

Discourse plurals in an update semantics¹

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Abstract. Quantification famously licenses later reference to the set of individuals quantified over, as in “Almost every student wrote an extra-credit paper. They wanted to improve their grades.” Reference is also possible to other sets of individuals introduced in the quantification, as in “Almost every student wrote an extra-credit paper. They are sitting in a pile on my desk.” Previous work has cast doubt on whether such reference is available within the quantification itself, but this paper argues that it is available internally, as in “Most North Atlantic countries signed a treaty declaring that an attack on one of them an attack on all of them,” where the pronouns ‘them’ refer only to the signatory countries. Once such internal reference is allowed, it turns out that several other difficult phenomena can be captured without further machinery, such as reciprocals, cumulative readings, and sentences with ‘same’ and ‘different.’

Keywords: dynamic semantics, plural logic, update semantics, discourse plurals

1. Introduction

This might seem like a paper about plural pronouns – and it is! – but it’s part of a larger project to explore just how simple our theory of semantics can be. Can we get away with fewer rules of interpretation? Can we represent meanings via simpler data structures, e.g. sets instead of sets of sets? I take these questions to be inherently interesting, but if you disagree, just concentrate on the narrow analysis of plurals and translate the framework into your favorite system.

The main empirical claims of this paper pertain to so-called *discourse plurals*:

- (1) a. Most students in my class wrote a final paper.
b. The rest of them completed a different final project. [Restrictor Set]
- (2) a. Most students in my class wrote a final paper .
b. They left them in a pile on my desk. [Nuclear Scope Set] [Dependent Set]

Quantified sentences like (1a) and (2a) license subsequent plural pronouns referring to various sets related to the quantification. The pronoun *them* in (1b) refers to the *restrictor set*, the set of individuals defined only by the quantifier’s restriction, *students in my class* in this case. The pronoun *they* in (2b) refers to the *nuclear scope set*, the subset of the restrictor set that satisfies the nuclear scope, e.g., just those students who wrote a final paper in (2). (I indicate this by highlighting the whole DP.) Finally, pronouns may also target individuals introduced within the quantification, such as the papers in (2), as illustrated by *them* in (2b). I will call such sets *dependent sets*.

The discourse plural pronouns presented above all occur outside the quantified sentence that licenses them, and I will therefore call these *external* discourse plurals. The empirical question I will pursue below is whether there are *internal* discourse plurals, accessible within the very sentences that define them. Kamp and Reyle (1993) and Nouwen (2003, 2007) argue that internal reference to discourse plurals is impossible, based on the following example sentence:

¹Thanks to Jakub Dotlačil and Jeremy Kuhn for extra helpful discussion!

- (3) a. Most lawyers hired a secretary they (jointly) interviewed.
 b. ≠ A majority of (the) lawyers hired a secretary they jointly interviewed.

(3b) has a salient reading where *they* refers to just that majority of lawyers *L* who (each) hired a secretary whom *L* all interviewed together. This is presumably due to the indefinite nature of the subject *A majority of lawyers*, which introduces a plural discourse referent, which I called *L* above, which is restricted by the nuclear scope and available for pronoun reference, even within the nuclear scope.

When the indefinite *a majority of lawyers* is replaced by a quantifier, as in (3a), though, this reading disappears. In other words, the pronoun *they* in (3a) cannot refer to the nuclear scope set, but rather must refer to some other group that interviewed the secretaries hired by the lawyers in the nuclear scope set. This is due to the subject being a true quantifier rather than an indefinite. The two sentences therefore have logical forms something like (4):

- (4) a. For most lawyers *l* [*l* hired a secretary that *X* jointly interviewed.]
 (where *X* is previously defined)
 b. There's a majority of the lawyers *L* and
 $\forall l \in L [l \text{ hired a secretary that } L \text{ jointly interviewed.}]$

Despite this evidence, I will argue that internal readings of discourse plurals do in fact exist. As a preview, consider the following:

- (5) a. Most of the lawyers in the firm just hired a secretary for a new labor pool they are forming (together).
 b. $\exists L [L \text{ comprises a majority of the lawyers in the firm and}$
 $\forall l \in L [l \text{ hired a secretary for the pool } L \text{ are forming together.}]$

(5a) does have a reading, sketched in (5b), where the pronoun *they* refers to the nuclear scope set. I use cases like this to motivate machinery within a dynamic logic that easily captures such internal discourse plurals. As it turns out, such machinery can provide simpler representations of some other constructions of interest to semanticists, such as reciprocals, dependent indefinites, sentences involving *same* and *different*, and cumulative readings of modified numerals.

2. Empirical Data

But first, let's survey the empirical landscape.

