Two Kinds of Modified Numerals

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Abstract
In this paper I argue that there are two kinds of numeral modifiers: (Class-A) those that express the comparison of a certain cardinality with the value expressed by the numeral and (Class-B) those that express a bound on a degree property.

1 Introduction

The landscape of modified numerals is strikingly diverse. Apart from what I will call *comparative quantifiers*, like "more than 100", "fewer than 100", "less than 100", "no more than 100", "no fewer than 100", etc., there are *superlative quantifiers* like "at least 100" and "at most 100"; *disjunctive quantifiers* like "100 or more", "100 or fewer", etc.; *prepositional quantifiers* like the locative "under 100", "over 100" and "between 100 and 200" and the directional "from 100" and "up to 100"; and a whole range of other quantifiers based on operators expressing a bound, such as "minimally 100", "maximally 100" or "100 tops".

Recently, a number of studies have tried to explain this variation. Such investigations usually concern the specific quirks of certain modified numerals.[1] While I believe that it is important to have a semantic analysis of modified numerals on a case by case basis, I also believe that what is lacking from the literature so far is a view of to what extent the various modified numerals involve similar semantic structures. In this paper, I will attempt to reach a generalisation along these lines by claiming that there are two kinds of modified numerals: (A) those that relate the numeral to some (specific) cardinality and (B) those that place a bound on the cardinality of some property. The difference will be made clear below. The most obvious examples of (A) are comparative modified numerals like "more/fewer than". Most other kinds of modified numerals fall in the second class.

[1] See, e.g., Geurts and Nouwen (2007); Umbach (2006); Corblin (2007); Krifka (2007) on *at least/most*, Corver and Zwarts (2006) on locative quantifiers, Nouwen (2008a) on directional quantifiers, Nouwen (2008b) on negative comparative quantifiers. See also Nouwen (2008c) for an overview.

I will start by making clear what distinguishes the two classes of modified numerals by presenting a body of data that sets them apart. Then, in section 3, I introduce a well-founded decompositional treatment of comparative quantifiers (Hackl, 2000), which I take to represent the proper treatment of class A modifiers. In section 4, I propose that class B modifiers are operators that indicate maxima/minima. Section 5 concludes by discussing a remaining problem as well as some speculations about which modifiers belong to which class.

2 Class A and class B modified numerals

It is a striking feature of comparative quantifiers that they can be used to assert extremely weak propositions. For instance, (1) is acceptable, even though it expresses a rather under-informative truth.

(1) A hexagon has fewer than 10 sides. A

This example contrasts strongly with the examples in (2), which are all unacceptable. (Or, alternatively, one might have the intuition that they are *false*).

(2) a. #A hexagon has at most 10 sides. B
b. #A hexagon has maximally 10 sides. B
c. #A hexagon has up to 10 sides. B

Why is this so? A naive theory might have it that (1) states that the number of sides in a hexagon is strictly smaller than 10 (i.e. $<$10), and that the only difference with (2) is that, there, it is stated that this number is smaller *or equal* to 10 (i.e.$\leq$ 10). Clearly, 6 is both $<$ 10 and $\leq$ 10. So why are not both kinds of examples under-informative but true?

Let's call quantifiers that are acceptable in such examples *class A* quantifiers and those that are like (2) *class B* quantifiers. As the contrast between (3) and (4) shows, the distinction is also visible with lower bound quantifiers. That is, (3) is under-informative, yet true and acceptable, while the examples in (4) are unacceptable/false.

(3) A hexagon has more than 3 sides. A

(4) #A hexagon has { at least / minimally } 3 sides. B

What I think is the underlying problem of examples involving class B expressions is that such quantifiers are incapable of relating to definite amounts. Imagine, for instance, that we are talking about my new laptop and that we are concerned with how much internal memory it has. Say, it has 1GB of memory and that I know that it has so much memory. In case, for instance, you just told me that your laptop has 2GB of memory, then I can assert (5).

(5) I know exactly how much memory my laptop has and it is less than 2GB.

Or, if your computer has a mere 512MB of memory, I can boast that:

(6) My laptop has more than 512MB of memory.

