_{P,t}$ or $c_{P,t}$, depending on the linguistic context. For example, in statements like *Dan is taller than Sam*, the adjective *tall* denotes $f_{tall,t}$, while in statements like *Dan is tall*, *tall* (or its projection) denotes $c_{tall,t}$ (Kennedy, 1999).

While in standard vagueness models, supervaluations represent different cutoff points for vague adjectives like *tall*, in the present proposal, they serve to represent different measuring conventions for gradable adjectives. For example, while in a total context t_1 the values of $f_{tall,t1}$ may correspond to the outcome of measuring entities' heights with a centimeter ruler (so, e.g., the meter is mapped to the number 100), in another total context t_2 the values of $f_{tall,t2}$ may correspond to the outcome of measuring entities' heights with an inch ruler (so, e.g., the meter is mapped to 39.4).

We need a representation of vagueness pertaining to degrees, because some information is *not* encoded by adjectival degree functions. In particular, ordering dimensions (height, heat, happiness, etc.) are typically mass noun interpretations, although we cannot directly count quantities of the 'stuff' denoted by such nouns. No given quantity of water, height, heat or happiness is unequivocally associated with a given (context-invariant) value like 1 or 2 or 345. Thus, objects d with a non-zero quantity of, e.g., height, should be mapped to different numbers in different total contexts in T_c of an actual context c ($\exists t_1,t_2 \in T_c$: $f_{tall,t1}(d) \neq f_{tall,t2}(d)$).

In fact, many functions (many types of rulers, if you like) adequately represent heights. Any function that maps equally tall entities to the same number and that maps the concatenation of n equally tall entities to n times that number, is *additive with respect to height*, i.e. adequately represents differences and ratios between entities' heights. For example, the mapping of two equally tall entities, d_1 and d_2, and their concatenation, $d_1 \oplus_{height} d_2$, to the values 5, 5 and 10, respectively, is additive. But so is their mapping to 2,2, and 4, respectively, and so is their mapping to 100, 100 and 200, respectively, etc. Each mapping corresponds to (the outcome of measuring entities' heights with) some possible ruler (inch, centimeter, meter, etc.)

2.2 Information about degree ratios and the interpretation of unit names

We see that many different functions may be associated with adjectives like *tall*. Despite this intrinsic vagueness regarding the mapping of entities to degrees, some information *is* encoded by adjectival degree functions. In particular, all the functions that may be associated with adjectives like *tall* adequately represent height ratios. This means that they share the same ratios between degrees, e.g., since the height of $d_1 \oplus_{height} d_2$ is twice the height of d_1 in all the examples just given, the ratio between their degrees is the number 2 in all these examples (**2**×5 = 10; **2**×2 = 4 and **2**×100 = 200). In fact, all rulers (meter rulers, inch rulers, etc.) specify the same ratios between entities' degrees (precisely the ratios between the entities' heights). As these ratios are easily accessible to us (they are unequivocally determined numbers, identical in all the additive measuring systems), in any t, $f_{tall,t}$ should adequately represent them. Thus, in every $t \in T_c$, entities with n times d's height are mapped to the number $n \times f_{tall,t}(d)$. All *tall*'s functions in T_c, then, yield the same ratios between entities' degrees (these ratios are context-invariant).[1]

The moral to be drawn from the above observations is the following. It is not the case that, say, *Dan is 2 meters tall* is true in a context c iff in every total context t of T_c, $f_{tall,t}$ maps Dan to 2. In any actual context c, the value to which $f_{tall,t}$ maps Dan varies across accessible total contexts, rendering Dan's value undetermined ($\neg\exists n \in \Re$: $\forall t \in T_c$, $f_{tall,t}([[Dan]]_t) = n$). Rather, the truth of statements with numerical degree

[1] Surely, degree functions of adjectives like *tall* map entities to numbers per a given total context *and* time point, and entities' height ratios are identical in every total context *per* a given time point. It is only for simplicity that I omit indices such as those representing time points.

predicates must be determined based on information regarding height ratios. In particular, directly based on measurement theory (cf. Klein, 1991), I propose that an entity d falls under the predicate *2 meters tall* iff the ratio between d's degree in *tall* and the degree in *tall* of the original meter stick in Paris or any other entity that is one meter tall ('a meter unit-object'), $r_{m,t}$, is 2:

(3) $[[\text{Dan is two-meters tall}]]_c = 1$ iff $\forall T \in T_c$: $f_{tall,t}([[\text{Dan}]]_t) = 2 \times r_{m,t}$.

