# Divergent Approximators＊ 

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#### Abstract

We often mark uncertainty in our utterances with words like maybe， but when we mark uncertainty on numerals，some surprising effects，including approximation，result．This paper describes these unexpected effects and pro－ vides a possible world semantics analysis．This analysis will in turn inform our view on other scalar modifiers，like approximately．Additionally，it will help identify a complication for so－called slack regulators（e．g．loosely speaking，ex－ actly），pointing to the unexplored importance of modality in differentiating ap－ proximators．I will propose that some approximators，like maybe，have modal components and behave differently from non－modal approximators，like approx－ imately，most noticeably in their ability to accommodate contextual information．


## 1 Introduction

Language allows us to express ourselves with varying degrees of precision（i．a． Lakoff 1973；Lasersohn 1999；Krifka 2009）．Some words like tall have a mean－ ing that varies with context，while other words like heap lack a precise mean－ ing altogether．Furthermore，there are terms with precise meanings that can be used imprecisely，where context allows．For example，the numeral twenty can be used to refer to something that costs $\$ 20.00$ exactly，or where contextually appropriate a speaker could round and use twenty to refer to something that cost $\$ 19.50$ ．Additionally，there are countless modifiers that affect precision， such as roughly，more－or－less，and exactly．

Various authors have highlighted these different types of imprecision in their analyses．For example，Sauerland \＆Stateva（2007）distinguish forms with a precise meaning from those which lack a fixed precise meaning．This sepa－ rates the numeral twenty，which has a precise meaning of $20 . \overline{0}$ ，from the ad－ jective tall and the noun heap，which have no such meaning．Similarly，Syrett， Kennedy \＆Lidz（2010）distinguish forms that tolerate imprecision pragmat－ ically from those that are semantically context dependent．This separates the

[^0]numeral twenty, which can be used in a round sense where context allows, from the adjective tall, which varies depending on what it modifies (e.g. tall person vs. tall building), but does not vary depending on the level precision appropriate to the context the way twenty does.

Here I propose an additional distinction, one that differentiates between modal and non-modal approximators. I will illustrate this by first examining the modal maybe as a marker of uncertainty, which, in the right context (viz. when applied to a scalar), leads to an approximate reading. We will then compare this type of approximation to that which arises from the use of the nonmodal approximately. The differences between these two approximators will follow largely from their differing modal statuses, with the most striking difference being their differing abilities to accommodate contextual information. We will then compare these types of approximation to pragmatic halos (Lasersohn 1999), which suffice for non-modal approximators, but which cannot sufficiently describe approximators with a modal component.

## 2 Uncertainty and Approximation

An approximate reading can arise when scalars are marked as uncertain. This can be seen most clearly with scalar numerals combined with the modal maybe.

### 2.1 Uncertain Numerals

When we use words like maybe to mark our uncertainty with respect to an item, our interlocutor might entertain alternatives to this uncertain item. For example, in the exchange in (1) Bill thinks John won the race, but he is not certain, which he expresses through his response maybe John.
(1) a. Ann: Who won the race?

Bill: Maybe John.
b. \{John, Mary, Peter\}

As a result of Bill's uncertainty, Ann may entertain other likely winners, as sketched in (1b).

When the uncertain item is a numeral, we find a strong tendency for the set of alternatives to resemble approximation, as in (2).
(2) a. Ann: How many people competed?

Bill: Maybe twenty.
b. $\{18,19,20,21,22\}$

This approximation becomes even more salient if we consider a similar response Bill could have made, namely approximately twenty, where the alterna-
tives entertained by Ann would again look like (2b).
Approximation, however, does not occur with all uncertain numerals, as demonstrated in (3). When giving the number of the player with the most fouls, Bill indicates his uncertainty with maybe, again uttering maybe twenty.
(3) a. Ann: Which player has the most fouls?

Bill: Maybe twenty.
b. $\{20,6,77,15\}$

Here, however, this uncertain numeral is unlikely to give rise to approximation. Instead, Ann may consider other players likely to have numerous fouls, independently of their number.

