# One many, many readings1

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**Abstract.** This paper pursues a unified analysis of the different readings that the so-called quantifier *many* gives rise to. Adopting a degree-based semantics account, *many* is decomposed into a gradable cardinality predicate and the positive operator POS. The different readings are argued to result from different scope of POS and association with focus. This improves on alternative accounts, which either employ more than one lexical entry for *many* or do not specify a compositional implementation.

Keywords: quantifier decomposition, degree semantics, positive operator

#### 1. Introduction

It is well known that the quantifier *many* gives rise to several readings. Partee (1989) distinguishes the so-called cardinal and proportional reading of *many*. Under the cardinal reading, a sentence with *many* is true iff the number of individuals that fall in the intersection of the restrictor and the nuclear scope counts as large in the given context. Sentence (1) under the cardinal reading, for instance, is true in a scenario where the number of students who took Intro to Semantics is considered large, e.g. compared to other courses or compared to previous years.

(1) Many students took Intro to Semantics.

Under the proportional reading, sentence (1) can be paraphrased as 'A high proportion of all the students took Intro to Semantics' and comes out as true if the ratio of the number of students taking Intro to Semantics to the total number of students counts as high in the given context. Under the proportional reading, the truth of a sentence does not only depend on the number of individuals in the intersection, but also on the number of individuals in the denotation of the NP. In contrast to the cardinal reading, we also learn something about the students who did not take Intro to Semantics, namely that their number is small.

Besides the cardinal and proportional reading, Westerståhl (1985) noted an additional reading of *many*. This reading is illustrated by his celebrated sentence (2), which is intuitively considered true in the scenario in (3) (describing the actual state of affairs at the time Westerståhl's paper was written), but false both under the cardinal and proportional reading – neither the absolute number 14 can be considered large nor the ratio of 14 to millions of Scandinavians.

- (2) Many Scandinavians have won the Nobel Prize in literature.
- (3) Of a total of 81 Nobel Prize winners in literature, 14 come from Scandinavia.

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Intuitively, sentence (2) is true in the scenario in (3) because 14 out of a total of 81 Nobel Prize winners in literature is considered a high proportion. In this case the relevant proportion is determined not with respect to the NP denotation, but with respect to the denotation of the VP. Since this reading can be paraphrased as 'Many of the Nobel Prize winners in literature are Scandinavians', with the arguments of *many* reversed in comparison to the sentence (2), this reading has become known as the reverse reading.

One way of coping with this three-way ambiguity would be to employ three different lexical entries for *many*, each corresponding to one of the readings discussed above. In the framework of Generalized Quantifier Theory (GQT), we would then have the following determiners:<sup>2</sup>

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(4) a. [[many_{CARD}]] = \lambda P.\lambda Q. | P \cap Q | > n, where n is a large number CARDINAL b. [[many_{PROP}]] = \lambda P.\lambda Q. | P \cap Q | : | P | > k, where k is a large fraction PROPORTIONAL c. [[many_{REV}]] = \lambda P.\lambda Q. | P \cap Q | : | Q | > k, where k is a large fraction REVERSE
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The lexical entry for  $many_{REV}$  in (4c) is particularly noteworthy, as pointed out by Westerståhl (1985). Since it makes reference to the cardinality of the VP denotation, it violates the Conservativity Universal (Barwise and Cooper, 1981; Keenan and Stavi, 1986). According to this semantic universal, determiners in natural languages are conservative, which means that the extension of the second argument of a determiner counts only insofar as it overlaps with the extension of the first argument, corresponding to the restrictor. But for reverse many in (4c), the entire extension of the second argument enters into the truth conditions.

Even leaving aside the issue of non-conservativity raised by the lexical entry for reverse *many*, there are obvious reasons to be dissatisfied with the meaning rules in (4). Employing a separate lexical entry for each reading is not very insightful and a more parsimonious way of dealing with the ambiguity that *many* gives rise to would be welcome.

The aim of the present paper is to explore a unified analysis in which the different readings are derived from a single lexical entry of *many*. Recent advancements in semantic theory give reasonable hope to such an endeavor. There is a growing body of work that decomposes the expressions GQT labels as quantifiers into smaller meaningful parts and derives the interpretations these expressions give rise to in a fully compositional way (Hackl, 2000, 2009 among many others). The analysis developed in the present paper builds in particular on Romero (2015, 2016), who shows that the reverse reading can be explained under the assumption that the so-called quantifier *many* is the positive form of a gradable determiner *many*, which is analysed in a degree-based semantic framework in analogy to gradable adjectives. While the present paper follows Romero in adopting a degree semantics account and her analysis of the positive operator, the goal of the investigation is to take the analysis a step further and derive all three readings from a single lexical entry in a compositional manner. The key idea is that *many* itself is a gradable cardinality predicate that combines with the positive operator POS,

<sup>&</sup>lt;sup>2</sup> Cardinal *many* has also been treated as a cardinality predicate rather than as a genuine quantifier (Hoeksema, 1983; Partee, 1989, among others). Motivation for this comes from the observation that cardinal *many* patterns with indefinites and other so called weak determiners in terms of distribution in existential *there*-sentences and in subject position of individual-level predicates (Milsark, 1977); see section 4.2. We will use a version of this cardinality predicate analysis that makes the parallel between *many* and gradable adjectives transparent.

and the different readings result from different scope of POS (cardinal vs. proportional readings) and (free) association with focus (regular vs. reverse readings).

The paper is organized as follows: Section 2 lays out the ingredients that go into the analysis: on the one hand *many* as gradable cardinality predicate, on the other the semantics of degree operators, in particular of the positive operator. The analysis is presented in Section 3, showing how the different readings can be derived using these ingredients. Section 4 takes a closer look at reverse readings. In Section 5 it is shown how distributional restrictions on the different readings can be explained in terms of the proposed analysis. Section 6 concludes.

