

Why *regret* and *realize* can embed false beliefs¹

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Abstract. Verbs of emotion (e.g., *regret*) and discovery (e.g., *realize*) presuppose factivity, but they can felicitously follow false-belief statements that suspend their factivity (Klein, 1975; Gazdar, 1979), in contrast to *know*. This paper addresses this puzzle by proposing a *believe*-based account with two ingredients: (i) a lexical semantics of *regret* and *realize* that contains a backgrounding operator turning at-issue meanings into presuppositions, and (ii) a theory of presupposition projection in attitude contexts (Karttunen, 1974; Heim, 1992; Sudo, 2014). The factivity of *regret* and *realize* is derived as a context-sensitive pragmatic implicature, while the factivity of *know* is a lexical presupposition that holds across contexts.

Keywords: factivity, presupposition, projection, attitude predicates.

1. Introduction

Factive clause-embedding predicates (e.g. *know*, *regret*, *realize*) trigger the inference that their complements are true (Kiparsky and Kiparsky, 1970), as illustrated in (1a). Evidence for this factive inference is provided in (1b): the proposition embedded under these verbs cannot be explicitly denied.

- (1) a. John **knows/regrets/realizes** that it is raining. \rightsquigarrow *It is raining*
b. #It is not raining, but John **knows/regrets/realizes** that it is raining.

However, in contrast to the cognitive factive *know*, emotive predicates (e.g., *regret*, *be glad*, *be surprised*) and verbs of discovery (e.g., *realize*, *discover*, *find out*) can embed false beliefs (Klein, 1975; Gazdar, 1979; Huddleston and Pullum, 2002; Schlenker, 2005; Egré, 2008; Klinedinst, 2016; Djärv et al., 2018; Djärv, 2019; Abrusán, 2023). For example, in (2), the factivity of *regret* and *realize* is suspended in false-belief contexts.

- (2) a. John falsely believes that Mary got married, and he **regrets/#knows** that she is no longer single. (adapted from Egré 2008)
b. Falsely believing that he had inflicted a fatal wound, John **realized/#came to know** that he was a murderer. (adapted from Gazdar 1979)

This paper presents a *believe*-based account to explain why verbs of emotion and discovery can embed false beliefs. I assume that *regret* and *realize* are lexically encoded with a backgrounding operator, which turns at-issue meanings into presuppositions. Adopting a theory of presupposition projection in attitude contexts (Karttunen, 1974; Heim, 1992; Sudo, 2014), I derive factive inferences as context-sensitive pragmatic implicatures.

The rest of this paper is organized as follows. Section 2 presents the puzzles. Section 3 introduces the lexical entries of *regret* and *realize*, incorporating a *believe*-based semantics with a backgrounding operator. Section 4 reviews theories of presupposition projection in attitude

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contexts and argues for the Karttunen-Heim-Sudo view. Section 5 explains the puzzles by combining the lexical semantics and the projection theory. Section 6 concludes.

2. Puzzles

Factive predicates trigger a factive inference (e.g., *it is raining* in (1a)). This inference is presupposed: first, it is backgrounded information that can project from a family of non-entailing contexts (Chierchia and McConnell-Ginet, 1990), as in (3); second, it is taken-for-granted information that passes the *hey-wait-a-minute* test (von Stechow, 2004), as diagnosed in (4).

- (3)
- a. John knows/regrets/realizes that it is raining.
 - b. John does not know/regret/realize that it is raining.
 - c. Does John know/regret/realize that it is raining?
 - d. Perhaps John knows/regrets/realizes that it is raining.
 - e. If John knows/regrets/realizes that it is raining, he will stay home.
 - f. \rightsquigarrow *It is raining*
- (4)
- A: John knows/regrets/realizes that it is raining.
 B: Hey wait a minute, I didn't know that it is raining.

Emotive factives and verbs of discovery indeed presuppose the truth of their complements in a 'simple-contradiction context', which involves an explicit denial of the complement. This has been shown by the infelicity of (1b).

