Proportional comparatives and relative scales[1]

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Abstract. This paper examines the proportional interpretation of comparatives such as *More residents of Ithaca than New York City know their neighbors*, taking these as the starting point for an investigation of relative or proportional measurement more generally. Two mechanisms for deriving mappings from individuals to degrees of proportion are considered, the first based on posited proportional entries for *many/few*, the second involving proportional measure functions. It is shown that only the measure function approach adequately accounts for the distribution of proportional expressions and proportional interpretations. A further consequence is to eliminate the need to analyze words such as *many* as lexically ambiguous.

Keywords: degree, measurement, comparative, proportion, partitive.

1. Introduction

The sentences in (1)-(2) exemplify a curious type of comparative construction that to my knowledge was first discussed by Partee (1989):

(1) More residents of Ithaca than New York City know their neighbors.
(2) More Norwegians than Brazilians have university degrees.

At first glance, examples of this sort are entirely unobjectionable, but on further reflection, one realizes there is something rather odd about them. The number of residents of Ithaca, NY (population 30,000) who know their neighbors is without doubt smaller than the number of inhabitants of New York City (population 8 million) who do; and likewise, in absolute terms there are certainly a smaller number of Norwegians than Brazilians with university degrees. Nevertheless, both of these sentences have salient interpretations on which they are true.

Intuitively, on their true readings the above examples express comparisons of proportions: it is the proportion of Ithaca residents who know their neighbors that is greater than the corresponding proportion of New York City residents (and similarly, *mutatis mutandis*, for proportions of Norwegians versus Brazilians with degrees). The goal of this paper is to develop a compositional semantic analysis of examples such as (1)-(2) on their proportional readings, and in doing so to investigate the semantic structures underlying the interpretation of proportional expressions more generally.

In a degree-based semantic framework, comparatives are analyzed as expressing relations between degrees on a scale corresponding to some dimension of measurement (Cresswell 1977; von Stechow 1984; Heim 1985; Kennedy 1997; see Beck 2011 for a recent review). Taking this approach, we might loosely represent the meaning of (1) as follows:

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(3) $max\{d : d\text{-many Ithaca residents know their neighbors}\} \succ$
$max\{d : d\text{-many NYC residents know their neighbors}\}$

If we take the degrees *d* here to range over values on the scale of cardinality – that is, natural numbers – we get only the false absolute reading. To get the true proportional reading, it is necessary for *d* to range over degrees of proportion. That is, we require a scale that tracks proportions of a totality. In the context of current work in degree semantics, this is an unusual idea. As has been observed by Cresswell (1977), Wellwood (2015) and many authors in between, comparative *more* can operate over a variety of dimensions, including not just cardinality (e.g. (4a)) but also mass quantity dimensions such as volume (e.g. (4b)) as well as adjectival dimensions such as intelligence (e.g. (4c)).

(4) a. Sue owns more books than John. (cardinality)
b. Sue drank more wine than John. (volume)
c. Sue is more intelligent than John. (intelligence)

The existence of proportional scales, however, is not generally recognized. I will argue here that precisely such scales are a necessary part of the degree ontology, and that they play a role beyond comparative constructions of the sort exemplified above.

The structure of the paper is as follows: Section 2 introduces two possible approaches to the semantics of proportional comparatives, the first based on a well known ambiguity in the interpretation of the vague quantity expressions *many* and *few*, the second making use of the notion of domain-restricted measure functions, one instantiation of which is a proportional measure function. Section 3 turns to other sorts of proportional expressions, comparing how the two proposed solutions to proportional comparatives fare in accounting for their distribution. Section 4 returns to consider the readings available to *many* and *few*, and demonstrates that only the measurement-based approach allows a principled explanation for the constraints on the availability of proportional readings of their positive and comparative forms. Finally, Section 5 wraps up with some conclusions and questions still left open.

2. Two accounts of proportional comparatives

2.1. Ambiguous *many* and *few*

Since Bresnan (1973) it has been common to analyze *more* and *fewer* as the comparative forms of *many/much* and *few*, respectively. As discussed by Milsark (1977) and especially Partee (1989), *many* and *few* themselves appear to be ambiguous between two distinct readings which have come to be called ‘cardinal’ and ‘proportional’. For instance, (5) might be interpreted to mean simply that a large (small) number of aspens burned (the cardinal reading) or instead that a large (small) proportion of some contextually relevant set of aspens burned (the proportional reading).

(5) Many (few) aspens burned.

The cardinal/proportional ambiguity is easiest to detect in the case of *few*. On its proportional interpretation, *few* can be paraphrased as 'a small proportion of'. But on the cardinal reading, *few Ns* can actually be <u>all</u> of the relevant *Ns*, as demonstrated by the following example (from Partee 1989):

(6) Few egg-laying mammals were found in our survey, perhaps because there <u>are</u> few.

Partee argues that the existence of these two readings is more than a matter of radical context dependence but instead involves a true lexical ambiguity, which corresponds to a more basic distinction in semantic type; specifically, she proposes to analyze *many/few* as ambiguous between an adjectival entry with cardinal semantics and a quantificational entry with proportional semantics.

