Common nouns as variables: Evidence from conservativity and the temperature paradox[1]

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Abstract. Common nouns and noun phrases have usually been analyzed semantically as predicates. In quantified sentences, these predicates take variables as arguments. This paper develops and defends an analysis in which common nouns and noun phrases themselves are treated as variables, rather than as predicates taking variables as arguments. Several apparent challenges for this view will be addressed, including the modal non-rigidity of common nouns. Two major advantages to treating common nouns as variables will be presented: Such an analysis predicts that all nominal quantification is conservative, rather than requiring conservativity to be stipulated as a constraint on determiner denotations; and it makes possible some improvements to the analysis of the temperature paradox, allowing for quantificational examples without adding a spurious layer of modal variability.

Keywords: common nouns, variables, rigidity, conservativity, temperature paradox

1. Introduction

Common nouns are frequently analyzed semantically as predicates. Beginning logic students are trained to represent sentences like (1)a. using formulas like (1)b., in which the 1-place predicate M seems to correspond directly to the common noun *man*, just as the 1-place predicate S corresponds to the verb *smiles*:

(1) a. Every man smiles.
    b. $\forall x[M(x) \rightarrow S(x)]$

M and S here are syntactically and semantically similar in every way: Both combine with an argument to form a formula; both are the sort of thing which can be truthfully or falsely predicated of an individual; and therefore both are naturally analyzed as denoting the set of individuals of which they can be truthfully predicated (or almost equivalently, the characteristic function of this set).

Montague (1973) treated common nouns as belonging to logical type $\langle\langle s, e\rangle, t\rangle$ — that is, as 1-place predicates of "individual concepts." Following Bennett (1975), most subsequent literature in the Montague-derived tradition (including major textbooks such as Dowty, et al. (1981), Gamut (1991), Heim and Kratzer (1998) and many others) has analyzed them as belonging to type $\langle e, t\rangle$ — as 1-place predicates of individuals. In both approaches, common nouns are treated identically to intransitive verbs. Exceptions are sometimes made for relational nouns such as *mother*, *brother*, *top*, *bottom*, etc., which are then treated as being of type $\langle e, \langle e, t\rangle\rangle$; but this still gives such nouns the status of predicates — just 2-place predicates rather than 1-place.

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Proper names, in contrast, are most often treated as being of type *e*. That is, each proper name is analyzed in such a way that it denotes some particular individual, rather than "holding true" of all the individuals in some class. This results in a sharp division of semantic function with proper names on one side but verbs and common nouns together on the other. This division is preserved even in most analyses which do not treat proper names as individual-denoting — for example by assimilating proper names to quantifiers, as in Russell's (1910) theory of proper names as "disguised definite descriptions" or in more modern treatments of names and quantifiers as second order predicates (whether of type $\langle\langle e, t\rangle, t\rangle$ or Montague's more baroque $\langle\langle s, \langle\langle s, e\rangle, t\rangle\rangle, t\rangle$).

Occasionally, proper names have been analyzed as predicates (Quine (1960), Fara (2015)), but then, of course, common nouns are treated as predicates too: verbs, common nouns and proper names are all treated similarly, with no clear semantic correlate to the morphosyntactic distinction between verbs and nouns (whether proper or common).

What I will argue in this paper is that common nouns are similar in semantic type to proper names, and differ in type from verbs. The morphosyntactic distinction between nouns and verbs thus corresponds directly to a difference in semantic type. More particularly, I will argue that common nouns are *variables*, in roughly the same sense as the variables of predicate logic. *Man* is more like the *x* in (1)b. than the *M*.

This is not an entirely new idea. Lepore and Ludwig (2007), for example, say "In 'All men', 'men' functions as if it were a variable restricted to taking on as values only men…" But this suggestion appears only in their informal discussion. In their formalization, common nouns are not treated this way — or analyzed at all, really: Lepore and Ludwig give formalized rules only for interpreting a simplified artificial version of English which does not contain sentences like (1)a. but only predicate-logic-like formulas such as (2):

(2) [Every x : x is a man](x smiles)

No interpretation rule is given for the single word *man* (or any other common noun), but only for the whole open formula 'x is a man'. If any semantic analysis of the single word *man* is intended, it is not made explicit; and the notation here, with separate elements 'x' and 'man', does not suggest that the noun itself is a variable, despite Lepore and Ludwig's informal discussion.

To my knowledge, the idea that common nouns are variables has never been developed or defended in detail.[2] The idea faces a number of technical and theoretical challenges: How can we deal with relational nouns? With mass nouns? With quantification? With intensionality? Can such an analysis be made compositional? Even if all these challenges are met, is there any *advantage* to treating common nouns as variables, or does this idea turn out to be equivalent (or inferior) to a treatment of common nouns as predicates?

[2] Aside from Ludwig and Lepore, an interesting comparison may be made to Luo (2012). The idea that common nouns are variables is also, I suspect, the motivating intuition behind Breul (2013), but I must confess an inability to make sense of Breul's formalism.