Additionally, we find that when this approximation effect occurs, the range of alternatives depends on the numeral. For example, if twenty in (2) is replaced with twenty-seven, the range of alternatives tends to be smaller.
(4) a. Ann: How many people competed?

Bill: Maybe twenty-seven.
b. $\{26,27,28\}$

In summary, uncertain numerals lead to three questions: why do uncertain numerals give rise to approximative readings, as in (2), why do some uncertain numerals fail to give rise to approximative readings, as in (3), and why do some uncertain numerals give rise to more approximate readings than others, as in (2) vs. (4)?

### 2.2 Uncertain Numerals Explained

These puzzles can be given a formal explanation using a possible world semantics, such as the one described in Kratzer (1991), along with Krifka's (2009) conception of numerals. Here we will consider alternatives to be sets of possible worlds (i.e. worlds consistent with the epistemic modal base). These sets of worlds will be ordered in terms of their plausibility by an ordering source, as sketched in figure 1 .


Figure 1: Alternatives as sets of worlds, ordered according to a modal base. Here, for example, $w_{\text {John }}$ represents the set of worlds where John won the race, and $w_{20}$ represents the set of worlds where twenty people competed.

Following Krifka (2009) we will assume that numerals represent a range which can be characterized as the values which fall within one standard deviation $(\sigma)$ of the uttered numeral $(\mu)$ on a normal distribution over the number line. For example, twenty in the sentence This book cost twenty dollars used in a relatively imprecise context can be considered to have $\sigma=2$, such that twenty then represents values in the range $\mu \pm \sigma$ or [18-22], as shown in fi-gure 2 . The normal distribution represents the probability of each value being true, with probability decreasing as the distance from $\mu$ increases. Beyond one standard
deviation (i.e. beyond the shaded area in figure 2), probability is assumed too low for consideration. ${ }^{1}$


Figure 2: A normal distribution centered at 20 with $\sigma=2$

This range information can be expressed as the propositions $p_{\sigma}$ in (5), which picks out worlds where the value intended by the speaker $(y)$ falls within one standard deviation $(\sigma)$ of the uttered numeral $(\mu)$, and a family of functions $p_{x}$ in (6), which picks out worlds where the intended value ( $y$ ) falls within $\sigma-x$ of that number $(\mu)$ for $0<x<\sigma .{ }^{2}$ We will let $y$ assign to any world the numeric value intended by the speaker in that world, representing public uncertainty about what value the speaker intends.
(6) $\quad p_{x}=\lambda w \cdot y(w) \in\{\llbracket \mu-x \rrbracket, \ldots, \llbracket \mu+x \rrbracket\}, 0<x<\sigma$

We can see how this works in the example sentence from above This book cost

[^1]twenty dollars with $\mu=20$ and $\sigma=2$. Here $p_{\sigma}=\lambda w . y(w) \in\{\llbracket 20-2 \rrbracket, \ldots$, $\llbracket 20+2 \rrbracket\}$ (i.e. picks out set of worlds where the value $y$ intended by the speaker in that world is between 18 and 22) and $p_{x}=\lambda w . y(w) \in\{\llbracket 20-x \rrbracket, \ldots, \llbracket 20+x \rrbracket$ $\}, 0<x<2$.

As for maybe, I will treat it as involving an epistemic modal possibility operator. For uncertain numerals (e.g. maybe twenty), the modal base will contain the sets of worlds consistent with $p_{\sigma}$ (i.e. worlds within $\sigma$ of $\mu$ ) and the ordering source will contain the worlds consistent with the propositions in $p_{x}$ for $0<x<\sigma$ (i.e. will order more closely worlds where the value is closer to $\mu)$.