However, the factivity of these predicates can be suspended in a 'false-belief context', where the attitude holder holds a mistaken belief. This observation is supported by numerous examples from the literature, as shown in (5) and (6).^{2,3}

- (5) **Emotive factives**
- a. Falsely believing that he had inflicted a fatal wound, Oedipus **regretted** killing the stranger on the road to Thebes. (Klein, 1975)
 - b. Ed believed that he had offended his parents and very much **regretted** that he had done so, but it turned out that he had been mistaken: they had not in the least been offended. (Huddleston and Pullum, 2002)
 - c. Sally misremembered not having left a tip and **regretted** it. (Karttunen, 2016)
 - d. John believes that Mary left and he **regrets** that. She never did go. (Zeevat, 1992)
 - e. Mary, who was under the illusion that it was Sunday, **was glad** that she could have a long lie-in. (Klein, 1975)
 - f. Jane misheard that the dinner was free and she **was glad** not to have to pay. (Abrusán 2023, after Karttunen 2016)
- (6) **Verbs of discovery**
- a. Mary, who was under the illusion that it was Sunday, **realized** that she could stay in bed. (Gazdar, 1979)
 - b. Believing that he had inflicted a fatal wound, Oedipus slowly **became aware** that he was a murderer. But the wound was not mortal. (Abrusán 2023, after Gazdar 1979)

²This observation holds regardless of whether the false-belief statement is syntactically a gerund or a main clause.

³These examples are preceded by discussions in Frege (1892), Moravcsik (1973), and Delacruz (1976).

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The contrast between simple-contradiction and false-belief contexts is further supported by Beaver and Zeevat's (2007) observations in (7) and (8). Both predicates may embed a false proposition only when the attitude holder's false belief is explicitly asserted.

- (7) a. Jane's keys were sitting in her bag. However, **she believed that her friend had them**, and she regretted that her friend had them.
b. Jane's keys were sitting in her bag. #However, she regretted that her friend had them.
- (8) a. John had never killed anyone. However, **he believed that he had inflicted a fatal wound**, and he realized that he was a murderer.
b. John had never killed anyone. #However, he realized that he was a murderer.

These contrasts raise two puzzles. The first is *the context puzzle*: why are *regret* and *realize* felicitous in false-belief contexts but infelicitous in simple-contradiction contexts? The second is *the factive puzzle*: why do *regret* and *realize* allow for the suspension of their factivity, in contrast to *know*?

3. Proposal

3.1. A non-factive lexical semantics of *regret* and *realize*

Emotive factives and verbs of discovery presuppose a belief and disbelief relation between the attitude holder and the embedded proposition, respectively. Specifically, *x regrets p* presupposes that the attitude holder *x* believes the proposition *p*. This inference is backgrounded, as shown by the projection test in (9), and it is taken-for-granted, as shown in (10).

- (9) a. John regrets that it is raining.
b. John does not regret that it is raining.
c. Does John regret that it is raining?
d. \rightsquigarrow *John believes that it is raining*
- (10) A: John regrets that it is raining.
B: Hey wait a minute, I didn't know that John believes that it is raining.

Furthermore, *x realizes p* presupposes that, for all time prior to the time of realization, *x* did not believe *p*. This inference also passes the diagnostics for presuppositions, as in (11) and (12).

- (11) a. John realizes that it is raining.
b. John does not realize that it is raining.
c. Does John realize that it is raining?
d. \rightsquigarrow *John did not previously believe that it is raining*
- (12) A: John realizes that it is raining.
B: Hey wait a minute, I didn't know that John didn't believe that it is raining.

In contrast, *x knows p* does not presuppose either a belief inference (*John believes that it is raining*) or a disbelief inference (*John does not believe that it is raining*). Both inferences fail to project in non-entailing contexts, such as in polar questions, as seen in (13c).

- (13) a. John knows that it is raining. \rightsquigarrow *x believes p*; $\not\rightsquigarrow$ *x doesn't believe p*
b. John does not know that it is raining. $\not\rightsquigarrow$ *x believes p*; \rightsquigarrow *x doesn't believe p*

- c. Does John know that it is raining? $\not\rightarrow x$ believes p ; $\not\rightarrow x$ doesn't believe p

Drawing on this contrast, I propose a *believe*-based lexical semantics for *regret* and *realize* in (14) and (15). Both predicates express two layers of meaning: a presupposition about the (dis-)belief relation and an assertion about the attitude holder's emotive or doxastic state.⁴

- (14) x regrets p
 a. Presupposition: x believes p (to be revised)
 b. Assertion: x has a negative attitude toward p
- (15) x realizes p
 a. Presupposition: x did not previously believe p (to be revised)
 b. Assertion: x currently believes p

Since *know* does not presuppose a (dis)belief relation, I assume that *know* lexically presupposes the truth of its complement and asserts the doxastic attitude of the attitude holder, as in (16).