2.1. Excursus: Bare Plurals

Sentences where a bare plural follows *most* (like Kamp & Reyle's counterexample in (3a)) are most easily understood as generic and in fact are resistant to episodic readings (Cooper, 1996; Matthewson, 2001; Crnić, 2009). Simply adding *of the* to such examples (to form a *partitive*) greatly improves the episodic readings:

- (6) a. Most linguists ✓are millionaires / #went to New Zealand last year.
 (Matthewson, 2001)
 b. Most of the linguists ✓are millionaires / ✓went to New Zealand last year.

Such sentences also resist collective and cumulative readings (Nakanishi and Romero, 2004):

- (7) a. Most #(of the) boys lifted the piano (together) (Crnić, 2009)
 b. Most #(of the) boys ate three pizzas (between them).

The collective reading in (7a), where the boys in question worked together to perform a single action, is degraded when the subject is simply *most boys* but improves when the subject is *most of the boys*. Similar results hold for the cumulative reading in (7b), where you count up the total number of pizzas eaten by the group represented by the subject of the sentence.

Crnić (2009) points out, though, that the episodic, collective, and cumulative readings reappear when the bare plural following *most* cannot be easily construed as a kind:

- (8) a. Most linguists I ran into yesterday went to New Zealand last year.
 b. Most boys taking a break right now just lifted a piano together.
 c. Most boys at my party last night ate three pizzas (between them).

I will therefore be careful below to either use partitives or non-kind-denoting restrictors for quantifiers like *most*.

2.2. Collective and Cumulative Readings

Once the problem of bare plurals is addressed, and some context is added, internal readings to nuclear scope sets improve quite substantially. We will concentrate first on collective and cumulative readings:

- (9) Collective Readings
 a. Most (of the) lawyers in my firm gathered at the dock reserved for the private cruise they had jointly paid over \$1M for.
 b. Almost every woman I know rented a coach they rode in together.
- (10) Cumulative Readings
 a. Most (of the) boys in my study group ate three pizzas (between them) that they had brought for themselves.
 b. Almost every kid I know could name at most five Senators between them even if they compared notes.

Predicates like *gather* require and those like *rent a coach (together)* prefer collective readings – the subjects of (9) performed the action in question together as a group. In (9a) the lawyers gathered as a group for a single million-dollar cruise. In (9b), the women all rented one coach and rode in it together. In both cases, the pronoun *they / they* refers to this whole group. Similarly, the subjects in (10) perform actions that combine to yield some total result – eating three pizzas or naming five Senators. The whole group again can easily be referred to via the pronouns *them / they / themselves* in (10a) and *them / they* in (10b).

Hey, wait a minute!, you might be thinking. Couldn't these collective and cumulative readings be evidence that the subjects of the sentences are acting more like indefinites? In other words, couldn't *most of the lawyers in my firm* mean something more like *a majority of lawyers in my firm* and couldn't *almost every kid I know* mean *a group comprising almost every kid I*

know? Well, maybe. But as I show in the next section, even truly quantificational readings have plausible internal readings of nuclear scope sets.

2.3. Distributive Readings

I will take distributive readings to be a sign of true quantification, at least enough that it should rule out internal readings, if Kamp and Reyle (1993) are correct. And yet, consider the following sentence, which illustrates internal reference to the nuclear scope set within a distributive reading:

- (11) Most of the employees in my division received \$1,000 (apiece) in bonus pay this year for their group effort to retain our largest client.

Here the overall predicate of the VP is clearly distributive: each employee in the group received their own thousand-dollar bonus, as indicated by the acceptability of *apiece*. And yet, the pronoun *their* refers to the group as a whole. A single person may not have a “group effort,” as shown in (12), so the pronoun must not be singular *they* but rather refer to the plurality in question.

- (12) #I received a bonus for my group effort to retain a top client.

Further examples are shown below, with evidence of distributivity highlighted in each case:

- (13) a. Most North Atlantic countries authorized a high-ranking national diplomat to sign the new treaty declaring that an attack on one of them is considered an attack on all of them. [\checkmark *they* = only the signatory countries]
- b. Almost every woman in town drove herself alone to a field where they all gathered to discuss a secret plot. [\checkmark *they* = only the drivers]
- c. Most lawyers in the firm just hired a personal secretary for a new labor pool they are forming (together). [\checkmark *they* = only the contributing lawyers]
- d. Almost every friend of mine is one part-owner of a house they all share. [\checkmark *they* = only the cohabitating friends]
- e. More than half my teammates were fined \$50 (apiece) after one of them tattled on their late-night shenanigans. [\checkmark *them / their* = only the rule-breaking teammates]

In each case, despite the clear distributivity, the nuclear scope set is available for pronoun reference within the nuclear scope itself. Furthermore, sometimes the nuclear scope set pronoun itself is embedded inside the clearly distributive portion of the verb phrase, ruling out an analysis where a separate distributive operator scopes lower than the entire VP. For instance, in (13a), *them / they* are within an argument of the distributive VP *authorized a high-ranking national diplomat to sign ...*. And in (13d), *they* clearly falls within the distributive noun phrase *part-owner of a house they all share*. This evidence calls into question the generalization posited by Kamp and Reyle (1993) and Nouwen (2003, 2007).

