

# Pied-piping in the composition of degrees<sup>1</sup>

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**Abstract.** The internal make-up of degree DPs like *the number of books that Amy read* introduces a challenge for semantic composition. One solution, given in Scontras (2017), builds on the assumption that degrees are kinds in the construal of Chierchia (1998). An alternative solution links degree DPs to degree questions. Building on Heim (1987, 1992), this solution posits pied-piping in the syntax of degree DPs and hypothesizes that it tracks pied-piping in degree questions. As an argument for the pied-piping analysis, we note that it correctly predicts scope options in degree DPs that are not predicted under the degrees-as-kinds analysis.

**Keywords:** degree DPs, degrees, kinds, degree questions, scope, pied-piping.

## 1. Introduction

In the DP in (1), the preposition *of* joins a “degree noun”, *number*, with an “object noun”, *books*, to form what we will call the “core” of the DP, *number of books*. We will refer to any DP with such a core as a “degree DP”. In (1), the core is followed by a “gapped clause”, *that Amy read*, where the gap appears to saturate a predicate of objects, *read*. Focusing on degree DPs with this particular profile, which is further illustrated by the examples in (2), we will ask how this gapped clause enters into the semantic composition.

- (1) the **number** of **books** *that Amy read*
- (2) a. the **amount** of **apples** *that Bill ate* (Scontras, 2017)  
b. the **quantity** of **eggs** *that you sold*  
c. the **volume** of **water** *that you wasted*  
d. the **range** of **shoes** *that you tried*  
e. the **share** of **adults** *who live in middle-class households*  
f. the **proportion** of **teenagers** *that own a flip phone*

This question turns out to introduce a challenge with two solutions emerging from the literature. One, due to Scontras (2017), parses the core as an NP headed by the degree noun, and exploits a view of degrees as kinds, construed as in Chierchia (1998). The other, inspired by Heim (1987, 1992), takes the core to be an NP headed by the object noun, hypothesizes a link to degree questions, and crucially invokes pied-piping. Based on scope data, we will present an argument for the pied-piping approach over the degrees-as-kinds approach. Using the degree DP in (1) for illustration, we will in the remainder of this introductory section preview the challenge, the two competing solutions, and our argument.

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### 1.1. The challenge

In the degree DP in (1), there are two possible attachment sites for the gapped clause *that Amy read*. It can attach high, as in (3a), combining with the entire core. The core then constitutes an NP. Alternatively, the gapped clause can attach low, as in (3b), modifying only the object noun. In that case, the core does not form a constituent.

- (3) a. *High attachment:*  
       [DP the [RS [NP **number of books**] [GC that Amy read — ] ] ]  
       b. *Low attachment:*  
       [DP the [NP number of [NP **books**] [CP that Amy read — ] ] ] ]

At this point, when discussing high attachment in (3a) we intentionally avoid calling the gapped clause, the sequence that follows the core, a “relative clause.” We also do not wish to commit to syntactic category of this sequence, for now. This is why we use the pre-theoretic term “gapped clause” to refer to this sequence, and accordingly use the stand-in label “GC” in (3a). Likewise, we do not at this point wish to commit to the category of the larger constituent that comprises the core and the gapped clause, and that restricts the determiner, hence the label “RS” in (3a), for “restrictor.” The reasons for this temporary lack of commitment will become clear shortly.

The high attachment of the gapped clause in (3a) presents a challenge. If the DP as a whole denotes a degree, then the restrictor (RS) must itself be a predicate of degrees. This, in turn, implies that both the core NP and the gapped clause (GC) must likewise be degree predicates. For the core, this is unproblematic: if it is headed by the degree noun, the core as a whole is expected to be a predicate of degrees. For the gapped clause, however, this requirement clashes with its surface form, which suggests that GC denotes a predicate of *objects*, not degrees.

No such clash arises under low attachment in (3b). Here the restrictor can be a degree predicate in virtue of being an NP headed by the degree noun, while the gapped clause can be analyzed as a relative clause CP that modifies just the object noun. Composition should then converge. However, low attachment falls short in other data. Consider the variant of (1) in (4), with an intensional operator in the gapped clause, and its two conceivable parses in (5).

- (4) the number of books that Amy **has to** read
- (5) a. *High attachment:*  
       [DP the [RS [NP **number of books**] [GC that Amy **has to** read — ] ] ]  
       b. *Low attachment:*  
       [DP the [NP number of [NP **books**] [CP that Amy **has to** read — ] ] ] ]

This degree DP is judged to permit a *non-specific* reading, a reading that does not imply the existence of any *particular* books that Amy has to read. It is not clear how such a reading could arise from the low-attachment parse in (5b), where the modified NP is expected to be a predicate of particular books that Amy has to read. This suggests that the non-specific reading is instead sourced in the high-attachment parse in (5a). If so, we must ultimately explain how high-attachment structures can be semantically composed. Thus, there is no escape from the kind of clash described above for (3a). The clash must be resolved, and in a way that makes it possible for a non-specific reading to be associated with a surface parse like (5a).

## 1.2. Two approaches

Two solutions emerge from previous literature. Both converge on the view that, despite initial appearances, the gapped clause under high attachment somehow yields a relative clause that denotes a predicate of *degrees* rather than of *objects*. But they disagree on the internal make-up of the relative clause and, correspondingly, on *how* it comes to be a predicate of degrees.

In one approach, due to Scontras (2017), the core NP is headed by the degree noun, and degrees are construed as kinds. Adopting assumptions about kinds and kind predication from Chierchia (1998), Scontras (2017) suggests that object predicates like *read* can compose with degree-denoting expressions. This allows for the gap in cases like (3a) to range over degrees. Accordingly, a gapped clause in such data can then be a relative clause ranging over degrees. As we will see, the analysis in particular derives non-specific readings for structures like (5a).