## 2. Ingredients of the analysis

In the analysis I propose, what has been labeled quantifier *many* is analysed as the positive form of a gradable cardinality predicate. There are thus two ingredients that go into the analysis, each having been motivated and proposed independently. The first is the assumption that *many* is not a determiner, but rather a gradable cardinality predicate. The second is the semantics of degree operators, specifically the semantics of the positive operator.

## 2.1. Many as a gradable cardinality predicate

Since *many* shows an obvious analogy to gradable adjectives – it can be put in the comparative form *more* as well as in the superlative *most* – it has also been proposed that it should be analysed in analogy to gradable adjectives (Schwarz, 2006; Hackl, 2009, among many others). In this type of analysis, *many* is a cardinality predicate with the semantics in (5).<sup>3</sup>

(5) 
$$[[many]] = \lambda d. \lambda x. |x| \ge d$$

The lexical entry in (5) follows the usual semantics assumed for gradable adjectives (e.g., Heim, 2001) and is formulated in analogy to that of gradable adjectives like high in (6). Note in particular that this semantics for gradable adjectives is downward monotonic, in the sense that a specific individual is not only associated with a single degree, but with a set of degrees: high is a relation between an individual x and all degrees up to x's exact height, i.e. all degrees contained in the interval (0, HEIGHT(x)].

(6) 
$$[[high]] = \lambda d. \lambda x. HEIGHT(x) \ge d$$

The dimension that *many* targets is that of cardinality. In a mereological framework of plural semantics (Link, 1983), *many* relates a plural individual with the number of atomic parts it consists of, i.e. |x| is the cardinality of  $\{y: atom(y) \& y \le x\}$ .<sup>4</sup>

Under this view, many is a predicate rather than a determiner. Quantificational force, therefore, has to come from somewhere else. I adopt the common assumption that there is a

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<sup>&</sup>lt;sup>3</sup> I use a semantics in which the cardinality measure function is part of the meaning of *many*. There are, however, reasons to believe that the measure function is separate and not lexically contained in *many* (see Solt, 2009, 2015). For the purpose of the present paper, nothing hinges on this difference.

<sup>&</sup>lt;sup>4</sup> I will sometimes sloppily talk about the cardinality of a plurality.

phonetically empty determiner  $\varnothing$  with the semantics of an existential quantifier. I assume that  $\varnothing$  is present whenever *many* occurs without an overt determiner.<sup>5</sup>

Just as gradable adjectives, *many* can combine with a range of degree operators: when it combines with the comparative operator it surfaces as *more*, and as *most* when it combines with the superlative operator (Bresnan, 1973; Hackl, 2000, 2009). In the cases we are interested in, where *many* occurs without a overt degree modifier, a phonetically empty positive operator POS is assumed. Before we discuss the semantics of POS in detail, let us first look at degree operators more generally.

## 2.2. Scope of degree operators and association with focus

In order to introduce some key features of degree operators, which will be put to use in the analysis of *many*, let us first look at the semantics of superlatives and the influential analysis in Heim (1999). The analysis starts out from the observation that superlatives are ambiguous between absolute and relative readings (Szabolcsi, 1986). Consider the following sentence.

## (7) John climbed the highest mountain.

Under the absolute reading, (7) is true iff John climbed a mountain higher than any other relevant mountain. In a situation where we are talking about mountains in Scotland, for instance, (7) under the absolute reading is tantamount to saying that John climbed Ben Nevis. Under the relative reading, (7) is true iff John climbed a higher mountain than any other relevant person. It could be true even if John climbed a mountain of rather modest height, as long as no other person under consideration climbed a higher mountain. Put differently, whereas the heights of mountains are at stake under the absolute reading, the mountain climbing achievements of people are being considered under the relative reading.

In the relative reading, focus plays an important role in determining truth-conditions. This is illustrated in the following contrast:<sup>6</sup>

- (8) a. BILL got the best grade in semantics.
  - b. Bill got the best grade in SEMANTICS.

(8a), with focus on the subject, is true in a situation where out of all the students taking semantics, Bill's grade is the best. If focus is on *semantics*, in contrast, the sentence describes a situation where out of all the courses Bill took, his best grade is in semantics.

To account for the ambiguity of superlatives and the role focus plays, Heim (1999) proposes the following analysis: Superlatives are formed by a superlative operator *-est* which is semantically a quantifier over degrees. This operator is restricted by a covert variable C, which provides a comparison class and is resolved contextually. If focus is present and makes

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<sup>&</sup>lt;sup>5</sup> It has been observed that *many* can be accompanied by an overt definite article, as in *Jane regrets the many mistakes she has made*. This is in line with the view that *many* is a predicate rather than a determiner. See Solt (2009, 2015) for detailed discussion of the distribution of *many*.

<sup>&</sup>lt;sup>6</sup> I use capital letters to indicate semantic focus independently of its intonational realisation.

available alternatives, these alternatives are factored in when C is resolved and C has to be a subset of the focus alternatives. In the terms of Beaver and Clark (2008), this means that *-est* shows free association with focus. The lexical entry in (9) is proposed by Heim to factor in the contribution of focus in the usual way via the squiggle operator following Rooth (1992).

(9) 
$$\llbracket -\operatorname{est} C \rrbracket = \lambda D_{\leq d,t}$$
.  $\exists d [D(d) \& \forall D' \in C [D' \neq D \to \neg D'(d)]$ 

Note that in this lexical entry for *-est*, the comparison class C is a set of degree properties (type <<d,t,>t>). Being a quantifier, *-est* undergoes QR and can take local scope, i.e. internal to the host DP, or raise to a DP-external position to take non-local scope. Crucially, it is assumed that the material in the scope of *-est* determines the relation relative to which individuals are compared, which means that the members of C must have the same form as the LF sister of *-est*. With these assumptions, the two readings of superlatives discussed above can be derived as resulting from different scope of *-est*.

