Modified numerals revisited: The cases of *fewer than 4* and *between 4 and 8*[1]

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Abstract. There seems an interplay between (i) the interpretation of sentences containing non-increasing modified numerals (e.g., *fewer than 4*, *between 4 and 8*) and (ii) the type of predicates in those sentences. For example, (i) *fewer than 4 boys smiled*, where the predicate is distributive, has no existential entailment, but an upper-bound reading; while (ii) *fewer than 4 boys lifted the piano together*, where the predicate is collective, has an existential entailment, but no upper-bound reading. Following Brasoveanu (2013)'s post-supposition-style account for modified numerals, here I propose that (I) the semantic contribution of non-increasing modified numerals is twofold: (i) introducing a maximal referent as at-issue meaning in the derivation, and (ii) adding a cardinality constraint as a secondary dimension of meaning, and that (II) it is the type of predicates (distributive vs. collective) that determines the scope of maximization and where to evaluate this secondary dimension of meaning – at the sentential level or within a group-denoting DP.

Keywords: modified numerals, distributivity, collectivity, maximality, post-supposition.

1. Introduction

The semantics of **non-increasing modified numerals** has been a hot topic in current formal linguistic research. It has been noticed for decades that there seems an interplay between the interpretation of sentences containing this kind of modified numerals and the type of predicates therein.

As illustrated in (1), when a **downward-entailing (DE)** modified numeral (here *fewer than 4*) combines with a **distributive** predicate (here *smile*), the sentence has an upper-bound reading, but no existential entailment. I.e., sentence (1) is compatible with a situation in which no boys smiled. In contrast, as illustrated in (2), when *fewer than 4* combines with a **collective** predicate (here *lift the piano together*), the sentence has an existential entailment. I.e., if there were no boys at all involved in piano-lifting, sentence (2) is judged infelicitous according to our intuition. However, if there were two groups, one of 3 boys and the other of 8 boys, and each group collectively lifted the piano, sentence (2) is true and felicitous in this scenario – it simply means that a certain group of fewer than 4 boys collectively lifted the piano. I.e., there is no upper-bound reading for sentence (2): it does not rule out the possibility of more boys involved in piano-lifting.

(1) Fewer than 4 boys smiled. *smile*: **distributive** predicate
  a. ✓**Upper-bound reading**: ruling out the possibility of more boys smiling.
  b. # **Existential entailment**: asserting the existence of smiling boys.

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(2) Fewer than 4 boys lifted the piano together. *lift the piano together*: **collective** predicate
  a. # **Upper-bound reading**: ruling out the possibility of more boys involved in piano-lifting.
  b. ✓**Existential entailment**: asserting the existence of piano-lifting boys.

Thus (3) summarizes the basic empirical observations with regard to interpreting sentences containing DE modified numerals. This pattern was initially noted in Winter's work (see, e.g., Winter 2001; Ben-Avi and Winter 2003) and much studied recently by Buccola and Spector (see Spector 2014; Buccola 2015a, b; Buccola and Spector 2016; Buccola 2016). As noted in these works, the generalization in (3) can also be extended to the cases of **non-monotone** modified numerals, such as *between 4 and 8* (see (4), except that here there is always an existential entailment).

(3) Generalization on interpreting sentences containing DE modified numerals:[2]

	Distributive predicate	**Collective predicate**
Upper-bound reading	✓	#
Existential entailment	#	✓

(4) *Between 4 and 8* + **distributive / collective** predicates:
  a. Between 4 and 8 boys smiled. ✓ Upper-bound reading
  b. Between 4 and 8 boys lifted the piano together. # Upper-bound reading

To account for the pattern shown in (3), Buccola and Spector have proposed an 'over-generation + pragmatic blocking' approach, which, I argue, suffers from empirical and conceptual problems. In this paper, by following Brasoveanu (2013)'s post-supposition-style approach, which was originally developed to analyze the cumulative reading of sentences containing *exactly*-type modified numerals (e.g., *exactly three boys saw exactly five movies*), I propose a purely semantic account that generates exactly the attested readings summarized in (3). In a nutshell, I propose that **(I)** the semantic contribution of non-increasing modified numerals is twofold: (i) introducing a maximal referent as **at-issue meaning** in the derivation, and (ii) adding a cardinality constraint as a **secondary dimension of meaning**, and that **(II)** it is the type of predicates (distributive vs. collective) that determines the scope of maximization and where to evaluate this secondary dimension of meaning – at the sentential level or within a group-denoting DP.

Section 2 presents Buccola and Spector (2016)'s 'over-generation + pragmatic blocking' approach that is based on the notion of number-based maximality, and I discuss empirical and conceptual challenges to it. Based on Brasoveanu (2013)'s study, my new account is presented in Section 3. Section 4 discusses cumulative readings. Section 5 discusses another approach developed in Buccola and Spector (2016), which is based on the notion of informativity-based maximality. Section 6 compares the current account with the main idea of Solt (2006). Section 7 concludes the paper.

[2]The lack of existential entailment for distributive-reading sentences with DE quantifiers has also been called *Van Benthem's problem* (see Van Benthem 1986).

2. Buccola and Spector (2016)'s 'over-generation + pragmatic blocking' approach

Buccola and Spector (2016) have presented and discussed four specific accounts, each involving some notion of **maximality**. Overall, all these accounts are based on the idea of 'over-generation + pragmatic blocking'. They differ along two parameters: (i) the **type** of maximality (number-based vs. informativity-based) and (ii) the **source** of maximality (lexically encoded vs. non-lexical). In this section, I focus on the two accounts using a number-based maximality operator, and those using an informativity-based maximality operator will be discussed in Section 5.

2.1. Buccola and Spector (2016): Number-based maximality operator + pragmatic blocking rule

The basic idea of this approach includes two parts. First, in natural language semantics, there is a maximality operator max of type $\langle dt,d\rangle$ involved in the interpretation of modified numerals. Crucially, there is scope interaction between this operator max and existential closure $\exists$, leading to potential ambiguity, i.e., over-generating sentence readings. Second, there is a pragmatic rule, which rules out any readings in which the semantic contribution of a numeral n is trivial.

(5) shows the definition of the number-based maximality operator max in Buccola and Spector (2016). Essentially, max takes a totally ordered set P (of type $\langle dt\rangle$ – d stands for *degree*, an element in a totally ordered set) as input and returns its upper bound n (of type d) as output. In Buccola and Spector (2016), inputs of max are almost always convex sets of natural numbers.[3] To avoid any potential existential entailment problems, Buccola and Spector (2016) assume that an empty set can also serve as the input of max, and in this case, the output is simply 0.[4]

(5) Number-based maximality operator: max Buccola and Spector (2016)

$$[\![\mathsf{max}(P_{\langle dt\rangle})]\!] = \begin{cases} \iota n.P(n) \wedge \forall m[P(m) \rightarrow m \leq n] & \text{if } \exists n P(n) \\ 0 & \text{otherwise} \end{cases}$$

Buccola and Spector (2016) provide two ways to implement max: **L(exical)Max** and **SMax**.[5] As (6) shows, within the analysis of LMax, max is part of the lexical semantics of *fewer than*. *Fewer than* takes a number n (of type d) as input and returns a generalized quantifier over

[3] Briefly speaking, a **convex** totally ordered set is a totally ordered set P such that for any elements a and b in the set, if $a \leq b$, then any element x such that $a \leq x \leq b$ is also in the set. E.g., $\{x|x > 0\}$, $\{x|x < 4\}$, and $\{x|4 \leq x < 8\}$ are all convex sets, while sets such as $\{x|x \leq 5 \vee x > 8\}$ are not convex.