In these examples, I am comparing the definite amount of 1GB to some given contrasting amount 2GB (512MB) by means of "less than" ("more than"). This is something class A quantifiers can do very well, but something that is unavailable for class B modified numerals:

(7) I know exactly how much memory my laptop has...
  a. ...and it is { #at most / #maximally / #up to } 2GB.
  b. ...and it is { #at least / #minimally } 512MB.

In contrast to (7), class B quantifiers are acceptable when what is 'under discussion' is not a definite amount, but rather a range of amounts, as in (8).

(8) a. Computers of this kind have { at most / maximally / up to } 2GB of memory.
  b. Computers of this kind have { at least / minimally } 512MB of memory.

In other words, it appear that class B quantifiers relate to ranges of values, rather than to a single specific cardinality. This intuition is supported (9).

(9) Jasper invited maximally 50 people to his party.

We normally interpret (9) to indicate that the speaker does not know how many people Jasper invited. That is, it is unacceptable for a speaker to utter (9) if s/he has a definite amount in mind, which is why the rider in (10) is infelicitous. (Cf. Geurts and Nouwen (2007), Corblin (2007)).

(10) Jasper invited maximally 50 people to his party, #namely 43.

By assuming that the speaker does not know the exact amount, (9) is interpreted as being about the range of values possible from the speaker's perspective. The speaker thus states that there is a bound on that range. The same intuition occurs if we substitute "maximally 50" by any other class B quantifier.

3 Hackl's semantics for comparative modifiers

In this section, I discuss the semantics for comparative modified numerals as developed in Hackl (2000). I will assume that this represents the proper treatment of class A numeral modifiers. I also extend the framework slightly by adding a way to account for the ambiguity of non-modified numerals.

3.1 Class A modifiers as degree quantifiers

What is the semantics of a class A quantifier? It is tempting to think that class A quantifiers correspond to the well-known generalised quantifier-style determiner denotations such as the one in (11).

(11) $[\text{fewer than } 10] = \lambda PQ.|P \cap Q| < 10$

In the past decade it has become clear that it is important to have a closer look at these modified numerals (Krifka, 1999; Hackl, 2000). In what follows, I will assume the following semantics of "fewer than", which is based on the arguments in Hackl (2000).

(12) $[\![\text{fewer than } 10]\!] = \lambda M.\ \max_n(M(n)) < 10$

The workings of this definition will become clear below, but one of the main motivations for an analysis along this line can be pointed out immediately. The semantics in (12) is simply that of a comparative construction, where cardinalities are seen as a special kind of degrees. That is, like the comparative, it involves a degree predicate M and a maximality operator that applies to this predicate (Heim, 2000). In other words, (12) is completely parallel to *other* comparatives, like (13).

(13) $[\![\text{shorter than } d]\!] = \lambda M.\max_{d'}(M(d')) < d$

Hackl assumes that argument DPs containing a (modified) numeral always contain a silent counting quantifier "many":

(14) $[\![\text{many}]\!] = \lambda n \lambda P \lambda Q.\exists x[\#x = n \ \&\ P(x)\ \&\ Q(x)]$

(15) 10 sushis $\rightsquigarrow$ [$_{\text{DP}}$ [10 many] sushis]

In this framework, the numeral is an argument of the quantifier "many". By applying [10 many] to the noun (phrase), the standard generalised quantifier denotation of "10 sushis" is derived: $\lambda Q.\exists x[\#x = 10\ \&\ \text{sushi}(x)\ \&\ Q(x)]$. The structure of a DP containing a modified numeral does not differ essentially. Modified numerals are also the argument of a counting quantifier, as illustrated in (16).

(16) fewer than 10 sushis $\rightsquigarrow$ [$_{\text{DP}}$ [[fewer than 10] many] sushis]

"Fewer than 10", however, is a (degree) quantifier, not a number constant. Thus, for type purposes, the modified numeral in (16) has to move, leaving a degree trace and creating a degree property.

(17) Jasper ate fewer than 10 sushis.
$\rightsquigarrow$ [[fewer than 10] [λn [Jasper ate n many sushis]]]

This leads to the following interpretation, which results in the desired simple truth-conditions.