I think that each of these challenges can be answered, and that there are in fact several advantages to treating common nouns as variables, over treating them as predicates. Showing this in detail is a larger project than can be presented in this short paper, so I will focus here on meeting just one of the "challenges" and two of the "advantages." Specifically, I will show how we can reconcile an analysis of common nouns as variables with the fact that typical common nouns are modally non-rigid; and I will argue that such an analysis predicts that all nominal quantification is conservative, so that conservativity does not need to be stipulated as a condition on determiner denotations; and that it makes possible some improvements to the analysis of the temperature paradox, allowing for quantificational examples without adding a spurious layer of modal variability.

2. First pass: Common nouns as variables

What does it mean to say that common nouns are variables? This will depend on exactly what a "variable" is, of course. Unfortunately, there is no universally agreed-on, standard definition of variables, so our main thesis is somewhat obscure at the outset. We shall therefore have to begin with a technique for the semantic analysis of variables which I think most readers will at least find familiar, and show how common nouns can be treated as variables using this technique. In the end, I will adopt a somewhat different approach to the semantics of variables; but starting with a more familiar technique should at least clarify the intuition underlying the claim that common nouns are variables.

In this familiar technique, expressions are assigned denotations relative to a series of parameter values, including an *assignment of values to variables*. Semantic rules are given in such a way that one can derive equations of the form in (3), where α is a linguistic expression, g is an assignment of values to variables, and the three dots abbreviate whatever other parameters denotations are relativized to:

(3) $[\![\alpha]\!]^{\ldots,g} = a$

Variables are expressions whose denotations are fixed directly by the assignment of values to variables. That is, α is a variable iff for all g (and all ways of filling in the three dots):

(4) $[\![\alpha]\!]^{\ldots,g} = g(\alpha)$

Variable *binding* is analyzed as the assignment of denotations relative to a given assignment g based on denotations relative to assignments which agree with g in what they assign to all variables other than the one being bound.[3] For example, we can define standard variable binding operators like ∀ and ∃ as in (5):

(5) a. $[\![\forall\alpha\varphi]\!]^{\ldots,g} = 1$ iff $[\![\varphi]\!]^{\ldots,h} = 1$ for all h agreeing with g on all variables other than α.
b. $[\![\exists\alpha\varphi]\!]^{\ldots,g} = 1$ iff $[\![\varphi]\!]^{\ldots,h} = 1$ for at least one h agreeing with g on all variables other than α.

[3] This phrasing does not imply that the relevant assignments differ from g in what they assign to the variable being bound. Any assignment g agrees with itself on all variables other than α (for any variable α).

With this understanding of what variables and binding are as background, how can we treat common nouns as variables?

Instead of giving lexical stipulations like those in (6), we may specify the meanings of common nouns as part of the definition of an assignment of values to variables, as in (7):

(6) a. ⟦*student*⟧ = $\lambda x \,.\, x$ is a student
a. ⟦*professor*⟧ = $\lambda x \,.\, x$ is a professor

(7) *g* is an *assignment of values to variables* iff
a. *g* is a function with domain $\{x \mid x$ is a common noun token[4] or …$\}$;
b. If α is a token of *student*, then g(α) is a student;
c. If α is a token of *professor*, then *g*(α) is a professor;
d. etc.

It should be noted that under (7), *student* and *professor* are of type *e*, not ⟨*e*, *t*⟩.

We shall have to revise the definition in (7) several times before the end of this paper; but let us adopt it for the moment, and explore how it may function in a larger theory of quantification and binding for English.

I assume that LF representations are derived via an operation of Quantifier Raising, which adjoins a quantifier phrase to its scope, leaving a trace in its original position. I will also assume here that Quantifier Raising is clause-bounded. This means we normally cannot use it to obtain transparent readings for quantifier phrases in opaque contexts; but alternative techniques are available, and perhaps better motivated anyway.[5]

By the *antecedent* of a trace, let us mean the NP of the DP whose trace it is. That is:

(8) Where δ is a determiner token and κ is a common noun phrase token: antecedent(trace(δ κ)) = κ.

Note that this is a slightly non-standard use of the term *antecedent*. In [*every professor* [*e smiles*]], *professor*, not *every professor*, is the antecedent of *e*. We do not assume that traces are co-indexed with their antecedents (or with the DPs containing their antecedents). The analysis is compatible with such co-indexation, but does not require it, provided there is some way of identifying the antecedent of each trace.

Traces will be interpreted as variables, with a requirement that (relative to any assignment of values to variables *g*) a trace co-denotes with its antecedent:

(9) *g* is an *assignment of values to variables* iff
a. *g* is a function with domain $\{x \mid x$ is a common noun token, trace token or…$\}$;
b. If α is a token of *student*, then *g*(α) is a student;
c. If α is a token of *professor*, then *g*(α) is a professor;

[4] Instead of assigning values to expression tokens, we could assign values to pairs of an expression type and a context of use. In the long run, this may be the preferable option; but a consideration of the issue here would force a long digression from our central concerns in this paper, so I assign values to tokens here in the interest of simplicity.
[5] See, e.g., Cresswell (1990).

d. etc.;
e. If ε is a trace token, then $g(\varepsilon) = g(antecedent(\varepsilon))$.