The absolute reading obtains from an LF where *-est* takes local scope inside its host DP, as shown in (10).<sup>8</sup>

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(10) a. LF: John climbed [DP the [2 [ [-est C] [1 [t_2 t_l-high mountain]]]]] b. climb(j, t_i [ \exists d [mountain(t_i) & HEIGHT (t_i) \geq d & \forall D' \in C [ D' \neq [\lambda d'. x is a d'-high mountain] \rightarrow \neg D'(d)]]]) c. C = \{ \lambda d'. Ben Nevis is a d'-high mountain, \lambda d'. Ben Macdui is a d'-high mountain, \lambda d'. Braeriach is a d'-high mountain, ...}
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In this configuration, the heights of contextually relevant mountains are compared. This is so because in the LF, the sister of -est is [1 [ $t_2t_1$ -high mountain]]. In consequence, the members of the comparison class C are all of the form ' $\lambda d$ '. x is a d'-high mountain'. Following Szabolcsi (1986), it is assumed that x ranges over contextually relevant individuals to which the property of being a d'-high mountain can be felicitously applied, i.e. mountains. Under the downward monotonic semantics for high in (6), the members of C are sets of degrees consisting of all the degrees up to the respective mountain's height, e.g. C in (10c) contains the intervals (0, HEIGHT(Ben Nevis)], (0, HEIGHT(Ben Macdui)], etc. The denotation of the entire superlative DP is then the unique individual that figures in the member of C with the greatest maximum, i.e. the mountain whose height is the greatest.

Relative readings obtain from an LF where *-est* takes non-local scope and its LF sister does not only comprise the DP but crucially also the focused constituent. Now we also have to factor in the contribution of focus. In the relative reading of example (7) focus is on the subject, as indicated in (11).<sup>9</sup>

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<sup>&</sup>lt;sup>7</sup> In fact, C has to be a set of degree property intensions, i.e. of type <<<s,<d,t>>,t>. This is necessary in order to distinguish different degree properties that happen to have the same extensions. For now, I use the extensional version for the sake of readability, but I will return to this issue in footnote 11.

<sup>&</sup>lt;sup>8</sup> This derivation assumes that landing sites for QR can be created inside DPs by PRO-movement (see Heim and Kratzer, 1998, chap. 8).

<sup>&</sup>lt;sup>9</sup> Following Szabolcsi (1986), I assume that the definite article is interpreted as an indefinite in relative readings of superlatives.

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a. LF: [ [-est C] [1 [JOHN climbed A t<sub>I</sub>-high mountain]] ~C]
b. ∃d [∃x[climb(j,x) & mountain(x) & HEIGHT (x) ≥ d &
∀D'∈ C [ D'≠ [ λd'. John climbed a d'-high mountain] → ¬D'(d)]]]
c. C ⊆ {λd'. John climbed a d'-high mountain, λd'. Bill climbed a d'-high mountain, λd'. Sam climbed a d'-high mountain, ...}
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As before, the form of the members of C is determined by the material inside the LF sister of -est. In addition, the squiggle operator adds the constraint that C has to be a subset of the focus alternatives, which are of the form ' $\lambda d$ '. x climbed a d'-high mountain', with x ranging over contextually relevant individuals. This results in a comparison class C comparing relevant persons in terms of the height of the mountains they climbed. Since the truth conditions in (11b) are fulfilled iff the maximum of the degree set given by ' $\lambda d$ '. John climbed a d'-high mountain' is higher than the maximum of any other degree set in C, the sentence comes out as true iff John climbed a higher mountain than any other relevant person.

In summary, the ambiguity of superlatives between absolute and relative readings can be analysed as a scope ambiguity of the superlative operator *-est*. In relative readings, focus affects truth conditions.

## 2.3. The positive operator POS

Building on Schwarz (2010), Romero (2015) argues that the properties discussed in the preceding subsection for the superlative operator also hold of the phonetically empty positive operator Pos. Motivation for this comes from the observation that adjectives in the positive give rise to an ambiguity between absolute and relative readings, too. This is illustrated in the following example discussed by Schwarz (2010).

#### (12) Mia has an expensive hat.

Under the absolute reading, (12) can be paraphrased as 'Mia has a hat that is expensive for a hat'. Here the price of Mia's hat is compared to the price of hats in general. Under the relative reading, which can be paraphrased as 'Mia has a hat that is expensive for somebody like Mia to have', Mia is compared to other persons in terms of the expensiveness of their hats. Romero (2015) proposes that this ambiguity can be derived from DP-internal vs. DP-external scope of POS in analogy to absolute and relative readings of superlatives. She moreover argues that the positive form of adjectives shows the same kind of focus sensitivity as the superlative. Therefore, crucial features of the analysis of *-est* carry over to the analysis of POS. These are summarized in (13).

## (13) The positive operator POS:

- (i) The positive operator POS is restricted by a covert variable C providing a comparison class, which is resolved contextually.
- (ii) The effects of focus are factored in when C is resolved (free association with focus in Beaver and Clark's 2008 terms).
- (iii) POS can take local (DP-internal) or non-local scope (DP-external).
- (iv) The material in the scope of POS determines the form of the members of C.

To implement this analysis, I use the following lexical entry for POS, which is parallel to that of *-est* in (9).

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(14) [POS C] = \lambda D_{\langle d,t \rangle}. \exists d [D(d) \& d > \theta_C]
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POS requires the degree property D serving as its argument to hold to a degree exceeding a contextual standard. The standard is computed by the function  $\theta$ , which maps a comparison class C to a degree, taking into account the distribution of values in C (see Solt, 2011a). As for superlatives, I assume that the comparison class C is a set of degree properties.

With these assumptions, the two readings of sentence (12) above can be derived in the following way. The absolute reading is derived from an LF where POS takes DP-internal scope, as shown in (15).