2.4. Internal Reference to other Discourse Plurals

Restrictor sets and dependent sets may also be the targets of pronouns inside the nuclear scope of the quantifiers that create them. Examples are shown below, again with *evidence of distributivity* highlighted:

- (14) a. Most of the partners invested at most \$50,000 in the firm they all started together.
 [✓ *they* = all the partners, restrictor set]
- b. Most farmers who own a donkey had gathered them in the public field to graze.
 [✓ *them* = the gathered donkeys, nuclear-scope dependent set]

(14a) shows restrictor set anaphora inside the nuclear scope. (14b) shows anaphora inside the nuclear to a dependent set whose antecedent *a donkey* is in the restrictor. In this case, the anaphora is dependent on the nuclear scope set – only donkeys that gathered are included in the pronoun *them*.²

3. An Update Semantics

Following Kamp and Reyle (1993), existing plural semantic systems do not allow internal reference to discourse plurals. In this section, I will describe a plural dynamic logic that can capture the new empirical data described in the previous section. I will call this logic *PLUS* for *PLural Update Semantics*.

As mentioned above, though, an auxiliary goal of this paper is to test how simple our semantic system can be. To that end, we will describe a dynamic update logic whose states are single assignments, leveraging the fact that update semantics operate over sets of states in order to derive a plural semantics. The dominant existing plural logics, e.g. those following van den Berg (1996), are relational systems, rather than update systems. Relational systems give denotations to formulas that are relations over states, rather than functions over sets of states. For a simple singular logic like Dynamic Predicate Logic (Groenendijk and Stokhof, 1991: henceforth *DPL*), this means that the denotations of formulas are relations over assignments and for van den Berg's plural logic, the denotations are relations over sets of assignments. Converting each of these logics straightforwardly to update systems would instead involve sets of assignments for DPL and sets of sets of assignments for the plural logic of van den Berg (1996). Therefore, an update semantics whose states are single assignments (and thus whose denotations are sets of assignments) is roughly at the level of the singular logic DPL in terms of state complexity.

²Some types of discourse plural do not seem possible:

- (i) a. #Most teachers of (all) their kids ...
 b. Most people who teach all their children ...

(ia) is an attempt at referencing the restrictor set within the restrictor itself, which runs into an apparent i-within-i violation. Although (ib) sounds much better, it's likely that *their* is bound by the relative pronoun or operator rather than established via a discourse plural. In fact, it's quite hard to even test this type of discourse plural due to these issues.

3.1. Preliminaries

PLUS is based on first-order predicate logic, and as such involves formulas with predicates and variables (semi-colons represent conjunction):

$$(15) \quad \text{farmer}(f); \text{donkey}(d)$$

The interpretation of the variables in a formula requires an *assignment*, a function mapping variables to individuals, like $g = [f \rightarrow \text{farmer_sue}, d \rightarrow \text{daisy}]$. For instance, if (15) is interpreted relative to g , the result has the same denotation as $\text{farmer}(\text{farmer_sue}); \text{donkey}(\text{daisy})$.

Dynamic logics track information about possible values for variables in a formula and hence information about the model/world. In PLUS, this information is tracked via a set of assignments, each representing a possible combination of values for variables that satisfies the formula so far. I will call single assignments *states* of PLUS and these sets of assignments *information states*. An idealized discourse begins with a special information state: the singleton set containing the empty assignment (the assignment which maps no variables to any values, i.e., the empty set), represented by the “►” symbol:

$$(16) \quad \blacktriangleright = \{[\]\} = \{\emptyset\}$$

This starting information state combines with formulas via the update function $+$. For instance, an expression $[x]$ in PLUS essentially declares the variable x , making it available for later use in the formula. Immediately after its declaration, x may have any value in the domain \mathcal{D} :

$$(17) \quad \text{start} + [f] = \{ [f \rightarrow a] : a \in \mathcal{D} \} \\ = \left\{ [f \rightarrow \text{farmer_sue}], [f \rightarrow \text{farmer_bob}], \dots \right\} \\ \left\{ [f \rightarrow \text{daisy}], [f \rightarrow \text{bessie}], \dots \right\}$$

The information state in (17) contains all assignments whose domains are just $\{f\}$ and which map f to a value in \mathcal{D} . Subsequent predicate literals, such as $\text{farmer}(f)$, narrow down the values for f by eliminating assignments from this set:

$$(18) \quad \blacktriangleright + [f]; \text{farmer}(f) = \{ [f \rightarrow \text{farmer_sue}], [f \rightarrow \text{farmer_bob}], \dots \}$$

Notice that this set now reflects something about the world/model: we can recover the set of farmers by extracting all possible values for f in the component assignments of the set.