Another approach ties the interpretation of degree DPs of the relevant format to the interpretation of degree questions. Heim (1987, 1992) notes that a question like *How many books did Amy read?* is interpreted as though just the *wh*-word *how* had been displaced, rather than *how many books*. Heim credits this to pied-piping in the surface syntax, which is undone for semantic interpretation. A parallel interpretation can be posited for degree DPs. Matching [*how many*] *books*, the core NP in (3a) can be parsed as [*wh number*] *of books*, with the object noun as the head and a silent degree *wh*. Extending the parallel, composition in degree DPs can be hypothesized to fully track composition in degree questions. This yields the desired degree predicate for the restrictor in (3a), and it moreover predicts non-specific readings for structures like (5a).

## 1.3. A case for pied-piping

While both approaches derive non-specific reading for degree DPs like (4), we will identify a related test case for which the pied-piping analysis predicts a type of reading that is not predicted by the degrees-as-kinds analysis. In data of the form in (6), overt *wh*-movement leaves a gap in the scope of one operator, *Op*<sub>2</sub>, which in turn is in the scope of a second operator *Op*<sub>1</sub>.

(6) *Form of test case:*

the [<sub>RS</sub> number of books [<sub>GC</sub> ... **Op**<sub>1</sub> [... **Op**<sub>2</sub> [... — ... ] ] ] ]

The pied-piping analysis predicts that the reconstruction site for the core *number of books* can be in the interpreted syntax *between* *Op*<sub>1</sub> and *Op*<sub>2</sub>. This derives an *intermediate scope* reading—a reading that is neither fully specific nor fully non-specific. Such a reading is not predicted by the degrees-as-kinds analysis. Under the degrees-as-kinds analysis, high attachment is expected to derive a fully non-specific reading only. Our central observation is that intermediate scope readings are indeed attested. This observation indicates that the degree-as-kinds analysis undergenerates. By the same token, the observation constitutes an argument for the compositional pathway posited under the pied-piping analysis.

In the remainder of this paper, we will spell out this argument in more detail. For ease of exposition, we will continue to illustrate with definite degree DPs with the degree noun *number*, like (1) or (4), though our central argument fully extends to degree DPs at large. Section 2 presents the degrees-as-kinds approach and Section 3 the pied-piping approach. Section 4 discusses spe-

cific readings of degree DPs with intensional operators, to prepare the ground for identifying and testing the predictions about intermediate readings in Section 5. Section 6 concludes.

## 2. The degrees-as-kinds solution

A central ingredient to the composition challenge that we hinted at above is the content of the gapped clause in the relevant sort of degree DP. Consider again the “proto-structure” for (1) shown in (3a) and reproduced here as (7), with the stand-in labels “RS” and “GC”.

(7) *Proto-structure for (1):*

[DP the [RS [NP number of books] [GC that Amy read \_ ] ] ]

The problem, we noted, is that one might expect the gapped clause to be a predicate of objects. After all, the gap internal to the clause appears to be the argument of the object predicate *read*, and abstraction over an object variable inevitably yields a predicate of objects. Scontras (2017), however, holds that degrees *can* saturate object predicates and that, as a consequence, the gapped clause in a degree DP like (1) *can* be a predicate of degrees. In Scontras’ approach, this is made possible by a particular assumption about what degrees *are*, which in turn conspires with a particular assumption about composition. The two assumptions allow for the structure in (7) to be elaborated as in (8).

(8) *Interpreted structure for (1):*

[DP the [NP [NP [N number] of books] [CP wh<sub>1</sub> [C<sup>o</sup> that Amy read t<sub>1,D</sub> ] ] ] ]

In this structure, GC and RS in the proto-structure are identified with CP and NP, respectively, and *wh* is a covert relative pronoun in the specifier of C that is assumed to trigger abstraction. Taking D to be the type of degrees, this abstraction yields a predicate of degrees.

To explain in full how the structure in (8) is interpreted in Scontras’ approach, we will first report on the proposed construal of degrees, and then turn to the assumed principle of composition that capitalizes on this construal.

### 2.1. Building degrees from objects

Under the view that degrees are primitives—on a par with ordinary objects, but sortally distinct (e.g. Heim (2000))—there seems to be no principled way to compose an object predicate with a degree denoting expression. An alternative, however, is to treat degrees not as primitives, but as entities derived from ordinary objects. On a classic formulation, a degree in a given dimension is a set of ordinary objects that are “equimensural” (i.e. have the same measurement) in that dimension. For any given dimension, the degrees in that dimension can be understood as the equivalence classes defined by the equimensurability relation in that dimension (Cresswell, 1976; Klein, 1980; Bale, 2006). Natural numbers, for instance, could be viewed as equivalence classes of pluralities with the same number of atomic parts. The number 2, say, would then be the set of all pairs of objects, that is, all pluralities of objects with exactly two atomic parts.

Building on ideas in Chierchia (1998), Scontras (2017) presents a novel elaboration of this approach. For Scontras, instead of equimensural *sets* of objects, degrees are *nominalized* equimen-

sural *properties* of objects. With a property of objects understood as a function from possible worlds to sets of objects, an equimensural property is a property that for any input world, outputs an equimensural set of objects. Crucially, the members of such an output set need not cover the entire set of objects on the domain, but could be limited to, say, the pluralities of books in the input world. So the logical space of equimensural properties of objects includes, say, the property of being a pair of books, which we can render as in (9).

- (9) *An equimensural property:*  
 $\lambda w. \lambda x. \text{BOOKS}(w)(x) \wedge \#(x) = 2$

Scontras proposes that any equimensural property of objects can be turned into a *degree* through “nominalization” in the sense of Chierchia (1998). Nominalization in this sense is an operation that maps properties to their “individual correlates”, individuals of sorts, which are like primitive members of the domain of individuals in their ability to serve as arguments in composition. Letting  $\cap$  be the nominalization operation, we can render the degree that corresponds to the property in (9) as shown in (10).