We write "$g \sim_\kappa h$" to mean that assignment h agrees with assignment g on all common noun phrases other than κ. Now we can define *every* syncategorematically:

(10) $[\![every\ \kappa\ \varphi]\!]^g = 1$ if $\forall h\ [g \sim_\kappa h \rightarrow [\![\varphi]\!]^h = 1]$;
0 if $\exists h[g \sim_\kappa h\ \&\ [\![\varphi]\!]^h = 0]$.

It is easy to confirm that this gives the correct result that *Every professor e smiles* is true iff for every x such that x is a professor, x smiles. Other quantifiers can be defined analogously (but revisions will be necessary to deal with more complex cases).

It is perhaps worth noting at this point that if we analyze quantification in the way just sketched, there is no semantic motivation any more to have QR move whole DPs, rather than just determiners, or to have QR leave traces. We could get the same effect simply by moving the determiner, and leaving the NP in situ. But let us leave that issue aside, in order to concentrate on semantic matters rather than syntax.

3. First advantage: Conservativity is predicted

Even though our analysis is not in its final form, we can already see one advantage to treating common nouns in this way, rather than treating them as predicates: The analysis predicts that all nominal quantification is conservative.[6]

This claim must be clarified, because the relevant notion of conservativity here is not exactly the traditional textbook sense. The standard definition is in (11):

(11) D is conservative iff for all A, B: D(A, B) iff D(A, A∩B)

But if we take D to be a determiner denotation, A to be a common noun phrase denotation, and B to be a predicate denotation, this definition presupposes that common nouns denote sets. Analyses which treat common nouns in some other way will *never* claim that determiners are conservative in exactly this sense. In order to compare theories, we need a more general notion of conservativity, which is not tied so tightly to a particular approach to the analysis of common nouns and determiners.

A more useful way to conceptualize conservativity for our current purposes is to recognize that the intuitive content of the standard definition is that A functions as a *domain of quantification*. That is, in ascertaining whether D(A, B), one need not consider those members of B which are not in A. Put differently (and more sloppily and English-specifically): in determining the truth value of a sentence of the form D N VP, you only need to consider the N's: Which N's does VP apply to and which N's doesn't VP apply to? You never need to consider the truth value that results from applying VP to something which isn't an N.

[6] The suggestions made in this section may be interestingly compared to proposals like those in Fox (2002), Sauerland (2004), Sauerland (2007), Romoli (2015); but such a comparison will have to wait for another occasion.

In any analysis, some distinction must be drawn between the things which a given noun accurately describes and those which it doesn't. If common nouns are analyzed as denoting sets, these are the members of the set denoted by the noun; if common nouns are analyzed as denoting functions of type $\langle e, t\rangle$, these are the things mapped onto 1 by the function denoted by the noun; if common nouns are analyzed as variables, these are the things assigned to the noun by the various assignments of values to variables. No matter which approach is taken, a theory claims that determiner quantification is conservative iff whenever a determiner-noun combination combines with a predicate *P* to form a sentence, the truth value of that sentence can be ascertained by considering only how *P* applies to the things accurately described by the noun, so that the truth values which result from applying *P* to things not accurately described by the noun are irrelevant.

Now we can see how conservativity falls out from the general approach to determiner quantification just outlined. Assignments of values to variables, in this approach, are functions from common noun phrases (and traces) to individuals. For any such function, the individual assigned to a given noun N is something which "is an N" — something which would be a member of the extension of N in a more conventional analysis. As one considers a class of assignments which differ from one another at most in what they assign to N, therefore, one is effectively considering the things which are accurately described by the noun. As long as object-language quantification over individuals is analyzed in terms of metalanguage quantification over assignments of values to variables, and as long as determiner-noun combinations are interpreted by quantifying over those assignments which agree on all variables other than the noun with which the determiner combines, conservativity is automatic.

To give just a little more detail: Assume that interpretation rules for quantifiers conform to the following general template, based on the rule for *every*:

(12) $[\![\delta\ \kappa\ \varphi]\!]^g = 1$ if for δ-many assignments of values to variables h such that $g \sim_\kappa h$, $[\![\varphi]\!]^h = 1$;
$= 0$ if it is not the case that for δ-many assignments of values to variables h such that $g \sim_\kappa h$, $[\![\varphi]\!]^h = 1$.

Rules conforming to this template derive a value for $[\![\delta\ \kappa\ \varphi]\!]^g$ based on $[\![\varphi]\!]^h$, for various assignments h which differ from g at most in what they assign to κ (and any trace with κ as its antecedent). But $h(\kappa)$ will always be something which is accurately described by κ, and so will $h(\varepsilon)$, where *antecedent*$(\varepsilon) = \kappa$. There simply *are* no assignments of values to variables relative to which ε receives a value which is not accurately describable by κ, so it makes no sense to ask whether the truth value of φ is relative to such assignments is relevant to the truth value of the whole sentence $[\delta\ \kappa\ \varphi]$.

That is, quantification is just over those individuals which are accurately described by the noun, which is to say, the quantification is conservative — and this follows from the general architecture of the theory. In contrast, if we assume simply that determiners denote 2-place relations between sets (or functions of type $\langle\langle e, t\rangle, \langle\langle e, t\rangle, t\rangle\rangle$), nothing guarantees conservativity; it must be independently stipulated.[7]

4. Adding intensionality: Non-rigid variables

Analyzing common nouns as variables will not work properly in an intensional semantics, if we treat variables in the traditional way. Standard versions of intensional logic and quantified modal logic incorporate a principle like that in (13):

(13) If α is a variable, then $[\![\alpha]\!]^{M,w,g} = g(\alpha)$.