Partee's notion of a lexical ambiguity has been adopted by many later authors (e.g. Herburger 1997; Cohen 2001; though see Greer 2014 for an opposing view). Recently, Romero (2015) has developed an updated version of the ambiguity account in a degree-based framework, according to which *many/few* are ambiguous between cardinal and proportional entries, each of which has the type of a gradable quantifying determiner, type $\langle d,\langle et,\langle et,t\rangle\rangle\rangle$:[2]

(7) a. $[\![many_{CARD}]\!] = \lambda d \lambda P_{\langle et\rangle} \lambda Q_{\langle et\rangle}.|P \cap Q| \succeq d$

b. $[\![many_{PROP}]\!] = \lambda d \lambda P_{\langle et\rangle} \lambda Q_{\langle et\rangle}.\dfrac{|P \cap Q|}{|P|} \succeq d$

(8) a. $[\![few_{CARD}]\!] = \lambda d \lambda P_{\langle et\rangle} \lambda Q_{\langle et\rangle}.|P \cap Q| \prec d$

b. $[\![few_{PROP}]\!] = \lambda d \lambda P_{\langle et\rangle} \lambda Q_{\langle et\rangle}.\dfrac{|P \cap Q|}{|P|} \prec d$

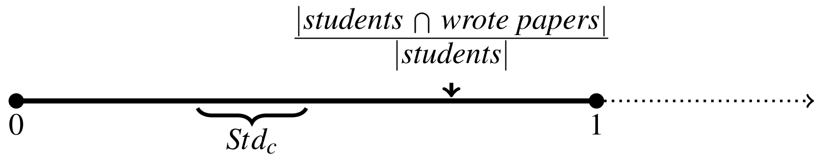
In both cases, the initial type *d* argument can be saturated or bound by a degree modifier, just as in the case of gradable adjectives such as *tall*. When *many/few* appear in their unmodified form (e.g. as in (5)), it is proposed that this role is played by a null degree operator *POS*, again in parallel to a standard analysis of the unmodified forms of gradable adjectives. For example, if *POS* is taken to introduce a standard of comparison in the form of a 'neutral range' Std_c relative to some context *c* (von Stechow, 2009), a simple example such as (10a) on its proportional reading receives the interpretation in (10b), depicted in (10c):

(9) $[\![POS]\!]^c = \lambda I_{\langle dt\rangle}.Std_c \subset I$

(10) a. Many students wrote papers.
POS$_1$ [[t_1-many$_{PROP}$ students] wrote papers]

b. $Std_c \subset \{d : \dfrac{|students \cap wrote\ papers|}{|students|} \succeq d\}$

[2] Romero further notes that *few* might be decomposed into a degree-negation operator plus *many*. Note also that while she does not discuss quantificational *much* and *little*, these might be handled by replacing the cardinality operator with a measure function suitable for portions of matter.

c. 

Returning to the topic of the present paper, the proportional entry for *many* in (7b) in combination with standard assumptions about the syntax and semantics of comparatives give us what we need to derive the proportional reading of our original examples. Specifically, we can take the comparative morpheme *-er* to express a relation between two sets of degrees:

(11) $[\![\textit{-er}]\!] = \lambda I_{\langle dt\rangle}\lambda J_{\langle dt\rangle}.max(J) \succ max(I)$

Syntactically, an example such as (12) is taken to be a (covert) clausal comparative, with the *than*-clause containing an elided copy of *many* whose degree argument is bound by a null operator *OP*. The comparative morpheme *-er* plus the *than* clause as a whole originate within the noun phrase, but must undergo quantifier raising (QR) for type-driven purposes, yielding the LF in (13).

(12) More Ithaca residents than New York City residents know their neighbors.

(13) [-er than [OP$_2$ [t_2 ~~many~~ NYC residents ~~know their neighbors~~]]]$_1$ [t_1 many Ithaca residents know their neighbors]

The first argument of *-er* is saturated by the *than* clause, while the second is provided by the matrix clause via lambda abstraction over the trace of the raised comparative morpheme, yielding the following interpretation:

(14) $$max\{d : [\![d\text{-}many_{PROP}]\!]([\![Ithaca\ residents]\!])([\![know\ neighbors]\!])\} \succ$$
$$max\{d : [\![d\text{-}many_{PROP}]\!]([\![New\ York\ City\ residents]\!])([\![know\ neighbors]\!])\}$$

$$= max\{d : \frac{|Ithaca\ residents \cap know\ neighbors|}{|Ithaca\ residents|} \succeq d\} \succ$$
$$max\{d : \frac{|NYC\ residents \cap know\ neighbors|}{|NYC\ residents|} \succeq d\}$$

As desired, this expresses a comparison of the proportions of residents of the two cities who know their neighbors. That is, we are able to derive the proportional reading of our original example (1).

2.2. Domain-restricted measure functions

The account developed in the previous section relies crucially on the analysis of *many/few* themselves as parameterized quantificational determiners, the same semantic type (once their initial type *d* argument is saturated) as quantifiers such as *every*. While this is a common view, which can be traced back to classic works such as Barwise and Cooper (1981), there is evidence that it cannot be correct. The issue is that words of the *many* class have uses on which they do not quantify over individuals, a prime example being the differential use (e.g. *Many fewer / few more than 100 students attended the lecture*). Instead, authors including Rett (2006, 2008) and Solt (2009, 2015) propose that *many/few* have degree-based interpretations.

To formalize this, we make the following assumptions about the semantics of degree and measurement. A **scale** is a triple of the following form:

(15) $S = \langle D, \succ, DIM \rangle$, where
- D is a set of degrees;
- $\succ$ is an ordering relation on that set;
- DIM is a dimension of measurement.

This formalization has the effect of establishing a one-to-many relation between dimensions of measurement and the scales that track them (as a simple example, think of height measured in inches vs. centimeters). Entities are mapped to degrees by **measure functions**, where μ_{DIM} is a function that maps members of the domain of individuals D_e to degrees on some scale corresponding to dimension DIM.