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(15) a. LF: Mia has [DP \ a \ [2 \ [POS \ C] \ [1 \ [t_2 \ t_l] - expensive hat ]]]] b. \exists x \ [have(m,x) \& \exists d \ [hat(x) \& Expensiveness(x) \ge d \& d > \theta_C ]] c. C = \{ \lambda d' \ h_1 \ is \ a \ d' - expensive hat, \lambda d' \ h_2 \ is \ a \ d' - expensive hat, ... \}
```

In this configuration, the LF-sister of POS is  $[1 \ [t_2 \ t_l]$ -expensive hat ]] and consequently, the members of C are of the form ' $\lambda d$ '. x is a d'-expensive hat', with x ranging over hats. The contextual standard that the semantics of POS refers to is determined relative to this comparison class, i.e. relative to the price of hats in general.

In the absolute reading of (12), the price of Mia's hat is compared to the price of hats owned by persons comparable to her, e.g., 3-year-old girls. This reading is derived from an LF where POS takes DP-external scope and focus is on the subject, as shown in (16). In analogy to the derivation of the relative reading of superlatives discussed above, C is a subset of the focus alternatives, which are of the form ' $\lambda d$ '. x has a d'-expensive hat', with x ranging over contextually relevant individuals.

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(16) a LF: [ [POS C] [1 [MIA has a t_l-expensive hat ]] \sim C]
b. \exists d \ [\exists x \ [ have(m,x) \& hat(x) \& Expensiveness(x) \ge d] \& d > \theta_C ]
c. C \subseteq \{ \lambda d'. Mia has a d'-expensive hat, \lambda d'. Emma has a d'-expensive hat, \lambda d'. Hannah has a d'-expensive hat, ...}
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Romero (2015) further observes that in relative readings, the focus associate of POS can not only be external to the host DP, as in (16), but that it can also be internal to the host DP. She discusses example (17), which has a reading under which it is true in the scenario in (18).

- (17) (For what he has been giving her, now) Rockefeller gave Kate an inexpensive CAR.
- (18) Scenario: Rockefeller just gave Kate a very expensive car. Still, his present compares poorly to his previous astronomically expensive presents (e.g. apartment in Manhattan, island in the Pacific, etc.)

The relevant reading of (17) can be paraphrased as 'Rockefeller gave Kate a car and this present is inexpensive compared to his other presents to her'. The comparison class is thus

made up of sets of degrees to which other presents that Rockefeller gave Kate are inexpensive. We get this comparison class from an LF where POS takes DP-external scope while the focus associate is *car* inside the host DP, as shown in (19).

```
(19) a. LF: [ [POS C] [1 [Rockefeller gave Kate a t<sub>I</sub>-inexpensive CAR ]] ~ C]
b. ∃d [∃x [ give(r,k,x) & car(x) & Expensiveness(x) ≤ d ] & d < θ<sub>C</sub> ]
c. C ⊆ {λd'. Rockefeller gave Kate a d'-inexpensive car,
λd'. Rockefeller gave Kate a d'-inexpensive apartment in Manhattan,
λd'. Rockefeller gave Kate a d'-inexpensive island in the Pacific, ...}
```

Let us now apply this analysis of the positive operator to the cases of quantifier *many* under the assumption that we are dealing with the positive form of a gradable cardinality predicate.

## 3. Analysis

This section proposes an analysis of the quantifier *many*, where it is decomposed into three different components: *many* itself is treated as a gradable cardinality predicate, which combines with the phonetically empty positive operator. Quantificational force comes from a covert existential quantifier. It is shown that the different readings of *many* discussed in Section 1 can be derived under the assumptions about the semantics of *many* and the positive operator POS introduced in the previous section. The different readings are argued to result from different scope of POS and association with focus.

## 3.1 Cardinal reading

Cardinal readings are in a way the default readings that arise under the present analysis. With the assumptions introduced in the preceding section, cardinal readings result from an LF where POS takes DP-external scope. This is illustrated in (21) for sentence (20).

- (20) Many students took Intro to Semantics.
- (21) a. LF: [POS C] [1 [ [DP  $\emptyset$  [ $t_1$ -many students]] took Intro to Semantics ]] b.  $\exists d$  [  $\exists x$  [students(x) &  $|x| \ge d$  & take(x, Intro-to-Semantics)] &  $d > \theta_C$ ] c.  $C = \{ \lambda d'. d'$ -many students took Intro to Semantics,  $\lambda d'. d'$ -many students took Intro to Syntax,  $\lambda d'. d'$ -many students took Intro to Phonology, ...}

In the in LF (21a) [POS C] is extracted out of the host DP and takes sentential scope. The value of the comparison class C is contextually resolved, possibly restricted by focus. Assuming that the number of students taking Intro to Semantics is assessed with respect to the number of students in other comparable courses, C might look as shown in (21c). According to the truth conditions in (21b), the sentence then comes out as true iff the number of students taking Intro to Semantics exceeds the number that would be expected on the basis of other comparable courses. This is similar to the meaning one would get using the determiner  $many_{CARD}$  in (4), but differs from the analysis in GQT-style in one important respect: we are now more explicit regarding the standard relative to which the number of students is

considered large. It is computed on the basis of a comparison class, which is (partly) determined by the compositional semantics.

## 3.2 Proportional reading

We just saw that DP-external scope of POS yields cardinal readings of *many*. I now show that the proportional reading of *many* can be obtained from an LF where POS takes DP-internal scope.<sup>10</sup> Following the analysis of the proportional reading of *most* in Hackl (2009), I assume that in this configuration the comparison class C consists of the cardinalities of the pluralities denoted by the NP sister of *many*. In example (22), for instance, pluralities consisting of books on the reading list are compared in terms of how many atomic parts they have.

(22) John read many books on the reading list.

```
a. LF: John read [Ø [2 [POS C] [1 [t₂ t₁-many books on the reading list]]]]
b. ∃x [read(j,x) & ∃d [|x| ≥ d & books-otrl (x) & d > θ<sub>C</sub>]]
c. C = {λd'. x are d'-many books on the reading list: x is a plurality consisting of books on the reading list}
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In the LF (23a), the sister of [POS C] is [1 [ $t_2$   $t_1$ -many books on the reading list]]. Thus, the members of C all have the form ' $\lambda d$ '. x are d'-many books on the reading list'. In analogy to absolute readings of superlatives, it is assumed that x ranges over individuals to which the property of being d'-many books on the reading list can be felicitously applied. Following Hackl (2009), I assume that C consists of the cardinalities of all the pluralities in the denotation of 'books on the reading list'. In order to see what it means that a certain degree exceeds a standard computed relative to this comparison class, let us inspect the values in C and their distribution more closely. Assuming for illustration, that there are eight books on the reading list,  $b_1, b_2, \ldots, b_8$ , the pluralities in the denotation of 'books on the reading list' and their cardinalities are shown in (24). Consequently, C in this case looks as in (25).