This gives the effect that for all w, w', $[\![\alpha]\!]^{M,w,g} = [\![\alpha]\!]^{M,w',g}$. That is, variables are modally rigid. But now recall (9)b.: "If α is a token of *student*, then $g(\alpha)$ is a student." In which world must $g(\alpha)$ be a student? Since no world is mentioned, we standardly assume the condition is meant to apply in the *actual* world $w_@$. Since variables are rigid, this gives the effect that for any w, $[\![student]\!]^{M,w,g}$ is a student in $w_@$ — but nothing requires $[\![student]\!]^{M,w,g}$ to be a student in w. This is obviously wrong — it results in incorrect truth conditions for sentences like (14):

(14) *John believes that every student e smiles*

Assuming our current semantics including (9), (10) and (13), combined with a standard possible-worlds analysis of *believe* along the lines of Hintikka (1969), (14) is assigned a reading which is true (in the actual world $w_@$) iff in every world w compatible with John's belief state (in $w_@$), everyone who is a student in $w_@$ smiles in w. It is not assigned a reading which is true (in $w_@$) iff in every world w compatible with John's belief state (in $w_@$), everyone who is a student in w smiles in w. In other words, the analysis wrongly predicts that all common nouns should be interpreted transparently.

To allow opaque readings, we need to allow *modally non-rigid variables*.[8] This requires us to revise our definition of an assignment of values to variables:

(15) g is an assignment of values to variables iff
   a. g is a partial[9] function with domain $\{x \mid x \text{ is a common noun token, trace token or}\ldots\} \times W$;
   b. If α is a token of *student*: for all w, if there exists an x such that $x = g(\alpha, w)$, then $g(\alpha, w)$ is a student in w;
   c. If α is a token of *professor*: for all w, if there exists an x such that $x = g(\alpha, w)$, then $g(\alpha, w)$ is a professor in w;
   d. etc.;
   e. If ε is a trace token, then for all w, $g(\varepsilon, w) = g(antecedent(\varepsilon), w)$.

[7] Our discussion in this section raises the question of how to deal with *only* and other quantifiers which appear not to be conservative. I assume in such cases, the quantifier binds a variable which is constructed in part based on intonational focus or other factors, rather than simply binding the NP (if any) which serves as its syntactic complement. That is, the quantification in such cases is not strictly "nominal." A detailed consideration of such cases will have to await another occasion.

[8] These have rarely been suggested before, but see e.g. Hughes and Cresswell (1968: 195–201).

[9] We allow g to be a partial function because in some worlds, there aren't any students or professors.

Making this change necessitates corresponding revisions to our definition for the double brackets, and to our definition of what it means for one assignment to agree with another on all values other than a particular one:

(16) If α is a variable, then for all worlds w and assignments g: $[\![\alpha]\!]^{w,g} = g(\alpha, w)$.

(17) $g \sim_{\kappa,w} h$ iff
   a. there exists some x such that $h(\kappa, w) = x$; and
   b. for all common noun phrases ν and worlds w': if $\nu \neq \kappa$ then $g(\nu, w') = h(\nu, w')$.

We also need to revise our rule for *every* to reflect the change made in (17):

(18) $[\![every\ \kappa\ \varphi]\!]^{w,g} = 1$ if $\forall h\ [g \sim_{\kappa,w} h \rightarrow [\![\varphi]\!]^{w,h} = 1]$;
   0 if $\exists h[g \sim_{\kappa,w} h\ \&\ [\![\varphi]\!]^{w,h} = 0]$.

It is easy to confirm that our rules now give the desired modal profile for sentences like *Every student e smiles*.

However, our revisions have introduced another problem. Some variables *should* be rigid — specifically *pronouns*, assuming these can be bound across the borders of intensional contexts.

For example, consider (19), assuming that *professor* is the antecedent of *she*.

(19) [*Every professor e believes* [*that she smiles*]]

If we treat the pronoun here the way we have treated traces — as codenoting, in each world w, with its antecedent — we assign the wrong truth conditions. Intuitively, (19) is true in $w_@$ only if for every x such that x is a professor in $w_@$, in every world w compatible with x's belief state in $w_@$, x smiles in w. But if we require, for every assignment g, world w and pronoun π, that $g(\pi, w) = g(antecedent(\pi), w)$ — like we did with traces — we do not get that reading, because nothing requires that $g(she, w) = g(professor, w_@)$. Quantifying on assignments that agree with g on all values other than $g(professor, w_@)$ will not quantify over values for *she* in the "belief worlds".

To solve this problem, we must again revise our definition of assignment functions, so that each one is keyed to a particular world:

(20) g is an assignment of values to variables for w (or "w-assignment") iff
   a. g is a partial function with domain $\{x \mid x$ is a common noun token, trace token, pronoun token, or…$\} \times W$;
   b. If α is a token of *student*: for all w': if there exists an x such that $x = g(\alpha, w')$, then $g(\alpha, w')$ is a student in w';
   c. If α is a token of *professor*: for all w': if there exists an x such that $x = g(\alpha, w')$, then $g(\alpha, w')$ is a professor in w';
   d. etc.;

e. If ε is a trace token, then $g(\varepsilon, w') = g(antecedent(\varepsilon), w')$;
f. If π is a pronoun token, then $g(\pi, w') = g(antecedent(\pi), w)$.