[4] Notice that if a set has no upper bound (e.g., $\{x|x \geq 0\}$, which has an infinite endpoint) or the upper bound is open (e.g., $\{x|0 \leq x < 5\}$, in which the endpoint 5 is not included), then for any number x in the set, there is aways another number y such that (i) $y > x$ and (ii) y is also in the set. Thus this kind of sets are undefined in Buccola and Spector (2016)'s max. In a footnote, Buccola and Spector (2016) mention the maximality failure issue for sentences such as *fewer than 5 prime numbers are odd*, which is intuitively judged false, not undefined, but according to their analysis, the set of numbers $\lambda n.[\exists x.|x| = n \wedge \textsc{prime-number}(x) \wedge \textsc{odd}(x)]$ is undefined for their max in (5). This issue is left unsolved in their paper, but it does not arise with the analysis proposed in this paper (see Footnote 10).

[5] According to Buccola and Spector (2016), SMax can be understood as either SyntacticMax or SeparateMax.

degrees (i.e., something of type $\langle dt, t\rangle$) as output.[6] Since [[fewer than n]] is analyzed as a generalized quantifier, there can be scope interaction between [[fewer than n]] and existential closure $\exists$ in deriving sentence meanings. (7) shows the derivation of two (potential) readings for the sentence *fewer than 4 boys smiled*. In (7a), *fewer than 4* raises to a position that takes scope over the rest of the sentence. Since max takes wider scope over the existential closure and it can potentially return 0 as output, the interpretation thus derived has no existential entailment. In contrast, in (7b), *fewer than 4* raises to a position that is still inside of the scope of existential closure (actually inside of AP), thus the wide scope of existential closure guarantees the existential entailment. Since *smile* is a distributive predicate (see (8a)), the semantic contribution of *fewer than 4* in (7b) is in effect vacuous.

(6) The implementation of maximality in **LMax**:
[[fewer than]]$_{\langle d,\langle dt,t\rangle\rangle} \stackrel{\text{def}}{=} \lambda n_d.\lambda P_{\langle dt\rangle}.[\mathsf{max}(P) < n]$

(7) Using LMax to derive [[fewer than 4 boys smiled]]:
a. [[fewer than 4]] takes wider scope over $\exists$: $\rightsquigarrow$ Upper-bound reading
[fewer than 4][λn.[$\exists$[n boys smiled]]]
$\Leftrightarrow \mathsf{max}([\lambda n.[\exists x[|x| = n \wedge \textsc{boys}(x) \wedge \textsc{smile}(x)]) < 4$
i.e., the maximal value that is equal to the cardinality of smiling boys is less than 4.
b. $\exists$ takes wider scope over [[fewer than 4]]: $\rightsquigarrow$ Existential entailment
[$\exists$[λx.[[fewer than 4][$\lambda n.|x| = n$]]]boys]smiled
$\Leftrightarrow \exists x[\mathsf{max}(\lambda n.|x| = n) < 4 \wedge \textsc{boys}(x) \wedge \textsc{smile}(x)]$
$\equiv \exists x[|x| < 4 \wedge \textsc{boys}(x) \wedge \textsc{smile}(x)]$ (see (8a))
$\equiv \exists x[\textsc{boys}(x) \wedge \textsc{smile}(x)]$ i.e., there are smiling boys. $\rightsquigarrow$ No upper-bound reading

(8) **Distributivity**: P is distributive iff $\forall x, y[P(x) \wedge y \leq_{\text{part}} x \rightarrow P(y)]$.
a. ✓*Smile*: if a group of boys smiled, it follows that each boy smiled.
b. # *Lift the piano together*: if a group of boys collectively lifted the piano, it doesn't follow that a subset of these boys formed a group and lifted the piano together.

The unattested reading shown in (7b) means that insofar as there are smiling boys, there always exists a subset of smiling boys whose cardinality is less than 4. I.e., the numeral 4 can be replaced by any other numerals without affecting the truth condition of this reading. Thus, Buccola and Spector (2016) propose the pragmatic constraint (9) to rule out the reading in (7b).

(9) Buccola and Spector (2016)'s pragmatic economy constraint:
An LF ϕ containing a numeral n is infelicitous if, for some m distinct from n, ϕ is truth-conditionally equivalent to $\phi[n \mapsto m]$ (the result of substituting m for n in ϕ).

Similar to (7), (10) illustrates the derivation of two readings for a sentence containing collective

[6] The definition in (6) is slightly different from the one given in Buccola and Spector (2016), which is like this: [[fewer than]]$_{\langle d,\langle dt,t\rangle\rangle} \stackrel{\text{def}}{=} \lambda n_d.\lambda P_{\langle dt\rangle}.\exists m[m < n \wedge \mathsf{max}(P) = m]$. I.e., in their original definition, there is also an existential closure, the semantic contribution of which is actually vacuous and not motivated in their paper.

predicates. In (10b), since *lift the piano together* is a collective predicate, obviously, if fewer than 4 boys formed a group and lifted the piano together, it doesn't follow that a subset of these boys also formed a group to lift the piano together (see (8b)). Thus, the semantic contribution of *fewer than 4* in (10b) is not trivial, and there is no need to apply the pragmatic rule in (9). In this analysis, (10b) naturally has an existential entailment, but no upper-bound reading. The derivation in (10a) essentially leads to the unattested reading that no groups of boys with a cardinality equal to or larger than 4 lifted the piano together, but in Buccola and Spector (2016), the accounts of LMax and SMax do not include a mechanism to block this unattested reading.

(10) Using LMax to derive [[fewer than 4 boys lifted the piano together]]:

a. [[fewer than 4]] takes wider scope over $\exists$: $\rightsquigarrow$ Upper-bound reading
[fewer than 4][λn.[$\exists$[n boys lifted the piano together]]]
$\Leftrightarrow \mathsf{max}([\lambda n.[\exists x[|x| = n \wedge \textsc{boys}(x) \wedge \textsc{lift-the-piano-together}(x)]]) < 4$
i.e., the max. value that is equal to the cardinality of piano-lifting boys is less than 4.

b. $\exists$ takes wider scope over [[fewer than 4]]: $\rightsquigarrow$ Existential entailment
[$\exists$[λx.[[fewer than 4][$\lambda n.|x| = n$]]]boys]lifted the piano together
$\Leftrightarrow \exists x[\mathsf{max}(\lambda n.|x| = n) < 4 \wedge \textsc{boys}(x) \wedge \textsc{lift-the-piano-together}(x)]$
$\equiv \exists x[|x| < 4 \wedge \textsc{boys}(x) \wedge \textsc{lift-the-piano-together}(x)]$ (see (8b))
$\not\equiv \exists x[\textsc{boys}(x) \wedge \textsc{lift-the-piano-together}(x)]$

In the account of SMax, the maximality operator max is independent of the lexical semantics of modified numerals, and the use of max is rather optional. Consequently, the scope of max – depending on the landing site of *fewer than n* in quantifier raising – can be flexible. Thus, all the readings shown in (7) and (10) can be generated, and the pragmatic rule (9) is needed to block the unattested 'existential entailment' reading shown in (7b). Moreover, since max is optional, when it is not applied, as shown in (12), the effect is equivalent to $\exists$ taking wide scope over max.