(18) $[\lambda M.\max_n(M(n)) < 10]\ (\ \lambda n.\exists x[\#x = n\ \&\ \text{sushi}(x)\ \&\ \text{ate}(j,x)])$
$= \max_n(\exists x[\#x = n\ \&\ \text{sushi}(x)\ \&\ \text{ate}(j,x)]) < 10$

If, like degree operators, modified numeral operators can take scope, we expect to find scope alternations that resemble those found with degree operators (Heim, 2000). As Hackl observed, this is borne out by examples like the following.[2] The two readings of (19) are an upper bound reading, and one which is very weak, stating simply that

[2] I show here the scope interaction with weak modality. There is a similar interaction with strong modals. The example in (i), for instance is ambiguous with (i-a) and (i-b) as its two readings. See Hackl (2000) for details.

values below the numeral are within what is permitted, without stating anything about the permissions for higher values.

(19) John is allowed to bring fewer than 10 friends.
  a. 'John shouldn't bring more than 9 friends'
  b. 'It's OK if John brings 9 or fewer friends (and it might also be OK if he brings more)'

Following Heim (2000), Hackl analyses this ambiguity as resulting from alternative scope orderings of the modal and the comparative quantifier.

(20) $\max_n(\Diamond\exists x[\#x = n \;\&\; \text{friend}(x) \;\&\; \text{bring}(j,x)]) < 6$
[[fewer than 6] [λn [allow [John invite n-many friends]]]]

(21) $\Diamond[\max_n(\exists x[\#x = n \;\&\; \text{friend}(x) \;\&\; \text{bring}(j,x)]) < 6]$
[allow [[fewer than 6] [λn [John invite n-many friends]]]]

The reader may check that Hackl's predicted readings in (20) and (21) are indeed the attested ones.

3.2 Class B modifiers are different

These analyses are strongly supportive of an approach which treats comparative quantifiers as comparative constructions. The question now is whether class B quantifiers should be given a similar treatment. In other words, will the semantics in (22) do?

(22) $[\![\text{up to / maximally / at most / etc... } 10]\!] =^? \lambda M.\ \max_n(M(n)) \leq 10$

Choosing a semantics that is parallel to that of "fewer than" is partly unintuitive since the class B quantifiers are not comparative constructions. Yet, cases like "maximally 10" suggest that the crucial ingredient of the semantics is the same, namely a maximality operator. The unsuitability of the analysis in (22) becomes immediately apparent, however, if we investigate examples with class B modified numerals embedded under a weak modal: these turn out not to be ambiguous. Class B modifiers like *maximally*, *up to* and *at most* always yield an upper bound on what is allowed and resist the weaker reading that was found with comparative modifiers.

(23) John is allowed to bring { up to / at most / maximally } 10 friends.
#But more is fine too.

A further interesting property of the interaction of class B modified numeral quantifiers and modals is that weak modals intervene with the inferences about speaker knowledge that we found for simple sentences. Above, I observed that (24) licenses the inference

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(i) (Bill has to read 6 books.) John is required to read fewer than 6 books.
  a. 'John shouldn't read more than 5 books'
  b. 'The minimal number of books John should read is fewer than 6'

For reasons explained in Heim (2000), structural ambiguity arising from degree quantifiers and intensional operators like modals is only visible with non-upward entailing quantifiers.

that the speaker does not know how many friends Jasper invited. In contrast, (25) does not license any such inference; it is compatible with the speaker knowing exactly what is and what is not allowed.

(24) Jasper invited maximally 50 friends.

(25) Jasper is allowed to invite maximally 50 friends.

These observations add to the data separating class A from class B quantifiers. Summarising, the distinctions are then as follows. First of all, class B quantifiers, but not class A quantifiers, resist definite amounts, except when embedded under a weak modal. Second, class B quantifiers, but not class A quantifiers, resist weak readings when embedded under a weak modal.

In the next section I will argue that the peculiarities of class B quantifiers can be explained if we assume that they are quite simply maxima and minima indicators. Basically, what I propose is that the semantics of "maximally" ("minimally") is simply the operator $\max_d$ ($\min_d$). This might be perceived as stating the obvious. What is not obvious, however, is how such a proposal accounts for the difference between class A and class B quantifiers. I will argue that the limited distribution of class B modifiers is due to the fact that they give rise to readings that are in competition with readings available for non-modified structures. I will show that, in many circumstances, the application of a class B modifier to a numeral yields an interpretation which is equivalent to one that was already available for the bare numeral. Before I can explain the proposal in detail, I therefore need to include an account of bare numerals in the framework.