- (16) x knows p
 a. Presupposition: p is true
 b. Assertion: x believes p

3.2. A novel ingredient: the backgrounding operator

The lexical entries in (14) and (15) do not yet predict that factivity arises from *regret* and *realize* in any contexts. To derive their factivity, I propose that their presuppositions contain a backgrounding operator, which turns at-issue meanings into presuppositions (Yimei Xiang, p.c.). This operator, as defined in (17), applies to a proposition p , turns the assertive meaning of p into a presupposition, and asserts a semantically vacuous tautology represented by **T**.

- (17) $\pi := \lambda p : p.\mathbf{T}$

This backgrounding operator is parallel to the *A*(ssertion)-operator (Bochvar, 1939; Link, 1986), but they serve opposite semantic functions: while the π -operator pushes at-issue meanings into the background, the *A*-operator asserts and foregrounds presuppositional meanings.⁵

⁴See a similar decomposition of emotive predicates in Beaver (2001); Schueler (2013). Beaver (2001: 158) suggests that *regret* is 'so intrinsically intensional that it is almost impossible to isolate a purely mental component for it in ordinary English.' In his attempt to describe the emotive attitude expressed by *regret*, Beaver suggests that *regret* asserts that the attitude holder has negative vibes arising from the belief about the embedded presupposition.

⁵Here I compare the π -operator with the *A*-operator (Bochvar, 1939; Link, 1986) and the presupposition ∂ -operator (Beaver, 2001). All three operators can be defined within a trivalent semantic system, which uses three truth values: 1 for truth, 0 for falsity, and # for undefinedness. The *A*(ssertion)-operator, as defined in (ia), asserts that ϕ is true and returns falsity otherwise. A failed presupposition that would have resulted in undefinedness now only yields falsity. In contrast, the presupposition ∂ -operator, as defined in (ib), asserts that ϕ is true and returns undefinedness otherwise. An unsatisfied asserted meaning now leads to undefinedness, as if the meaning were presupposed.

(i) a. $\llbracket A(\phi) \rrbracket = 1$ iff $\llbracket \phi \rrbracket = 1$; $\llbracket A(\phi) \rrbracket = 0$ otherwise.
 b. $\llbracket \partial(\phi) \rrbracket = 1$ iff $\llbracket \phi \rrbracket = 1$; $\llbracket \partial(\phi) \rrbracket = \#$ otherwise.

The semantic effect of π differs from that of *A* but coincides with that of ∂ . However, while the ∂ -operator typically marks presuppositions in dynamic semantics frameworks, the π -operator is defined in a static account and can be embedded under linguistic operators.

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I propose that, in the revised lexical presuppositions of *regret* and *realize*, the backgrounding operator π applies to the embedded propositions p . The revision is implemented in (18) and (19): the presuppositions now state a (dis)belief relation between the attitude holder x and $\pi(p)$ (cf. p in the previous version). The *believe*-complement $\pi(p)$ yields a tautology presupposing the truth of p , which I write as \mathbf{T}_p .

- (18) x *regrets* p
a. Presupposition: x *believes* $\pi(p)$ (final version)
b. Assertion: x *has a negative attitude toward* p
- (19) x *realizes* p
a. Presupposition: x *did not previously believe* $\pi(p)$ (final version)
b. Assertion: x *currently believes* p

The presuppositions of *regret* and *realize* now contain an embedded factive presupposition. Crucially, this presupposition is embedded under an attitude predicate *believe*, and it is expected to project in the same way as other presuppositions project in attitude contexts. In the next section, I discuss existing theories on how presuppositions project in attitude contexts.

4. Presupposition projection in attitude contexts

4.1. E-inferences and i-inferences

When a presupposition trigger is embedded under an attitude verb (e.g., *believe*, *imagine*, *hope*), it typically generates two inferences (Geurts, 1998): an ‘e(xternal)-inference’, which states that the presupposition holds in the actual world, and an ‘i(nternal)-inference’, which states that the presupposition is true according to the attitude holder’s belief.

For example, in (20), the verb *believe* embeds a cleft construction, which triggers an existential presupposition, namely, *someone came to the party*. Without additional contextual information, the sentence carries two inferences: (20a) that someone indeed came to the party in the actual world, and (20b) that, according to John’s belief, someone came to the party.