Following Solt (2015), vague quantity words such as *many* can then be interpreted as gradable quantifiers over degrees (where *many/few* are distinguished from *much/little* in that the former select for degrees of cardinality, while the latter are blocked in the cardinality case):

(16) a. $[\![\textit{many/much}]\!] = \lambda d \lambda I_{\langle dt \rangle}.d \in I$ (Solt, 2015)
b. $[\![\textit{few/little}]\!] = \lambda d \lambda I_{\langle dt \rangle}.\neg d \in I$

On their (apparently) quantificational uses, the degrees over which these expressions operate are introduced by a phonologically null functional head *Meas*, which encodes an underspecified measure function μ^c_{DIM}:

(17) $[\![Meas]\!]^c = \lambda P_{\langle et \rangle} \lambda x.[P(x) \wedge \mu^c_{DIM}(x) = d]$

The specific dimension introduced by *Meas* is contextually determined, as is the individual scale that tracks that dimension; but we require that the dimension so introduced be **monotonic** on the part-whole relationship between individuals (Schwarzschild, 2006), as defined below:[3]

(18) **Monotonicity constraint:** $\forall x, y \in D_e,\ x \sqsubset y \Rightarrow \mu^c_{DIM}(x) \prec \mu^c_{DIM}(y)$

Once their first degree argument is saturated by a degree expression or its trace, *many/few* have the type of degree operators, which like other such operators (e.g. *POS* in (9) or *-er* in (11)) must undergo QR to be interpreted. A simple quantificational example thus has the LF in (19a); assuming that quantification over individuals arises via existential closure, the corresponding interpretation is that in (19b).[4]

[3] For discussion on the source of this constraint, see Rett (2014).
[4] See Solt (2015) for details of the compositional derivation.

(19) a. Many students wrote papers.
POS$_2$ [t_2 many]$_1$ [[t_1 *Meas* students] wrote papers]
b. $Std_c \subset \{d : \exists x[students(x) \wedge \mu^c_{DIM}(x) = d \wedge wrote\text{-}papers(x)]\}$

If μ^c_{DIM} is set to be a cardinality function, it is the cardinal reading that most naturally emerges: the number of students who wrote papers exceeds some value or range that serves as the threshold for what counts as 'many' in the context. On this approach to words of the *many* class, it is then tempting to treat the proportional reading of sentences such as this contextually, by specifying the standard range introduced by *POS* in such a way that values that exceed it correspond to a large proportion of the relevant totality (here, the students). But this is not sufficient to derive the proportional reading of comparative constructions, for which we need a scale that directly tracks proportions.

To address this, I propose to introduce a new type of measure function, namely one that is restricted to measuring subparts of some entity:[5]

(20) A **domain-restricted measure function** $\mu^c_{DIM;x} \rightarrow S^c_{DIM}$ is a function that maps parts of some entity $x \in D_e$ to degrees on some scale S tracking dimension DIM.

Since the domain of a function of the form in (20) is restricted, its range is likewise restricted to degrees that are measures of some part of the domain, which might constitute a proper subpart of the full scale S.

I further propose that one particular variety of domain-restricted function that the grammar makes available is one that maps parts of an entity to values that encode the proportion they represent of the totality (with respect to the dimension in question):

(21) A **proportional measure function** is a function of the following form:

$$\text{For } y \sqsubseteq x : \mu^c_{DIM;x}(y) = \mu^c_{DIM\text{-}prop;x}(y) = \frac{\mu^c_{DIM}(y)}{\mu^c_{DIM}(x)}$$

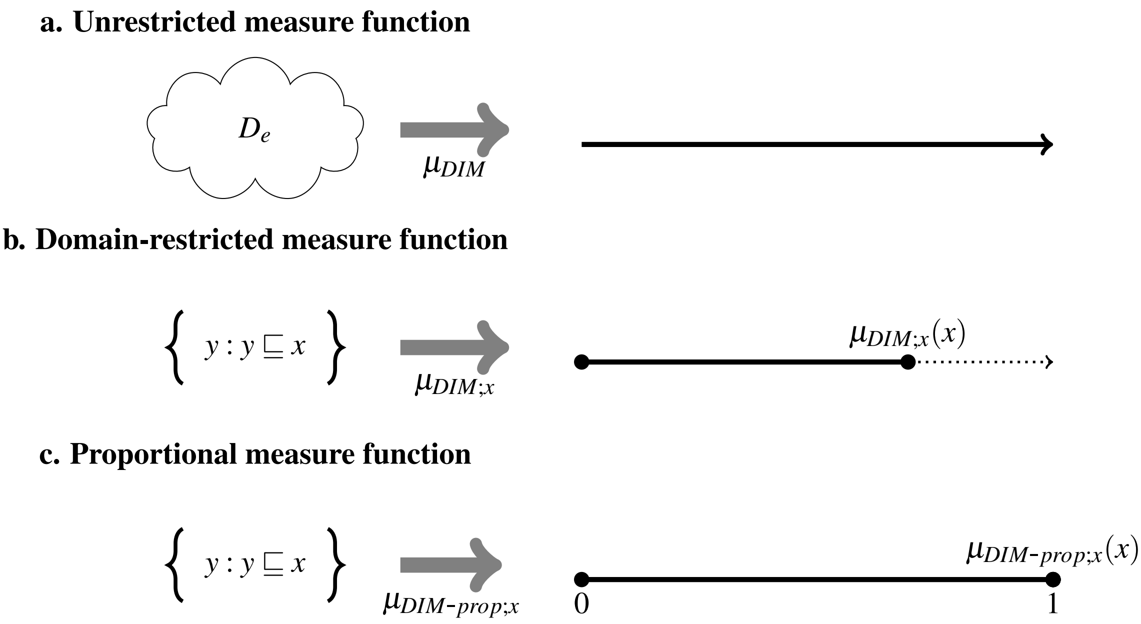
The three types of measure functions discussed here – ordinary unrestricted, domain-restricted and proportional – are depicted graphically in Figure 1.