3.3 The semantics of numerals

Above, I adopted the semantics of Hackl (2000) for comparative modified numerals. An important part in that framework is played by the counting quantifier *many*. I will rename this operator $many_1$, for, in what follows, I assume that for any (modified) numeral there are two counting quantifiers available. These two options are to account for the two meanings of numerals that may be observed: the existential weak lower-bounded meaning and the doubly bound strong meaning. An example like (26), for instance, is ambiguous between (26-a) and (26-b).

(26) Jasper read 10 books.
  a. the number of books read by Jasper ≥ 10
  b. the number of books read by Jasper = 10

I assume that the meaning in (26-b) is semantic and not the result of a scalar implicature that results from (26-a). See e.g. Geurts (2006) for a detailed ambiguity account, and for some compelling arguments in favour of it.

In the current framework, that of Hackl (2000), the weak reading in (26-a) is due to a weak semantics for the counting quantifier: i.e. $many_1$. I propose that the strong reading, (26-b), is accounted for by an alternative quantifier $many_2$ (taking inspiration from Geurts (2006).)

(27) $[\text{many}_1] = \lambda n \lambda P \lambda Q.\exists x[\#x = n \;\&\; P(x) \;\&\; Q(x)]$
$[\text{many}_2] = \lambda n \lambda P \lambda Q.\exists! x[\#x = n \;\&\; P(x) \;\&\; Q(x)]$

Not only does this option suffice to account for the ambiguity of bare numerals, it is moreover harmless with respect to the semantics of comparative quantifiers. That is, it is important to note that the account of the ambiguity of bare numerals does not predict further ambiguities to arise for modified numerals, since such ambiguities do not appear to exists. It is instructive to see in somewhat more detail why the availability of two counting quantifiers changes nothing for our account of comparative quantifiers. The structure in (28) is exemplary of any simple sentence with a modified numeral. As explained earlier, the modified numeral applies to the degree predicate that is created by moving the quantifier out of the DP.

(28) [MOD *n* [λd [Jasper read *d* $\text{many}_{1/2}$ books]]]

Of course, the denotation of the degree predicate depends on which of the two counting quantifiers is chosen. The predicate in (29) is the result of a structure containing *many*$_1$; the predicate in (30) is based on *many*$_2$. If, in the actual world, Jasper read 10 books, then (29) denotes $\{1,2,3,4,5,6,7,8,9,10\}$. When, however, the predicate contains the *many*$_2$ quantifier, the denotation is a singleton set: $\{10\}$ if Jasper reads 10 books. This is because only the maximal group of books read by Jasper is such that it is the unique group of that kind of a certain cardinality. In general, the *many*$_2$-based degree predicate extension is a singleton set containing the maximum of the values in the denotation of the *many*$_1$-based degree predicate.

(29) $\lambda d.\exists x[\#x = d \;\&\; \text{book}(x) \;\&\; \text{read}(j,x)]$

(30) $\lambda d.\exists! x[\#x = d \;\&\; \text{book}(x) \;\&\; \text{read}(j,x)]$

As discussed above, comparative quantifiers involve maximality operators. However, the maximal values for degree predicates like (29) and (30) are always equivalent. In simple sentences based on a structure like (28), the option of having two distinct counting quantifiers does therefore not result in any ambiguity.

When we turn to cases where the degree predicate is formed by moving the modified numerals over a strong modal operator, something similar can be observed. If Jasper is required to read (exactly) 10 books, then the structure in (31) yields, again, the set $\{1,2,3,4,5,6,7,8,9,10\}$. Once more, the structure which contains the bilateral counting quantifier, the one in (32), yields the set containing the maximum of its weaker counterpart.

(31) [λd [require [Jasper read *d* many_1 books]]]
$\rightsquigarrow \lambda d.\Box\exists x[\#x = d \;\&\; \text{book}(x) \;\&\; \text{read}(j,x)]$

(32) [λd [require [Jasper read *d* many_2 books]]]
$\rightsquigarrow \lambda d.\Box\exists! x[\#x = d \;\&\; \text{book}(x) \;\&\; \text{read}(j,x)]$

Given that the relation between (32) and (31) is once again one of a set and its maximal value, no ambiguities can be expected to arise when comparative quantifiers are applied to these two predicates. This is as is desired.