(11) **SMax** – max is not contained in the lexical semantics of *fewer than*: (cf. (6))
$[\![\text{fewer than}]\!]_{\langle d,\langle dt,t\rangle\rangle} \stackrel{\text{def}}{=} \lambda n_d.\lambda P_{\langle dt\rangle}.\exists m[m < n \wedge P(m)]$

(12) SMax: when max is not applied:

a. [[fewer than 4 boys smiled]] = $\exists x[|x| < 4 \wedge \textsc{boys}(x) \wedge \textsc{smile}(x)]$ = (7b)

b. [[fewer than 4 boys lifted the piano together]]
$= \exists x[|x| < 4 \wedge \textsc{boys}(x) \wedge \textsc{lift-the-piano-together}(x)]$ = (10b)

Buccola (2016) points out that due to the extra flexibility in using max, SMax is superior to LMax in analyzing the generic reading of the sentence *fewer than 4 boys can together lift that piano*. As shown in (13), under this **generic** reading, the sentence means that for a certain number n, in general, n boys can collectively lift that piano, and this number n is smaller than 4. Crucially, this reading can be naturally derived via SMax, with no use of max. However, since max is a necessary part in the semantics of *fewer than* in LMax, LMax fails to generate this reading.

(13) The **generic** reading of *fewer than 4 boys can together lift that piano*:
$\exists n[n < 4 \wedge \forall_{\text{GEN}} x[|x| = n \wedge \textsc{boys}(x) \rightarrow \diamond\textsc{together-lift-that-piano}(x)]]$
(Context: Al asks: 'How many boys do we need to lift that piano?' Then Bill answers: 'Well, I'm not sure. But I believe fewer than 4 boys can together lift that piano.')

To sum up, Buccola and Spector (2016) propose to use a maximality operator that is applied to sets of **numbers**, and the use of this maximality operator helps create ambiguity (via scope interaction and/or optionality). Their pragmatic rule helps to block some of the over-generated readings.

2.2. Challenges to the approach of Buccola and Spector (2016)

Here I present empirical and conceptual challenges to these two accounts LMax and SMax. First of all, as already mentioned, both LMax and SMax generate an unattested upper-bound reading for sentences containing collective predicates (see (10a)), and their pragmatic constraint (9) is not applicable in this case to rule out this reading. Similarly, the sentence shown in (13) has two other readings that can be generated in LMax and SMax: (i) any group of boys with a cardinality less than 4 can, in general, lift that piano together (see (14a)); (ii) the maximal number n such that in general, n boys can together lift that piano is less than 4 (see (14b)). While it is doubtful whether (14a) is an available reading of the sentence, the reading (14b) is certainly unavailable.

(14) Two other readings of *fewer than 4 boys can together lift that piano*:
a. $\forall_{\text{GEN}} x[|x| < 4 \wedge \textsc{boys}(x) \rightarrow \textsc{can-lift-the-piano-together}(x)]$
(Either max takes a narrow scope within *fewer than 4 boys* or it is not used at all.)
b. $\mathsf{max}(\lambda n.\forall_{\text{GEN}} x[|x| = n \wedge \textsc{boys}(x) \rightarrow \textsc{can-lift-the-piano-together}(x)) < 4$
(max takes a wide scope over the rest of the sentence.)

Second, since the accounts of LMax and SMax rely heavily on a pragmatic rule that targets specifically the use of numerals (see (9)), these accounts fail to relate the pattern shown in (3) with some other data discussed in the semantics literature. Solt (2007) has noticed that the use of *few* interacts with the type of predicates in affecting the grammaticality of sentences. As shown in (15) and (16), the DE quantifier *few* is compatible with distributive predicates, but incompatible with collective predicates. Moreover, Solt (2007) has pointed out that to express the notion of a small quantity in sentences containing collective predicates, *a few* should be used, instead of *few*, as illustrated by the contrast between (16) and (17). Intriguingly, when we compare (15) and (17), the attested readings show exactly the same pattern as in (3). Obviously, a unified account that can explain both the patterns with the use of **non-increasing modified numerals** and with the use of ***few/a few*** would be more favorable, and this kind of accounts cannot rely on rules constraining the interpretation of numerals, because no numeral is involved in (15) – (17).

(15) Few boys smiled. *few* + **distributive** predicate
a. ✓ **Upper-bound reading** b. # **Existential entailment**

(16) *Few boys lifted the piano together. Ungrammatical: *few* + **collective** predicate

(17) A few boys lifted the piano together. *a few* + **collective** predicate
a. # **Upper-bound reading** b. ✓ **Existential entailment**

Third, as already discussed by Buccola and Spector (2016), the over-generation approach makes wrong predictions on NPI (negative polarity item) licensing. As shown in (18), NPI licensing is possible in (18a), suggesting that *fewer than 4 boys* is involved in creating a DE environment when combined with a distributive predicate; in contrast, NPI licensing is impossible in (18), suggesting that no DE environment is created when *fewer than 4 boys* combines with a collective predicate. This kind of contrast should be unexpected if there were indeed an over-generation mechanism underlying the interpretation of modified numerals. Actually, these data on NPI licensing suggest that some part of the semantic contribution of *fewer than 4 boys* (e.g., in creating a DE environment) is initially unspecified, but later determined by the type of predicates it combines with.[7]

(18) NPI licensing:
a. Fewer than 4 boys ate **any** soup. **Distributive** predicate ⇝ NPI licensing ✓
b. *Fewer than 4 boys surrounded **any** table.
Collective predicate ⇝ NPI licensing #
c. Fewer than 4 boys surrounded a table.

Fourth, there is an important factor overlooked by Buccola and Spector (2016): whether the noun that combines with a non-increasing modified numeral is a **group noun** (e.g., *army*, *team*, *committee*) or a **non-group noun** (e.g., *boy*, *dog*). Therefore, their accounts fail to fully characterize the exact mechanism that governs the semantic composition between distributive / collective predicates and the rest part of sentences. As (19) illustrates, even though here the predicate is collective, the sentence has an upper-bound reading (i.e., it is false if 4 or more than 4 groups of boys lifted the piano together), but no existential entailment (i.e., it is compatible with a situation in which no groups of boys lifted the piano together after all). Moreover, (20) shows that NPI licensing is possible here despite the use of a collective predicate (cf. (18b)). These data show that the interpretation of '[[DE modified numeral + **group** noun] + collective predicate]' is parallel to that of '[[DE modified numeral + **non-group** noun] + distributive predicate]', suggesting that the notion of group should also be taken into consideration.