Note the difference between clauses e. and f. Relative to each world *w′*, a trace is required to denote the same thing as the denotation of its antecedent in *w′*, but a pronoun is required to denote the same thing as the denotation of its antecedent in *w*, the world to which the assignment is keyed. This guarantees that relative to any given assignment, the denotation of a pronoun is modally rigid.

Now we revise the tilde notation so that agreeing assignments must be keyed to the same world:

(21) $g \sim_{\kappa,w} h$ iff
a. *g* and *h* are both *w*-assignments;
b. there exists some *x* such that $h(\kappa, w) = x$; and
c. for all common noun phrases ν and worlds w': if $\nu \neq \kappa$ then $g(\nu, w') = h(\nu, w')$.

Our rule for *every* in (18) can remain notationally the same, but operates differently due to the replacement of (17) with (21). In (18), *g* and *h* are now both required to be *w*-assignments — so relative to *h*, any pronouns in φ will rigidly denote what κ denotes relative to *h* in *w*. This ensures that (19) will be true relative to a world *w* only if for every *x* such that *x* is a professor in *w*, for every *w′* compatible with *x*'s belief state in *w*, *x* smiles in *w′*. The person who smiles in each professor's belief worlds is correctly required to be that same person who is a professor in *w*, and holds the belief. There is no requirement that every professor must be a professor in all her belief worlds.

5. Second advantage: The temperature paradox

We can now turn to a second major advantage to treating common nouns as variables: It allows some improvements in how we analyze the temperature paradox. The paradox and its proposed solution in Montague (1973) are familiar to most readers and will only be briefly sketched here: The argument in (22) is of a form which seems naturally representable as in (23), but (23) is a valid argument and (22) is not.

(22) The temperature is rising.
The temperature is 90.
Therefore, 90 is rising.

(23) $\exists x[\mathbf{temperature}(x) \;\&\; \forall y[\mathbf{temperature}(y) \rightarrow x = y] \;\&\; \mathbf{rise}(x)]$
$\exists x[\mathbf{temperature}(x) \;\&\; \forall y[\mathbf{temperature}(y) \rightarrow x = y] \;\&\; x = n]$
$\therefore \mathbf{rise}(n)$

Montague's solution is to treat *rise* as a predicate of functions from indices (including times) to individuals, but treat *is* as holding (at a given index) between two such functions iff they return the same value for that index.[10] Under this analysis, (22) can be paraphrased as (24), which is easily seen as invalid:

(24) The unique temperature function is a rising function.
The unique temperature function and the 90 function currently yield the same value.
Therefore the 90 function is a rising function.

The analysis is made more formal through the use of ^- and ᵛ-operators, defined as in (25) and (26):

(25) $[\![{}^\wedge\alpha]\!]^{M,w,t,g}$ = that function f with domain $W \times T$ such that for all $w' \in W$, $t' \in T$: $f(w', t') = [\![\alpha]\!]^{M,w',t',g}$

(26) $[\![{}^\vee\alpha]\!]^{M,w,t,g} = [\![\alpha]\!]^{M,w,t,g}(w, t)$

Now the argument may be represented as (27) rather than (23):

(27) $\exists x[\mathbf{temperature}(x)\ \&\ \forall y[\mathbf{temperature}(y) \rightarrow x = y]\ \&\ \mathbf{rise}(x)]$
$\exists x[\mathbf{temperature}(x)\ \&\ \forall y[\mathbf{temperature}(y) \rightarrow x = y]\ \&\ {}^\vee x = {}^\vee n]$
$\therefore \mathbf{rise}(n)$

In (27), the variables x and y are of type $\langle s, e\rangle$, not of type e. Hence **rise** and **temperature** must both be of type $\langle\langle s, e\rangle, t\rangle$. In the case of **rise**, this is intuitive and appropriate. What counts as rising at a time t depends on how things are at times other than t. It is impossible to ascertain whether the temperature is rising at a moment by examining a photograph of a thermometer taken at that moment; one needs multiple photographs, taken at different times. It therefore makes sense to treat *rise* as taking functions from times to numbers as its arguments, so that *rise* can "see" what is going on at other times; there is a clear intuitive basis for treating *rise* as temporally intensional.

But the situation with **temperature** is quite different: To tell whether a particular value is the temperature at a given moment, a single photograph taken at that moment suffices, and photographs taken at other times are irrelevant. There isn't the same intuitive basis for letting *temperature* take functions from times to numbers as its arguments as there is for *rise*; *temperature* is not temporally intensional in its conditions of application like *rise* is.

This feature of Montague's analysis leads to a problem pointed out by Anil Gupta.[11] Intuitively, the argument in (28) is valid. But it is translated into Intensional Logic as in (29), and (29) is *not* a valid argument:

(28) Necessarily, the temperature is the price.
The temperature is rising.
Therefore, the price is rising.