Thus, I take unit nouns (e.g., *meter*) to be extensional, in the sense of being *directly* linked to a set of entities which, by virtue of a convention, are regarded as *unit objects*, e.g., the entities whose height we call 'one meter'. This set, then, does not vary across the total contexts of any given context c. For any unit name *unit*, let $D_u \subseteq D$ be the set of unit objects of *unit*. I claim that, the word *meters* in statements such as *Dan is two meters tall* is interpreted as equivalent to the predicate "$\lambda P.\lambda n.\lambda x$. x is n times as P as a meter unit-object". In this interpretation, in every total context t, the unit name denotes (the Schonfinkelized function of) a relation between (the degree function of) an adjective P (e.g. *tall, wide, long,* etc.), a number n, an and entity d in D, such that d's amount of P-hood (e.g., d's height, represented numerically by the value $f_{tall,t}(d)$) equals n times that of a meter unit-object (represented by the value $f_{tall,t}(d_m)$, for any meter unit object $d_m \in D_m$; Since all unit objects are equally tall, we can represent their degree as a constant, $r_{m,t} = \sigma(\{ f_{tall,t}(d_m): d_m \in D_m\})$, where σ is a function from singletons to their unique members). Thus, we can give a general interpretation rule for unit names and numerical degree predicates. Let us add to the language the category UNIT $\subseteq$ NOUN[3] that consists of words like *meter*(*s*)*, gram*(*s*), etc.:

(4) Unit names and numerical degree predicates
$\forall t \in T, \forall P \in ADJ, \forall u \in UNIT$:
a. $\exists r_{u,P} \in \Re, r_{u,P} \neq 0$: $D_u = \{d \in D: f_{P,t}(d) = r_{u,P}\}$.
b. $[\![\lambda n.\lambda x.u(P,n,x)]\!]_t = \lambda r \in \Re.\lambda d \in D.$ for some $d_u \in D_u$: $f_{P,t}(d) = r \times f_{P,t}(d_u)$.
In any t, the denotation of a numerical degree predicate that is based on u and P is a relation between numbers r and objects d, such that d's degree in P equals r times the degree of any unit-object d_u.

For example, $[\![\text{centimeters}]\!]_t = \lambda f_{P,t} \in \Re^D.\lambda r \in \Re.\lambda d \in D.$ for some $d_{cm} \in D_{cm}$: $f_{P,t}(d) = r \times f_{P,t}(d_{cm})$. Table 1 illustrates my proposal by means of a simplified model with three individuals and three total contexts. According to my proposal, we consider individuals' degrees in *tall* as specified because: (i) The ratios between (values representing) heights do not vary across total contexts, e.g., as the ostrich in Table 1 is twice as tall as the chicken, the ratio between the degrees of the ostrich and the chicken is 2 in every total context; (ii) A set of unit objects exists, $D_{cm} = \{d_{cm}\}$, s.t. d is *n centimeters (cm) tall* iff the ratio between d's degree and the centimeter unit objects' degree is n, e.g., as the ratio between the degrees of the ostrich and the centimeter unit object is 100 in every total context in our example, *the ostrich is 100 centimeters tall* is true in it.[2]

Table 1: An example of my proposal

	(i) $f_{tall,t}(d)$		**$r_{cm,t}$**	**(ii) $f_{tall,t}(d) = n \times r_{cm,t}$**	
	$d_{ostrich}$	$d_{chicken}$	$\mathbf{d_{cm}}$	$d_{ostrich}$ is *100cm tall*:	$d_{chicken}$ is *50cm tall*:
t_1	100	50	1	100 = **100** × 1	50 = **50** × 1
t_2	200	100	2	200 = **100** × 2	100 = **50** × 2
t_3	300	150	3	300 = **100** × 3	150 = **50** × 3

3 Consequences

In Part 3, I describe in detail the various consequences of the analysis just presented. I argue in detail that it captures not only the interpretation, but also the puzzling limited distribution of measure phrases. In particular, I have argued for (5):

(5) In actual contexts c, speakers feel they have information about entities' degrees in *tall* only because the following two preconditions hold:

a. *Precondition (i)*: The ratios between entities' degrees are context-invariant numbers ($\forall d_1,d_2 \in D, \exists n \in \Re: \forall t \in T_c, f_{tall,t1}(d_1) / f_{tall,t1}(d_2) = n$), and

b. *Precondition (ii)*: There is a consensus regarding a set of unit-objects (e.g., the meters) that serves as a reference point, so that any entity d is mapped to a context-invariant number, representing the ratio between d's degree and the unit objects' degree in *tall*.