(19) Fewer than 4 groups of boys lifted the piano together. **collective** predicate
a. ✓ **Upper-bound reading** b. # **Existential entailment**

(20) Fewer than 4 groups of soldiers surrounded any castle. ⇝ NPI licensing ✓

[7]This might be unsurprising if we reflect on the role the type of predicates plays in specifying actual interpretations in other cases. For example, (i) in *John and Mary left*, where the predicate is distributive, [[John and Mary]] is essentially $\lambda P.$[John $\sqcap$ Mary(P)], i.e., a set intersection, while in (ii) *John and Mary built a raft together*, where the predicate is collective, [[John and Mary]] essentially denotes [John $\oplus$ Mary]$_e$, i.e., a sum (or a group). See Zhang (2015); Champollion (2016), etc., that deal with this issue and give the semantics of coordination a unified account.

Finally, conceptually, according to LMax and SMax, *fewer than n* is analyzed as a generalized quantifier of degrees, and its type is $\langle dt,t\rangle$. As a consequence, it remains unclear how to extend this analysis of *fewer than n* to further derive the meaning of *no fewer than n* (see (21)).[8]

(21) No fewer than 4 boys smiled. $[\![\text{no fewer than 4}]\!] \approx [\![\text{at least 4}]\!]$

Overall, the discussion here provides motivation for a new account that (i) does not overgenerate, (ii) explains the parallelism between the interpretation pattern of *fewer than n* and (*a*) *few*, (iii) makes use of the notion of *group*, and (iv) avoids analyzing *fewer than n* as a generalized quantifier.

3. A new account à la Brasoveanu (2013)

Following Brasoveanu (2013), here I propose a post-supposition-style account for the semantics of non-increasing modified numerals, and implement my analysis in Brasoveanu (2013)'s modified version of Dynamic Predicate Logic (DPL, Groenendijk and Stokhof 1991). Brasoveanu (2013)'s version of DPL is slightly different from Groenendijk and Stokhof (1991)'s in two ways: (i) its ontology includes both **atomic** (e.g., *John*) and **non-atomic individuals** (e.g., *John and Bill*, *the men*); (ii) it makes a distinction between **at-issue meaning** and **a secondary dimension of meaning** and implements the latter as **post-suppositions**. In the following, I first introduce Brasoveanu (2013)'s framework and analyze sentences of the pattern 'DE modified numeral + distributive predicate' (Section 3.1). Based on this, I introduce the concept of **group** and account for sentences of the pattern 'DE modified numeral + collective predicate' (Section 3.2). I discuss more data (e.g., *between 4 and 8*) in Section 3.3 and some consequences in later sections.

3.1. The framework of Brasoveanu (2013) and the semantics of *fewer than 4 boys smiled*

The goal of Brasoveanu (2013) is to analyze the **cumulative** reading of sentence (22), and his basic idea includes two parts. First, the interpretation of '*exactly* n + NP' follows an existing generalization: unlike **bare numerals** (e.g., *4 boys*), **modified numerals** introduce **maximal referents** (see Szabolcsi (1997); de Swart (1999); Krifka (1999); Umbach (2006); Charlow (2014), etc). As illustrated in (23), to use *other boys* felicitously, its antecedent needs to be a contextually non-maximal referent. Thus, the incompatibility between *at least 4 boys* and *other boys* indicates that *at least 4 boys* introduces a maximal referent. Second, Brasoveanu (2013) proposes that the cardinality constraint contributed by modified numerals involves a secondary dimension of meaning (cf. at-issue meaning). To a certain extent, this is reminiscent of the semantic contribution of non-restrictive relative clauses (see the paraphrase in (22)). This secondary dimension of meaning is implemented as **post-suppositions** in a DPL framework by Brasoveanu (2013).

[8] Geurts and Nouwen (2007) have also argued that *more than n* should not be analyzed as a generalized quantifier.

(22) Exactly 3 boys saw exactly 5 movies. **Cumulative** reading
$\rightsquigarrow$ The contextually maximal set of boys, the cardinality of which is exactly 3, saw the contextually maximal set of movies, the cardinality of which is exactly 5.

(23) a. At least 4 boys smiled. (# Perhaps there were other boys smiling, but I forgot.)
b. Four boys smiled. (✓ Perhaps there were other boys smiling, but I forgot.)

The central idea of DPL is to analyze sentential meanings as **information change potential** (i.e., ways of changing (the representation of) the information of the interpreter), instead of truth conditions (Groenendijk and Stokhof 1991). Thus, the meaning of a sentence can be captured in terms of a relation between states. For our current purpose, it is analyzed as a pair of **assignment functions**.

Following Brasoveanu (2013), my DPL models have the structure $\mathfrak{M} = \langle \mathfrak{D}, \mathfrak{I} \rangle$, in which $\mathfrak{D}$ is the domain of individuals, and $\mathfrak{I}$ is the basic interpretation function such that $\mathfrak{I}(R) \subseteq \mathfrak{D}^n$, for any n-ary relation R. The domain $\mathfrak{D}$ consists of **atomic** and **non-atomic individuals**. **Atomic individuals** are singleton sets, e.g., {JOHN}; **non-atomic individuals** are sets with a cardinality larger than 1, e.g., {JOHN, BILL}. I also include the empty set $\varnothing$ in $\mathfrak{D}$. The **cardinality** of an individual x, written as $|x|$, means the count of its singleton subsets. The **sum** of two individuals $x \oplus y$ is the union of the sets x and y, e.g., {JOHN} $\oplus$ {BILL} = {JOHN, BILL}. The **part-of relation** over individuals $x \leq y$ means that x is a part of y, and formally it means that $x \subseteq y$. For any property P, the **cumulative closure** $*P$ is the smallest set such that $P \subseteq *P$ and if individuals $a, a' \in *P$, then $a \oplus a' \in *P$.

An **assignment function** g is a total function from the set of variables $\mathcal{V}$ to $\mathfrak{D}$. Sentence meanings are modeled as $\langle g, h \rangle$, i.e., a pair of assignment functions, in which g represents the current information state, and h the updated state. $h[x]g$ means that for any variable $v \in \mathcal{V}$, if $v \neq x$, then $h(v) = g(v)$, i.e., h differs from g at most with regard to the value it assigns to the variable x.

As shown in (24a), atomic formulas for lexical relations are **tests** (only unary relations, i.e., properties, are shown in this paper), i.e., the input and output assignment functions g and h are equal and the value of certain variables is checked. Cardinality constraints are tests as well, as illustrated in (24b) with the use of '='. Dynamic conjunction and random assignment are shown in (24c) and (24d). (25a) shows an example, in which the introduced discourse referent is marked in green, the restrictor in red and the nuclear scope in blue, and it can be decomposed in the way shown in (25b).