Importantly, if a measure function μ^c_{DIM} tracking dimension DIM satisfies the monotonicity constraint (18), so too do corresponding domain-restricted measure functions $\mu^c_{DIM;x}$, including the proportional measure function $\mu^c_{DIM\text{-}prop;x}$. This means that the latter two sorts are possibilities for the measure function introduced by the functional head *Meas*. Thus I take it that *Meas* can have the following form, where the relevant totality is that formed by summing over entities in the denotation of the common noun phrase:

(22) $[\![Meas]\!]^c = \lambda P_{\langle et \rangle} \lambda x.[P(x) \wedge \mu^c_{DIM;\oplus P}(x) = d]$

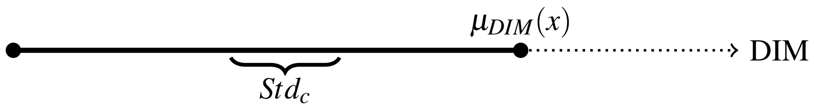
[5] Kennedy (2007) utilizes domain-restricted measure functions of a different sort to account for facts in the adjectival domain. I leave it as an open question whether there might be a connection between the two cases.

Figure 1: Varieties of measure functions



Taking advantage of the possibilities offered by (22), we are able to derive the proportional readings of *many/few* and their comparative forms. Starting with the unmodified case, the proportional reading of examples such as *Many (few) aspens died* can be derived by assuming a domain-restricted measure function, and making one more very plausible assumption, namely that whenever the measure function introduced is of this variety (either an ‘ordinary’ domain-restricted function or a proportional one), the standard range introduced by *POS* is fully contained within the segment of the scale that is the range of this function, as illustrated below (where x is the relevant totality):

(23)



This has the effect of ensuring that *many Ns* is a suitably large proportion of all the *Ns*, and crucially that *few Ns* cannot be all of the *Ns*. While this may seem to be a stipulation, note that essentially the same stipulation is required under the ambiguity analysis (see (10c) above), where it must be assumed that the standard range is fully contained in the interval [0,1].

The proportional interpretation of comparatives can now be derived by taking the contextually determined measure function to be a proportional one $\mu^c_{DIM\text{-}prop;\oplus P}$. Maintaining the earlier assumptions regarding the syntax and semantics of comparative constructions, a relevant example has the LF in (24), and the interpretation in (25), where the underlying dimension that forms the basis for the calculation of proportions is cardinality, indicated by #.

(24) [-er than [OP$_4$ [t_4 ~~many~~$_3$ [t_3 *Meas* NYC residents ~~know their neighbors~~]]]]$_2$ [t_2 many$_1$ [t_1 *Meas* Ithaca residents know their neighbors]]

(25) $$max\{d : \exists x[Ithaca\text{-}res(x) \wedge \mu^c_{\#\text{-}prop;\oplus Ithaca\text{-}res}(x) = d \wedge know\text{-}neighbors(x)]\} \succ$$
$$max\{d : \exists x[NYC\text{-}res(x) \wedge \mu^c_{\#\text{-}prop;\oplus NYC\text{-}res}(x) = d \wedge know\text{-}neighbors(x)]\}$$

$$= max\{d : \exists x[Ithaca\text{-}res(x) \wedge \frac{\mu^c_{\#}(x)}{\mu^c_{\#}(\oplus Ithaca\text{-}res)} = d \wedge know\text{-}neighbors(x)\} \succ$$
$$max\{d : \exists x[NYC\text{-}res(x) \wedge \frac{\mu^c_{\#}(x)}{\mu^c_{\#}(\oplus NYC\text{-}res)} = d \wedge know\text{-}neighbors(x)\}$$

Crucially, the function introduced by *Meas* in the matrix clause maps pluralities of Ithaca residents to the proportion they represent of the totality of Ithaca residents, while its copy in the *than* clause correspondingly maps pluralities of NYC residents to the proportion they make up of all NYC residents. Thus as above we get the desired comparison of proportions.

2.3. Summary

In this section we have seen two possible approaches to the analysis of proportional comparatives. Both relate the reading of interest to the proportional interpretations of *many* and *few*, and both feature some mechanism for mapping individuals to scalar values that represent proportions. Where they differ is in how the required proportional scale is derived. The first analysis localizes the calculation of proportions in the semantics of *many* and *few* themselves on their proportional interpretations, one of two distinct lexical entries for these items. The second assumes a simpler and unambiguous semantics for *many/few*, with measurement instead encoded by an underspecified functional element *Meas*, which as one possibility can introduce a mapping from elements in the denotation of a nominal expression to a degree representing their proportion of the totality of such entities. Both of these analyses are able to capture the facts seen so far. In the remainder of the paper we will examine how they fare in accounting for a wider range of data. Previewing the findings, we will see that only the measurement-based analysis provides a satisfactory explanation for certain facts relating the the distribution of proportional expressions and proportional interpretations.

3. The semantics of percent

Words such as *many* are of course not the only sorts of expressions with a proportional meaning, the other obvious case being those of the form *n percent*, as in the following:

(26) a. Twenty percent of the students wrote papers.
b. Sixty percent of the wine we sell is German.