3.2. PLUS v1.0

Here is our first working definition of PLUS, meant to replicate DPL (Groenendijk and Stokhof, 1991):

$$(19) \quad \begin{array}{ll} \underline{\text{Domains}} & \\ \mathcal{D} & \text{individuals } a, b, c, \dots \\ \mathcal{V} & \text{variables } x, y, z, \dots \\ \mathcal{G} = \mathcal{V} \rightarrow \mathcal{D} & \text{partial assignments } g, h, k, \dots \text{ (partial functions from } \mathcal{V} \text{ to } \mathcal{D}) \\ \mathfrak{G} = \wp(\mathcal{G}) & \text{information states } G, H, K, \dots \text{ (subsets of } \mathcal{G}) \end{array}$$

- (20) Predicates
For any n -place predicate P , P' is a relation over n plurals – i.e., $P' \subseteq [\wp(\mathcal{D})]^n$
- (21) Terms v1.0
 $x^g = g(x)$
- (22) Formulas v1.0

$$G + [x] = \{h : \exists g \in G (g[x]h)\}$$

$$G + P(x, y, \dots) = \{g \in G : \langle x^g, y^g, \dots \rangle \in P'\}$$

$$G + \phi ; \psi = (G + \phi) + \psi$$

$$G + \neg \phi = G \quad \text{iff } G + \phi = \emptyset$$
where $g[x]h \Leftrightarrow g \setminus \{ \langle x, g(x) \rangle \} = h \setminus \{ \langle x, h(x) \rangle \}$
- (23) Truth
A formula ϕ is true iff $\blacktriangleright + \phi \neq \emptyset$

PLUS works with two basic domains, \mathcal{D} for individuals and \mathcal{V} for variables. The domain of assignments, \mathcal{G} , is defined as the set of partial functions from \mathcal{V} to \mathcal{D} . The domain of information states, \mathfrak{G} , is defined as the power set of \mathcal{G} (sets of assignments). Predicates in PLUS are interpreted via the $'$ operator, returning a set of n -tuples for each n -place predicate as shown in (20). Thus, models in PLUS would be pairs $\langle \mathcal{D}, ' \rangle$. Note that predicates are only defined over sets of individuals not individuals themselves – this will allow us to define plural predicates. Singular predicates will simply take singleton sets to represent singular values (and the set braces around such singletons will often be omitted below).

The only terms (arguments to predicates) in PLUS v1.0 are variables, which are interpreted relative to an assignment as shown in (21). The three basic formula types are random assignments $[x]$, literals $P(x, y, \dots)$, conjunctions $\phi ; \psi$, and negations $\neg \phi$. The denotations for these formulas are shown in (22), and their truth conditions are given in (23).

PLUS v1.0 is basically equivalent to DPL, so I will not go over it in detail here.

3.3. PLUS v2.0: Variable Sums

The first new machinery we will add to PLUS is the eponymous “+” operator, which retrieves all values in the information state for some particular variable. This requires a major change to our term interpretation and a slight adjustment to our literal formula interpretation:

- (24) Terms v2.0

$$x^{g,G} = g(x)$$

$$+x^{g,G} = G(x) \quad \text{where } G(x) \stackrel{\text{def}}{=} \{g(x) : g \in G\}$$
- (25) Formulas v2.0 Same as v1.0 except:

$$G + P(t_1, t_2, \dots) = \left\{ g \in G : \langle t_1^{g,G}, t_2^{g,G}, \dots \rangle \in P' \right\}$$

In short, we now interpret terms relative to both an assignment and an information state. There are two kinds of terms in PLUS v2.0: (plain) variables as before and variable sums written with a + before a variable. Only these new sums make use of the information state in their

interpretation, denoting the set of all values for the given variable in the (assignments within the) given information state. An example is now in order, but first some helpful definitions:

(26) Helpful Abbreviations

- a. $\blacktriangleright \stackrel{\text{def}}{=} \{\emptyset\}$
- b. $\mathcal{G}_x \stackrel{\text{def}}{=} \blacktriangleright + [x]$, i.e. $\{x\} \rightarrow \wp(\mathcal{D})$, i.e. $\{[x \rightarrow X] : X \subseteq \mathcal{D}\}$
- c. $P_x \stackrel{\text{def}}{=} \mathcal{G}_x + P(x)$, i.e. $\{x\} \rightarrow P'$
- d. $[x=t] \stackrel{\text{def}}{=} [x]; x=t$

We've seen the starting information state \blacktriangleright before, but \mathcal{G}_x is new: it denotes the result of just declaring the variable x , i.e., the set of all singleton assignments mapping x to some value in \mathcal{D} . And P_x is the result updating such a \mathcal{G}_x with $P(x)$. Finally, we introduce an abbreviation for when we introduce variables.