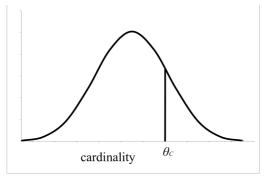
$(24)$ $b_1, b_2,, b_8,$	8 of cardinality 1
$b_1 \oplus b_2$ , $b_1 \oplus b_3$ , $b_1 \oplus b_4$ ,, $b_7 \oplus b_8$ ,	28 of cardinality 2
$b_1 \oplus b_2 \oplus b_3, \ldots, b_6 \oplus b_7 \oplus b_8,$	56 of cardinality 3
$b_1 \oplus b_2 \oplus b_3 \oplus b_4, \ldots, b_5 \oplus b_6 \oplus b_7 \oplus b_8,$	70 of cardinality 4
$b_1 \oplus b_2 \oplus b_3 \oplus b_4 \oplus b_5, \ \dots, \ b_4 \oplus b_5 \oplus b_6 \oplus b_7 \oplus b_8,$	56 of cardinality 5
$b_1 \oplus b_2 \oplus b_3 \oplus b_4 \oplus b_5 \oplus b_6, \ \dots, \ b_3 \oplus b_4 \oplus b_5 \oplus b_6 \oplus b_7 \oplus b_8,$	28 of cardinality 6
$b_1 \oplus b_2 \oplus b_3 \oplus b_4 \oplus b_5 \oplus b_6 \oplus b_7, \dots, b_2 \oplus b_3 \oplus b_4 \oplus b_5 \oplus b_6 \oplus b_7 \oplus b_8,$	8 of cardinality 7
$b_1 \oplus b_2 \oplus b_3 \oplus b_4 \oplus b_5 \oplus b_6 \oplus b_7 \oplus b_8$	1 of cardinality 8

(25)  $C = \{ \lambda d'. b_1 \text{ are } d'\text{-many books on the reading list,} \\ \lambda d'. b_2 \text{ are } d'\text{-many books on the reading list,} ..., \\ \lambda d'. b_1 \oplus b_2 \oplus b_3 \oplus b_4 \oplus b_5 \oplus b_6 \oplus b_7 \oplus b_8 \text{ are } d'\text{-many books on the reading list} \}$ 

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<sup>&</sup>lt;sup>10</sup> See Krasikova (2011) and Solt (2011b) for similar proposals.

Note that the distribution of values in C shows a particular pattern, as indicated in (24): Because there are 8 pluralities of cardinality 1, the set of degrees with maximum 1 is represented 8 times; there being 28 pluralities of cardinality 2, the set of degrees with maximum 2 is represented 28 times, and so on. In general, if there are n atomic individuals of a certain kind, the number of pluralities with cardinality k is given by the binomial coefficient  $\binom{n}{k}$ . The distribution of values in a comparison class representing the cardinalities of the pluralities in the NP denotation corresponds to a bell curve, as illustrated in Figure 1.



**Figure 1**: Distribution of values in a comparison class representing the cardinalities of the pluralities in the NP denotation

Given this distribution of values in C, the standard for 'many'ness relative to C,  $\theta_C$ , is likely to be located somewhere in the rightmost third of the area enclosed by the curve, as indicated in Figure 1. In effect, the truth conditions in (23b) express that John read a plurality of books whose cardinality is high relative to the cardinalities of the members of the power set of the set of books on the reading list. This mirrors the meaning assigned to proportional *many* by the analysis in the framework of Generalized Quantifier Theory, but crucially without making reference to actual proportions.

#### 3.2 Reverse reading

For the derivation of the reverse reading, I follow Romero (2015, 2016) and assume that it is the role that focus plays in determining the comparison class that gives the impression of a

Using intensional degree properties is crucial to ensure that the degree properties corresponding to different pluralities with the same number of atomic elements are not equivalent. Note that e.g. (i-a) and (i-b) are different degree properties. Although the pluralities do not differ across worlds regarding the number of atomic elements, the degree properties (ii-a) and (ii-b) differ with respect to the worlds at which they are defined and are thus not equivalent (e.g., (ii-a) would be undefined at world w<sub>29</sub> if the individual b<sub>1</sub> does not exist in w<sub>29</sub>).

(ii) a.  $\lambda w. \lambda d'$ .  $b_1 \oplus b_2$  are d'-many books on the reading list in w b.  $\lambda w. \lambda d'$ .  $b_3 \oplus b_4$  are d'-many books on the reading list in w

 $<sup>^{11}</sup>$  In this case, it is crucial that the comparison class consist of intensional degree properties. C in (25) should more accurately be rendered as in (i):

<sup>(</sup>i)  $C = \{ \lambda w. \lambda d'. b_1 \text{ are } d'\text{-many books on the reading list in } w, \lambda w. \lambda d'. b_2 \text{ are } d'\text{-many books on the reading list in } w, \dots, \lambda w. \lambda d'. b_1 \oplus b_2 \oplus b_3 \oplus b_4 \oplus b_5 \oplus b_6 \oplus b_7 \oplus b_8 \text{ are } d'\text{-many books on the reading list in } w \}$ 

'reverse' reading. But in contrast to Romero, under the present analysis the reverse reading is derived as a special case of a cardinal reading.

As noted by Herburger (1997), the reverse reading is facilitated by placing focus on (part of) the NP sister of *many*. In Westerståhl's (1985) famous example focus is on 'Scandinavians', as indicated in (26).

(26) Many SCANDINAVIANS have won the Nobel Prize in literature.