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[10] I have simplified Montague's analysis here by setting aside some irrelevant complications with type assignment.
[11] Gupta's problem is outlined in Dowty, et al. (1981: Appendix III)

(29) $\Box\exists x[\mathbf{temperature}(x)\ \&\ \forall y[\mathbf{temperature}(y) \rightarrow x = y]\ \&\ \exists z[\mathbf{price}(z)\ \&\ \forall y[\mathbf{price}(y) \rightarrow z = y]\ \&\ {}^{\vee}x = {}^{\vee}z]$
$\exists x[\mathbf{temperature}(x)\ \&\ \forall y[\mathbf{temperature}(y) \rightarrow x = y]\ \&\ \mathbf{rise}(x)]$
$\therefore \exists x[\mathbf{price}(x)\ \&\ \forall y[\mathbf{price}(y) \rightarrow x = y]\ \&\ \mathbf{rise}(x)]$

To see that (29) is invalid, let T_1, T_2, T_3, P_1, P_2, P_3 be the functions given in (30). Now consider a model where **temperature** holds at $\langle w_1, t_1\rangle$ uniquely of T_1, at $\langle w_1, t_2\rangle$ of T_2, and at $\langle w_1, t_3\rangle$ of T_3, and where **price** holds at $\langle w_1, t_1\rangle$ uniquely of P_1, at $\langle w_1, t_2\rangle$ of P_2, and at $\langle w_1, t_3\rangle$ of P_3:

(30)

$T_1(w_1, t_1) = 99$	$T_2(w_1, t_1) = 89$	$T_3(w_1, t_1) = 79$
$T_1(w_1, t_2) = 100$	$T_2(w_1, t_2) = 90$	$T_3(w_1, t_2) = 80$
$T_1(w_1, t_3) = 101$	$T_2(w_1, t_3) = 91$	$T_3(w_1, t_3) = 81$
$P_1(w_1, t_1) = 99$	$P_2(w_1, t_1) = 91$	$P_3(w_1, t_1) = 83$
$P_1(w_1, t_2) = 98$	$P_2(w_1, t_2) = 90$	$P_3(w_1, t_2) = 82$
$P_1(w_1, t_3) = 97$	$P_2(w_1, t_3) = 89$	$P_3(w_1, t_3) = 81$

It is easy to see that the first premise of (29) is true: at each world-time pair there is a unique temperature function, and a unique price function, and the two functions return the same current value — 99 at $\langle w_1, t_1\rangle$, 90 at $\langle w_1, t_2\rangle$, and 81 at $\langle w_1, t_3\rangle$. The second premise of (29) is also true at all indices, under an intuitive definition of **rise** and an assumption that t_1 precedes t_2 and t_2 precedes t_3: At each index, the unique temperature function is a rising function, returning a higher value at each successive time. For example, at $\langle w_1, t_2\rangle$ the unique temperature function is T_2, which returns 89 at $\langle w_1, t_1\rangle$, 90 at $\langle w_1, t_2\rangle$, and 91 at $\langle w_1, t_3\rangle$. However, the conclusion of (29) is false at every index, since at every index the unique price function is a falling function, not a rising one.

In Lasersohn (2005) I suggested a solution to this problem, which unfortunately did not work. I argued that the source of the problem was in treating *the* as a variable-binding quantifier. In the sentence *The temperature is rising*, this quantificational analysis of *the* requires *temperature* to take the same variable as its argument as *is rising* does, which in turn forces *temperature* into the intuitively unjustified type $\langle\langle s, e\rangle, t\rangle$, which allows different temperature functions (and not just different temperature values) at different times, which leads to Gupta's problem. But if we switched to a non-quantificational analysis of *the*, I argued, we could avoid using a single variable both as argument to *temperature* and as argument to *is rising*. Specifically, if we represent *the temperature* as ιx **temperature**(x), where **temperature** is of type $\langle e, t\rangle$ and the iota-operator is interpreted as in (31), then *The temperature is rising* can be represented as in (32):

(31) $[\![\iota x\ P(x)]\!]^{M,w,t,g}$ = the unique element u such that $[\![P]\!]^{M,w,t,g}(u) = 1$ if there is such a unique element, undefined otherwise.

(32) $\mathbf{rise}({}^{\wedge}\iota x\ \mathbf{temperature}(x))$

At each index, ιx **temperature**(x) denotes the unique temperature (value) at that index. By the semantics of the ^-operator in (25), $^{\wedge}\iota x$ **temperature**(x) will rigidly denote the function mapping each index onto the unique temperature at that index. In this way, we derive just one temperature function — the same one at every index — and just one price function. Hence, if at every index the temperature function yields the same value as the price function (so they are really the very same function), and the temperature function is a rising function, then the price function must be a rising function too. Gupta's problem is eliminated.

However, this solution cannot be correct, as Romero (2008) was quick to point out. By tying the solution to a non-quantificational analysis of *the*, I made it impossible to deal with more clearly quantificational examples in the same way. But such examples exist; in fact, Montague himself gave such an example, perhaps because he anticipated the kind of analysis I was proposing:

(33) A price rises.
Every price is a number.
Therefore, a number rises.