We now have an explanation for the approximation that arises from uncertain numerals: only worlds where values close to the uttered numeral are true will be epistemically accessible, so only these values will be plausible alternatives. We also have an explanation for why approximation does not always occur with uncertain numerals: this effect only happens with scalar numerals, like in (2), not with numerals acting in a non-scalar labeling capacity, as in (3), which do not represent ranges and are therefore not associated with $p_{\sigma}$ and $p_{x}$ like scalars are.

And finally, if we consider Krifka's pragmatic preference for simple expressions, we have an explanation for why the range of alternatives depends on the numeral, as we see when maybe twenty in (2) leads to a wider range of alternatives than maybe twenty-seven in (4). A pragmatic preference for simple expressions leads more complex numerals like twenty-seven to represent smaller ranges (i.e. induce smaller $\sigma s$ ) than simpler numerals like twenty. Since twentyseven has a smaller $\sigma$, its $p_{\sigma}$ allows a smaller range of possible worlds, leading to its narrower interpretation as an uncertain numeral (see Krifka (2009) for details).

To summarize the explanations offered here, first, uncertain numerals give rise to approximative readings because they involve $p_{\sigma}$ and $p_{x}$ in their modal base and ordering source respectively, so possible worlds are those in which the numeral is close to the uncertain numeral. Some uncertain numerals fail to give rise to approximative readings because they are not scalar and therefore are not associated with $p_{\sigma}$ and $p_{x}$. Some uncertain numerals give rise to more approximate readings than others because they are associated with larger $\sigma \mathrm{s}$, so $p_{\sigma}$ allows a wider range of possible worlds.

### 2.3 Uncertain Scalars

Numerals are not unique in expressing ranges, so this approximation effect should not be unique to numerals either. Specifically, we expect that whenever some appropriately range-representing expression is marked as uncertain, it
gives rise to approximation. And this is indeed the case.
We can see this, for example, with uncertain colors, as in (7) and (8). Here, if blue is interpreted as a scalar representing a range of colors within the blue spectrum (i.e. as not necessarily representing one specific hue), a range of colors (here with wavelengths from purple to green) might be entertained as alternatives.
(7) a. A: You say you got a good look at John's car. What color is it?

B: Maybe blue.
b. $\quad\{425 \mathrm{~nm}, \ldots, 525 \mathrm{~nm}\}$

We even see an effect of complexity, much like we did with twenty vs. twentyseven. Here, the more complex color term cyan gives rise to a smaller range of alternatives (here with wavelengths within the light blues) than the simpler color term blue, since the more complex cyan will be pragmatically associated with a smaller $\sigma$.
(8) a. A: You say you got a good look at John's car. What color is it? B: Maybe cyan.
b. $\quad\{450 \mathrm{~nm}, \ldots, 500 \mathrm{~nm}\}$

This approximation effect can be seen with any item that is used scalarly, including such an unlikely term as beef stroganoff. To see this, consider a scalar interpretation of beef stroganoff, like the one required in the sentence It was only approximately beef stroganoff. Using this scalar interpretation, consider the sentence What John cooked was maybe beef stroganoff. This gives the reading that what John cooked was somewhere near the ideal of beef stroganoff, or approximately beef stroganoff. ${ }^{3}$

So, the same phenomena we saw with uncertain numerals happen with other scalars, and the same explanation applies: the scalar represents some range, and when combined with the modal maybe this range information represented in $p_{\sigma}$ and $p_{x}$ enters into the modal base and ordering source such that

[^2](i) Judgments from Sauerland \& Stateva (2007)
a. What John cooked was definitely/maybe beef stroganoff.
b. \# What John cooked was exactly/approximately beef stroganoff.

Here, I suggest that beef stroganoff can in fact be scalar, and when it is, maybe beef stroganoff and approximation beef stroganoff give rise to a similar type of scalar approximation. This is much like the similarity between maybe twenty and approximately twenty discussed above.
scalarly-close items are more likely alternatives.

### 2.4 Other Uses of maybe

At the beginning of section 2.3 it was remarked that any appropriately rangerepresenting expression gives rise to approximation when marked as uncertain. The qualifier appropriately is present to exclude certain readings, especially those involving epistemic vagueness and uncertain labels, described below.