Of course, it could be that the actual situation is not one containing a specific requirement, but one with for instance a minimality requirement. Say, for instance, Jasper has to read at least 4 books. In that case, (32-a) denotes the set $\{1,2,3,4\}$. The extension of (32-b), however, is the empty set. (In such a context, there is no specific n such that Jasper has to read exactly n books.) Clearly, the maximal value for the predicate is undefined in such a case. This means the LF based on many$_2$ will not lead to a sensible interpretation and, so, we again do not expect to find ambiguity.

The case of predicates that are formed by abstracting over a weak modal operator is illustrated in (33) and (34). If Jasper is allowed to read a maximum of 10 books, then the two predicates are equivalent, both denoting the set $\{1,2,3,4,5,6,7,8,9,10\}$.[3]

(33) $\lambda d.\Diamond\exists x[\#x = d \;\&\; \text{book}(x) \;\&\; \text{read}(j,x)]$

(34) $\lambda d.\Diamond\exists! x[\#x = d \;\&\; \text{book}(x) \;\&\; \text{read}(j,x)]$

In sum, the option of two counting quantifiers *many*$_1$ and *many*$_2$ is irrelevant when combined with a comparative quantifier. This is because the comparative quantifier is based on maximality and the degree predicates containing the different counting quantifiers do not differ in their maximum value.

4 The semantics of class B quantifiers

I now turn to the main proposal: class B quantifiers are maxima/minima indicators. I start with the upper-bounded modifiers.

4.1 Upper bound class B modifiers

In the formula in (35) MOD generalises over any of the class B modifiers "at most", "maximally", "up to", etc.

(35) $[\text{MOD}] \;=\; \lambda d.\lambda M.\ \text{max}_n(M(n)) = d$

If the semantics of upper bound class B quantifiers is as in (35), then why is their distribution so limited? What I think is the reason for the awkwardness of a lot of examples with class B quantifiers is the fact that in many cases (35) is a vacuous operator. I think it will be important to find out for which degree predicates M, stating that $\text{max}_d(M(d)) = n$ is equivalent to simply stating $M(n)$. The easy answer is that this equivalence holds when M denotes a singleton set. This observation has profound consequences for when it actually makes sense to state that the maximum of a degree predicate equals a certain value. To see this let us carefully go through the following examples.

We know from the discussion above that one of the interpretations available for (36) is (37).

(36) Jasper invited 10 people.

[3]If there is in addition a lower bound, the two predicates are no longer equivalent, but their maximum will be.

(37) $\exists!x[\#x = 10 \;\&\; \text{people}(x) \;\&\; \text{invite}(x)]$

Now consider (38), which is interpreted either as (39) or as (40).

(38) Jasper invited maximally 10 people.

(39) [maximally 10 [λd [Jasper invited d many_1 people]]]
$\rightsquigarrow \max_n(\exists x[\#x = n \;\&\; \text{people}(x) \;\&\; \text{invite}(j,x)]) = 10$

(40) [maximally 10 [λd [Jasper invited d many_2 people]]]
$\rightsquigarrow \max_n(\exists!x[\#x = n \;\&\; \text{people}(x) \;\&\; \text{invite}(j,x)]) = 10$

The interpretations in (39) and (40) are equivalent. In fact, just like we do not expect ambiguities to arise with comparative quantifiers on the basis of the $many_1/many_2$ choice, we do not expect any ambiguities to arise with upper-bound class B quantifiers, for the simple reason that both such operators involve a maximality operator and that the maximal values of predicates based on $many_1$ are always those of predicates based on $many_2$. In what follows, we will therefore gloss over the two equivalent options by representing the semantics following the general scheme in (41).

(41) [maximally 10 [λd [Jasper invited d $\text{many}_{1/2}$ people]]]
$\rightsquigarrow \max_n(\exists(!)x[\#x = n \;\&\; \text{people}(x) \;\&\; \text{invite}(j,x)]) = 10$

Importantly, the single reading of (38) is equivalent to (37), the strong reading of (36). The example in (36), however, reaches this interpretation by means of a much simpler linguistic form; one which does not involve a numeral modifier. I propose that this is why the reading in (41) of (38) does not surface: it is blocked by (36).