- (10) *A nominalized equimensural property:*  
 $\cap \lambda w. \lambda x. \text{BOOKS}(w)(x) \wedge \#(x) = 2$

We will sometimes use the term “rich degrees” to refer to degrees as construed by Scontras—nominalized equimensural properties of objects. It is such rich degrees that are taken to populate the domain of type D, referenced in the structure in (8) above. Generalizing from (10), rich degrees can be described with expressions of the form in (11), where  $\mu$  is a measure function (mapping objects to measurements),  $m$  is a measurement, and  $P$  is a property of objects. To facilitate exposition, we will write “ $m_p$ ” as a shorthand for expressions of the form in (11), thus we can write “ $2_{\text{BOOKS}}$ ” to abbreviate the expression in (10).

- (11) *Format of rich degrees:*  
 $\cap \lambda w. \lambda x. P(w)(x) \wedge \mu(x) = m$

Having introduced Scontras’ construal of degrees, we now turn to reviewing how this construal supports a solution to the problem of high attachment in degree DPs.

## 2.2. Degrees in composition

By viewing degrees as nominalized properties of objects, Scontras posits an important link between degrees and *kinds* as construed in Chierchia (1998). In fact, Scontras portrays the construal of degrees as a direct extension of Chierchia’s construal of kinds in terms of nominalized properties of objects. Crucially, this unification of degrees and kinds opens up a way of understanding the compositional interpretation of structures like (8) above.

Chierchia (1998) updates a classic proposal in Carlson (1977) by positing a composition principle of “Derived Kind Predication”. This principle crucially relies on the assumption that the nominalization operation can be reversed in composition. The nominalizing function  $\cap$  is taken to have an inverse, symbolized as  $\cup$ . The *instances* of a kind  $k$  in a possible world  $w$  are then

the members of the set of objects  ${}^Uk(w)$ . Referencing the instantiation relation reconstructed in this way, Chierchia's definition of Derived Kind Predication can be stated as in (12).

(12) *Derived Kind Predication:*

For any world  $w$ , if  $\alpha$  is a predicate of objects and  $\beta$  denotes a kind, then:

$$\llbracket \alpha \beta \rrbracket^w \Leftrightarrow \exists x \in D_e \left[ \llbracket \alpha \rrbracket^w(x) \wedge {}^U\llbracket \beta \rrbracket(w)(x) \right]$$

Thus, Derived Kind Predication posits that a kind-denoting expression can compose with an object predicate to yield the statement that the predicate applies to at least one instance of the kind in the world of evaluation.

Assuming Derived Degree Predication, the structure in (8), repeated below, will be assigned the intended referent under unexceptional assumptions. With the *wh*-trace ranging over rich degrees (type D), the relative CP is expected to have the denotation in (13), the set of all rich degrees that Amy read an instance of in the world of evaluation. As for the core NP, Scontras proposes that it denotes a set of rich degrees  $m_p$  such that  $m$  is a measurement in the dimension given by the degree noun, and  $P$  is the property given by the object noun. As stated in (14), for the core NP in (8), this yields the set of all rich numbers based on the property of being a book.

(8) *Interpreted structure for (1):*

[DP the [NP [NP [N number] of books] [CP wh<sub>1</sub> [C' that Amy read t<sub>1,D</sub> ] ] ] ]

$$(13) \llbracket [CP wh_1 [C' that Amy read t_{1,D} ] ] \rrbracket^w = \{y \in D_D: \exists x \in D_e [{}^Uy(w)(x) \wedge \text{READ}(w)(x)(\text{Amy})]\}$$

$$(14) \llbracket [NP \text{ number of books} ] \rrbracket = \{1_{\text{BOOKS}}, 2_{\text{BOOKS}}, 3_{\text{BOOKS}}, \dots\}$$

Suppose now that Amy read exactly two books in  $w$ . In that case, the intended rich degree referent for (8) would be  $2_{\text{BOOKS}}$ . This is indeed what the composition will naturally derive. Assuming that the relative CP and the core NP compose intersectively, the main NP in (8) will in  $w$  denote the doubleton  $\{1_{\text{BOOKS}}, 2_{\text{BOOKS}}\}$ , from which the definite article can plausibly be taken to select  $2_{\text{BOOKS}}$  as the maximal member.

Crucially, this approach can also derive non-specific readings for degree DPs like (4). In parallel to (3a) and (8), the high attachment structure in (5a) can be elaborated as in (15). The expected denotation of the relative CP in any given world  $w$  is the set of rich degrees shown in (16).

(15) *Interpreted structure for (4):*

[DP the [NP [NP number of books] [CP wh<sub>1</sub> [C' that **has** [VP Amy to read t<sub>1,D</sub> ] ] ] ] ]

$$(16) \llbracket [CP wh_1 [C' that \textbf{has} [VP Amy to read t_{1,D} ] ] ] \rrbracket^w = \{y \in D_D: \forall w' \in \text{Acc}(w) \left[ \exists x \in D_e [{}^Uy(w')(x) \wedge \text{READ}(w')(x)(\text{Amy})] \right]\}$$

Suppose now that in a world  $w$ , Amy has to read two or more books, with the choice of book up to her. In that case, the intended rich degree referent for the degree DP in (4) would again be  $2_{\text{BOOK}}$ , and, indeed, under the relevant assumptions, this is once again what is derived. The intersection of the CP denotation in  $w$  with the denotation of the core NP in (14) will again be the doubleton  $\{1_{\text{BOOKS}}, 2_{\text{BOOKS}}\}$ , which the definite article can map to  $2_{\text{BOOKS}}$ .

In sum, by assimilating degrees to kinds as construed in Chierchia (1998) and adopting Chierchia's principle of Derived Kind Predication, the analysis of degree DPs in Scontras (2017)

renders high attachment parses of degree DPs interpretable and derives viable DP denotations. Crucially, the analysis in particular captures non-specific readings for degree DPs with an intensional operator inside the gapped clause.