(24) a. $[\![P(x)]\!]^{\langle g,h \rangle} = \mathbb{T}$ iff $g = h$ and $h(x) \in *\mathfrak{I}(P)$. **Atomic formula**
b. $[\![|x| = n]\!]^{\langle g,h \rangle} = \mathbb{T}$ iff $g = h$ and $|h(x)| = n$. **Cardinality constraint**
c. $[\![\phi \wedge \psi]\!]^{\langle g,h \rangle} = \mathbb{T}$ iff there is a k s.t. $[\![\phi]\!]^{\langle g,k \rangle} = \mathbb{T}$ and $[\![\psi]\!]^{\langle k,h \rangle} = \mathbb{T}$. **Dynamic conjunction**
d. $[\![[x]]\!]^{\langle g,h \rangle} = \mathbb{T}$ iff $h[x]g$. **Random assignment**

(25) a. $[\![\text{A}^x \text{ wolf came in}]\!] \Leftrightarrow \exists x[|x| = 1 \wedge {*}\textsc{wolf}(x)]({*}\textsc{come-in}(x))$
b. $\exists x[|x| = n \wedge \phi](\psi) := [x] \wedge |x| = n \wedge \phi \wedge \psi$ $[x]$: storing a discourse referent (dref)

Brasoveanu (2013)'s maximization operator σ introduces a maximal referent (see (26)). As (27) illustrates, basically, here x stores **all** the atomic entities that satisfy the restrictor and nuclear scope.

(26) $[\![\sigma x(\phi)]\!]^{\langle g,h\rangle} = \mathbb{T}$ iff **Maximization operator σ**
(i) $[\![[x] \wedge \phi]\!]^{\langle g,h\rangle} = \mathbb{T}$ and (ii) there is no h' such that $[\![[x] \wedge \phi]\!]^{\langle g,h'\rangle} = \mathbb{T}$ and $h(x) < h'(x)$.

(27) $[\![\text{Exactly three}^x \text{ wolves came in}]\!] \Leftrightarrow \exists |x| = 3[{*}\textsc{wolf}(x)]({*}\textsc{come-in}(x))$
$\Leftrightarrow \sigma x({*}\textsc{wolf}(x) \wedge {*}\textsc{come-in}(x)) \wedge |x| = 3$ (The post-supposition part is to be revised.)

In Brasoveanu (2013), post-suppositions are implemented as **tests on the output context**, i.e., they are introduced at certain points in meaning derivation, but checked after the at-issue meaning of a formula is evaluated. Thus, tests need to be added into our representation of input and output states, and the meaning of a sentence is written as $\langle g[\zeta], h[\zeta']\rangle$, where g and h are assignment functions, and ζ and ζ' are (possibly empty) sets of tests such that $\zeta \subseteq \zeta'$. As (28) shows, a post-supposition ϕ does not update the input assignment function g; it is only added into the input set of tests ζ. Based on this, some adjustments are made in defining DPL concepts, as shown in (29).

(28) $[\![{}^{\phi}]\!]^{\langle g[\zeta],h[\zeta']\rangle} = \mathbb{T}$ iff ϕ is a test, $g = h$ and $\zeta' = \zeta \cup \{\phi\}$ **Post-supposition ϕ**

(29) a. ϕ is **true** relative to an input context $g[\varnothing]$ ($\varnothing$ is an empty set) iff there is an output assignment h and a (possibly empty) set of tests $\{\psi_1, ..., \psi_m\}$ such that
(i) $[\![\phi]\!]^{\langle g[\varnothing],h[\{\psi_1,...,\psi_m\}]\rangle} = \mathbb{T}$ and (ii) $[\![\psi_1 \wedge ... \wedge \psi_m]\!]^{\langle h[\varnothing],h[\varnothing]\rangle} = \mathbb{T}$. **Truth**
b. $[\![P(x)]\!]^{\langle g[\zeta],h[\zeta']\rangle} = \mathbb{T}$ iff $g = h$, $\zeta = \zeta'$ and $h(x) \in {*}\Im(P)$. **Atomic formula as test**
c. $[\![\phi \wedge \psi]\!]^{\langle g[\zeta],h[\zeta']\rangle} = \mathbb{T}$ **Dynamic conjunction**
iff there is a k and a ζ'' such that $[\![\phi]\!]^{\langle g[\zeta],k[\zeta'']\rangle} = \mathbb{T}$ and $[\![\psi]\!]^{\langle k[\zeta''],h[\zeta']\rangle} = \mathbb{T}$.
d. $[\![[x]]\!]^{\langle g[\zeta],h[\zeta']\rangle} = \mathbb{T}$ iff $h[x]g$ and $\zeta = \zeta'$. **Random assignment**
e. $[\![\sigma x(\phi)]\!]^{\langle g[\zeta],h[\zeta']\rangle} = \mathbb{T}$ iff (i) $[\![[x] \wedge \phi]\!]^{\langle g[\zeta],h[\zeta']\rangle} = \mathbb{T}$ **Maximization operator σ**
and (ii) there is no h' such that $[\![[x] \wedge \phi]\!]^{\langle g[\zeta],h'[\zeta']\rangle} = \mathbb{T}$ and $h(x) < h'(x)$.

Finally, to analyze the distributive reading of sentences, I also follow Brasoveanu (2013) and define a distributivity operator δ. As shown in (30), when we distributively interpret a formula ϕ relative to a plural individual $g(x)$, we check that ϕ is satisfied by each atom a that is a part of $g(x)$.[9] The (ii) part of this definition means that the distributivity operator δ discharges all the post-suppositions contributed by ϕ within its scope. Thus distributivity is externally static.

[9] This is rather a simplified picture. Basically, $\delta x(\phi)$ is treated as a test here, and I don't consider any quantificational dependencies introduced by or within the scope of δ. Things become much more complicated for this kind of example: *Every year, John wrote a longer novel, which is about 50 pages longer than the previous one* (See Bumford 2015).

(30) $[\![\delta x(\phi)]\!]^{\langle g[\zeta],h[\zeta']\rangle} = \mathbb{T}$ iff $g = h$, $\zeta = \zeta'$, and for all atoms $a \leq g(x)$, if we let g' be such that $g'[x]g$ and $g'(x) = a$, then there is a k and a (possibly empty) set of tests $\{\psi_1, ..., \psi_m\}$ such that (i) $[\![\phi]\!]^{\langle g'[\zeta],k[\zeta \cup \{\psi_1,...,\psi_m\}]\rangle} = \mathbb{T}$ and (ii) $[\![\psi_1 \wedge ... \wedge \psi_m]\!]^{\langle k[\zeta],k[\zeta]\rangle} = \mathbb{T}$

On the base of this DPL framework developed by Brasoveanu (2013), the semantics of sentences of the pattern 'DE modified numeral + distributive predicate' can be accounted for straightforwardly.