The non-numeral scalars discussed above (e.g. cyan) are subject to another kind of approximation, termed epistemic by Sauerland \& Stateva (2007). This concerns uncertainty regarding the precise meaning of the word in question, as can be seen with the word heap. Saying This pile of rice is maybe a heap may mean that the speaker knows exactly how many grains of rice there are, but is unsure if together they constitute a heap. Similarly, The color is maybe cyan may mean that the speaker knows exactly which hue they have in mind, but is unsure if it can be called cyan. This is not the kind of vagueness I am concerned with here. Rather, I am interested in the case where the speaker does not know the hue, but believes it to be close to cyan.

Another case of uncertainty is the uncertain label discussed in the context of (3), where the word modified by maybe is acting as a label, not a scalar. It should be kept in mind that this type of label reading is available for all the examples above and can cause them to lose their approximate reading, which again is only available when they are interpreted as scalars.

An interesting case related to this labeling reading can be seen in (9) and (10), as pointed out by Stephanie Solt (p.c.).
(9) Context: Ann organized, but did not attend, a party last night and hopes that at least 75 people attended. Bill attended the party and does not know exactly how many people were there, but believes the number to be 40 , give or take 10 .
Ann: How many people were at the party?
Bill: Maybe fifty.
Here, it seems that Bill chose his response to best fit Ann's expectations, rather than to reflect the number he really thought was most likely, 40 . He chose the highest likely value, 50, to minimize Ann's disappointment. This seems to be an instance of labeling. Bill had several answers he could have given, and for pragmatic reasons he chose the one called fifty.

Note that this 'exaggeration' can go down as well as up, so long as is in the direction of the hearer's goals. Compare (10a), where Ann is hoping for a low temperature, with (10b), where Charlie is hoping for a high temperature.
(10) Context: Bill thinks that the temperature is around freezing $\left(32^{\circ} \mathrm{F}\right)$.
a. Ann: I hope it's cold enough to go ice skating. How cold is it? Bill: Maybe 30.
b. Charlie: I hope it's too warm to go ice skating. How cold is it? Bill: Maybe 35.

This exaggerative use of maybe does not seem to impede communication, since it is intonationally distinguished from the non-exaggerative use. In particular, this maybe is typically stressed and drawn out, and is often accompanied by a grimace. Therefore, it is unlikely that Ann or Charlie would interpret Bill's response as a scalar (i.e. as representing a range of values centered around the expressed numeral, as in (11c)) such that the actual value could be even closer to their goal value. Instead, they should recognize this as an exaggerative used and entertain alternatives along the lines of (11b).
(11) Context: Bill thinks that the temperature is around freezing $\left(32^{\circ} \mathrm{F}\right)$.
a. Ann: I hope it's cold enough to go ice skating. How cold is it? Bill: Maybe 30.
b. $\{30,31,32,33,34\}$
c. \#\{28, 29, 30, 31, 32$\}$

### 2.5 Alternatives as Approximation

Considering the similarity in interpretation between maybe twenty and approximately twenty, we might wonder if the interpretation of maybe John in (1) could be thought of as approximation too. This seems quite possible, provided that we are able to determine the appropriate scales to range over. To see this, we can think of John as representing a point on some set of scales. Alternatives to John then are like John in certain relevant respects (e.g. speed, predisposition to race, and susceptibility to performance anxiety) and represent points on these relevant scales that fall close enough to John to be considered likely. There is a marked difference between maybe/approximately twenty and maybe/approximately John, which will be discussed in section 3.2.

## 3 Other Approximators

While we have seen that uncertain numerals can give rise to approximation, many other modifiers give rise to approximation as well, and the analysis of uncertain numerals here can inform the way we think of these other approximators.