There is surprisingly little work on the semantics of such relative or proportional measures, notable exceptions being Ionin et al. (2006) and more recently Ahn and Sauerland (2015, 2017). The latter authors propose the following as the lexical entry for *percent*:

(27) $$[\![percent]\!] = \lambda x_e \lambda n_d \lambda P_{et}.\left[\frac{\mu(x \sqcap \oplus P)}{\mu(x)} = \frac{n}{100}\right]$$

On this analysis, examples such as (26) do not involve the direct measurement of proportions. Rather, the measure function μ maps entities to values on ordinary quantity scales. In (26a), for example, μ is interpreted as a cardinality function; in (26b) it is a function mapping portions of wine to some appropriate measure of liquids, perhaps volume in liters. The computation of proportions itself is lexicalized by *percent*. Assuming *of* to be semantically vacuous, (26a) receives the following analysis:

(28) $$\frac{\mu_{\#}(\textit{the-students} \sqcap \oplus(\textit{individuals who wrote papers}))}{\mu_{\#}(\textit{the-students})} = \frac{20}{100}$$

Although this approach is appropriate for examples such as (26), there is reason to believe that in other cases expressions of the form *n percent* have interpretations on which they directly denote degrees. The strongest evidence for this is that they occur in the same syntactic positions as do numerals when these are analyzed as degree denoting. Three such contexts are illustrated below:

(29) a. Twenty times ten equals two hundred.
b. More/fewer than twenty of the students wrote papers.
c. We sold twenty fewer houses this year than last year.

(30) a. Twenty percent of ten percent equals two percent.
b. More/fewer than twenty percent of the students wrote papers.
c. We sold twenty percent fewer houses this year than last year.

The (a) examples above are mathematical statements, which appear to express relations between degrees. The (b) examples involve comparative quantifiers. It is common (e.g. Nouwen, 2010) to analyze the comparative morpheme in this case as a degree quantifier that takes an initial number or degree argument, as in (31). We must then conclude that *twenty percent* in (30b), just like *twenty* in (29b), can denote a degree that can saturate this argument.

(31) $[\![\textit{-er}]\!] = \lambda d \lambda I.max(I) \succ d.$

Finally, the (c) examples illustrate the differential use of measure expressions. In (29c), there is quite obviously no plurality of houses which is asserted to have cardinality twenty. Rather, the numeral describes the distance between two values on the number line; that is, the sentence is true if it holds that the difference between $|\textit{houses we sold last year}|$ and $|\textit{houses we sold this year}|$ equals 20. It seems that a parallel analysis must be extended to (30c).

Importantly, in none of the examples in (30) can *twenty percent* be given an analysis based on the proposed lexical entry for *percent* in (27). Instead, we require it in each case to be interpreted as denoting a degree, i.e. $[\![\textit{twenty percent}_d]\!] = 20\%$. This provides further evidence that we require a scale of proportion on which such degrees are situated.

Let us consider how this set of facts might be accounted for under the two approaches to proportional comparatives introduced in the previous section, each of which provides in one way or another a mapping to scalar values that represent proportions of a totality.

In the ambiguity account of Section 2.1, the calculation of proportions is part of the semantic function of the proportional versions of *many* and *few*. While we do not assume measure functions that map individuals to proportions, the first arguments of *many/few* on their proportional readings (7b), (8b) can be viewed as ranging over degrees of proportion. This leads us to predict that we will detect degrees of proportion – and expressions that refer to them – only in the presence of *many/few* (or *much/little*). This makes proportional expressions in mathematical statements such as (30a) somewhat unexpected, though we might propose to construe them as answers to covert *how* $many_{PROP}$*?* questions. Expressions of proportion in comparative quantifier examples such as (30b) can be handled by taking these to be based on comparative forms of proportional *many* and *few*, i.e. essentially the same analysis given above to proportional comparatives. But differential examples such as (30c) are problematic. The only kind of proportion that can be derived from the proportional entries of *many/few* is the proportion that a plurality represents of some totality that it is a part of. But *twenty percent* in (30c) does not have this sort of interpretation. In fact, as noted above, in this example there is no plurality whose measure is reported. Rather, the percentage expression describes the difference between two values in proportional terms. This value cannot be calculated on the basis of the proportional entry for *few* proposed in Section 2.1. It is necessary to posit some other mechanism; for example, we might start with an interpretation of the comparative morpheme that allows differential degrees (see e.g. Beck 2011), and modify it to allow these to express proportions.

Note also that an account based on proportional interpretations for *many/few* and degree-denoting interpretations for expressions of the form *n percent* does not in itself provide an analysis of simple partitive examples such as (26), because there is apparently no element present that introduces degrees of proportions. There are several ways this issue might be addressed. As one possibility, we might propose that *n percent* has a second interpretation based on Ahn & Sauerland's entry for *percent* in (27). Alternately, we might assume that such partitive examples include a covert counterpart of $many_{PROP}$, or that a comparable function is encoded in the semantics of partitive *of*. All of these potential solutions add duplication to the system as a whole; below, we will see some further issues that they present.

Now let us turn to how the same data can be handled by the measurement-based account developed in Section 2.2. Recall that on this analysis, quantity measurement is encoded by a null functional element *Meas*, which introduces a contextually determined measure function. One potential option for this function is a domain-restricted function, and more specifically a proportional one. Thus in contrast to the previous approach, this analysis assumes a measure function whose range is a scale of proportion (see Fig. 1). It is thus not surprising that there are linguistic expressions that refer to points on this scale: just as *twenty* can refer to a point on the number line and *twenty inches* to a point on a scale of height, so too can *twenty percent* refer to a point on a scale of proportion.