Because focus alternatives are factored in when the comparison class C is computed, focusing *Scandinavians* has an important effect on the truth-conditions. Consider the LF (27) that we get for sentence (26) under the assumption that POS takes DP-external scope.

```
(27) a. LF: [[POS C] [1 [\varnothing [ t_1-many SCANDINAVIANS ]] have won the NP] \sim C] b. \exists d [ \exists x[Scandinavians(x) & |x| \ge d & won(x, NP)] & d > \theta_C] c. C \subseteq \{ \lambda d'. d'-many Scandinavians have won the NP, \lambda d'. d'-many Mediterraneans have won the NP, \lambda d'. d'-many Eastern Europeans have won the NP, ... }
```

This configuration is parallel to the adjectival case in (19) discussed in Section 2.3 above. Because [POS C] takes sentential scope and the comparison class C is a subset of the focus alternatives triggered by focus on *Scandinavians*, we get the comparison class C in (27c). With this comparison class, different world regions are compared in terms of the number of Nobel Prize winners they have produced. The truth conditions in (27b) are fulfilled in case the number of Nobel Prize winners from Scandinavia is large compared to the number of Nobel Prize winners from other parts of the world. This is the case if the Scandinavians make up a significant proportion of all the Nobel Prize winners, as is the case in the scenario in (3), discussed in Section 1, where 14 out of a total of 81 Nobel Prize winners in literature come from Scandinavia (although this is in fact hard to say unless we know where the other 67 winners come from).

Before closing this section, I would like to add a remark regarding conservativity. Westerståhl's (1985) example triggered a lot of discussion in the semantics literature, because it has been regarded as a counter-example to the generalization that determiners in natural languages are always conservative (Barwise and Cooper, 1981; Keenan and Stavi, 1986). Under the approach taken here *many* is not a determiner, so the question whether *many* obeys conservativity does not even arise.

To summarize, this section showed that the different readings can be derived under a single lexical entry of *many* as a cardinality predicate that combines with the positive operator POS. The proportional reading is derived from an LF where POS takes DP-internal scope. When POS takes DP-external scope, cardinal readings result. Reverse readings are a special case of cardinal readings that arise if (part of) the NP-sister of *many* is focused.

## 4. A closer look at reverse readings

Above, the reverse reading was argued to arise as a special case of a cardinal reading, where the NP sister of *many* is focused. We now compare the truth conditions derived for the reverse reading in (27) above to the ones derived from a determiner *many*<sub>REV</sub> in GQT-style, and to the truth conditions derived under the analysis of Romero (2015, 2016).

Let us first compare the truth conditions derived under the present analysis to the truth conditions in (28b) obtained from a GQT-style analysis with the determiner  $many_{REV}$  with the lexical entry in (28a).

```
(28) a. [[many_{REV}]] = \lambda P.\lambda Q. |P \cap Q| : |Q| > k, where k is a large fraction b. |\{x: Scandinavian(x)\} \cap \{x: NP-winner(x)\} | : |\{x: NP-winner(x)\}| > k
```

Comparing (28b) and the truth conditions derived under the present analysis in (27b) above, a difference can be noted concerning the question whether it makes a difference where the non-Scandinavian Nobel Prize winners come from. In (27b) the number of Nobel Prize winners per world region enters into the computation of the standard relative to which a number counts as 'many'. The nationality of the non-Scandinavian Nobel Prize winners can therefore make a difference to the truth conditions. For (28b), in contrast, it does not seem to matter where the Nobel Prize winners who are not Scandinavians come from, since it only relates the number of Scandinavian Nobel Prize winners to the total number of Nobel Prize winners.

But we have to keep in mind a further difference between the present analysis and the one in GTQ-style concerning the question how the standard for 'many'-ness is determined. While the present analysis holds that the comparison class, which is partly determined by the grammar, plays a crucial role in computing this standard, the GTQ-style analysis is vague and just assumes that the threshold value k, above which a fraction counts as large, is determined by the context. Depending on the assumptions we make about what contextually given facts are taken into account in determining the standard, the conditions under which (28b) is true get more similar to the truth conditions in (27b). If we assume that the decision what counts as a large proportion of a set S is also influenced by the composition of S, then the nationality of the non-Scandinavian Nobel Prize winners becomes again relevant: if we are aware that Scandinavia has produced more Nobel Prize winners than most other parts of the world, the proportion of Scandinavian Nobel Prize winners to the total number of Nobel Prize winners is probably considered large. To conclude, it is not straightforward to compare the truth conditions derived under the two types of analyses, as long as the GTQ-style analysis is vague about how the contextual standard is determined.

We now turn to the analysis of Romero (2015, 2016). Following Hackl (2000), Romero analyses *many* as a gradable determiner, i.e. a determiner with an extra degree argument. But in contrast to Hackl, Romero employs two different lexical entries for *many*, MANY<sub>CARD</sub> for cardinal readings and MANY<sub>PROP</sub> for proportional readings, as shown in (29).

(29) a. 
$$[[MANY_{CARD}]] = \lambda d_d \cdot \lambda P_{\langle e,t \rangle} \cdot \lambda Q_{\langle e,t \rangle}$$
.  $|P \cap Q| \ge d$ , where  $d$  ranges over natural numbers b.  $[[MANY_{PROP}]] = \lambda d_d \cdot \lambda P_{\langle e,t \rangle} \cdot \lambda Q_{\langle e,t \rangle}$ .  $(|P \cap Q| : |P|) \ge d$ , where  $d$  ranges over fractions between 0 and 1 or percentages

For the derivation of the reverse reading, Romero uses the proportional determiner Many<sub>PROP</sub>. Leaving intact the rest of the analysis from above, which was in fact borrowed from Romero, the reverse reading results from an LF where POS takes DP-external scope and the NP sister of *many* is focused. Combining this with the proportional determiner Many<sub>PROP</sub> yields the truth conditions in (30b) where the standard is determined relative to a comparison class C that looks like (30c).

```
(30) a. LF: [[POS C] [1 [[t_l-MANYprop SCANDINAVIANS ] have won the NP]] \sim C ] b. \exists d [ | {x: Scandinavian(x)} \cap {x: NP-winner(x)} | : | {x: Scandinavian(x)} | \geq d & d > \theta_C ] c. C \subseteq \{ \lambda d'.(|\{x: Scand.(x)\} \cap \{x: NP-winner(x)\}| : |\{x: Scandinavian(x)\}|) \geq d', \lambda d'.(|\{x: Mediterranean(<math>x)} \cap {x: NP-winner(x)} | : |{x: Mediterranean(x)} \cap {x: NP-winner(x)} | : |{x: Eastern Europ.(x)} | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x |
```