In the following, I show that in languages that allow adjectives to combine with measure pharses, an adjective does not license unit names and numerical degree predicates iff at least one of these preconditions is violated.

3.1 Violations of precondition (ii): No consensus about unit-objects

My proposal predicts that the absence of conventional unit objects will result in vagueness concerning the mapping of entities to numbers. I propose that some adjectives have no unit names associated with them because it is impossible to determine a convention for them regarding a set of unit objects. Consequently, we have the impression that these adjectives do not map entities to numerical degrees.

[2] Some unit names (like *Celsius*) are interpreted by other interpretation rules. Yet we will see in Section 3.4 that speakers often wrongly presuppose that the unit name *Celsius* is interpreted by the above given rule (4). Thus, this rule is productively used by speakers, while other rules invented by scientists are not.

Consider, for instance, *happy*. Emotions are internal states. It is hard to come up with conventions as to which emotional extent should be mapped to degree 1, 2, 3, etc. Even if one speaker treats a certain internal state as a unit object, no other speaker has access to this state. So no object d can be *agreed upon by all the community of speakers* to constitute a unit object. This is the case even if any one of the speakers associates with *happy* internal (subjective and non-conventional, but nonetheless actual) means of additively measuring happiness intensities (including a suitable 'concatenation' relation for such intensities).

Similarly, while *weight* can be measured by kilograms, the internal states of speakers when they lift objects (their feeling of the objects being *heavy*, *light*, etc.) cannot be measured by conventionally established unit names. If a language maps a predicate to the latter type of degrees, the predicate will *not license* unit names and numerical degree modifiers. But this does not show that predicates do not map entities to numbers. In fact, when no unit name is explicitly mentioned, it is rather meaningless to say that something is tall to degree 456 (456 what? Kilometers? Meters? Inches?) In adjectives like *happy,* this is always the situation.

This proposal improves upon non-numerical theories (cf. Moltmann, 2006) because it accounts for the compatibility of *happy* with ratio and difference modifiers. For example, *Dan is twice as happy as Sam* is a claim concerning the ratios between the arguments' *happiness* degrees (with no reference to unit objects):

(6) $[\![\text{Dan is twice as happy as Sam}]\!]_c = 1$ iff
$\forall t \in T_c$: $f_{happy,t}([\![\text{Dan}]\!]_t) = 2 \times f_{happy,t}([\![\text{Sam}]\!]_t)$.

In addition, we can present a unified analysis of comparative morphemes with and without numerical degree modifiers, whereby the interpretation of these morphemes is mediated by a difference operation, creating difference modifiers:

(7) a. $[\![\text{Dan is } \textit{2 meters taller} \text{ than Sam}]\!]_c = 1$ iff
$\forall t \in T_c$: $f_{tall,t}([\![\text{Dan}]\!]_t) - f_{tall,t}([\![\text{Sam}]\!]_t) = 2 \times r_{m,t}$.

b. $[\![\text{Dan is } \textit{happier} \text{ than Sam}]\!]_c = 1$ iff
$\forall t \in T_c, \exists r \in \Re, r>0$: $f_{happy,t}([\![\text{Dan}]\!]_t) - f_{happy,t}([\![\text{Sam}]\!]_t) = r$.

We see that speakers do not need to know the degrees of entities they refer to, only the ordering or ratios between their degrees. These are available to them (cf., Section 2.2).

Notice that speakers often assert, for instance, that they are *twice as happy*, as a manner of speech – a figurative way of stating that they are much happier. However, this does not show that *twice as happy* is ungrammatical. Presumably, we may not be familiar nor understand the nature of any ratio-scale measering means for *happy*. Still, there is no a-priory reason to think that such measurements are impossible (we do not possess information according to which perceptual and emotional measurements of our experiences *must* be non-ratio-scale). For this reason we do not judge utterances of expressions such as *twice as happy* ungrammatical, even if we do not completely understand what they mean (we will see in the next Section that this is not the case for expressions like *twice as short*).