As observed above, we can nevertheless make sense of (38) once we interpret the sentence to be about what the speaker holds possible. So, a further possible reading for (38) is that in (42).

(42) $\max_n(\Diamond\exists(!)x[\#x = n \;\&\; \text{people}(x) \;\&\; \text{invite}(j,x)]) = 10$

Crucially, this interpretation is not equivalent to (43), which is the result of interpreting (36) from the perspective of speaker possibility.

(43) $\Diamond\exists!x[\#x = 10 \;\&\; \text{people}(x) \;\&\; \text{invite}(j,x)]$

In other words, the meaning in (42) for (38) is not blocked by the bare numeral form in (36) since (36) lacks this reading.

What is crucial is that degree predicates based on weak modals denote non-singleton sets even when the counting quantifier associated with the numeral is $many_2$. This entails that saying that the maximum value for such a predicate is n is not equivalent to saying that the predicate holds for n. As a result, whenever an upper bound class B modifier scopes over a weak modal, no blocking from the simpler bare numeral form will be able to take place. In other words, the application of an upper bound class B quantifier to a degree predicate is only felicitous if the resulting readings are not readings that can be expressed just as well by omitting the class B modifier. This is the case when a weak modal has scope inside the degree predicate.

Treating upper bound class B quantifiers as maxima indicators also explains why in the interaction with weak modality, no weak meanings occur. Consider (44).

(44) Jasper is allowed to invite maximally 10 people.

If "maximally 10" is taken to have wide scope over the modal, then we arrive at the reading that says that the maximum number of people Jasper is allowed to invite equals 10. This is not a semantic interpretation that is available for (45). Its many$_2$ reading, for instance, says that inviting exactly 10 people is something that Jasper is allowed to do. This is much weaker than our interpretation for (44). The only way we can arrive at an equally strong reading for (45) is by means of implicature.

(45) Jasper is allowed to invite 10 people

Given the fact that (45) yields weak semantic meanings, (44) lacks such weak meanings. If we take the modal in (44) to have widest scope, the resulting interpretation is one in which inviting exactly 10 people is allowed for Jasper. This is the reading for (45) discussed above, and so it is blocked.

An interesting side to the account presented here is that the upper bound class B quantifiers do not encode the $\leq$ relation. As maxima indicators, their application only makes sense if what they apply to denotes a range of values. Otherwise, using the strong reading of the bare numeral form will do just as well.[4]

4.2 Lower-bound class B modifiers

Lower-bound class B modifiers correspond to minimality operators. Let MOD correspond to any of the class B expressions *at least*, *from*, *minimally*, etc.

(46) $[\![\text{MOD}]\!] = \lambda d.\lambda M.\, \min_n(M(n)) = d$

Note first that minimality operators are sensitive to the many$_1$ / many$_2$ distinction. Say, we once more consider the degree predicate [λd. John read d many$_{1/2}$ books] and, say,

[4]Interestingly, the approach also predicts that some of the examples I discussed above do not only result in a blocking effect, but could moreover be predicted to be false. For instance, according to the approach set out above, the meaning of (i-a) is that in (i-b).

(i) a. #A triangle has maximally 10 sides.
b. '*the maximum number of sides in a triangle is 10*'

The reading in (i-b) is not only blocked by "A triangle has 10 sides", but it is moreover plainly false. I believe that this predicts that (i-a) should be expected to have a somewhat different status from (ii), which strictly speaking has a true interpretation, but one that can be expressed by simpler means.

(ii) #A triangle has maximally 3 sides.

It is difficult to establish whether this difference in status is borne out, or even how this difference can be recognised. However, my own intuition tells me that while (i) is never acceptable, (ii) could be used in a jokingly fashion. Native speakers inform me that (iii) is marginally acceptable:

(iii) "?A triangle has minimally and maximally 3 sides".

that John read 10 books. In the $many_1$ version of the LF, the minimal degree equals 1. In fact, independent of how many books John read, as long as he read books, the minimal degree will *always* be 1. In the $many_2$ version of the LF, the predicate denotes a singleton set, $\{10\}$ if John read 10 books. The minimal degree in that case is, of course, 10.

These observations already straightforwardly account for our intuitions for an example like (47).

(47) John read minimally 10 books.