### 3.1 Approximately

Approximately gives rise to approximate readings (e.g. approximately twenty people), but not the way maybe does. Instead, approximately expresses that something falls within a range (e.g. that the number of people falls within some range around twenty), with a denotation like (12). ${ }^{4}$

$$
\begin{equation*}
\llbracket \text { approximately } \rrbracket=\left[\lambda n \cdot \lambda y . \exists z \in\left\{\llbracket \mu_{n}-\sigma_{n} \rrbracket, \ldots, \llbracket \mu_{n}+\sigma_{n} \rrbracket\right\} \mid \# y=z\right] \tag{12}
\end{equation*}
$$

Here again $\mu$ corresponds to the uttered numeral, and approximately takes a scalar $n$ and some $y$ and returns true if the location of $y$ is within the con-textually-determined $\sigma$ of $n$ on the relevant scale.

For example, in approximately twenty people,
【approximately twenty people】
$=\left[\lambda n . \lambda y . \exists z \in\left\{\llbracket \mu_{n}-\sigma_{n} \rrbracket, \ldots, \llbracket \mu_{n}+\sigma_{n} \rrbracket\right\} \mid \# y=z\right](\llbracket$ twenty $\rrbracket)(\llbracket$ people $\rrbracket)$
$=\left[\exists z \in\left\{\llbracket \mu_{20}-\sigma_{20} \rrbracket, \ldots, \llbracket \mu_{20}+\sigma_{20} \rrbracket\right\} \mid \#\right.$ people $\left.=z\right]$
and if we again assume $\mu_{20}=20$ and $\sigma_{20}=2$, this yields

$$
=[\exists z \in\{18, \ldots, 22\} \mid \# \text { people }=z]
$$

(i.e. there is some number in the range [18-22] which is equal to the number of people, i.e. the actual number of people is in the range [ $18-22$ ]).

Approximately shows the same range effects as maybe, as can be seen by replacing maybe with approximately in (2) and (4) (note that approximately cannot replace maybe in (3) to give rise to a reading like (2b), since twenty here is not scalar). These approximative effects are captured in the denotation in (12), which incorporates $\sigma$ to determine its range.

This denotation also captures an important difference, shown in (13).
(13) a. It's Susan's birthday today, and she's maybe thirty.
b. \# It's Susan's birthday today, and she's approximately thirty.

Here approximately in (13b) is unable to accommodate the fact that it is Susan's birthday (i.e. that ages like 28 and three months are impossible). ${ }^{5}$ With maybe in (13a), on the other hand, this information can easily be accommodated in the modal base, excluding incompatible ages from consideration. This difference is reflected in the denotation above in (12), where $z$ is drawn from a continuous range. Note that approximately is still technically consistent with it being Su-

[^3]san's birthday, but it suggests that intermediate values are possible. This results in strangeness, requiring a certain amount of work on hearer's behalf in order to fit the utterance to the context.

So, through associating scalars with the kind of information described by Krifka, the similarities between maybe and approximately, as well as their differences, can be captured. These are summarized briefly in (14).

| approximately | maybe |
| :--- | :--- |
| - non-modal | - modal |
| - does not accommodate con- | - accommodates contextual in- |
| textual information | formation |
| - uses $\sigma$ for range | - uses $\sigma$ for modal base |

Since approximately is not modal, it is unable to accommodate contextual information, but since it draws on $\sigma$ in determining range, it gives rise to the same roundness effects as maybe.

### 3.2 Atomicity in Approximation

This discussion of approximately brings up a new question: why is approximately twenty people as a response in (2) is less offensive than approximately thirty in (13b)? More specifically, why does approximately twenty people not mean that there may have been, say, 21.7 people? The solution here is atomicity. In particular, people are considered atomic, and so only integer-increments of people are considered in (2). Years, on the other hand, are readily divisible, so non-integer increments are considered in (13b).