Furthermore, even when speakers are willing to accommodate the presupposition that additive measuring systems exist for happiness, they cannot always be precise about degree ratios, e.g., "on Monday I was twice as happy as I was on Sunday" is a very precise conclusion to reach through introspection. Speakers may be reluctant to commit themselves to this level of precision regarding their emotions.

In sum, I propose that it is for these reasons, and not because it is ungrammatical in the literal sense, that *twice as happy* is used as a manner of speech. We *can* reason with statements like *I am ten times happier now than I used to be ten years ago* or *I am twice as happy now as I was ten minutes ago*. In fact, the 'figurative' use probably emerges by virtue of the fact that the literal sense does exist. Thus, one source for cross linguistic variation in the licensing of measure phrases with positive adjectives is formed by differences in the measure type associated with (translations of) an adjective, e.g., languages may vary as to whether predicates like *heavy* or *warm* are associated with measures of external or internal states or both. With internal measures, measure phrases are ruled out due to violations of condition (ii) (absence of conventional unit objects; cf. (5b)), i.e. regardless of whether the given adjectival measure encodes degree ratios or not.

3.2 Violations of precondition (i): No information about degree ratios

While we may acknowledge the ratios between, say, our degrees of happiness on separate occasions, we can hardly ever acknowledge the ratios between degrees of entities in predicates like *short*. This is illustrated by the fact that ratio modifiers are less acceptable with *short* (#*twice as short*) than with *tall* or with *long* (for similar contrasts in other antonym pairs see Kennedy, 2001). Sassoon (2008) empirically supports the claim that ratio modifiers are less often used with negative adjectives (e.g., *short*) than with their positive antonyms (e.g., *tall*), based on a study of Google search-results of equative comparisons and ratio comparisons with pairs of antonym adjectives.

Accordingly, the present analysis predicts that, in the absence of a specification of (or information concerning) ratios between degrees, numerical degree predicates are not licensed, i.e. we directly explain why negative adjectives fail to combine with measure phrases to form numerical degree predicates, e.g., the infelicity of #*two meters short*.

3.3 Measure phrases in comparison statements

Still, numerical degree predicates *are* acceptable in the comparative form of either positive or negative adjectives (cf. Kennedy, 1999), as ilustrated by the contrast between #*Dan is two meters short* and *Dan is two meters shorter than Sam*. In fact, in actual contexts, we can positively say that Dan's degree in *short* is n meters bigger than Sam's, iff Sam's degree in *tall* is n meters bigger than Dan's.

In Sassoon (2008), I show that we can predict these facts by assigning any negative adjective, in any index t, a degree function that linearly reverses and linearly transforms the degrees of its positive antonym. In other words, I propose that for any total context $t \in T_c$, there is a constant $Tran_{short,t} \in \Re$, such that **$f_{short,t}$ assigns any d in D the degree ($Tran_{short,t} - f_{tall,t}(d)$)**. Let me breifly motivate this proposal.

The motivation for assuming that degree-functions of negative adjectives are reversed compared to the functions of their positive antonyms is rather straightforward. This assumption represents the fact that, e.g., Dan is taller than Sam iff Sam is shorter than Dan, i.e. the ordering between the degrees assigned to any two entities by *short* is reversed in comparison with the ordering between the degrees assigned to them by *tall*.