The $many_1$ LF of (47) will be rejected, for it will always be false. The minimal value for any simple $many_1$-based degree predicate is always 1. The $many_2$ LF of (47) will be rejected too, for it will correspond to an interpretation saying that John read (exactly) 10 books. This reading is blocked by the bare numeral. (In fact, (47) in the $many_2$ variant is equivalent to *John read maximally 10 books*, which is equally blocked.)

We can save (47) by sticking in a weak modal. This yields two readings:

(48) a. $\min_d(\Diamond\exists x[\#x = d \ \&\ \text{read}(j,x) \ \&\ \text{book}(x)]) = 10$
b. $\min_d(\Diamond\exists! x[\#x = d \ \&\ \text{read}(j,x) \ \&\ \text{book}(x)]) = 10$

The form in (48-a) is once more a contradiction: the minimal degree for which it is deemed possible that John read d-$many_1$ books is always 1. The reading in (48-b) is much more informative. It says that that the minimal number for which it is thought possible that John read exactly so many books is 10. In other words, this says that it is regarded as impossible that John read fewer than 10 books. This is exactly the reading that is available.[5]

5 Outlook and conclusion

The analysis presented in the last section is just the tip of the iceberg. In fact, beyond what I have shown so far loom some problems which are hugely problematic. As Hackl (2000) observed, there is an interesting interaction between modified numerals and modals. I have extended these observations by showing how weak modals have a tight connection to class B modifiers in that they license their (otherwise blocked) existence. What I have not discussed at all is how class B modifiers interact with strong

[5]Some words are in order on the interaction of numeral modifiers with non-modal operators. Given the current proposal, any property that involves existential quantification would license the use of a class B modifier. However, it is known that degree operators (which we take modified numerals to be) cannot take scope over nominal quantifiers (see for instance Heim (2000)). This explains why (i) does not have the reading in (ii).

(i) Someone is allowed to invite maximally 50 friends.

(ii) the person who is allowed to invite most friends is allowed to invite 50 friends

As observed above, however, bare plurals do interact with class B quantifiers, as in for instance example (8). This would suggest that some intensional/modal analysis of the readings involved in such examples is in order. (Thanks to Maribel Romero for pointing this out to me.) At this point, however, I have no worked out theory of how to deal with such examples in detail.

modals. It turns out that this part of the story is not straightforward at all.[6] To sketch the problem, consider (49).

(49) To please his mother, John should read minimally 10 books.

According to my proposal, (49) means the following: 10 is the minimal number of books such that in every world in which John pleases his mother, John reads that number of books. This makes no sense. Say that we are in a situation in which John's mother is pleased in case John reads 5 or more books. In such a situation, every world in which John pleases his mother is a world in which John reads (at least) 5 books. Yet, every such world is also a world in which John reads a book. This means that the minimal number of books read in every *pleased-mother* world is 1 and not 5. In fact, in any situation, this minimal number will be 1.

The proposal, then, makes the wrong prediction: (49) is predicted to be non-sensical, when in reality it has a clear and intuitive meaning.

Nevertheless, I think that this problem does not undermine the proposal above. The reason is that the same problem occurs with different examples, that are completely independent of modified numerals. According to my theory, (49) means exactly the same as (50).

(50) The minimal number of books John must read to please his mother is 10.

This example wears its semantic analysis on its sleeve: (50) corresponds to (51).

(51) $\min_d(\Box_{\text{please John's mother}} \exists x[\#x = d \;\&\; \text{book}(x) \;\&\; \text{read}(j,x)]) = 10$

The example in (50) does not have a modified numeral in it. Still, its analysis, that of (51), runs in exactly the same trouble as did my proposal for (49): the form in (51) is non-sensical, for the minimal *d* alluded to will always be 1. I conclude from this that the problem is evidence for a puzzle which is independent of modified numerals, one which involves the interaction of modality and degree generally. Until we solve this bigger puzzle, there is no point in passing judgement on the current proposal.

⋆

The main point of this paper is that there are two kinds of modified numerals. I leave it an open question exactly which quantifiers belong to which class. Nevertheless, I can already offer some speculations on this. Class A, for instance, could very well contain non-comparative quantifiers. Possible candidates are certain locative prepositional modifiers. Compare (52-a) and (52-b).

(52) a. You can get a car for under 1000 euros.
b. You can get a car for maximally 1000 euros.