With this comparison class, different world regions are compared with respect to the number of Nobel Prize winners in relation to the total number of inhabitants of that region. That yields a crucial difference to the truth conditions derived under the present proposal in (27) above: in (30) not only the number of Nobel Prize winners from a certain region is taken into account, but also the size of the overall population. In effect, the smaller the total population of a region is, the fewer Nobel Prize winners are sufficient to make the corresponding sentence true. Cohen (2001) and Romero (2015) argue that this move is necessary to adequately capture the truth conditions of the sentence. Contrasting the truth conditions of the minimally different sentences in (31), Cohen (2001: 48) remarks: "It may be sufficient for as few as two or three Andorrans to have won the Nobel Prize in literature for [(31b)] to be true; but such a small number would not be enough for the truth of [(31a)]."

- (31) a. Many SCANDINAVIANS have won the Nobel Prize in literature.
  - b. Many ANDORRANS have won the Nobel Prize in literature.

Different intuitions, however, have been reported regarding the precise conditions under which Westerståhl's Nobel Prize winner-sentence is true, and in particular regarding the question whether the total number of individuals in the NP denotation is taken into account. According to Eckardt (1999:175), it does not matter: "The sentence is even true if the nation in question is the biggest on earth, and the ratio of winners per nation is even rather bad."

Given these diverging intuitions about the truth conditions of Westerståhl's example, without an in depth empirical investigation it is impossible to decide whether an adequate semantics for the reverse reading of a sentence of the form  $many\ A$  are B does indeed have to make reference to the proportion  $|A\cap B|:|A|$ . But I would like to point out a possibility how to accommodate the impression that the total number of individuals in the NP denotation matters to the truth conditions of Westerståhl's example, if it turns out to be correct. So far, we have made only one specific assumption about the function  $\theta$  determining the contextual standard to which POS makes reference, namely that it does so on the basis of a comparison class C. And one position would indeed be that the context dependency of the truth conditions of sentences with positive forms of adjectives all boils down to the comparison class C: once C is fixed, the standard is also fixed. Under this view,  $\theta_C$  always yields the same degree for all contexts (for discussion of a contextually-stable threshold function  $\theta$  used in the semantics

of many see Fernando and Kamp, 1996; Schöller and Franke, 2015). But an alternative view could maintain that  $\theta_C$  is itself context-dependent, i.e. what value  $\theta$  yields for the same C might differ from context to context. If we assume such a contextually-variable function  $\theta_C$  considerations about how many Nobel Prize winners one would expect to come from a certain region could enter into the computation of the standard for 'many'ness, such that the standard would be higher for populous parts of the world than for parts with smaller population.

In any case, there are examples that clearly have a reverse reading where the total number of individuals in the NP denotation does not matter. A case in point is sentence (32a), which can be paraphrased as 'A high proportion of the things Peter got for his birthday are books'. Corresponding truth conditions are derived from an LF where POS takes DP-external scope and the comparison class C is restricted by focus on the NP sister of many, as in (27) above. In general, attributing reverse readings to focus effects can account for types of reverse readings that the GQT-style analysis cannot accommodate. This is the case in (32b), where focus is inside the temporal modifier. The sentence has a reading under which a sensible person could still board an airplane, namely that most airplanes that crash do so when landing. This reading can be derived with the tools the present analysis makes available, namely by letting the focus inside the temporal modifier restrict the comparison class C, which will then compare the number of crashes in different stages of a flight.

- (32) a. Tom got many BOOKS for his birthday.
  - b. Many airplanes crash when LANDING.

A fact supporting the view that the reverse reading arises as a cardinal reading rather than a proportional reading comes from the observation that the reverse reading arises in the same grammatical contexts as cardinal readings, and is impossible in contexts where only proportional readings are available. The grammatical constraints governing the distribution of the different readings is the topic of the next section.

#### 5. Distributional restrictions of the different readings

It is well known that the cardinal and proportional reading of *many* are restricted to certain grammatical contexts, as are the interpretations of other so-called weak quantifiers, like cardinal numerals and *some* (Milsark, 1977; Partee, 1989 among many others). When a *many*-phrase serves as the subject of an individual-level predicate, as in (33a), only the proportional reading is available. For *many*-phrases as subject of stage-level predicates and objects, in contrast, both the cardinal and the proportional reading is possible, cf. (33b) and (33c). *Many*-phrases in *there* sentences, finally, receive only the cardinal reading, cf. (33d).

```
(33) a. Many Huskies have blue eyes.

b. Many students took Intro to Semantics.

c. Carl invited many semanticists.

d. There are many children in the garden.

#cardinal reading, √proportional reading

√cardinal reading, √proportional reading

√cardinal reading, √proportional reading
```

Under the present analysis of *many*, a possible way to handle the distribution of cardinal and proportional readings in the spirit of Diesing (1992) would be the following. Let us assume,

following Diesing, that genuine quantifiers are obligatorily interpreted at the level of IP, while a default existential closure operator binds any unbound (individual) variables at the level of vP. At LF, subject of stage-level predicates and objects can either occupy their base generated position inside vP, or, if they are genuine quantifiers, move to the level of IP. Subjects of individual-level predicates are base generated at the level of IP, whereas subjects of there-sentences are obligatorily interpreted vP-internally. Combining these assumptions with the present analysis, the quantificational force of many-phrases can come from different sources, depending on their structural position. Keeping the assumption from above that there is a phonetically empty existential determiner  $\emptyset$ , many-phrases that are interpreted at the level of IP involve  $\emptyset$ . In contrast, many-phrases that occupy a vP-internal position at LF get their existential force from the default existential closure operator.