The basic motivation for transformation values is the following (Sassoon 2008). We can positively say that an adjective like *tall*, which is linked to conventional additive measuring systems, maps entities with no height to zero. The outcome of measuring entities with no height, such as the surface of the floor, with a ruler (just any possible ruler) is systematically the number zero.[3] So in every $t \in T_c$, $f_{tall,t}$ maps entities with no height (abstract entities; surfaces) to 0 (it's additive). However, consider the adjective *short*. If, in every total context t of T, $f_{tall,t}$ is additive (it maps entities with no height to 0), and *short* is not transformed in a context c ($Tran_{short,t} = 0$ in every total context t of T_c, i.e. $f_{short,t} = - f_{tall,t}$), then the degree of entities d with no height in *short* is predicted to be 0 in c (because in every $t \in T_c$ it is $- f_{tall,t}(d) = - 0 = 0$). But is this so? Can we positively say that *short* maps entities with (almost) no height, such as the surface of the floor, to (almost) zero? (or, in other words, that the surface of the floor is short to degree zero?) Not really. As *tall* does not have a maximum point (we cannot tell which entities are the tallest), the antonym *short* does not have a minimum point (a zero). Some semantic theories (cf., von Stechow, 1984b; Kennedy, 1999) endorse the view that entities with (almost) no height are mapped to (a degree that approximates) infinity (formally, they are mapped to the largest interval $(0,\infty)$, not the zero interval $(0,0)$). Therefore, in these theories, too, the degree function of *short* transforms height quantities by a non-zero constant. When I ask speakers to examine their intuitions regarding this issue, they are puzzled. Our intuitions about the point of 'zero shortness', so to speak, are completely blurred. I propose that this is the natural sign of an unspecified transformation value. Hence, the degree function of *short* transforms height quantities by a non-zero constant, $Tran_{short}$. We know nothing about this constant. It may be any number (it varies across total contexts in T_c), rendering the zero point undetermined ($\neg\exists n \in \Re$: $\forall t \in T_c$, $Tran_{short,t} = n$).

But if $Tran_{short,t} \neq 0$, $f_{short,t}$ is not additive – it does not represent ratios between entities' heights. If, e.g., $f_{tall,t}(d_1)=f_{tall,t}(d_2)=5$, then by additivity $f_{tall,t}(d_1 \oplus_{height} d_2)=10$. But, say, a function f_{1-f} that maps each d to $(1- f_{tall,t}(d))$ is such that $(f_{1-f}(d_1)= f_{1-f}(d_2)=-$

[3] Additive height-functions, as opposed to transformed ones, must map entities with no height to zero (and entities *with* height to degrees other than zero), for otherwise they will not adequately represent height ratios. In order to see this, consider, for example, a function f, such that f maps some entity d_0 with no height (say, the surface of the floor) to some number other than zero, say, 1/2 and f maps a meter unit-object to the number 1. The ratio between d_0's value and the value of a meter unit-object is then the non-zero number 1/2 (it is half a meter tall), while the ratio between d_0's height and the height of a meter unit-object (or any other object) is 0. Thus, f does not adequately represent height ratios.

4) and ($f_{1-f}(d_1 \oplus_{height} d_2) = 1 - f_{tall,t}(d_1 \oplus_{height} d_2) = -9 \neq (2 \times -4)$). The ratio between the degrees of $d_1 \oplus_{height} d_2$ and d_1 is 9/4, while the ratio between their heights is 8/4.

The fact that negative adjectives fail to represent ratios has consequences.

First, ratio modifiers are less acceptable when combined with negative adjectives than when combined with their positive antonyms (cf. the infelicity of, e.g., *#Dan is twice as short as Sam*).

Second, negative adjectives do not license unit names. The semantic value of unit names, e.g. *inches*, crucially relies on the fact that the function denoted by its adjectival argument P in each context of use encodes the ratios between the amounts of P-hood in entities (so to speak) and the amount of P-hood in a unit object. Thus, adjectives whose functions do not encode these ratios cannot form arguments for unit names. A unit name like *inches* exists for the adjective *tall* by virtue of the fact that the degree function of *tall* does encode the given ratios. Had the degree function of *tall* been transformed, like the degree function of *short*, inches would not constitute possible units for *tall*.

Nor can we use negative adjectives in the positive form with numerical degree phrases construed of their positive antonyms and their unit names (as in **two meters$_{tall}$ short*). Why? If, for instance, in c, *tall* maps some d to 2 meters ($\forall t \in T_c$, $f_{tall,t}(d) = 2 \times r_{m,t}$), *short* maps d to $Tran_{short}$ – 2 meters ($\forall t \in T_c$, $f_{short,t}(d) = Tran_{short,t} - 2 \times r_{m,t}$). As the transformation value of *short*, $Tran_{short,t}$, varies across total contexts in T_c, we cannot say which entities are *2 meters short* in c ($\neg \exists d$: $\forall t \in T_c$, $f_{short,t}(d) = 2 \times r_{m,t}$).