The example in (52) is somewhat strange, since it claims that the most expensive car you can buy is 1000 euros. The example in (51), in contrast, makes no such claim. It clearly

[6] There is a precedent. In an earlier theory of "at least", Geurts and Nouwen (2007), the correct predictions regarding its relation to strong modals are arrived at by an essentially non-compositional mechanism. Krifka (2007) deals with this issue in a similarly non-standard way.

has a weak reading: there are cars that are cheaper than 1000 euros and there might be more expensive ones too. Such weak readings occur with class A and not with class B quantifiers. Furthermore, "under" seems perfectly compatible with definite amounts, such as in (53).

(53) The total number of guests is under 100. (To be precise, it's 87.)

Class A is then not restricted to comparative constructions only. In fact, other locative prepositions seem to behave similarly to "under".

(54) The total number of guests is between 100 and 150. (It's 122.)

The locative complex preposition "between ... and ... " contrasts with its directional counterpart "from ... (up) to ... ", which behaves like a class B modifier: it is incompatible with definite amounts, as in (55), but felicitous if it relates to a range of values.

(55) #The ticket to the Stevie Wonder concert that I bought yesterday cost from €100 to €800.

(56) Tickets to the Stevie Wonder concert cost from €100 to €800.

"Over" parallels "under". In (57), "over 100" is clearly relating the precise weight 104kg with 100kg. Note in (58) how this contrasts with the directional "100 ... and up", which is made felicitous by embedding it under a weak modal.

(57) He weighs over 100 kg. To be precise, he weighs 104 kg.

(58) a. #He weighs 100 kg and up.
b. He is allowed to weigh 100 kg and up.

Data like these suggest that even more generalisations about the two kinds of modified numerals are still to be discovered. (It seems, for instance, that locative prepositions end up as class A modifiers, whereas directional ones are members of class B). Just like the precise formulation of how to account for the interaction of modifiers and strong modality, however, a detailed investigation of the generalisations governing the class A/B divide is left to further research.

Acknowledgment

I am grateful to the audience of Sinn und Bedeutung in Stuttgart for many thought-provoking comments. This work was supported by a grant from the Netherlands Organisation for Scientific Research NWO, which I hereby gratefully acknowledge.

References

Corblin, F. (2007). Existence, maximality and the semantics of numeral modifiers. In Comorovski, I. and von Heusinger, K., editors, *Existence: Semantics and Syntax*, volume 84 of *Studies in Linguistics and Philosophy*.

Corver, N. and Zwarts, J. (2006). Prepositional numerals. *Lingua* **116**(6):811–836.

Geurts, B. (2006). Take five: the meaning and use of a number word. In Vogeleer, S. and Tasmowski, L., editors, *Non-definiteness and plurality*, 311–329. Benjamins, Amsterdam/Philadelphia.

Geurts, B. and Nouwen, R. (2007). At least et al.: the semantics of scalar modifiers. *Language* **83**(3):533–559.

Hackl, M. (2000). *Comparative Quantifiers*. PhD thesis, Department of Linguistics and Philosophy, Massachusetts Institute of Technology.

Heim, I. (2000). Degree operators and scope. In *Proceedings of SALT 10*, Ithaca, NY. CLC Publications.

Krifka, M. (1999). At least some determiners aren't determiners. In Turner, K., editor, *The semantics/pragmatics interface from different points of view*, volume 1, 257–291. Elsevier.

Krifka, M. (2007). More on the difference between more than two and at least three. Paper presented at University of California at Santa Cruz.

Nouwen, R. (2008a). Directionality in modified numerals: the case of 'up to'. In Friedman, T. and Ito, S., editors, *Proceedings of SALT 18*, Ithaca, NY. CLC Publications.

Nouwen, R. (2008b). Upper-bounded *no more*: the implicatures of negative comparison. *Natural Language Semantics* **16**(4):271–295.

Nouwen, R. (2008c). What's in a quantifier? In Lentz, T., de Mulder, H., Nilsen, Ø., and Zondervan, A., editors, *Theoretical Validity and Psychological Reality*. Forthcoming.

Umbach, C. (2006). Why do modified numerals resist a referential interpretation? In *SALT XV Proceedings*, 258 – 275. Cornell University Press.