Before moving on, we note that Scontras suggests a parallel composition for “kind DPs”. Such DPs have the form as degree DPs, except that they contain the noun *kind* instead of a degree noun. As a case in point, the DP in (17) is the kind DP counterpart of the degree DP in (1).

(17) the **kind** of **book** *that Amy read*

Scontras does not analyze kind DPs with gapped clauses in full detail. But if kinds are nominalized properties, as Scontras assumes, then (17) plausibly allows for a composition that traces the steps of composition shown above for (1). We note that this invites a broader hypothesis about degree DPs, one that is not actually tied to Scontras’s specific assumptions reviewed above: that the composition for kind DPs, however explicated, is mirrored by the composition for degree DPs. We will briefly return to this more general hypothesis in the concluding section.

### 3. The pied-piping solution

An alternative analysis of high attachment in degree DPs is suggested by a parallel with degree questions. To introduce the approach, consider the *how many*-question in (18). The unexceptional surface structure in (19) parses the fronted sequence *how many books* as a DP whose D head is not overtly filled. The NP complement of D is taken to be headed by the object noun *books*. Aligned with the so-called adjectival theory of numerals (e.g. Landman (2004)), this head noun, in turn, is modified within NP by *how many*, parsed as AP.

(18) How many books did Amy read?

(19) *Surface structure for (18)*

[CP [DP [NP [AP how many] [NP books] ] ] [C' did Amy read \_ ] ]

As Heim (1987, 1992) notes, a phrase that could occupy the gap in the surface form of a degree question does not in general yield a direct answer to the question. For example, *Amy read Dracula and Emma*, does not intuitively count as a direct answer to (18). Evidently, a direct answer is instead given by numerals like *two*, as in *Amy read two books*, and degree questions at large are intuited to ask for degrees. This intuition leads Heim to conclude that in a *how many*-question, the fronting seen in surface form results from *how* having “pied-piped” the entire DP in which it is contained, and that this pied-piping is undone for semantic interpretation.

Heim suggests, more specifically, that the undoing of pied-piping can be understood in terms of covert syntactic reconstruction. In this view, the surface form in (18) would yield a semantically interpreted structure like the one sketched in (20), where *how* alone is fronted, binding a trace in its DP, which has been restored to its base position.

(20) *Interpreted structure for (18)*

[CP how<sub>1</sub> [C' did Amy read [DP ∃ [NP [AP t<sub>1,d</sub> many] [NP books] ] ] ] ]

Attending to other elements of the structure in (20) and its interpretation, note first that the trace left by *how* is marked as ranging over type *d*. Contrasting with the type *D* introduced earlier, we can take the domain of *d* to be the set of measurements, including numbers, understood as primitive elements that populate the domain of individuals alongside ordinary objects. The trace bound by *how*, in turn, combines with *many* into AP. Again in line with the adjectival theory of numerals, this AP can be taken to denote a set of objects, hence to function as an intersective modifier. Concretely, assuming the lexical entry in (21), the AP will, for any given number *n* denoted by the trace, denote the set of all pluralities of objects with exactly *n* members.

$$(21) \quad \llbracket \text{many} \rrbracket = \lambda d_d. \lambda x_e. \#(x) = d$$

Finally, (20) shows the head of DP as occupied by a covert element  $\exists$ , positing existential force for DP. This assumption too is familiar from the adjectival theory of numerals, which for any numeral *n*, analyzes *Amy read n books* as expressing an existential proposition, conveying that *there is* a plurality of books that Amy read with exactly *n* atomic members. In a well-motivated view, adopted by Heim (1987, 1992), it is such existential propositions that constitute the semantic answers to (18). All told, the structure in (20) supports a viable compositional interpretation.

Turning now to degree DPs, the pied-piping analysis of degree DPs with a high attaching gapped clause posits a tight link to degree questions. It holds that the restrictors of such DPs have a surface syntax that parallels that of degree questions, and it hypothesizes that the process of undoing pied-piping that enables semantic interpretation in degree DPs is the very same process that also applies in degree questions to render them interpretable.

To spell out the assumptions about degree DPs that this hypothesis calls for and confirm its viability, we return to the degree DP in (1), and consider again the proto-structure for high attachment in (7), repeated below. In a first step, we use the *how many*-question structure in (19) as a blueprint for a conceivable elaboration of (7). This leads us to the structure for (1) shown in (22).

(7) *Proto-structure for (1):*

[DP the [RS [NP number of books] [GC that Amy read — ] ] ]

(22) *Surface structure for (1):*

[DP the [CP [DP [NP [MP wh number] of [NP books] ] ] [C' that Amy read — ] ] ]

According to this elaboration of (7), the gapped clause is of category *C'*, and the restrictor of the degree DP as a whole is a CP. The core is understood as a DP with a covert *D*, which sits in the specifier of *C* on the grounds of having moved there from the position of the gap within *C'*. Within the core NP, the degree noun is assumed to combine with a covert *wh*-phrase to form a measure phrase (MP). The core NP as a whole is assigned a so-called pseudopartitive structure, parallel to, say, the structure for *two pounds of nuts* in (23).

(23) *A pseudopartitive structure:*

[DP [NP [MP two pounds] of [NP nuts] ] ]

Notably, in virtue of being a pseudopartitive structure, (23) parses the object noun, here *nuts*, as the head of NP, rather than the unit noun, here *pounds*. This is motivated by the fact the relevant phrases can saturate object predicates, like *buy* in *Amy bought two pounds of nuts*.

Building on (22) as the surface structure for (1), we now suggest that an analysis of high attachment becomes available under the central assumption that pied-piping applies in degree DPs in the same way that it applies in degree questions. Under this assumption, (1) is predicted to permit a semantically interpreted structure like (24).