- (20) John believes that **it is** Andy **who** came to the party. (adapted from Sudo 2014)
a. e-inference: *Someone came to the party*
b. i-inference: *John believes that someone came to the party*

This pattern holds across presupposition triggers. Based on the classification of presupposition triggers in the literature (Zeevat, 1992; Cummins, 2019; Roberts, 2006), I next consider four classes of triggers: resolution triggers (e.g., *she*, *the ghost*, *his sister*), bookkeeping triggers (e.g., *too*, *again*), background implicature triggers (e.g., *only*, *almost*), and lexical triggers (e.g., *stop*, *still*).

4.1.1. Resolution triggers

Resolution triggers collect established entities and associate them with new information. For example, a pronoun presupposes the existence of an entity with matching phi-features (Sudo, 2012); a definite description presupposes the existence of an entity that meets the nominal description (Ahn, 2024); a possessive construction presupposes a possessive relation.

When embedded in attitude contexts, these triggers generate both e-inferences and i-inferences.

For example, in (21), the embedded definite description implies the existence of a man holding a martini, both in the actual world and in Barney's belief.

- (21) Barney believes that **the man with a martini** is drunk.
- a. e-inference: *Some individual satisfies the description of 'being a man with a martini' in the actual world*
 - b. i-inference: *Barney believes that some individual satisfies the description of 'being a man with a martini'*

However, nominal expressions in intensional contexts exhibit an ambiguity between *de re* ('about the thing') and *de dicto* ('about what is said') interpretations. For example, the NP in (21) can be read *de re*, referring to an actual man with a martini, or receive a *de dicto* reading, referring to a man who Barney believes is holding a martini.

The *de re* reading of these resolution presupposition triggers suspends the i-inference, whereas the *de dicto* reading suspends the e-inference (Geurts, 1998). In (22), the definite description is interpreted *de re*. Barney believes that John is drunk, but does not conceptualize John as the man with a martini. Thus, the i-inference is blocked.

- (22) (Context: Barney believes that John is drunk, but Barney doesn't know that John is holding a martini right now.)
Barney believes that **the man with a martini** is drunk.
- a. ✓ e-inference: *Some individual satisfies the description of 'being a man with a martini' in the actual world*
 - b. ✗ i-inference: *Barney believes that some individual satisfies the description of 'being a man with a martini'*

In contrast, in (23), the definite description is interpreted *de dicto*. John falsely believes that John is holding a martini, so the e-inference is absent.⁶

⁶This observation extends to possessive constructions and pronouns. As Geurts (1998) observes, the possessive construction *his sister* in (i) presupposes that Barney has a sister. When this DP is embedded under *believe*, it triggers an e-inference and an i-inference.

- (i) Barney believes that **his sister** is cute. (adapted from Geurts 1998)
- a. e-inference: *Barney has a sister*
 - b. i-inference: *Barney believes that he has a sister*

The *de re* and *de dicto* readings of this DP block the i-inference and the e-inference, as in (ii) and (iii), respectively.

- (ii) (Context: Barney doesn't know that he has a sister *x*. He is acquainted with *x* and believes that *x* is cute.)
Barney believes that his sister is cute. ↯ i-inference: *Barney believes he has a sister*
- (iii) (Context: Barney believes that *x* is his sister and *x* is cute. However, *x* is actually not his sister.)
Barney believes that his sister is cute. ↯ e-inference: *Barney has a sister*

Pronouns similarly trigger an e-inference and an i-inference in intensional contexts, as shown in (iv).

- (iv) Billy believes that **she** is cute.
- a. e-inference: *The referent satisfies the property of 'being female' in the actual world*
 - b. i-inference: *The referent satisfies the property of 'being female' in Billy's belief*

However, pronouns generally prefer the *de re* interpretation and only allow a *de dicto* use when the *de re* reading is contextually unavailable (Sudo, 2012). For example, in (v), when the speaker cannot confirm that the phonologist in Bill's story actually exists (thus eliminating the possibility of a *de dicto* reading), they are unlikely to endorse that the phonologist is female. Consequently, the e-inference does not hold.

- (v) (Context: Bill said that he attended a talk by some phonologist with John. We have no clue whether they actually attended such a talk. He might be telling us a lie.) (adapted from Sudo 2012)
Bill believes that John criticized her after her talk.

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- (23) (Context: Barney believes that John is holding a martini and he is drunk, but John is actually holding a martini glass full of water.)
Barney believes that **the man with a martini** is drunk.
- ✗ e-inference: *Some individual satisfies the description of 'being a man with a martini' in the actual world*
 - ✓ i-inference: *Barney believes that some individual satisfies the description of 'being a man with a martini'*

I return to the *de re/de dicto* ambiguity in Section 4.3.1.