(31) $[\![$Fewer than 4^x boys smiled$]\!] \Leftrightarrow \exists^{|x|<4}[*\text{BOYS}(x)]\delta(*\text{SMILE}(x))$
$\Leftrightarrow \sigma x(*\text{BOY}(x) \wedge \delta x(*\text{SMILE}(x)))\wedge^{|x|<4}$
$\Leftrightarrow \sigma x(*\text{BOY}(x) \wedge \delta x(*\text{SMILE}(x))) \wedge |x| < 4$
i.e., the maximal set x such that (i) it consists of boys and (ii) each atomic part of x smiled has a cardinality smaller than 4.[10]

As shown in (31), both the predicates in the restrictor and nuclear scope are distributive, thus the at-issue meaning contributed by modified numerals, i.e., the maximization operator, takes scope over the whole sentence. On the base of this, the cardinality constraint is evaluated as a second dimension of meaning, and thus *fewer than 4* has the effect of taking a pseudo wide scope over the whole sentence, resulting in the upper-bound reading. Notice that I have also included the empty set $\varnothing$ in the domain $\mathfrak{D}$, thus it is not guaranteed that the introduced maximal referent is a non-empty set, which accounts for our intuition that this sentence has no existential entailment.[11]

3.2. The notion of group and the semantics of *fewer than 4 boys lifted the piano together*

Following Barker (1992) (see also Schwarzschild 1996; Winter 2001; Champollion 2010), I consider **groups** a special kind of **atomic individuals**. E.g., *a certain committee constituted by John and Bill* means a singleton set, and I write it as $\uparrow\{\text{John}, \text{Bill}\}$ in the following. In fact, the atoms of an individual X (i.e., the singleton subsets of a non-empty set X) can form many different groups, depending on context. E.g., the atoms of $\{\text{John}, \text{Bill}\}$ (i.e., $\{\text{John}\}$ and $\{\text{Bill}\}$) can form a beach volleyball team, a jazz duo, a job search committee, etc. Therefore, $\uparrow$ can be considered an operator that is the function composition of (i) a context-dependent **choice function** and (ii) a **group-generating operator** that takes a non-empty set X as input and returns the set containing all the groups constituted by the atoms in X. As a consequence, for $\uparrow X$ to be well-defined, X needs to be a non-empty set. Moreover, the operator $\uparrow$ is externally static, i.e., any secondary-dimension meaning introduced within the scope of $\uparrow$ needs to be discharged therein.

As (32) illustrates, there is an interaction between the type of nouns and predicates: group nouns are compatible with collective predicates, but incompatible with distributive predicates;

[10] Obviously, the maximal referent introduced by modified numerals can be an infinite set, and sentences such as *fewer than 5^x prime numbers are odd* cause no problems (cf. Buccola and Spector 2016, see also Footnote 4).

[11] However, in *Fewer than 4^x boys smiled; they^x solved all the problems*, the use of *they* – a refset anaphora (see Charlow 2014) referring to the boys who smiled – brings the presupposition that the maximal referent introduced previously has a cardinality above zero, and its interpretation fails if the accommodation of presupposition fails.

while the pattern with non-group nouns is exactly the opposite.[12] Thus, I assume the meaning postulates shown in (33) for these predicates: basically, they all require their agents be atomic individuals, but more specifically, distributive and collective predicates have complementary requirements. The upshot here is that the agent of *lift the piano together*, a collective predicate, has to be a group. Based on this, the semantics of sentences of the pattern 'DE modified numeral + collective predicate' can also be accounted for straightforwardly, as shown in (34).

(32) The interaction between the type of nouns and predicates:
a. * Every soldier formed a circle. b. ✓ Every soldier smiled.
c. ✓ Every army formed a circle. d. * Every army smiled.

(33) Meaning postulates for distributive and collective predicates:
a. *Smile* requires its agent be a **non-group atomic individual**.
b. *Form a circle* and *lift the piano together* require their agent be a **group**.

(34) [[Fewer than 4^x boys^Y lifted the piano together]] $\rightsquigarrow$ *a fewer-than-4-boy group*
$\Leftrightarrow \exists Y[Y =\uparrow$ [[fewer than 4^x boys]]]$(*\textsc{lift-the-piano-together}(Y))$
$\Leftrightarrow [Y] \wedge Y =\uparrow \{i \in x | \sigma x(*\textsc{boy}(x)) \wedge^{|x|<4}\} \wedge *\textsc{lift-the-piano-together}(Y)$ (see (25b))
$\Leftrightarrow [Y] \wedge Y =\uparrow \{i \in x | \sigma x(*\textsc{boy}(x)) \wedge |x| < 4\} \wedge *\textsc{lift-the-piano-together}(Y)$
i.e., a group of boys, the total number of which is smaller than 4, lifted the piano together.

In (34), Y stores a dref, which has to be a group, and here *fewer than 4 boys* provides some description for this dref, i.e., it consists of a set of boys, the cardinality of which is smaller than 4.[13] Evidently, the dref Y is not within the scope of any maximization operator, indicating that the meaning of the sentence is compatible with situations in which there are other groups, and thus there is no upper-bound reading for this sentence. Moreover, since $\uparrow$ requires its input set be non-empty, it is guaranteed that there are some boys, thus deriving the existential entailment.

To sum up, Sections 3.1 and 3.2 together account for the generalization shown in (3). The semantic contribution of *fewer than 4* remains constant, and there is no over-generation. In addition, each and every part of the proposed analysis here has been motivated independently in the literature.

3.3. Extensions of the current proposal

Between 4 and 8 The data in (4) show that sentences containing *between 4 and 8* pattern with those containing *fewer than 4* with regard to whether there is an upper-bound reading. This

[12] As pointed out by Barker (1992), expressions such as *the group of boys* is ambiguous between two readings: (i) group-as-individual, and (ii) group-as-set. Obviously, under the group-as-set interpretation, *the group of boys smiled* is a grammatical and felicitous sentence. I use **group** to mean Barker (1992)'s group-as-individual and treat groups as atomic individuals, and I use **non-atomic individual** to mean Barker (1992)'s group-as-set (see Section 3.1).

[13] This 'a fewer-than-4-boy group' analysis is somehow reminiscent of the contrast between (15) and (17): ***few*** *boys* is used along with distributive predicates, while ***a few*** *boys* is used along with collective predicates.

can be accounted for straightforwardly: in (35b), the predicate is collective, and *between 4 and 8 boys* refers to a group. Crucially, there is no maximization operator to limit the number of groups.

(35) a. $[\![\text{Between 4 and } 8^x \text{ boys smiled}]\!] \Leftrightarrow \sigma x(*\textsc{boy}(x) \wedge \delta x(*\textsc{smile}(x))) \wedge |x| \in [4,8]$

b. $[\![\text{Between 4 and } 8^x \text{ boys}^Y \text{ lifted the piano together}]\!]$
$\Leftrightarrow [Y] \wedge Y = \uparrow \{i \in x | \sigma x(*\textsc{boy}(x)) \wedge |x| \in [4,8]\} \wedge *\textsc{lift-the-piano-together}(Y)$

The use of group nouns As shown in the meaning postulates (33), both distributive and collective predicates require their agents be atomic individuals, but collective predicates specifically require their agents be groups. Obviously, this straightforwardly explains why the interpretation of '[[modified numeral + group noun] + collective predicate]' is parallel to that of '[[modified numeral + non-group noun] + distributive predicate]'. As illustrated in (36), here the collective predicate *surround the castle* distributes over groups (here *armies*), leading to an upper-bound reading.