(24) *Interpreted structure for (1):*

[<sub>DP</sub> the [<sub>CP</sub> wh<sub>1</sub> [<sub>C'</sub> that Amy read [<sub>DP</sub> ∃ [<sub>NP</sub> [<sub>MP</sub> t<sub>1,d</sub> number] of [<sub>NP</sub> books] ] ] ] ] ] ]

This structure relates to (22) in much the way that (20) relates to (19). Thus, the covert element *wh* is fronted on its own and binds a trace within its reconstructed DP, creating a derived predicate of degrees. Despite being a CP, rather than an NP, this derived predicate is assumed to serve as the restrictor of the definite article that heads the degree DP as a whole.

Once again, the reconstructed DP is taken to have existential force, as signalled by the covert head ∃ in (24). This assumption is motivated independently, by the fact that pseudopartitive DPs more broadly can have existential force. For example, *Amy bought two pounds of nuts* conveys that *there is* a plurality of nuts that Amy bought which measures two pounds.

Suppose now that, as stated in (25), the degree noun *number* has the very same denotation assigned to *many* in (21), and suppose that in a pseudopartitive structure, the measure phrase and the object noun compose intersectively (e.g. Nakanishi (2007)). In any given world *w*, the CP in (24) will then denote the set of degrees in (26).

(25)  $\llbracket \text{number} \rrbracket = \lambda d_d. \lambda x_e. \#(x) = d$

(26) *Denotation of CP in (24):*

$\{n \in D_d: \exists x \in D_e [ \text{BOOKS}(w)(x) \wedge \#(x) = n \wedge \text{READ}(w)(x)(\text{Amy}) ] \}$

This set will contain a natural number *n* just in case in *w*, there is a plurality of books with exactly *n* atomic parts that Amy read. Suppose now that Amy read exactly two books in *w*. On the view that the degree DP in (1) denotes a measurement, more specifically a natural number, its intuited referent in *w* is the number 2, and this is indeed what the analysis under consideration derives. In the world *w* considered, the set in (26) will be the doubleton {1, 2}. Assuming again that the definite article maps a set of degrees to its maximal member, the degree DP in (1) is predicted to denote the number 2.

This analysis moreover provides a path to deriving non-specific readings for degree DPs like (4). Under the pied-piping analysis of (1), the only additional assumption required is that reconstruction of a pied-piped DP can return this DP into the scope of an intensional operator. Under this assumption, (4) is predicted to permit the interpreted structure in (27).

(27) *Interpreted structure for (4):*

[<sub>DP</sub> the [<sub>CP</sub> wh<sub>1</sub> [<sub>C'</sub> that **has** [Amy to read  
[<sub>DP</sub> ∃ [<sub>NP</sub> [<sub>MP</sub> t<sub>1,d</sub> number] of [<sub>N</sub> books] ] ] ] ] ] ] ]

To confirm that this structure derives the intended interpretation, we can observe that in any given world *w*, the CP in (27) denotes the set of degrees in (28). Suppose now that in a world *w*, Amy has to read two or more books, with the choice of books up to her. In this case, the intended

referent for the degree DP in (4) would be the number 2. Under the present assumptions, this is again what is derived. The CP denotation in *w* will again be the doubleton {1, 2}, from which the definite article is expected to pick 2.

(28) *Denotation of CP in (27):*

$$\{n \in D_d: \forall w' \in \text{Acc}(w) \left[ \exists x \in D_e [ \text{BOOKS}(w')(x) \wedge \#(x) = n \wedge \text{READ}(w')(x)(\text{Amy}) ] \right] \}$$

Importantly, the pied-piping analysis of degree DPs is not merely *consistent* with the availability of non-specific readings for cases like (4). Given the central hypothesis that degree DPs track degree questions in their overt and interpreted syntax, the analysis *predicts* degree DPs to permit such readings. The reason is that, as is well known, degree questions, too, allow for non-specific readings (e.g. Rullmann (1995)). For example, the *how many*-question in (29) has a reading in which it does not suggest that there are any *particular* books that Amy has to read, and that permits potentially truthful answers like *two* that do not carry such an implication.

(29) How many books did Amy **have to** read?

In sum, by drawing a parallel with degree questions, the pied-piping analysis offers a viable route to interpreting degree DPs under high attachment of the gapped clause, a route which, in particular, succeeds in deriving non-specific readings. For the range of data considered so far, this analysis matches the effects of the degrees-as-kinds analysis reviewed in the last section. Having now laid out the two competing analyses, we turn to preparing the ground for identifying and testing a prediction that discriminates between them.

#### 4. Deriving specific readings

We have seen that both the degrees-as-kinds analysis and the pied-piping analysis of high attachment successfully account for the attested non-specific readings of degree DPs whose gapped clause contains an intensional operator. However, such DPs are also clearly judged to allow *specific* readings. In particular, the degree DP in (4), repeated here, permits an interpretation on which it refers to the number of particular books that Amy is required to read.

(4) the number of books that Amy **has to** read

How might this reading arise from composition? In this section, we will first review why low attachment structures are expected to be one source of specific readings. We will then point out that the pied-piping analysis, but not the degrees-as-kinds analysis, predicts that specific readings can *also* arise from high attachment structures.

##### 4.1. Specificity from low attachment

In Section 1, we entertained a low attachment structure for (4), reproduced in (30), and suggested that this structure could not derive a non-specific reading.

(30) *Surface structure for (4):*

$$[_{DP} \text{ the } [_{NP} \text{ number of } [_{NP} \text{ books } [_{CP} \text{ that Amy has to read } \_ ] ] ] ] ]$$

To confirm this assessment, we detail the expected interpreted structure under low attachment in (31). Here the gapped clause is analyzed as a relative clause CP, modifying the object noun. Relativization is assumed to yield abstraction over an object variable, deriving a predicate of objects. Intersective composition with the object noun would result in another predicate of objects, which in any given world  $w$ , is expected to denote the set in (32). To be a member of this set, an individual has to be a book that Amy is required to read. In virtue of deriving such a denotation for the NP headed by the object noun, the DP as a whole can at best denote the cardinality of a particular plurality of books that Amy is required to read. It is clear, therefore, that a low attachment structure cannot yield a non-specific reading.