We can now show the interpretation of the following formula:

- (27) All the farmers gathered (on the field)
 $[f]; \text{farmer}(f); \text{gathered}(+f)$

- (28) $\blacktriangleright + [f]$
 $= \{[f \rightarrow a] : a \in \mathcal{D}\}$
 $= \mathcal{G}_f$
 $\blacktriangleright + [f]; \text{farmer}(f)$
 $= \mathcal{G}_f + \text{farmer}(f)$
 $= \{[f \rightarrow a] : a \in \text{farmer}'\}$
 $= \text{farmer}_f$
 $\blacktriangleright + [f]; \text{farmer}(f); \text{gathered}(+f)$
 $= \text{farmer}_f + \text{gathered}(+f)$
 $= \{g \in \text{farmer}_f : \text{farmer}' \in \text{gathered}'\}$
 $= \begin{cases} \text{farmer}_f & \text{if } \text{farmer}' \in \text{gathered}' \\ \emptyset & \text{otherwise} \end{cases}$

Or more graphically:

- (29) $\blacktriangleright \xrightarrow{[f]} \mathcal{G}_f \xrightarrow{\text{farmer}(f)} \text{farmer}_f \dots$
 $\dots \xrightarrow{\text{gathered}(+f)} \text{farmer}_f \text{ iff } \text{farmer}' \in \text{gathered}'$

The result of this derivation is simply farmer_f (the set of all the smallest assignments that map f to a farmer) if all the farmers gathered on the field and \emptyset otherwise.

We can do plurals and generalized quantifiers now, too. Note that I will use a convention that

lowercase variables are only for singletons/singulars and uppercase variables only for plurals.

- (30) a. Five women gathered together.
 b. $[W]; \text{FIVE}(W); \text{women}(W); \text{gathered}(W)$
 c. $\blacktriangleright \xrightarrow{[W]; \text{FIVE}(W)} \text{FIVE}_W \xrightarrow{\text{women}(W)} \text{FIVE}_W \cap \text{women}_W \dots$
 $\dots \xrightarrow{\text{gathered}(W)} \text{FIVE}_W \cap \text{women}_W \cap \text{gathered}_W$

This information state contains all the smallest assignments that map W to a set of five women who gathered together.

For simple quantifiers, we can store previous sums in plural variables:

- (31) a. Most (of the) lawyers quit.
 b. $[L]; \text{lawyer}(L); [L=+L]; \text{quit}(L); [L'=+L]; \text{MOST}(L, L')$
 c. $\blacktriangleright + (b) = \left\{ g \in \text{lawyer}'_1 \cap \text{quit}'_1 : g(L) = \text{lawyer}' \ \& \ g(L') = (\text{lawyer}' \cap \text{quit}') \right\}$
 iff $|\text{lawyer}' \cap \text{quit}'| > \frac{1}{2} |\text{lawyer}'|$

This formula stores $+1$ in plural variables twice: once in L after a formula representing the restriction *lawyers* and once again in L' after the nuclear scope *quit*. This strategy for quantifiers works here, but we will refine it below.

3.4. PLUS v3.1: Maximality

Some issues arise if we want to continue using sum variables inside quantifications:³

- (32) a. A third of the students gathered (on the field).
 b. $[s]; \text{student}(s); [S=+s]; \text{gathered}(+s); [S'=+s];$
 $\text{ONE-THIRD}(S, S')$

The attempted translation in (32b) for (32a) does not capture the correct truth conditions. In particular, the formula requires that all the students gathered, in which case S' would be

³Another possibility would be to use simple plural variables, with universal quantification (or equivalent) distributing over these plurals when necessary:

- (i) a. Half the students (in my class) formed a rock band.
 b. $[S]; \neg([s \in S]; \neg \text{student}(s)); [S']; \text{formed-a-band}(S'); \text{HALF}(S, S')$,
 where $\text{HALF}(X, Y) \stackrel{\text{def}}{=} Y \subseteq X; |Y| \approx |X|/2$
- (ii) a. Half the band owned an instrument (of their own).
 b. $[B]; \text{band}(B); [B']; \neg([b \in B]; \neg([i]; \text{instrument}(i); \text{own}(b, i))); \text{HALF}(B, B')$

While this strategy gives us access to the restrictor set and the nuclear scope set, it blocks access to donkey anaphora and dependent sets, since they would be introduced under negation:

- (iii) a. Almost every student who owns an instrument brought it to school.
 b. $[S]; \neg([s \in S]; \neg(\text{student}(s); [i]; \text{instrument}(i); \text{owns}(s, i)));$
 $[S']; \neg([s \in S']; \neg \text{brought}(s, i)); \text{HALF}(S, S')$
- (iv) a. Almost every student who owns an instrument brought them to school.
 b. $[S]; \neg([s \in S]; \neg(\text{student}(s); [i]; \text{instrument}(i); \text{owns}(s, i)));$
 $[S']; \text{brought}(S', +i); \text{ALMOST-ALL}(S, S')$

identical to S . However, the formula later requires S' to be one third of S , so it will never be true. Instead, a correct formula would assign to S' the largest group of students to gather on the field.