(36) $[\![\text{Fewer than } 4^Y \text{ armies surrounded the castle}]\!]$
$\Leftrightarrow \sigma Y(*\textsc{army}(Y) \wedge \delta Y(*\textsc{surround-the-castle}(Y))) \wedge |Y| < 4$

Few / a few In the current account, nothing hinges on the numeral in modified numerals. Thus, sentences containing *few / a few* can be analyzed in the same way, simply by replacing the numeral in modified numerals with a context-sensitive threshold value of largeness, as illustrated in (37).

(37) a. $[\![\text{Few}^x \text{ boys smiled}]\!] \Leftrightarrow \sigma x(*\textsc{boy}(x) \wedge \delta x(*\textsc{smile}(x))) \wedge |x| < \textsc{Threshold}^{\textsc{Context}}$

b. $[\![\text{A few}^x \text{ boys}^Y \text{ lifted the piano together}]\!]$
$\Leftrightarrow [Y] \wedge Y = \uparrow \{i \in x | \sigma x(*\textsc{boy}(x)) \wedge |x| < \textsc{Threshold}^{\textsc{Context}}\} \wedge *\textsc{lift-the-piano-together}(Y)$

No fewer than 4 The current account can also be extended to give a compositional analysis for *no fewer than 4* (e.g., *no fewer than 4 boys smiled*). Similar to other modified numerals, *no fewer than 4* also (i) introduces a maximal referent and (ii) adds a cardinality constraint as a secondary dimension of meaning. Evidently, only this cardinality part needs to be slightly modified.

In the same spirit as Szabolcsi (1997); Geurts and Nouwen (2007), etc., which have pointed out that modified numerals provide quantity information and should not be analyzed as generalized quantifiers, here I follow Zhang and Ling (2015, 2017) and analyze this kind of quantity information in terms of intervals. Intervals are convex sets of degrees, and thus their type is $\langle dt \rangle$. We can write an interval – $\lambda\delta_d.\{\delta | D_{\text{lower-bound}} \leq \delta \leq D_{\text{upper-bound}}\}$ – in terms of its upper- and lower-bounds: $[D_{\text{lower-bound}}, D_{\text{upper-bound}}]$. An interval represents a range of possible values on a scale. In a certain sense, we can consider $(3,5)$ a vaguer value than $[4,5]$ or a singleton set $[4,4]$, because $(3,5)$ contains more possibilities. The operations on intervals are defined on the base of this idea (see Moore 1979). The compositional details of deriving the cardinality part of *(no) fewer than 4* are shown in (38). Notice that in (38e), I analyze the meaning of *no* as an operator on sets: it takes a set as input and returns its complement. As shown in (38f), the cardinality part of *no fewer than 4* is basically equivalent to what *at least 4* means, which is consistent with our intuition.

(38) The cardinality part of *(no) fewer than 4* is analyzed as a set of degrees of type $\langle dt\rangle$:

a. $[\![4]\!]_{\langle dt\rangle} \stackrel{\text{def}}{=} [4,4]$ I.e., a singleton set of degrees

b. $[\![\text{fewer}]\!]_{\langle dt\rangle} \stackrel{\text{def}}{=} (-\infty,0)$ I.e., a set denoting a differential of negative value

c. $[\![\text{than}]\!]_{\langle dt,\langle dt,dt\rangle\rangle} \stackrel{\text{def}}{=} \lambda I_{\text{COMP.-STANDARD}}.\lambda I_{\text{DIFFERENTIAL}}.\iota I_{\text{COMP.-SUBJ.}}[I_{\text{COMP.-SUBJ.}} - I_{\text{COMP.-STANDARD}} = I_{\text{DIFFERENTIAL}}]$

d. $[\![\text{fewer than 4}]\!]_{\langle dt\rangle} = \iota I.[I-[4,4]=(-\infty,0)] = (-\infty,4)$
(Interval subtraction: $[x_1,x_2]-[y_1,y_2]=[x_1-y_2,x_2-y_1]$) (see Moore 1979)

e. $[\![\text{no}]\!]_{\langle dt,dt\rangle} \stackrel{\text{def}}{=} \lambda I.\iota I'[I'=(-\infty,+\infty)\setminus I]$

f. $[\![\text{no fewer than 4}]\!]_{\langle dt\rangle} = \iota I'[I'=(-\infty,+\infty)\setminus(-\infty,4)] = [4,+\infty)$

Fewer than 4 boys can together lift that piano The generic reading of this sentence (see also Section 2.1) is derived in (39), with the assumption of a silent genericity operator $\forall_{\text{GEN}}$ binding Y.

(39) $[\![\text{Fewer than } 4^x \text{ boys}^Y \text{ can together lift that piano}]\!]$
$\Leftrightarrow \forall_{\text{GEN}}[\![\text{Fewer than } 4^x \text{ boys}^Y]\!][\![\text{can together lift that piano}]\!]$
$\Leftrightarrow \forall_{\text{GEN}}Y[Y =\uparrow \{i \in x|\sigma x(*\textsc{boy}(x)) \wedge |x| \in (-\infty,4)\} \rightarrow \diamond * \textsc{together-lift-that-piano}(Y)]$
I.e., in general, boy groups of a cardinality smaller than 4 can together lift that piano.

Crucially, the sentence meaning derived in (39) does not entail that boy groups of any cardinality smaller than 4 can together lift that piano. As I have mentioned above, $(-\infty,4)$ means a range of possible values, and thus it provides quantity information in a vague way. Therefore, the effect of using the information '*fewer than 4 boys*' in describing a group is analogous to the effect of describing a dish as *meat stew*: *meat stew* contains some kind of meat, but not any kind of meat, and due to our world knowledge, the possibility of, say, dinosaur meat, is certainly very improbable.

In sum, the data discussed in Section 3.3 can all be easily accounted for, which provides more empirical support for the current proposed account for modified numerals.

4. Sentences with a cumulative (or co-distributive) reading

Brasoveanu (2013) and Buccola and Spector (2016) approach cumulative-reading sentences differently. In Brasoveanu (2013)'s analysis of sentence (22) (repeated here as (40)), the same maximization operator globally binds both the variables x and y, introducing simultaneously two maximal sets. A consequence of this is that when there is a DE modified numeral, it is predicted that there is an upper-bound reading, but no existential entailment. The examples in (41) suggest that this prediction is borne out. On the other hand, according to Buccola and

Spector (2016)'s account for sentence (42), the predicate here is considered a collective one, and thus, the sentence has an existential entailment, but no upper-bound reading. Both cases can be handled by the current proposed analysis. However, there is a remaining issue for future research. For this kind of sentences containing multiple (modified) numerals, what factors determine their interpretation? Can there be examples showing a true ambiguity? Are animacy and agent/theme asymmetry involved here?