Degrees of this sort can form the basis for mathematical statements such as (30a), and can saturate the degree argument of the comparative quantifier version of *-er* in (31). An example such as (30b) can then be analyzed as involving degrees derived via a proportional instantiation of the measure function introduced by *Meas*, with *more/fewer* analyzed as the comparative forms of the degree quantifier entries in (16) above; again, this is parallel to the treatment of our original proportional comparative examples.

Putting aside for the moment the tricky example in (30c), let us consider proportional partitives such as (26). The simplest way to analyze such examples in the measurement-based framework is to treat partitive *of* as the overt instantiation of *Meas*. Such an approach goes back to Schwarzschild (2006), who was among the first to posit the existence of a null measuring element parallel to *of* underlying the interpretation of words of the *many* class. Specifically, we can analyze *of* in such cases as encoding a domain-restricted measure function:

(32) $[\![of]\!] = \lambda x \lambda d \lambda y.[\mu^{c}_{DIM;x}(y) = d]$

On this view, partitive *of* has a dual function, establishing the 'part of' relation (as in Ionin et al. 2006) and also associating entities with degrees.

Importantly, this approach to *of* allows an analysis of both numerical partitives such as (33a) and proportional partitives such as (33b). The first of these involves a domain-restricted measure function that maps subgroups of the relevant totality of students to a subsegment of the number line corresponding to the cardinalities of such subgroups. For example, if there are fifty relevant students, the corresponding scale segment is $[0,50]$. In the latter proportional case, the measure function is the proportional variety of a domain-restricted function, whose range is the proportions from 0% to 100%.

(33) a. Twenty of the students wrote papers.
$\mu_{\#;the\text{-}students} : x \sqsubseteq \textit{the-students} \rightarrow [0, \#\text{ of the students}]$

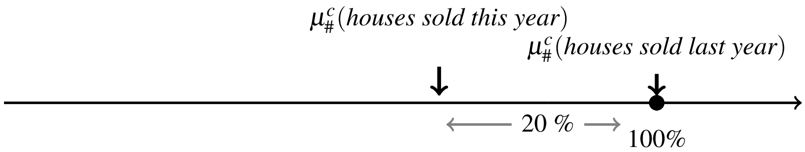
b. Twenty percent of the students wrote papers.
$\mu_{\#\text{-}prop;the\text{-}students} : x \sqsubseteq \textit{the-students} \rightarrow [0\%, 100\%]$

In both cases the domain – and therefore also the range – of the measure function is restricted. The difference is that in one case the values on the scale are construed as numbers (cardinalities), while in the other they are construed as proportions.

A nice consequence of unifying partitive *of* with *Meas* is that we correctly account for the fact that the same monotonicity constraint is present in both cases. As an example, *more copper than silver* – which we take to involve a comparative form of *much* – can describe a portion of copper that has a greater measure than the corresponding portion of silver in terms of volume or weight (both monotonic on the part-whole relation among entities), but not in terms of temperature or purity (both non-monotonic). By the same token, *twenty percent of the copper* can be a portion of copper whose measure in terms of volume or weight is 20% of that of the relevant totality of copper, but not one whose purity or temperature is 20% of that of the totality.[6] This parallel is not captured by an account on which the semantics of relative measures such as *twenty percent* are based on Ahn & Sauerland's entry for *percent* in (27), while *more* comparatives are based on generalized quantifier entries for *many/much* such as Romero's in (7).

Finally, let us return to the differential use exemplified in (30c), repeated below. While these might be handled in the same way suggested above for the ambiguity account, the measurement-based analysis offers another possibility. Recall that the semantics of *Meas* is underspecified: both the dimension of measurement and the particular scale tracking that dimension are contextually determined. Even when the dimension is set to be cardinality or number, there are multiple possible scales and corresponding measure functions. One such function is that which maps the totality of houses sold last year to the point 1 (or equivalently 100%), and other pluralities to values expressed in relation to this. In (30c), *twenty percent* describes the length of an interval on this scale:

(34) We sold twenty percent fewer houses this year than last year. (=(30c))



Note that the measure function here is not a proportional one (or for that matter a domain-restricted one of any sort); but it satisfies monotonicity, making it a licit choice.

To summarize, in this section the inquiry has been extended to include expressions of the form *n percent*. Contrary to the predictions of some current theories, it has been seen that certain uses of such expressions must be analyzed as degree denoting, thus providing further evidence for the existence of proportional scales. The measurement-based theory can handle these uses via the same mechanisms applied to proportional comparatives, while assuming a single degree-denoting entry for *n percent*. The ambiguity approach instead requires some sort of duplication to be added to the system, i.e. distinct interpretations for *n percent*, or multiple loci for the computation of proportions. Perhaps the most parsimonious enhancement to this account would be to assume a null proportional *many* in proportional partitives. Below it will be shown that this does not get the facts right.

4. Cardinal readings, proportional readings and their distribution

In this section we will examine more directly the proposal that *many* and similar words are actually lexically ambiguous between cardinal and proportional interpretations, and compare this view to one in which their different readings arise instead from underspecification in measurement, in conjunction with a distinction between ordinary and domain-restricted measure functions. The question to be addressed is which of these approaches best captures the distribution of the two readings in question.

[6]For related discussion on dimensions available in partitive constructions, see Krifka 1989.