However, in computing degree-differences, the transformation values of the two degrees cancel one another: $\forall t \in t_c$, d_2 is 2 meters taller than d_1 (i.e., $f_{tall,t}$ maps d_2 to some $n \in \Re$ and d_1 to $n - 2 \times r_{m,t}$; the difference between these degrees is $2 \times r_{m,t}$) iff $\forall t \in T_c$, d_1 is 2 meters shorter (i.e., $f_{short,t}$ maps d_2 to $Tran_{short,t} - n$ and d_1 to $Tran_{short,t} - (n - 2 \times r_{m,t})$; the difference between these degrees is still $2 \times r_{m,t}$: $(Tran_{short,t} - (n - 2 \times r_{m,t})) - (Tran_{short,t} - n) = \cancel{Tran}_{short,t} - \cancel{n} + 2 \times r_{m,t} - \cancel{Tran}_{short,t} + \cancel{n}) = \mathbf{2 \times r_{m,t}}$). So the differences between degrees are preserved under the reversal induced by $f_{short,t}$ in every total context t of T_c of actual contexts c. For this reason, we can felicitously say that entity-pairs stand (or do not stand) in, e.g., the relation '*two meters$_{tall}$ shorter*'.

In sum, facts pertaining to the licensing of measure phrases with negative adjectives receive a straightforward explanation if negative adjectives are analyzed as denoting interval-scale properties, i.e. mappings of entities to values that do not encode their height ratios, but do encode differences in height (cf. Sassoon, 2008).

3.4 Celsius

The interpretation of some units is not generated by the general 'linguistic' rule for the interpretation of unit names proposed in (4). Rather, their interpretation is derived from the interpretation of other unit names in some systematic way. For example, the interpretation of *Celsius* is complicated in that its derivation involves transformation of (additive) Kelvin degrees by a fixed, conventionally established value. For any n, entities that are "n Kelvin hot" are "n – 273 Celsius hot". So a box is *1 degree Celsius* iff it is *274 degrees Kelvin*, but a box is *1 degree Celsius more* than a shelf iff the box is *1 degree Kelvin more* than the shelf, not *274 degrees Kelvin* more.

The numbers that *Celsius* assigns to entities do not adequatly represent quantities of heat (or temperature), e.g., the fact that the heat in two cells together equals the sum of heat in the two separate cells (i.e. that for any t, $f_{hot,t}(d_1 \oplus d_2) = f_{hot,t}(d_1) + f_{hot,t}(d_2)$). For example, if cells d_1 and d_2 each contains the heat of 2 Kelvin degrees ($2\times r_{Kelvin}$), each falls under *(2 – 273) degrees Celsius*, and the heat contained in both of them together, the heat in 4 Kelvin unit objects ($4\times r_{Kelvin}$), falls under *(4 – 273) degrees Celsius*. But (2 – 273) + (2 – 273) = (4 – 546) ≠ (4 – 273). Thus, *Celsius* does not assign $d_1 \oplus d_2$ the sum of numbers it assigns to d_1 and d_2. The heat in any entity that is an instance of *2 degrees Celsius* is not twice the heat in an entity which is an instance of *1 degree Celsius*. In fact, handbooks of measurement theory are equipped with explanations as to why it is senseless to say that *4 degrees Celsius is twice as hot as 2 degrees Celsius*. However, despite these explanations, speakers cannot help judging this sentence to be perfectly acceptable (just like the sentence *4 meters is twice as long as 2 meters*). I submit that this further supports my proposal that speakers interpret unit names in terms of the interpretation rule in (4), which is, of course, erroneous for, e.g., *Celsius*. This mistake reveals the fact that a generative rule, such as the one in (4), is used productively when unit names are to be interpreted.

3.5 More on the infelicity of positive adjectives with measure phrases

Some positive adjectives resemble negative ones in terms of the licensing of measure phrases. For example, the statement *#The box is thirty degrees warm* resembles the statement *#The box is thirty degrees cold* in being somewhat awkward. Yet, *The box is 30 degrees warmer/colder than the shelf* is perfectly acceptable (Kennedy, 2001). So in terms of the licensing of numerical degree modifiers, *warm* resembles its negative antonym *cold* and not other positive adjectives. My proposal can capture these facts.