To solve this issue, PLUS v3.0 will introduce a maximality operator:

$$(33) \quad G + \langle \phi \rangle = G_{\phi} + \phi, \text{ where } G_{\phi} \stackrel{\text{def}}{=} \bigcup \{ H \subseteq G : H + \phi \neq \emptyset \} \quad [\text{provisional}]$$

Here G_{ϕ} is the largest subset of G that “satisfies” ϕ , i.e., does not yield \emptyset when updated on ϕ . (This definition assumes this maximal subset contains all other satisfying subsets, but other definitions would work just as well.) The operator $\langle \phi \rangle$ returns the result of updating this largest satisfying subset on ϕ . This lets us correctly capture the example above:

$$(34) \quad \begin{array}{l} [s]; \text{student}(s); [S=+s]; \langle \text{gathered}(+s) \rangle; [S'=+s]; \\ \text{ONE-THIRD}(S, S') \end{array}$$

Now, the clause $\langle \text{gathered}(+s) \rangle$ does not require all students s to have gathered; instead, it restricts the information state G so that $G(s)$ contains the largest subset of students who gathered on the field.

Another issue we can solve via a refinement of our maximality operator is the fact that variables introduced under quantification ought not to be referred back to via (singular) pronouns outside of quantification:

$$(35) \quad \begin{array}{l} \text{a. One third of the students gathered on the field. \#She was cold.} \\ \text{b. } [s]; \text{student}(s); [S=+s]; \langle \text{gathered}(+s) \rangle; [S'=+s]; \\ \quad \text{ONE-THIRD}(S, S'); \text{cold}(s) \end{array}$$

As PLUS stands, we are free to refer back to the quantificational variable s (as shown in (35b)) as if it were introduced by an indefinite. This does not seem to be allowed by natural language, though, as evidenced by the oddity of the second sentence in (35a).

To solve this issue, and simplify our formulas slightly, we will refine the maximality operator to “summarize” all the new variables introduced within its scope, replacing their individual values with their sum-variable values. These summarized values are appended to each assignment in the surrounding information state:⁴

$$(36) \quad G + \langle \phi \rangle = \{ g \cup h : g \in G \ \& \ h = [\lambda x \notin \text{dom}(g). (G_{\phi} + \phi)(x)] \} \quad [\text{final}]$$

The function h in the definition above takes any new variable x created in $G_{\phi} + \phi$ that were not in the original G . For such x , $h(x)$ returns the sum of all values for x in $G_{\phi} + \phi$, i.e., $(G_{\phi} + \phi)(x)$. The new information state returned by the maximality operator is the result of appending this h onto each g in the original G .

Thus defined, we can redo our translations so any variable introduced inside quantification will denote the sum of all its values inside its quantification when used outside its quantification.

⁴Technically, this should be

$$(i) \quad G + \langle \phi \rangle = \left\{ g \cup h : g \in G \ \& \ h = \left[\lambda x \in \left(\text{dom}(G_{\phi} + \phi) \setminus \text{dom}(g) \right). (G_{\phi} + \phi)(x) \right] \right\} \\ \text{where } \text{dom}(G) \stackrel{\text{def}}{=} \bigcup \{ \text{dom}(g) : g \in G \}$$

(Note that the variable s' is introduced in order to be summarized as the nuclear scope set for later use – this replaces the previous use of S' .)

(37) $\langle [s]; \text{student}(s); \langle \text{gathered}(+s); [s'=s] \rangle \rangle; \text{ONE-THIRD}(s, s')$

Here, any later clause involving s or s' will necessarily need to be plural, since this variable will hold all students and all students who gathered, respectively. This explains the oddity of a later sentence like *She was cold*; this should instead be *They were cold*. On the other hand, dependent sets are quite easy to capture, since our new definition of the maximality operator automatically summarizes all new variables:

(38) a. Every student who wrote a paper turned it in. They were fascinating.

$\langle [s]; \text{student}(s); [p]; \text{paper}(p); \text{wrote}(s, p);$

b. $\langle \text{turned-in}(s, p); [s'=s] \rangle$

$\rangle; \text{EVERY}(s, s'); \text{fascinating}(p)$

3.5. PLUS v3.2: Distributivity

Another item that PLUS does not yet capture is that a discourse plural can actually depend on the value of another variable (and hence be dependent on the local assignment g):

(39) a. A professor in the department conducts loud experiments. A third of the students gathered in the chair's office to protest about him.

$[p]; \text{professor}(p); \text{loud}(p);$

b. $\langle [s]; \text{student}(s); \langle \text{gathered-to-protest}(+s, p); [s'=s] \rangle;$
 $\text{ONE-THIRD}(s, s')$

Here we want different (summarized) s' for each possible value of p , but this is not possible, even with our maximality operators. The same issue arises even in cases with no sum variables internal to the quantification:

(40) a. A professor has recently retired. Most students of hers have been reassigned.