We can account for the observed restrictions on the readings of *many* if we make a further assumption about the possible interpretation sites of the positive operator POS. Assume that the positive operator POS cannot move out of the restrictor of the genuine quantifier  $\emptyset$ , while being a degree quantifier, it has to be interpreted at the IP-level and thus raise above the existential closure operator if it originates from a vP-internal *many*-phrase. With this assumption, there is a one-to-one correspondence between the reading and the source of the quantificational force of a *many*-phrase: since under the present analysis the proportional reading corresponds to DP-internal scope of POS, it results from a *many*-phrase involving  $\emptyset$ . The cardinal reading, corresponding to DP-external scope of POS, results from a vP-internal *many*-phrase that is subject to existential closure.

The pattern shown in (33) above now follows. Since *many*-phrases as subjects of individual-level predicates occupy a position at the IP-level, they obligatorily involve  $\emptyset$ .<sup>13</sup> This in turn means that POS can only be interpreted DP-internally, resulting in the proportional reading. *Many*-phrases as subjects of stage-level predicates and objects, in contrast, can occupy different positions at LF. If they involve  $\emptyset$ , they are interpreted at the IP-level, resulting in the proportional reading. Additionally they can also be interpreted vP-internally, in which

#### (i) Many people don't fit into this room.

degree operators.

The comparison with readings available for bare plurals raises the question why the phonetically empty existential determiner  $\emptyset$  does not seem to be available for bare plurals. Speculating on possible reasons goes beyond the scope of the present paper.

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<sup>&</sup>lt;sup>12</sup> This claim should not be restricted to POS, but should rather hold of degree operators in general. While it is difficult to judge whether this restriction also holds for the comparative operator *-er*, because scope of *-er* relative to ∃ does not make a truth-conditional difference, there is evidence that it applies to *-est*. First, it is well known that proportional *most* is a so-called strong determiner, i.e. *most*-phrases cannot occur in *there*-sentences. Second, only the proportional reading seems to be available for *most*-phrases as subjects of individual-level predicates. The different readings of *most* thus seem to show the same distributional restrictions as the readings of *many*. Under the analysis of Hackl (2009), where the proportional reading of *most* results from an LF where the superlative operator *-est* takes DP-internal scope, the restrictions of *most* would also follow from the present assumptions about the correspondence between the source of quantificational force and the scope options of

<sup>&</sup>lt;sup>13</sup> More precisely, *many*-phrases interpreted at the IP-level are assumed to involve  $\emptyset$  if they receive an existential interpretation. They can also be interpreted generically, as is well known from bare plurals. The generic cardinal reading (de Hoop, 1992) is illustrated in sentence (i), which expresses a statement about all groups of people reaching a considerable number.

case POS raises above the existential closure operator, giving rise to the cardinal reading. For *many*-phrases in *there*-sentences, only a vP-internal position is possible, making only the cardinal reading available. The structures assumed to underlie the proportional and the cardinal reading are schematized in (33).

(34) a. 
$$[IP [DP \varnothing [[POS C] [1 ... t_l-many ...]]] ...]$$
 proportional reading b.  $[IP ... [[POS C] [1 ... \exists_x [_{vP} ... t_l-many x ...]]]]$  cardinal reading

Let us now turn to the reverse reading. Herburger (1997) observes that the reverse reading is restricted to the same grammatical contexts as the cardinal reading. As illustrated in the following paradigm, the reverse reading cannot arise with individual-level predicates, whereas it is possible with stage-level predicates as well as for objects, and is perfectly available in *there*-sentences.

- (35) a. Many COOKS know how to make a soufflé.
  - "'A high proportion of the people that know how to make a soufflé are cooks."
  - b. Many COOKS applied.
    - 'A high proportion of the applicants are cooks.'
  - c. Carl interviewed many COOKS.
    - 'A high proportion of the people Carl interviewed are cooks.'
  - d. There are many speakers of Basque THAT ARE CITIZENS OF SPAIN.
    - 'A high proportion of Basque speakers are citizens of Spain.'

Under the present analysis, this distribution of the reverse reading is entirely expected, since the reverse reading is derived as a special instance of the cardinal reading, i.e. POS taking DP-internal scope, in which (part of) the NP-sister of *many* is focused. Under the assumption from above that the cardinal reading arises form the configuration in (33b), a *many*-phrase is predicted to give rise to the reverse reading only if it is interpreted in a vP-internal position, which is subject to existential closure. Under the analysis of Diesing (1992), a vP-internal interpretation is obligatory for subjects of *there*-sentences, possible for objects and subjects of stage-level predicates, but impossible for subjects of individual-level predicates. This accounts for the distribution of the reverse reading.

#### 5. Conclusion

This paper explored a uniform analysis of the so-called quantifier *many*, where *many* is decomposed into a gradable cardinality predicate and the positive morpheme POS. It was shown that the various readings that have been observed for *many* can be derived under this analysis. The different readings were argued to arise from different scope of POS and free association with focus. The proportional reading is generated from an LF where POS takes DP-internal scope. If POS takes non-local scope, cardinal readings are derived, with reverse readings being a special case arising from association with focus. This improves on existing accounts, which either employ more than one lexical entry for *many* (Romero 2015, 2016) or do not specify a compositional implementation (Solt 2009).

Under the present analysis, *many* always operates on a cardinality scale, which means that the degrees *many* takes as arguments are invariably natural numbers. This contrasts with the view

that proportional *many* operates on a scale of proportion, i.e. that the degrees it takes as arguments are rational numbers between 0 and 1, or alternatively percentages (Solt, 2018). More work is needed to investigate whether the kind of data that seem to require a scale of proportion can be accommodated under the present analysis.

The analysis presented in this paper contributes to the enterprise of quantifier decomposition. In the spirit of Hackl (2000, 2009), it is shown that ambiguities that are puzzling from a GQT perspective follow if expressions that appear to be quantifying determiners are decomposed and analysed with the tools made available in a degree-semantic framework.

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