The first observation to be made is that the availability of distinct cardinal and proportional readings for words of the *many* class is not an idiosyncratic fact about English: a similar 'ambiguity' has been documented for the corresponding forms in languages including German (Kobele and Zimmermann, 2012), Dutch (Ruys, 2017), Slovenian (Stateva and Stepanov, 2017), Basque (Etxeberria, 2008), Japanese (Tanaka, 2006) and Hausa (Zimmermann, 2008) (though Russian distinguishes the two meanings lexically; see Krasikova 2011). On an account that treats this as a lexical ambiguity, one would need to claim that the same essentially accidental pairing of meanings with a single surface form has arisen in a range of typologically unrelated languages, an unlikely situation. On the measurement-based approach a more general account suggests itself: the availability of the two interpretations derives from the interaction of the lexical semantics of vague quantity words with the semantics of quantity measurement, which we might propose to universally come in domain-restricted and non-restricted varieties. To further develop this idea, it would be beneficial to investigate in more depth the facts relating to proportional measurement cross-linguistically.

Returning to English, a second important observation is that cardinal and proportional readings are not universally available; rather, their distribution is constrained by both syntactic and semantic factors. One of these relates to predicate type. In particular, Milsark (1977), Partee (1989) and others have observed that in combination with individual-level predicates (Carlson, 1977) *many* and *few* allow only the proportional reading, and not the cardinal one. As discussed above, the availability of the cardinal interpretation is most easily diagnosed in the case of *few*. Recall that (6), repeated here as (35), is felicitous, evidence that the sentence has a cardinal reading on which *few Ns* can be all of the *Ns*. But when the stage-level predicate *identified in our survey* is replaced by the individual-level *suckle their young* in (36), the same continuation is infelicitous, indicating that only the proportional reading is available.

(35) Few egg-laying mammals were found in our survey, perhaps because there <u>are</u> few.
(36) Few egg-laying mammals suckle their young, #perhaps because there <u>are</u> few.

I cannot claim to offer a fully developed account of the source of this restriction, but one can imagine how this pattern might be approached from the perspective of the two analyses discussed in Section 2. On the ambiguity view, we might say that (for whatever reason) only the proportional entries for *many/few* can select an individual-level predicate as their second argument. On the measurement-based approach, the proposal would instead be that in the presence of such a predicate, the measure function introduced by *Meas* is necessarily of the domain-restricted variety; this might be related to a proposal by Ladusaw (1994) that such predicates participate in two-stage categorical judgments (Kuroda, 1972), in the first stage of which the judge's attention is drawn to an individual to which the predicate will be associated (see Solt 2009 for discussion).

Crucially, these two approaches make different predictions regarding the readings available to proportional comparatives. Consider again our original examples, repeated below:

(1) More residents of Ithaca than New York City know their neighbors.
(2) More Norwegians than Brazilians have university degrees.

Both of these feature individual-level predicates, and as such it is not surprising that they have proportional readings. But both also clearly have (false) cardinal readings, where it is absolute numbers of Ithaca vs. NYC residents and of Norwegians vs. Brazilians that are compared. On the ambiguity analysis, augmented as suggested to account for the facts relating to predicate type, this reading cannot be derived: if *more* in quantity comparatives is analyzed as the comparative form of *many*, and only the proportional version of *many* can surface with individual level predicates, then we predict only the proportional reading for these examples.

In the measurement-based account, a different picture emerges. Suppose, as suggested above, that in the presence of an individual-level predicate the measure function is necessarily a domain-restricted one $\mu_{DIM;\oplus P}$. As was demonstrated in Section 2.2, when this is interpreted in particular as a proportional measure function (Fig. 1c), we derive the proportional reading of the comparative. On the other hand, if it is interpreted as an 'ordinary' domain-restricted function (Fig. 1b), which maps pluralities to some bounded segment of the number line, the cardinal reading of the comparative results. But in either case, the standard range introduced by the positive morpheme *POS* is necessarily situated on the bounded segment of the scale (see the diagram in (23) above). As a result, bare *many* in a corresponding positive example such as (36) must be interpreted proportionally. That is, in the case of a domain-restricted quantity measure, we derive only the proportional reading of the unmodified positive form, but both cardinal and proportional readings of the comparative.

A second context in which only the proportional reading of *many/few* surfaces is the partitive (Milsark, 1977; Partee, 1989), and here again we observe a divergence between the positive and comparative. By way of example, (37) is false in a situation in which there <u>are</u> only a small number of first-year students and they all wrote papers, evidence that *few* must be interpreted proportionally. By contrast, (38) seems to allow a proportional reading, on which it is proportions of first- and second-year students that are compared; but it also fairly clearly has a cardinal reading, which expresses that the absolute number of first-year students who wrote papers is larger (smaller) than the corresponding number of second year students who did.

(37) Few of the first year students wrote papers.
(38) More/fewer of the first year students than the second year students wrote papers.

On the measurement-based analysis, this can be accounted for in the same way as with individual level predicates. Partitive *of* is analyzed as having a domain-restricted measure function as part of its semantic content (see (32) above), which necessarily yields a proportional reading for bare *many/few* due to a constraint on the scalar position of the threshold, but yields both readings of the comparative, depending on whether or not the measure function is specifically a proportional one. And with the ambiguity analysis, we run into the same issue that we did above: if only the proportional versions of *many/few* can occur with the partitive, we predict only the proportional reading of the corresponding partitive comparatives, a prediction that is not supported by examples such as (38).