$[p]; \text{professor}(p); \text{retired}(p);$

b. $\langle [s]; \text{student-of}(s, p); \langle \text{reassigned}(s); [s'=s] \rangle \rangle;$
 $\text{MOST}(s, s')$

Here, summarized s (and therefore s') should vary with the value of p , but it does not.

To solve this issue, we will introduce a distributive operator to be used in the translation of all indefinites:

$$(41) \quad G + \text{D}_x(\phi) = \bigcup \left\{ G_{x=g(x)} + \phi : g \in G \right\}$$

This operator calculates $G + \phi$ separately for each value of x , before taking the union of all resulting information states. Note that $G_{x=g(x)}$ is the set of all $h \in G$ where $h(x) = g(x)$.

This operation is quite useful for our purposes, since the summarization inherent in any embedded maximality operator clauses will be performed separately for each value distributed over. For instance, consider the following translations for the two sentences above:

(42) `[p]; Dp(professor(p); loud(p);
<[s]; student(s); <gathered-to-protest(+s,p); [s'=s]>>
>); ONE-THIRD(s,s')`

(43) `[p]; Dp(professor(p); retired(p);
<[s]; student-of(s,p); <reassigned(s); [s'=s]>>
>); MOST(s,s')`

Due to the distribution over `p` in both cases above, the final summarized versions of `s` and `s'` are potentially different for different values of `p`.

This translation for indefinites also solves the inverse pronoun accessibility problem to the one introduced in the section above: a variable introduced by an indefinite at the top level of a formula should not allow subsequent sum variables:

(44) a. A woman was eating in a crowded cafeteria.
b. #They all left. [intended: all the women eating in the cafeteria left]

(45) `[w]; woman(w); eating(w); in-cafeteria(w); left(+w)`

Without translating the indefinite *a woman* using the distributive operator, the sentence in (44b) should be fine, as represented in (45). This is avoided, though, if every indefinite introduces a distributive operator:

(46) `[w]; Dw(woman(w); eating(w); in-cafeteria(w); left(+w))`

This formula does indeed allow the sum variable `+w`, but this sum simply returns a single value, since the component assignments of the information state have been partitioned according to their value for `w`.

4. Consequences

Notice that we have justified the PLUS logic solely on the basis of plurals and collective predicates. It turns out that internal discourse plurals (among other phenomena) will fall out of the system without further changes.

4.1. Internal Reference to the Discourse Plurals

First, we now have the machinery necessary to refer to discourse plurals within the quantifications that define them. For instance, here is an internal reference to a dependent set, the set of all donkeys owned by farmers who gathered them to graze:

(47) a. Most farmers who own a donkey gathered them on the field to graze.
b. `<[f]; farmer(f); [d]; Dd(donkey(d); owns(f,d));
<gathered(f,+d); [f'=f]>>; MOST(f,f')`

Notice that the scope of the distribution over donkeys doesn't extend past the restriction, so this does not impede the later clauses of the formula. In particular, `+d` above will denote all donkeys in assignments satisfying the nuclear scope, due to the maximality operator around the nuclear scope.

Similarly, we can now refer to nuclear scope sets internal to the nuclear scope as well. Note that I assume that the indefinite *a house...* in (48a) is an exceptional wide-scope indefinite, represented here by scoping its random assignment `[h]` above the quantification:

- (48) a. Almost every friend of mine is one part-owner of a house they all share.
`[h]; Dh(<[f]; friend-of-mine(f);`
 b. `<house(h); part-owner(f, h); share(+f, h); [f'=f]>`
`>); ALMOST-ALL(f, f')`

The `+f` above will represent all friends sharing house *h* as co-owners.

4.2. Bonus constructions

Certain other historically problematic readings can easily be captured in the system presented so far. For instance, PLUS can capture reciprocals, such as *each other*, using the same mechanism for internal reference to nuclear scope sets (`+s` in this example):

- (49) a. Over half the students hated each other.
 b. `<[s]; student(s); <hated(s, +s\s); [s'=s]>>; OVER-HALF(s, s')`
 where $(t_1 \setminus t_2)^{g,G} = (t_1^{g,G}) \setminus (t_2^{g,G})$

(For previous dynamic analyses, see Murray, 2007, 2008; Dotlačil, 2012.)

In addition, cumulative readings of modified numerals (Brasoveanu, 2012; Charlow, 2017) can be captured by assuming these DPs do not introduce distributive or maximality operators:

- (50) a. Exactly three boys (total) saw exactly five movies (between them).
 b. `[b]; boy(b); ([m]; movie(m); saw(b, m); FIVE(+m)); THREE(+b)`

Cumulative DPs of the form “*N φ ψ*” translate as `[x]; φ; ψ; N(+x)`. I have placed parentheses around the translation of the nuclear scope of *exactly three boys* in (50b) in order to make this structure more clear. Without the maximality operator, the numerical restrictions on the sum variables are complete: `FIVE(+m)` limits all movies seen by a boy to total of exactly five and `THREE(+b)` requires there to be only three boys who saw a movie.