(31) *Interpreted structure for (4):*

[<sub>DP</sub> the [<sub>NP</sub> number of [<sub>NP</sub> books [<sub>CP</sub>  $wh_1$  [<sub>C'</sub> that **has** [Amy to read  $t_{1,e}$  ] ] ] ] ] ]

(32) *Denotation of lower NP in (31):*

$\{x \in D_e: \text{BOOKS}(w)(x) \wedge \forall w' \in \text{Acc}(w) [ \text{READ}(w')(x)(\text{Amy}) ] \}$

By the same token, as long as low attachment structures for degree DPs are well-formed and interpretable, they should be sources for *specific* readings. It in fact seems clear that low attachment structures *are* well-formed and interpretable. This is confirmed by the existence of degree DPs like (33), as in, say, *The number of recently published books is greater than 20*. The DP in (33) has the plausible structure in (34). Modulo word order, and the internal structures of AP versus CP, (34) parallels the structure that (31) posits for (4). If (34) is well-formed and interpretable, then arguably the same holds for (31).

(33) the number of recently published books

(34) *Structure for (33):*

[<sub>DP</sub> the [<sub>NP</sub> number of [<sub>NP</sub> [<sub>AP</sub> recently published] books ] ] ]

It seems reasonable to conclude, then, that specific readings can indeed arise from low attaching relative clauses. With this established, we now turn our attention back to high attachment.

#### 4.2. Specificity from high attachment?

Under the pied-piping analysis, high attachment of a gapped clause is compatible with a specific reading. The specific reading of (4) can be attributed to an interpreted structure where the pied-piped DP is restored to a position *above* the intensional operator, as in (35).

(35) *Interpreted structure for (4) under pied-piping analysis:*

[<sub>DP</sub> the [<sub>CP</sub>  $wh_1$  [<sub>C'</sub> that [<sub>DP</sub>  $\exists$  [<sub>NP</sub> [<sub>MP</sub>  $t_{1,d}$  **number**] of [<sub>N</sub> books] ] ] ] ] ]  
 $\lambda_2[\text{has} [\text{VP}$  Amy to read  $t_{2,e}$  ] ] ] ] ]

In this structure, CP denotes in any given world  $w$  the set in (36). For this set to contain a number  $n$ , this number must be the cardinality of some plurality of books in  $w$  that Amy is required to read. The definite article can be assumed to pick out the largest member of this set. Capturing the intuited referent of (4) in its specific reading, this degree is the total number of particular books that Amy is required to read.

(36) *Denotation of CP in (35):*

$$\{n \in D_d: \exists x \in D_e [ \text{BOOKS}(w)(x) \wedge \#(x) = n \wedge \forall w' \in \text{Acc}(w) [ \text{READ}(w')(x)(\text{Amy}) ] ] \}$$

Importantly, echoing our remarks above about non-specific readings, the pied-piping analysis is not merely *consistent* with such a compositional pathway to specific readings, but it actually *predicts* such a composition to be available. The reason for this is that, as reported in previous literature (e.g. Rullmann (1995)), degree questions likewise permit specific readings. Thus, apart from permitting a non-specific reading, the *how many*-question in (29), repeated here, can ask about the number of specific books that Amy was required to read.

(29) How many books did Amy **have to** read?

For *how many*-questions there is no equivalent to low attachment parses for degree DPs with gapped clauses. Therefore, the observation that (29) permits a specific reading establishes that pied-piping in degree questions can be restored into a position above the intensional operator. Hence, under the pied-piping analysis of high attachment, the same is predicted to be possible in degree DPs.

In contrast to the pied-piping analysis, the degree-as-kinds analysis of degree DPs can be argued to render high attachment *incompatible* with specific readings. To see why, let us consider what it would take under this analysis to reconcile high attachment with the composition of a specific reading for (4): it would require an interpreted structure that alters (15) above as shown in (37).

(37) *Interpreted structure for (4) under degrees-as-kinds analysis?*

$$[_{DP} \text{ the } [_{NP} [_{NP} \text{ number of books}] [_{CP} \text{ wh}_1 [_{C'} \text{ that } \mathbf{t_{1,D}} \lambda_2[\mathbf{has} [_{VP} \text{ Amy to read } t_{2,e} ] ] ] ] ] ] ]$$

This structure posits that the covert relative pronoun *wh* has moved to its final position in the specifier of C in two steps. In a first step, *wh* moves to a position just above the modal, and from there in a second step to its final position. Crucially, the two steps differ in terms of the type of the trace left behind by the movement of *wh*, and hence the type of the variable abstracted over: the first step of the movement leaves a trace of type *e* and derives a predicate of objects, while the second step leaves a trace of type *D*, the type of rich degrees, and therefore derives a predicate of rich degrees for the relative clause as a whole. Given these type assignments, (37) can be shown to indeed express the intended specific reading. Crucially, in contrast to (15) above, the principle of Derived Kind Predication stated in (12) will not be triggered at the point where the lexical object predicate *read* composes with its argument, but instead at the point where the object predicate derived by the first step of movement is saturated. As a result, the CP in (37) is a predicate of rich degrees with the denotation shown in (38).

$$(38) \quad \llbracket [_{CP} \text{ wh}_1 [_{C'} \text{ that } \mathbf{t_{1,D}} \lambda_2[\mathbf{has} [_{VP} \text{ Amy to read } t_{2,e} ] ] ] ] \rrbracket^w = \{y \in D_D: \exists x \in D_e [ \cup y(w)(x) \wedge \forall w' \in \text{Acc}(w) [ \text{READ}(w')(x)(\text{Amy}) ] ] \}$$

In any given world *w*, this set contains all those rich degrees that have specific instantiations in *w* for which it holds that Amy has to read *them*. In the composition for (37) as whole, this CP denotation will combine with the denotations of NP and the definite article to yield the rich degree referent intuited for the specific reading.