(40) Exactly 3^x boys saw exactly 5^y movies. **Cumulative** reading
$\sigma xy(*\textsc{boy}(x) \wedge *\textsc{movie}(y) \wedge *\textsc{see}(x,y)) \wedge |y| = 5 \wedge |x| = 3$

(41) These 10^x chickens laid <u>fewer than 20^y eggs</u> in total. ✓ Upper-bound; # Existential
<u>Fewer than 20^y eggs</u> were laid by these 10^x chickens. ✓ Upper-bound; # Existential
$\sigma xy(*\textsc{chicken}(x) \wedge *\textsc{egg}(y) \wedge *\textsc{lay}(x,y)) \wedge |y| < 20 \wedge |x| > 10$
(cf. <u>Fewer than 10 chickens</u> laid 20 eggs in total. # Upper-bound; ✓ Existential)

(42) Fewer than 4^x boys^Y drank 20 beers between them. # Upper-bound; ✓ Existential
$[Y] \wedge Y = \uparrow \{i \in x | \sigma x(*\textsc{boy}(x)) \wedge |x| < 4\} \wedge *\textsc{drink-20-beers-between-them}(Y)$
(cf. 4 boys drank <u>fewer than 20 beers</u> between them. ✓ Upper-bound; # Existential)

5. Buccola and Spector (2016)'s 'informativity-based maximality' approach

Buccola and Spector (2016) has also proposed an approach of informativity-based maximality. The basic idea is that for a degree n to be P-maximal in w, it satisfies two requirements: (I) the proposition $P(n)$ must be true in w, and (II) there is no degree m such that (i) $P(m)$ is true in w and (ii) $P(m)$ is more informative than $P(n)$. This idea is formally implemented as a $\mathsf{max}_{\text{inf}}$ operator, and there are two specific analyses: **L**(exical)**Max**$_{\textbf{inf}}$ (see (43)) and **S**(eparate)**Max**$_{\textbf{inf}}$.

(43) a. $[\![\text{fewer than}]\!]^w \stackrel{\text{def}}{=} \lambda n_d.\lambda P_{\langle s,\langle dt\rangle\rangle}.\exists m[m < n \wedge \mathsf{max}_{\text{inf}}(P)(w)(m)]$ LMax_{inf}
b. $[\![\text{fewer than 4}]\!]^w \stackrel{\text{def}}{=} \lambda P_{\langle s,\langle dt\rangle\rangle}.\exists m[m < 4 \wedge \mathsf{max}_{\text{inf}}(P)(w)(m)]$

This 'informativity-based maximality' approach has at least the following four problems, and the first two have already been mentioned and discussed in Buccola and Spector (2016). **(I)** First, scope interactions between a modified numeral and an existential closure still cause over-generation. Using a pragmatic rule like (9) can sometimes rule out some unattested readings, but, as I have discussed in Section 2.2, the use of this pragmatic rule has its own problems. **(II)** Second, although this approach naturally accounts for the (un)availability of upper-bound readings, there are non-trivial complications with regard to accounting for the (un)availability of existential entailment. Some stipulations are added in Buccola and Spector (2016) to deal with this issue. **(III)** Third, given semantic compositionality, a good account for the semantics of *fewer than 4* should be able to be extended to account for the semantics of *no fewer than 4*. However, LMax_{inf} cannot achieve this. Roughly speaking, in combining *no* with *fewer than (4)* to derive the semantics of *no fewer than 4*, since the part $\mathsf{max}_{\text{inf}}(P)(w)(n)$ in (43) should intuitively remain constant for both *fewer than 4* and *no fewer than 4*, it is unclear how *no* targets only the part $m < n$ or $m < 4$. **(IV)** Fourth, if SMax_{inf} is adopted, i.e., $\mathsf{max}_{\text{inf}}$ is independent of the semantics of modified numerals, then it raises the question why $\mathsf{max}_{\text{inf}}$ cannot be used in sentences containing bare numerals. If it is used, then since informativity-based maximality is now part of the truth conditions of sentences, in a situation where I ate 4 apples, the sentence *I ate 3 apples* is simply false, not under-informative.

However, there are another set of relevant data that seem to motivate the approach of informativity-based maximality (see Buccola 2015a; Buccola and Spector 2016), namely sentences expressing different scalarity entailment. E.g., from *fewer than 10 eggs can feed these people*, it follows that *fewer than 11 eggs can feed these people*, but from *fewer than 10 people can fit into this elevator*, it follows that *fewer than 9 people can fit into this elevator*. Whether the current proposed analysis can be extended to account for this kind of inference pattern is left for future research.

6. Brasoveanu (2013)'s approach vs. Solt (2006)'s 'decomposition + split scope' analysis

Solt (2006) has proposed a 'decomposition + split scope' account for *few*: *few* is decomposed into (i) a positive cardinality and (ii) a negation operator that takes the widest possible scope (see (44)).

(44) $[\![\text{few}]\!]_{\langle et\rangle} \overset{\text{def}}{=} \lambda x.[\textsc{large-value}^{\textsc{context}}(|x|)]$; storing a negation operator

a. [[**Few** students passed the test]]
$\Leftrightarrow \neg\exists x[\textsc{large-value}^{\textsc{context}}(|x|) \wedge {*}\textsc{student}(x) \wedge {*}\textsc{pass-the-test}(x)]$ $\neg > \exists$

b. [[**A few** students passed the test]]
$[\![\text{A few students}]\!] = \lambda P_{\langle et\rangle}.\exists x[\neg\textsc{large-value}^{\textsc{context}}(|x|) \wedge {*}\textsc{student}(x)]$ $\exists > \neg$

For sentences like *fewer than 4 boys lifted the piano together*, if we assume that there is also a silent existential closure within the subject DP (i.e., ***A*** *fewer-than-4-boy (group)*) and adopt a decompositional analysis à la Solt (2006), presumably the existential entailment and the lack of an upper-bound reading of this sentence can also be accounted for. Compared with Brasoveanu (2013)'s approach, Solt (2006)'s seems to have some disadvantages and an advantage. First, Solt (2006) cannot address the maximal reference issue. Second, in terms of compositionality, *fewer than 4* cannot be derived from (i) negation and (ii) *larger than 4* – '*equal to 4*' needs to be ruled out (see Zhang and Ling 2017). However, it seems that within the current proposed analysis, modified numerals like *fewer than 4* and *no fewer than 4* are analyzed in the same way (see Section 3.3), which raises the question of how to account for NPI licensing. This is also left for future research.

7. Summary and outlook

In this paper, based on existing well-motivated components of natural language semantics, I explain the interplay between (i) the interpretation of sentences containing non-increasing modified numerals and (ii) the type of predicates therein. In addition to what has been discussed in Sections 4 – 6, there are two remaining issues: (i) Whether there are other ways of implementing the secondary dimension of meaning of modified numerals, and whether/how they differ in terms of dynamicity/staticity; (ii) Whether/how the semantics of modified numerals interacts with modals.

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