Cumulatives can also vary based on the value of previous indefinites. This is also well captured in the system:

- (51) a. A famous film director (I know) is practically anonymous among his friends.
 Exactly three friends of his (total) have seen exactly five movies by him (total).
`[d]; Dd(director(d); famous(d); anonymous-to-friends(d);`
 b. `[f]; friend-of(f, d); ([m]; movie-by(m, d); saw(f, m); FIVE(+m));`
`THREE(+f)`

The cumulative is calculated as in (50), but separately for each possible such director. Inside the distributive `Dd(...)`, `+f` is only the friends of one specific `d` and `+m` only the movies by this `d` seen by one of his friends.

PLUS assigns reasonable translations to sentences involving *same* and *different*, too (Brasoveanu, 2011). We simply append a condition after their scope is complete. For *same* in (52), we can

require that the total number of hats worn by students is one:

- (52) a. Almost every student wore the same (type of) hat.
 <[s]; student(s);
 b. <[h]; Dh(hat(h); wore(s, h)); ONE(+h); [s'=s]>
 >; ALMOST-ALL(s, s')

And for *different*, we can require the number of hats to be the same as the number of students, using our apparatus for internal reference to the nuclear scope set (presumably *different* is anaphoric to this set):

- (53) a. Almost every student wore a different hat.
 <[s]; student(s);
 b. <[h]; Dh(hat(h); wore(s, h)); |+h|=|+s|; [s'=s]>
 >; ALMOST-ALL(s, s')

5. Conclusion

The machinery of PLUS was justified without reference to discourse plurals, and yet it correctly captures difficult cases involving such plurals, including internal reference to nuclear scope and dependent sets. Furthermore, once this internal reference is possible, other phenomena get straightforward analyses using the same machinery. Finally, the results here were achieved without using sets of assignments as states (only as information states). The update semantics lets us capture much of the same phenomena with these lower order data structures.

The system presented above does have one major failing versus existing plural logics: it cannot handle quantificational subordination, due to the “summarization” operation in the maximality operator. I have separately been working on a system to handle this in a similar framework, though, where new information states are simply appended to old ones, saving these previous states for reference in quantificational subordination (among other phenomena). Please see Keshet (2019) for details.⁵

References

- van den Berg, M. H. (1996). *Some aspects of the internal structure of discourse. The dynamics of nominal anaphora*. Ph. D. thesis, University of Amsterdam.
- Brasoveanu, A. (2011). Sentence-internal different as quantifier-internal anaphora. *Linguistics and philosophy* 34(2), 93–168.
- Brasoveanu, A. (2012, apr). Modified numerals as post-suppositions. *Journal of Semantics* 30(2), 155–209.
- Charlow, S. (2017). Post-suppositions and semantic theory. Ms. Rutgers University.
- Cooper, R. (1996). The role of situations in generalized quantifiers. In S. Lappin (Ed.), *The Handbook of Contemporary Semantic Theory*, pp. 65–86. Oxford, UK and Malden, MA: Blackwell.
- Crnić, L. (2009). On the (non-) cumulativity of cumulative quantifiers. In *Proceedings of Sinn und Bedeutung* 14, pp. 117–133.
- Dotlačil, J. (2012). Reciprocals distribute over information states. *Journal of Semantics* 30(4), 423–477.

⁵Available at <http://tinyurl.com/KeshetLSA2019>.

- Groenendijk, J. and M. Stokhof (1991). Dynamic predicate logic. *Linguistics and Philosophy* 14(1), 39–100.
- Kamp, H. and U. Reyle (1993). *From discourse to logic: Introduction to model theoretic semantics of natural language, formal logic and discourse representation theory*. Dordrecht,, The Netherlands: Kluwer Academic.
- Keshet, E. (2019). Partial updates to simplify plural dynamic logic. Unpublished poster presented at Linguistic Society of America 2019 Annual Meeting.
- Matthewson, L. (2001). Quantification and the Nature of Crosslinguistic Variation. *Natural Language Semantics* 9(2), 145–189.
- Murray, S. E. (2007). Dynamics of reflexivity and reciprocity. In *Proceedings of the Sixteenth Amsterdam Colloquium*, pp. 157–162.
- Murray, S. E. (2008). Reflexivity and reciprocity with (out) underspecification. In *Proceedings of Sinn und Bedeutung* 12, pp. 455–469.
- Nakanishi, K. and M. Romero (2004). Two constructions with most and their semantic properties. In *Proceedings of NELS 44*, Number 2, pp. 453–468.
- Nouwen, R. (2003). *Plural pronominal anaphora in context*. Ph. D. thesis, University of Utrecht.
- Nouwen, R. (2007). On dependent pronouns and dynamic semantics. *Journal of Philosophical Logic* 36(2), 123–154.