The issue of whether the degrees-as-kinds analysis allows for specific readings to arise under high attachment hinges on whether structures like (37) are well-formed. The question raised by a structure like (37) is whether the principles of grammar can be assumed to allow for an expression that ordinarily leaves a trace ranging over nominalized properties to *also* leave a trace that ranges over individuals. Chierchia (1998), whose proposals about kind predication undergird the degrees-as-kinds analysis developed in Scontras (2017), is unequivocal in answering this question in the negative, noting that “there is no rhyme or reason why a kind-level expression should leave behind a sorted object-level variable”. If Chierchia is right, then the assumed derivation of an object predicate by the first step of movement in (37) is not actually viable, hence (37) is not in fact available as an input to semantic interpretation.

Notably, the assumption that kind-level expressions cannot leave object-level traces is a central ingredient to Chierchia’s theory of kind predication. Updating proposals in Carlson (1977), Chierchia aims to derive Carlson’s generalization that bare plurals cannot take inverse scope. To illustrate, Carlson’s example in (39) can only be read with existential quantification scoping under negation, not with the reverse scope: the sentence conveys that I didn’t see any spots on the floor, not just that there are some that I didn’t see.

(39) I didn’t see spots on the floor.

Following Carlson, Chierchia proposes to explain this scope profile of bare plurals on the basis of the assumption that they denote kinds. Chierchia’s innovation, which Scontras’ degrees-as-kinds analysis capitalizes on, is the assumption that kinds are nominalized properties, and that kind denoting expressions can saturate object predicates via Derived Kind Predication. As Chierchia stresses, in this elaboration of Carlson’s perspective on bare plurals, it is essential to prohibit kind-level expressions from leaving object-level traces. Without this prohibition, sentence (39), for example, would be expected to permit the interpreted structure in (40), where covert movement of the bare plural is shown to have left an object-level trace. With Derived Kind Predication introducing existential quantification above negation, this structure would incorrectly derive for (39) an unattested inverse scope interpretation.

(40) *Interpreted structure for (39)?*  
[spots on the floor]  $\lambda_1$ [I didn’t see  $t_{1,e}$ ]

Crucially, if structures like (40) are unavailable on the grounds that kind-level expressions cannot leave object level traces, then in the degrees-as-kinds approach, conceivable interpreted structures like (37) should likewise be ill-formed.

If this is so, then the degrees-as-kinds approach predicts high attachment in degree DPs to preclude specific readings. In this regard, the degrees-as-kinds approach parts company with the pied-piping analysis, which we saw renders high attachment compatible with specific readings. Notably, this observation does not by itself yield a testable prediction that could be hoped to adjudicate between the two analyses. The reason is that the relevant degree DPs are predicted to permit specific readings regardless of *how* high attachment is taken to compose, as these readings can in any case be attributed to *low* attachment structures.

That said, if a way could be found to somehow render low attachment unavailable as a source of a specific reading, then the difference with regard to high attachment identified here would

translate into a testable prediction. It is this general line of thought that we will now pursue.

## 5. The test from intermediate readings

In the introduction to this paper, we presented the schematic degree DP format in (6), repeated below, where  $Op_1$  and  $Op_2$  stand in for intensional operators within the gapped clause. We announced that data with this profile can serve as a possible test case to distinguish empirically between the degree-as-kinds analysis and the pied-piping analysis.

(6) *Form of test case:*

the  $[_{RS}$  number of books  $[_{GC} \dots \mathbf{Op_1} [\dots \mathbf{Op_2} [\dots \text{---} \dots ] ] ] ]$

We are now in a position to see why the two analyses under consideration make different predictions about degree DPs that instantiate this profile. To do so, it will be useful to attend to a concrete degree DP of this form. Consider the degree DP in (41), where  $Op_1$  and  $Op_2$  in (6) are instantiated by the embedding verbs *hear* and *prohibit*, respectively.

(41) the number of books Bill **heard** Amy is **prohibited** from reading

To begin, we note that for reasons that will be clear from the last section, a low attachment parse for a degree DP with the profile in (6), shown in (42), is expected to yield a reading that is fully specific. To illustrate, low attachment in example (41) would result in a modified NP which denotes the set of all books of which Bill has heard that Amy is prohibited from reading *them*. Inevitably, the degree DP in (41) as a whole will then wind up referring to the cardinality of the plurality of all such books—a fully specific reading.

(42) *Low attachment parse:*

the  $[_{NP}$  number of  $[_{NP}$   $[_{NP}$  books  $[_{CP}$   $wh_1 \dots \mathbf{Op_1} [\dots \mathbf{Op_2} [\dots t_{1,e} \dots ] ] ] ] ]$

With this in mind, consider now the schematic structure in (43). This structure elaborates the profile in (6) by assuming high attachment of the gapped clause, and by showing the high attachment parse as construed under the pied-piping analysis.

(43) *Surface form under pied-piping analysis:*

the  $[_{CP} [_{DP} \text{wh number of books}] [_{C'} \dots \text{---} [ \dots \mathbf{Op_1} [ \dots \text{---} [ \dots \mathbf{Op_2} [\dots \text{---} \dots ] ] ] ] ] ] ]$

The three lines in (43) mark three positions within the relative clause that the undoing of pied-piping is expected to be able to target. First, leaving just *wh* in the specifier of C, reconstruction might return the fronted DP to its base position, marked by the rightmost line in (43). For (41), this would yield a reading in which the DP refers to a number *n* such that Bill has heard that Amy is prohibited from reading *n* books—a fully non-specific reading. It seems clear that (41) can indeed be so understood. Second, reconstruction might target the position marked by the leftmost line in (43). For (41), this would result in DP referring to a number *n* such that there exist a plurality of *n* books of which it holds that Bill heard Amy is prohibited from reading *them*—the same fully specific reading that also results from low attachment.

Finally, and most importantly, reconstruction might target the position marked by the line between the two operators in (43). With this intermediate reconstruction site, (41) is expected

to refer to a number  $n$  such that Bill heard that there are  $n$  books such that Amy is prohibited from reading *them*. We believe that, crucially, this reading is available as well. To confirm this assessment, consider the assumption stated in the scenario in (44).