Similarly, we can see that the alternatives arising from maybe John do not tend to be the same as the items that fall within the denotation of approximately John. Approximately John seems to point to some (possibly hypothetical) person who differs from John only slightly. Maybe John gives a more macroscopic reading, allowing for alternatives that differ more sharply from John. This difference may be due to contextual information accommodation: you are presumably searching for actual people, not hypothetical John-like people, so for maybe John the range ( $\sigma$ ) needs to be wider if it is to include any alternatives not already ruled out by world knowledge. For approximately John, on the other hand, the range will contain hypothetical entities even without widening.

## 4 Halos

The analysis presented above is reminiscent of Lasersohn's (1999) pragmatic halos. According to Lasersohn, some element $\alpha$ is surrounded by a halo of
elements which differ from $\alpha$ in pragmatically ignorable ways. ${ }^{6}$


Figure 3: $\alpha$ with its halos, containing $\alpha^{\prime}, \alpha^{\prime \prime}$, and $\alpha^{\prime \prime \prime}$, which differ from $\alpha$ only in pragmatically ignorable ways.

It would seem that the propositions in the modal base and ordering source above are the same as the information structuring these pragmatic halos (i.e. the information used to determine what is pragmatically ignorable and how to order based on similarity). However, one difference soon becomes apparent, which is seen most clearly through slack regulators.

Slack regulators like the hedges roughly, loosely speaking manipulate pragmatic halos, functioning to more-or-less expand $\llbracket \alpha \rrbracket$ to include its halo. ${ }^{7}$ For example, while $\llbracket t w e n t y \rrbracket$ is only true for 20 exactly, $\llbracket$ roughly twenty】 is true for values that differ from twenty in pragmatically ignorable ways.

To see how the information used in the possible worlds account differs from one using pragmatic halos, compare the use of maybe with the hedge roughly in (15).
(15) a. It's Susan's birthday today, and she's maybe thirty.
b. \# It's Susan's birthday today, and she's roughly thirty.

Again, maybe can readily accommodate the fact that it is Susan's birthday, but with roughly, this does not have the same effect on the halo, leading to infelicity. ${ }^{8}$ And this behavior is not specific to the term roughly. Even round numbers (e.g. twenty when it represents [18-22]) do not accommodate this kind of outside information. So, while there is overlap in the information structuring

[^4]pragmatic halos and the information structuring possible worlds, the overlap is not complete. Halos deal with precision $\left(p_{x}, p_{\sigma}\right)$ only, while modals accommodate precision as well as additional contextual information.


### 4.1 The Hedge like

Now that this distinction between modal (e.g. maybe) and non-modal (e.g. approximately) approximators has been noted, we may expect to find modal items like maybe which have been mis-classified as slack regulators. And indeed this seems to be the case for Siegel's (2002) like. In her analysis, like $\alpha$ denotes a variable corresponding either to $\alpha$ or an element within $\alpha$ 's halo. As can be seen in (16), however, like can accommodate outside information, just like maybe.
(16) It's Susan's birthday today, and she's, like, thirty.

In other words, like, like maybe and unlike approximately, is felicitous in contexts which require discontinuous sets of alternatives. This cannot be explained by halos and suggests that there is some modal semantic component to like such that outside information can be accommodated in its modal base, explaining the felicity of (16).

In summary, halos are similar to the present analysis in the way they determine the range of alternatives/approximation, but halos involve pragmatic precision only. An additional dimention, modality, is required to capture the differences highlighted in (13), (15), and (16). The means of approximation discussed here are divided as shown in (17).
a. Modal: maybe, like
b. Non-modal: approximately, roughly, pragmatic slack/halos/roundness

## 5 Summary

By examining constructions like maybe twenty I show that information associated with numerals can be incorporated into a possible worlds semantics. This analysis describes their approximating behavior as well as their divergence from constructions like approximately twenty. Scalars represent ranges, with closer values being more probable. In modal contexts (e.g. maybe twenty), this information is incorporated into the modal base and ordering source such that plausible alternatives are those scalarly close, resembling approximation. It can also be seen that, while this same information may be used in pragmatic halos, use of contextual information sets these types of approximation apart and suggests that certain hedges contain modal components. The approximators with a modal component can then accommodate contextual information, while non-modal approximators cannot.