In light of such examples, I concede the point that the temperature paradox does not really provide us with evidence for a non-quantificational theory of the definite article. Considerations of space preclude a review of Romero's very interesting analysis here, but it should be acknowledged that it accounts successfully for the validity of Gupta's argument.

However, it also retains an odd feature of Montague's original analysis, which seems to me to be very unintuitive: It allows nouns like *temperature* to denote different sets of functions relative to different times. That is, we can have temporal variation not just in what values get returned by the temperature function (or functions), but in which functions count as temperature functions in the first place.

Intuitively, that can't happen. Suppose we are talking about the temperature of one particular object, in one particular world. If at time t_1, the temperature function for that object in that world maps t_1 to 90, t_2 to 91, and t_3 to 92, then it cannot happen that at t_2, the temperature function for that object in that world maps t_1 to 85, t_2 to 84, and t_3 to 83. This kind of temporal variation is easy enough to rule out by meaning postulate (or some similar lexical stipulation), but it would be preferable, if possible, to find a more "architectural" solution, which prevented a case like this from coming up in the first place.

If we analyze common nouns as variables, and allow variables to be temporally non-rigid in the same way as we allowed them to be modally non-rigid, such a solution is available. First, to allow for temporal intensionality, we revise the definition of assignment functions in (20) so that assignments give values to variable-world-time *triples*, and add a clause for the noun *temperature*:

(34) g is an assignment of values to variables for w, t (or "w,t-assignment") iff

a. g is a partial function with domain $\{x \mid x$ is a common noun token or trace token or pronoun token or …$\} \times W \times T$

b. If α is a token of *student*: for all w', t': if there exists an x such that $x = g(\alpha, w', t')$, then $g(\alpha, w', t')$ is a student in w' at t';

c. If α is a token of *professor*: for all w', t': if there exists an x such that $x = g(\alpha, w', t')$, then $g(\alpha, w', t')$ is a professor in w' at t';
d. If α is a token of *temperature*: there is some pragmatically relevant object or location x, such that for all w', t': x exists at w', t' iff $g(\alpha, w', t')$ is the temperature of x in w' at t' (in the relevant scale — ˚F, ˚C, etc.)
e. etc.;
f. If ε is a trace token: for all w', t': $g(\varepsilon, w', t') = g(antecedent(\varepsilon), w', t')$;
g. If π is a pronoun token: for all w', t': $g(\pi, w', t') = g(antecedent(\pi), w, t)$.

Notice that according to (34)d., for each assignment function g and token of the noun *temperature* there is one particular object or location whose temperature at various times and worlds is tracked by the values which g assigns to that token (relative to those times and worlds). The value which g assigns to a token of *temperature* relative to a world w and time t is just a number — an entity of type e — not a function or set of functions. So for any given world and any given object, there will be just one function which picks out the temperature of that object in that world at each time — not different such functions at different times. This eliminates the source of Gupta's problem.

In a semantic theory using (34), the intension of a token of *temperature* relative to a given assignment g will naturally be a function mapping each world-time pair onto the temperature of one particular object or location in that world at that time.

Verbs like *rise* must remain at type $\langle\langle s, e\rangle, t\rangle$. But now we have a problem: If traces are required to co-denote with their antecedents, and nouns like *temperature* are type e, then the trace left by phrases like *the temperature* or *every price* will also be of type e, so it will not be of the right type to serve as argument to *rise*. Therefore, we now allow traces (and pronouns) of type $\langle s, e\rangle$. If a trace or pronoun is in position to fill an $\langle s, e\rangle$ argument place, the trace/pronoun must be of type $\langle s, e\rangle$. If it is in position to fill a type e argument place, it must be of type e. If a trace or pronoun is of type $\langle s, e\rangle$, its denotation must be the same as the *intension* of its antecedent, rather than its extension. More formally, we replace (34)f., g. with (35)a., (35)b.:

(35) a. If ε is a trace token: for all w', t': $g(\varepsilon, w', t') = g(antecedent(\varepsilon), w', t')$ or $g(\varepsilon, w', t')$ = that function f: $W \times T \rightarrow \mathbf{D}_e$ such that for all w'', t'', $f(w'', t'') = g(antecedent(\varepsilon), w'', t'')$, according as ε is of type e or of type $\langle s, e\rangle$;

b. If π is a pronoun token: for all w', t': $g(\pi, w', t') = g(antecedent(\pi), w, t)$ or $g(\pi, w', t')$ = that function f: $W \times T \rightarrow \mathbf{D}_e$ such that for all w'', t'', $f(w'', t'') = g(antecedent(\varepsilon), w'', t'')$, according as π is of type e or of type $\langle s, e\rangle$.

Finally, we need to make the obvious updates to the notation used in our rules and definitions, to reflect the fact that assignments are now to a time parameter in addition to a world. We replace (21) with (36):

(36) $g \sim_{\kappa,w,t} h$ iff
a. g and h are both w,t-assignments;
b. there exists some x such that $h(\kappa, w, t) = x$, and

c. for all common noun phrases ν, worlds w' and times t':
if $\nu \neq \kappa$, then $g(\nu, w', t') = h(\nu, w', t')$.