First, positive adjectives for which additive (ratio-scale) measures exist may have transformation values, too (even though their measures are not reversed). Temperature predicates are an example. Well established additive tools for measuring temperature, e.g. Kelvin thermometers, exist. However, in practice, more often than not, temperature is measured by transformed (interval-scale) thermometers. The reasons are pragmatic – while we never encounter entities with absolutely no temperature ('zero Kelvin hot'), we often experience events or entities that measure 273 Kelvin degrees. Thus, the use of transformed measures, such as the Celsius scale (which maps such entities to zero) is convenient. The existence of concepts like *Celsius* support the assumption that positive adjectives may be associated with transformed measures.

To summarize, the point of zero-Kelvin heat (i.e., – 273 Celsius degrees) is hardly ever relevant, experienced, or talked about by speakers who are not-scientists. Thus, for them, any choice of a zero is arbitrary (which, formally, means that in different total extensions of any actual context c, temperature predicates like *warm* are associated with different transformation values). Only when a unit name is explicitly used, must an additive interpretation be accommodated (by switching to an extension c' of c where the transformation value equals zero for any t of $T_{c'}$).

In addition to capturing our blurred intuitions regarding entities with no temperature, the association of *warm* with an unspecified transformation value renders *#2 degrees warm*, but not *2 degrees warmer*, infelicitous, as desired (cf., Section 3.3).

Thus, another source of cross linguistic variations regarding the licensing of numerical degree predicates is formed by the fact that languages may vary as to whether the measure associated with a given adjective will be transformed or not. 'Extent'-based analyses of antonymy[4] fail to capture these facts (for details see Sassoon, 2008). This is a serious problem given the pervasiveness of these phenomena. Thus, the present proposal improves upon 'extent' theories of antonymy in terms of the set of facts it adequately captures, while employing the simpler and more intuitive assumption whereby gradable adjectives map entities to single points, single real numbers $r \in \Re$.

Featherstone (to appear) discusses current trends in Psychophysics, suggesting that people *can generally build and use scales that encode differences between measures of stimuli, and sometimes also, but not necessarily, ratios*. Featherstone also makes an interesting new case for this claim, based on the experimental research of judgments of linguistic wellformedness. Featherstone shows that more accurate and informative results are obtained when subjects are encouraged to rank differences, rather than ratios, between the wellformedness of different linguistic structures, and when the data is processed accordingly. Featherstone's new view is in line with my assumption that the degree functions of many positive adjectives do not represent ratios, as they do not have a uniquely determined, agreed upon zero point (either in the first place, or because they are transformed) and that ratio statements are used only when additional information is presupposed (regarding the zero point, or regarding the transformation value being equal to zero).[5] The adjective *felicitous* forms an example of an adjective for which no zero point exists in the first place, yet differences between degrees accurately describe differences in felicity. Most plausibly, the majority of positive adjectives denote measures with all the properties of interval-scales (and not necessarily all the properties of ratio-scales) in the first place. When subjective judgments or internal states (e.g., degrees to which things feel heavy, loud, warm, tasty, funny, felicitous, nice, happy, organized, etc.) are at stake, the likelihood that speakers will regard the measure as additive (ratio-scale) is reduced. Speakers may use ratio-statements (or consider them acceptable) to the extent that their beliefs allow for the possibility that ratio measures exist (and they may use measure phrases if, in addition, conventions regarding unit objects exist).

[4] Examples include von Stechow (1984b) and Kennedy (1999, 2001).

[5] A common experimental practice in the social sciences is to present subjects with a numerical scale while instructing them that the differences between any two adjacent values are identical. When the scale has a zero point (representing complete absence of the measured property), data analysis relying on addition and multiplication (e.g., averaging, t-test, etc.) is considered appropriate, i.e. subjects are thought of as capable of producing ratio judgments in the given circumstances.

3.6 Conclusions

Part 3 presents compelling support for my proposal, whereby measurement theory's taxonomy of measures (cf. Part 1) and its conception of unit based measurements (cf. Part 2) apply to linguistics and explain a large number of semantic and distributional facts regarding unit names and measure phrases, including facts pertaining to adjectives that combine with unit names to form measure phrases (e.g., *tall*), adjectives that have no unit names as it is impossible to agree on conventional unit objects for them (e.g., *happy*), and adjectives that have no unit names as their degree function does not encode ratios (e.g., *short*), but whose comparatives can be modified by measure phrases combined from, e.g., their positive antonyms and their unit names. Finally, *Celsius* is an exceptional unit name, which nonetheless is interpreted by native speakers based on rule (4), thus further supporting the view that based on this rule, unit names are productively generated and interpreted.

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