(44) “*Intermediate scope*” assumption:

Bill heard that there are two books that Amy is prohibited from reading.

Intuitively, this assumption suffices to ensure that (41) can be understood as referring to the number 2. This is so even though (44) fails to guarantee that (41) refers to the number 2, or indeed any natural number, in either its fully specific or its fully non-specific reading. That is, the DP may be judged to refer to the number 2 even though there may not be any particular two books that Bill heard Amy is prohibited from reading, and even though Bill may not have heard that Amy is prohibited from reading any two books.

Importantly, again echoing our earlier remarks, the pied-piping analysis is not merely *consistent* with the derivation of an intermediate scope reading for degree DPs with the profile in (43). Given the baseline from degree questions, this reading is *predicted*. The reason is that degree questions of the relevant form likewise permit such intermediate scope readings. Concretely, consider the *how many*-question counterpart of (41) in (45). It seems intuitively obvious that the assumption stated in (44) suffices to ensure that this question can be answered truthfully and completely by referring to the number 2.

(45) How many books did Bill **hear** Amy is **prohibited** from reading?

Turning now to the degree-as-kinds analysis, the finding that degree DPs of the form in (6) permit intermediate scope readings suggests that this analysis undergenerates. Crucially, as noted, a low attachment structure in such cases yields a fully specific reading, rather than the intermediate scope reading. In this case, high attachment is the only conceivable source of the reading in question. More concretely, an elaboration of (6) that derives intermediate scope under high attachment would have to look as in (46).

(46) *Required interpreted structure under degrees-as-kinds analysis:*

the [NP [NP number of books] [CP *wh*<sub>1</sub> ... **Op**<sub>1</sub> [... *t*<sub>1,D</sub>  $\lambda_2$  [ **Op**<sub>2</sub> [... *t*<sub>2,e</sub> ... ] ] ] ] ]

In this structure, *wh* moved to the intermediate scope position in a first step, creating a predicate of objects, and from there moves past the higher operator to its final landing site to derive a predicate of rich degrees. This would derive the intermediate scope reading in virtue of ensuring that Derived Kind Predication applies at the intermediate scope site. However, as reviewed in the last section, such a derivation is incompatible with assumptions about kind predication motivated in Chierchia (1998). If these assumptions are to be maintained, then intermediate scope readings are beyond the reach of the degrees-as-kinds analysis.

## 6. Conclusion

Given the assumptions about kinds and kind predication that it is built on, the degree-as-kinds analysis in Scontras (2017) fails to generate intermediate scope readings for degree DPs with intensional operators in a gapped clause. In contrast, supported by baseline data from degree

questions, the pied-piping analysis predicts the full range of scope options observed in such cases. Of course, the degree-as-kinds analysis could perhaps be revised to avoid the undergeneration. Derived Kind Predication could be replaced with a mechanism that turns expressions denoting rich degrees into existential quantifiers with flexible scope. However, while this is conceivable, the question is whether such a revision can be motivated on principled grounds.

One possible approach to revision could question the pied-piping analysis of degree questions. One might reject the notion of pied-piping in degree questions as a process that requires undoing for semantic interpretation. Perhaps degree questions, too, are instead built from rich degrees—and this is why the range of readings they permit is mirrored in degree DPs. Indeed, the degrees-as-kinds analysis of degree DPs could be taken to suggest that the construal of degrees as kinds *should* cut across different domains of grammar. However, a challenge for this direction of analysis seems to come from syntactic considerations. Barring added stipulations, the approach would seem to require that in *how many books*, *many books* is a predicate of rich degrees that restricts *how*. Yet syntactic evidence suggests that, as indeed taken for granted in the literature, *how many* is a constituent, and hence that *many books* is not.

Another avenue for revision, invited by the discussion in Scontras (2017) and foreshadowed in Section 2, abandons the reliance on Derived Kind Predication while retaining a link between degree DPs and kind DPs, like *the kind of book that Amy read*. Much like the pied-piping analysis, this approach would centrally posit a correlation between two domains of data. While scope options for kind DPs remain to be studied, the hope would be that they may match those for degree DPs. If it could be shown that kind DPs call for an analysis in terms of nominalized properties, then the range of scope options for degree DPs could be argued to follow under a degrees-as-kinds analysis from whatever enables the range of scope option in kind DPs. This approach too seems principled. However, it is at present unclear whether kind DPs *do* require an analysis in terms of nominalized properties. Wilkinson (1995) argues that the noun *kind* can enter into pseudopartitive structures in much the same way degree nouns like *number* do under the pied-piping analysis. This suggests that kind DPs of the relevant form might again be amenable to an analysis in terms of pied-piping, which does not appeal to nominalized properties. The case for a degrees-as-kinds analysis of degree DPs would then remain unclear.

Importantly, though, Scontras (2017) identifies another central desideratum for an analysis of degree DPs, regarding the ways such DPs can compose with their *external* syntactic environment. Scontras (2017) notes that degree DPs can not only saturate degree predicates, like *be greater than twenty* in (47a), but also object predicates, like *read* in (47b).

- (47) a. [The number of books that Amy read] is greater than twenty.  
 b. Bill read [the number of books that Amy read].

Scontras presents the degrees-as-kinds analysis as an explanation for the sort of external object predication illustrated by (47b). Its ability to render such data expected is an important benefit of the degrees-as-kinds analysis. That said, this analysis will need to be compared to conceivable alternative approaches to external object predication. We previously attempted to argue that for degree DPs of the relevant form, *external* object predication is sourced in a pseudopartitive syntax obscured by ellipsis, paralleling the pseudopartitive structure of the core *internal* to the DP (Alonso-Ovalle and Schwarz, 2023, 2024). We suggest that the final conclusions about the composition challenge from degree DPs will depend in part on the success of those attempts.

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