And (18) with (37):

(37) $[\![every\ \kappa\ \varphi]\!]^{w,t,g} =$
1 if $\forall h\ [g \sim_{\kappa,w,t} h \rightarrow [\![\varphi]\!]^{w,t,h} = 1]$;
0 if $\exists h[g \sim_{\kappa,w,t} h\ \&\ [\![\varphi]\!]^{w,t,h} = 0]$.

Now our semantics (unlike that of Lasersohn (2005)) will assign a coherent interpretation to sentences like [*Every temperature* [*e rises*]]. In order to serve as argument to *rises*, the trace *e* must be of type ⟨*s*, *e*⟩. Therefore, relative to any assignment *g*, it must denote the intension of its antecedent, *temperature*, relative to *g*. Relative to *g*, the intension of *temperature* is a function tracking the temperature of one particular object or location — but relative to another assignment, the intension of the *temperature* might be a function tracking the temperature of some other relevant object or location. The sentence is true relative to an assignment *g* iff [*e rises*] is true relative to every assignment *h* agreeing with *g* on all nouns other than *temperature*. That is, it is true if the temperature function of each relevant object or location is a rising function.

The original temperature paradox argument correctly comes out invalid, even if we adopt a quantificational analysis of definites. We may define *the* as in (38):

(38) $[\![the\ \kappa\ \varphi]\!]^{w,t,g} = 1$ if $\exists h[g \sim_{\kappa,w,t} h\ \&\ \forall i\ [g \sim_{\kappa,w,t} i \rightarrow h = i]\ \&\ [\![\varphi]\!]^{w,t,h} = 1]$;
0 if $\neg\exists h[g \sim_{\kappa,w,t} h\ \&\ \forall i\ [g \sim_{\kappa,w,t} i \rightarrow h = i]\ \&\ [\![\varphi]\!]^{w,t,h} = 1]$.

Consider the truth condition assigned to (39) under this rule:

(39) [*The temperature* [*e rises*]]

In order for (39) to be true relative to an assignment *g* (in *w*, at *t*), there must be exactly one assignment *h* which agrees with *g* on all common noun tokens other than the token of *temperature* in this example. If *h* agrees with *g* in this way, there must be some pragmatically relevant object or location, whose temperature *h* assigns to this token at each world and time. Since *h* is unique, there can be only one function which maps a pragmatically relevant object or location onto its temperature at each world and time. That is to say, despite the fact that *temperature* is of type *e*, according to our rules (39) requires a unique temperature *function*, not just a unique temperature *value*. Of course this unique temperature function must also be a rising function in order for (39) to be true.

In contrast, (40) is true (relative to *w*, *t*, *g*) iff the value of the unique temperature function at *w*, *t* is 90; the trace here is of type *e*, and therefore denotes the extension, not the intension of its antecedent.

(40) [*The temperature* [*e is ninety*]]

Assuming that the intension of *ninety* is the constant function mapping each world-time pair onto 90, (41) will always be false, even if (39) and (40) are true:

(41) [*Ninety rises*]

It is also easy to see that Gupta's argument comes out valid. Sentence (42)a. equates, at all world-time pairs, the value of the unique temperature function with the value of the unique price function. If the unique temperature function yields the same value at every pair as the unique price function, then they are the same function; so if the temperature function is rising, the price function is rising.

(42) a. Necessarily, the temperature is the price.
b. The temperature is rising.
c. Therefore, the price is rising.

6. Conclusions

Common nouns which are usually analyzed predicates of type ⟨*e*, *t*⟩ may instead by analyzed as being of type *e* by treating them as restricted variables. Treating them this way, in combination with a standard understanding of what variable-binding is, predicts that all nominal quantification is conservative, and does so without any independent restriction on determiner meanings.

In order to assign correct truth conditions to sentences in which common nouns appear in intensional contexts, an analysis which treats them as variables must allow variables to be modally non-rigid. Certain other variables, notably pronouns, should continue to be analyzed as rigid.

Allowing modally non-rigid variables makes possible an analysis of the temperature paradox which can deal with quantificational examples, without requiring extra stipulations to rule out modal variation in which functions count as temperature functions, thus solving "Gupta's problem."

These arguments are not by themselves sufficient to establish that common nouns should be analyzed as variables. That would be a far larger project, requiring much more thorough and intensive investigation than can be accomplished in a single paper. Several important challenges to this view have not been addressed here: How can we deal with relational nouns? With complex noun phrases containing modifiers? With mass and plural nouns? Can the treatment of quantification be formulated in such a way that determiners are assigned denotations, instead of being interpreted syncategorematically, as they were in this paper? I believe all these challenges can be met; but demonstrating this will have to wait for another occasion.

Additional advantages to analyzing common nouns and noun phrases as variables also suggest themselves: Such an analysis may allow for a closer, more systematic correlation between syntactic categories and semantic types. If complex phrases like *farmers who own a donkey* can be treated as type *e* variables, a new pathway opens for the analysis of the "proportion problem" in sentences like *Most farmers who own a donkey beat it*. Treating common nouns as type *e* variables promises a more natural analysis of "collectivizing" conjunction of common nouns, as in the reading of *this man and woman* where it denotes a group whose members are a man and a woman. These and other topics must here be left to later investigation.

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