Note that the issue for the ambiguity analysis also extends to numerical partitive constructions such as *twenty percent of the students*. Recall that one way to approach these from the perspective of the ambiguity analysis would be to assume they include a null version of proportional *many* that introduces degrees of proportion to which an expression such as *twenty percent* can refer. But if we make the reasonable assumption that this null *many* obeys the same constraints as the overt one, specifically that in the partitive construction it must be interpreted proportionally, we seem to predict that only proportional measures will be possible, which is contradicted by examples such as *twenty of the students*. As discussed in Section 3, the measurement-based account can generate both absolute and proportional measures in partitives.

To summarize, we have seen two contexts in which there is a divergence in the interpretations available to positive and comparative forms: the former are necessarily interpreted proportionally, while the latter can but need not have this interpretation. This pattern is problematic for an ambiguity analysis of the form presented in Section 2.1, but not for the measurement-based approach of Section 2.2, because the latter offers a mechanism that the ambiguity approach does not, namely the possibility of quantity measurement that is relative (to a suitable totality) but not explicitly proportional.

Here a reader might object that the ambiguity analysis could be modified to address this objection, by incorporating the same semantic mechanism proposed to underlie the measurement-based account, and then replacing Romero's entries for cardinal and proportional *many* with something along the lines of the following:

(39) a. $[\![many_{CARD}]\!] = \lambda d \lambda P_{\langle et \rangle} \lambda Q_{\langle et \rangle} . \mu_{\#}(P \cap Q) \succeq d$

b. $[\![many_{PROP}]\!] = \lambda d \lambda P_{\langle et \rangle} \lambda Q_{\langle et \rangle} . \mu_{\#;\oplus P}(P \cap Q) \succeq d$

This is certainly true, but with this change, the difference between the two analyses reduces essentially to one of semantic type. On one approach (the 'ambiguity' analysis), *many* and like words are ambiguous between two quantifying determiners that differ in whether the measurement they encode is unrestricted or domain restricted (or we might even take them to have a single underspecified meaning that encompasses both possibilities). On the alternate approach (the 'measurement' analysis), an equivalent underspecification or ambiguity is localized in the semantics of a null measuring element (sometimes spelled out as *of*), while *many* words have a single simple interpretation as gradable predicates of sets of degrees. The data that would adjudicate between these two options falls outside the scope of the present paper; but as alluded to earlier, there are reasons to favor the degree-based analysis over the generalized quantifier approach, in that it allows a more general account of these items across the range of contexts in which they occur (see Solt 2015 for discussion). In any case it must be concluded that with regards to capturing the availability of distinct cardinal and proportional readings for *many* etc., the generalized quantifier approach does not have an advantage over the degree-based one.

5. Conclusion and Open Issues

The starting point for this paper was the proportional reading of quantity comparatives, which poses a challenge for a degree-based semantic framework. I have argued for an analysis based on the notion of domain-restricted and more specifically proportional measure functions, which map individuals to degrees that encode the proportion they represent of some totality they are part of. It has been demonstrated that this mechanism can be extended also to other sorts of proportional expressions, and that it offers advantages over an alternate approach based on explicitly proportional entries for gradable quantity words such as *many* and *few*. The conclusion is that proportional scales, and the corresponding measure functions, are a necessary part of the degree ontology. A secondary finding is that it is no longer necessary to analyze words of the *many* class as lexically ambiguous, a satisfying result given that the purported ambiguity is present in a range of typologically diverse languages.

At this stage there are, to be sure, some fairly significant questions left open. In that the account presented here depends crucially on underspecification in the linguistic encoding of quantity measurement, it faces the issue of overgeneration. As one aspect of this, we do not have an explanation for the observation (see e.g. Bale and Barner, 2009) that the default dimension for count noun comparatives is cardinality (number): *more students than teachers*, for example, must be a greater number of students, not, say, a group of greater weight. Interestingly, though, since a comparative such as this allows a proportional reading, we must consider proportion-of-number to be a variety of cardinality measurement. Perhaps more significantly, the restriction that quantity measurement be monotonic still allows an infinite number of specific measure functions for the representation of cardinality, or any other dimension. This proved useful in developing an analysis of proportional measures in differential position (see Section 3), but also has some less desirable consequences: for example, nothing would prevent selecting a measure function that maps pluralities to a value equal to twice their 'ordinary' cardinality, such that a group consisting of three students could be described as *six students*. I do not have a concrete proposal for how this can be avoided, though I suspect it might ultimately require a distinction to be made between counting and other forms of measurement, as proposed among others by Rothstein (2017).

Note finally that the present analysis does not yet capture all uses and interpretations of proportional expressions. In particular, I have not attempted to address non-conservative 'reverse' readings, which are available for both quantity words and percentage expressions (see Westerståhl 1985; Herburger 1997; Cohen 2001; Romero 2015 for the former and Ahn and Sauerland 2015, 2017 for the latter). For example, *Many Scandinavians have won the Nobel Prize in literature* has a reading on which it is true if Scandinavians make up a large proportion of winners, and *The firm hired 75% women* means that 75% of those hired were women (not that 75% of some group of women were hired). Recent works have analyzed these with reference to alternatives generated via focus, though in different ways: Romero proposes that such alternatives form a comparison class on the basis of which the standard for proportional *many* is derived, while Ahn & Sauerland analyze them as providing a set whose measure serves as the denominator for the calculation of (reverse) proportions. I leave it to be determined whether one of these approaches could be integrated with the account of proportional measurement developed in this paper. I have sought here to unify the analysis of the proportional reading of *many/few* and proportional measures of the form *n percent*, and it would be desirable to investigate if a unified treatment of their reverse readings is likewise possible.

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