## References

Kratzer, Angelika. 1991. Modality. In Arnim von Stechow \& Dieter Wunderlich (eds.), Semantik: Ein internationales Handbuch der zeitgenössischen Forschung, 639-650. Berlin: Walter de Gruyter.
Krifka, Manfred. 2009. Approximate interpretations of number words: A case for strategic communication 109-132. Stanford: CSLI Publications.
Lakoff, George. 1973. Hedges: a study in meaning criteria and the logic of fuzzy concepts. Journal of Philosophical Logic 2. 458-508.
Lasersohn, Peter. 1999. Pragmatic halos. Language 75. 522-551.
Sauerland, Uli \& Penka Stateva. 2007. Scalar vs. epistemic vagueness: Evidence from approximators. In Proceedings of SALT 17, Ithaca, NY: CLC Publications, Cornell University.
Siegel, Muffy E. A. 2002. Like: The discourse particle and semantics. Journal of Semantics 19. 35-71.
Syrett, Kristen, Christopher Kennedy \& Jeffrey Lidz. 2010. Meaning and context in children's understanding of gradable adjectives. Journal of Se mantics 27. 1-35.


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[^1]:    ${ }^{1}$ For simplicity we will assume strict cut-offs at $\mu \pm \sigma$. It seems, however, that the border should be fuzzy, which might result from difficulty determining a precise $\sigma$ from context. Alternatively, an applicable use of fuzzy sets is discussed in Lasersohn (1999).
    ${ }^{2}$ As described here, this results in a linear probability curve, not the Gaussian one described above, a problem which will not be addressed here.

[^2]:    ${ }^{3}$ Sauerland \& Stateva (2007) provide a different take on this kind of construction. They consider approximately beef stroganoff infelicitous in (i) because scalar approximators (exactly/approximately) can only combine with scalar items.

[^3]:    ${ }^{4}$ For approximately's counterpart, consider: $\llbracket \mathrm{exactly} \rrbracket=\left[\lambda n . \lambda y . \exists z \in\left\{\llbracket \mu_{n}-\sigma_{n} \rrbracket, \ldots, \llbracket \mu_{n}+\sigma_{n} \rrbracket\right\}\right.$ $\mid \# y=z$, defined if $\left.\sigma_{n}<\sigma_{c, n}\right]$ (takes a scalar $n$ and some $y$ and returns true if the location of $y$ is within the contextually-determined $\sigma$ of $n$ on the relevant scale, where $\sigma$ is less than some small contextually-determined value)
    ${ }^{5}$ Note that approximately is acceptable in a very precise context (e.g. Actually, she's 30 years 14 hours and 22 minutes), but this is not the reading that I am considering.

[^4]:    ${ }^{6}$ Lasersohn writes: "Given an expression $\alpha$ denoting some object x, I like to think of the set the context associates with x as arrayed around x in a sort of circular cluster, so I will call this set, together with its ordering relation, the PRAGMATIC HALO of x , or, extending the terminology, as the pragmatic halo of $\alpha$ ", (Lasersohn 1999: 527) and " $H_{C}(\alpha)$ is understood to be a set of objects which differ from $\llbracket \alpha \rrbracket^{M, C}$ only in ways which are pragmatically ignorable in $C ; \leq_{\alpha, C}$ is an ordering of $H_{C}(\alpha)$ according to similarity to $\llbracket \alpha \rrbracket^{M, C}{ }^{\prime}$, (Lasersohn 1999: 548).
    ${ }^{7} \llbracket$ loosely speaking $\Phi \rrbracket^{M, C}=\bigcup H_{C}(\Phi)-\llbracket \Phi \rrbracket^{M, C}$ (Lasersohn 1999: 545)
    ${ }^{8}$ Note that roughly (like approximately) is acceptable in a very precise context.