4.1.2. Bookkeeping triggers

Similar to resolution triggers, bookkeeping triggers require linguistic antecedents. However, the foregrounded and backgrounded information in sentences with bookkeeping triggers are independent—removing them does not render the sentence incomprehensible (Glanzberg, 2005).

In attitude contexts, bookkeeping triggers generate both e-inferences and i-inferences. For example, in (24), *too* presupposes that some individual other than Susan is present. When *too* is embedded under *believe*, this individual is implied to exist in both John's belief world and the actual world.

- (24) John believes that Susan is here **too**.
- e-inference: *Someone other than Susan is here*
 - i-inference: *John believes that someone other than Susan is here*

4.1.3. Background implicature triggers

Background implicature triggers express preconditions that are logically independent from the foregrounded information, but they tend to be more informative than bookkeeping triggers. For example, *only* presupposes the truth of its prejacent (Roberts, 2006). When embedded under an attitude verb, as in (25), *only* implies that the prejacent (*Mary came*) is true both in the actual world and within John's belief.

- (25) John believes that **only** Mary came.
- e-inference: *Mary came*
 - i-inference: *John believes that Mary came*

4.1.4. Lexical triggers

Lexical triggers refer to 'actions and states that trigger presuppositions about their logical prerequisites' (Cummins 2019: 82). For example, the sentence *it has stopped raining* presupposes that it was previously raining. If this condition were false, it would be difficult to evaluate whether it has stopped raining. In attitude contexts, lexical triggers generate both e-inferences and i-inferences. In (26), the verb *stop* presupposes that it was previously raining, which is implied to hold both in reality and in John's belief.

- (26) John believes that it has **stopped** raining. (Sudo 2014)
- e-inference: *It was raining*

↗ e-inference: *The referent satisfies the properties of 'being female' in the actual world*

- b. i-inference: *John believes that it was raining*

4.2. Existing accounts

The linguistic status of e-inferences and i-inferences is debated in the literature. This section reviews three competing theoretical approaches: multi-accommodation theories (Zeevat, 1992; Mandelkern, 2016; Singh, 2020), e-inference first theories (Geurts, 1998), and i-inference first theories (Karttunen, 1974; Heim, 1992; Schlenker, 2009; Sudo, 2014).

First, multi-accommodation theories propose that both e-inferences and i-inferences are presuppositions. Zeevat (1992) argues that a presupposition embedded in an attitude context can be accommodated both globally (triggering the e-inference) and locally (triggering the i-inference). More recently, Singh (2020) builds on Gazdar's (1979) notion of 'potential presuppositions', suggesting that both inferences project as presuppositions by default. Mandelkern (2016), adopting the Local Dissatisfaction theory, predicts that both inferences are presuppositions that project to the matrix level.

However, this account makes two problematic predictions. For presuppositions embedded under other desire attitude predicates, such as *hope* and *want*, this account incorrectly predicts that (27) presupposes (27a) rather than the observed inference (27b). Moreover, for quantified sentences, such as (28), this account also incorrectly predicts that (28) presupposes (28a) rather than the observed inference (28b).

- (27) Mary hopes that it will stop raining.
 a. $\not\rightarrow$ *Mary hopes that it is raining*
 b. \rightsquigarrow *Mary believes that it is raining*
- (28) No woman in that room loves her dog.
 a. $\not\rightarrow$ *No woman in that room has a dog*
 b. \rightsquigarrow *Every woman in that room has a dog*

Second, e-inference first theories argue that only the e-inference is presupposed, while the i-inference is derived pragmatically. One proposed mechanism is 'importation' (Geurts, 1998): if a presupposition holds in the actual world, conversational participants infer that the attitude holder also believes the presupposition. However, importation lacks clear licensing conditions—it is unclear when this rule applies (see also Blumberg 2023).

Third, i-inference first theories argue that the i-inference is a transformed presupposition resulting from the lexical semantics of *believe* (Karttunen, 1974; Heim, 1992), while the e-inference is pragmatically derived from a presupposed i-inference (Heim, 1992; Sudo, 2014). Specifically, *x believes ϕ_p* (where ϕ presupposes p) presupposes that *x believes p* (i.e., the i-inference). Heim (1992) suggests that the e-inference follows pragmatically: when the hearer accommodates the presupposed i-inference, they take *x believes p* to be 'uncontroversial' and 'unsurprising' and adjust the context such that p is also true. Sudo (2014) further elaborates on this pragmatic reasoning, as outlined in (29).

- (29) a. The proposition *x believes ϕ_p* presupposes the i-inference *x believes ϕ* , or $B(x, p)$.
 b. The hearer accommodates this presupposition and assumes that $B(x, p)$ is commonly believed by conversational participants, written as $CB(B(x, p))$.
 c. The hearer compares two contexts: $CB(p) \wedge CB(B(x, p))$ vs. $\neg CB(p) \wedge CB(B(x, p))$.

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- d. The hearer determines that when p is commonly believed ($CB(p)$), the i-inference is not assertion-worthy and is naturally accommodated.
- e. Given that p is now commonly believed, one infers that p is true (the e-inference).

4.3. Evidence for i-inference first theories

This section argues for the i-inference first theories based on two observations: (i) a conjunction of a *believe*-proposition with its e-inference presupposes the i-inference, while a conjunction of a *believe*-proposition with its i-inference does not presuppose the e-inference; (ii) e-inferences can be canceled in contexts where the attitude holder has a false belief or is considered unreliable, while i-inferences cannot. I discuss each observation in turn.

First, the i-inference first theories correctly predict the semantics of propositions that conjoin a *believe*-proposition with either its i-inference or e-inference. Consider (30), where *believe* embeds a cleft construction that triggers an existential presupposition.

- (30) John believes that it is Andy who won.
- a. e-inference: *Someone won*
 - b. i-inference: *John believes that someone won*

When the *believe*-proposition is conjoined with its e-inference, as in (31a), the conjunction presupposes the i-inference. In contrast, when the *believe*-proposition is conjoined with its i-inference, as in (31b), the conjunction does not presuppose the e-inference.

- (31) a. Someone won, and John believes it is Andy who won.
 \rightsquigarrow *John believes that someone won*
- b. John believes someone won, and he believes it is Andy who won.
 $\not\rightsquigarrow$ *Someone won*

This pattern extends to the presupposition trigger *too* (Heim, 1992). In (32), the *believe*-proposition triggers both an e-inference and an i-inference. When this *believe*-proposition is conjoined with the e-inference, the i-inference remains, as in (33a). However, when the *believe*-proposition is conjoined with a proposition that entails the i-inference, the e-inference disappears, as in (33b).

- (32) John believes that Susan is here too.
- a. e-inference: *Someone other than Susan is here*
 - b. i-inference: *John believes that someone other than Susan is here*
- (33) a. Mary is here, and John believes that Susan is here too.
 \rightsquigarrow *John believes that someone other than Susan is here*
- b. John believes that Mary is here, and he believes that Susan is here too.
 $\not\rightsquigarrow$ *Someone other than Susan is here*

To better understand presupposition projection in conjunctions, I follow Karttunen (1974) in assuming that the second conjunct of a conjunction is evaluated in its *local context*, which results from updating the discourse with the first conjunct. Consequently, when the presupposition of the second conjunct is entailed by the first conjunct, this presupposition is not inherited by the conjunction. For example, in (34a), the possessive construction presupposes that John has a daughter. When the proposition in (34a) is conjoined with and preceded by its presupposition,

as in (34b), the presupposition is entailed in the local context of the second conjunct and thus does not project.

- (34) a. John's daughter is cute. \rightsquigarrow *John has a daughter*
 b. John has a daughter and John's daughter is cute. $\not\rightarrow$ *John has a daughter*

Adopting this assumption, I take (31) and (33) as evidence for the i-inference first theories. To begin with, for the conjunction of the *believe*-proposition and its e-inference to presuppose the i-inference, the i-inference must be presupposed by one of the conjuncts. Since the e-inference does not presuppose the i-inference, the *believe*-proposition must be the source of this presupposition. This supports the i-inference first theories, which predict that the *believe*-statement presupposes its i-inference. Next, the conjunction of the *believe*-proposition and its i-inference does not presuppose the e-inference. There are two theoretical possibilities: (a) neither conjunct presupposes the e-inference, or (b) the e-inference is presupposed by the *believe*-proposition, but it is entailed in the local context by the i-inference. Since the i-inference neither entails nor presupposes the e-inference, the second possibility is excluded. Therefore, the *believe*-proposition does not presuppose the e-inference, which aligns with the predictions of the i-inference first theories.

Second, e-inferences can be canceled in contexts where the attitude holder is deemed unreliable or holds a false belief (Schueler and Reese, 2014). For example, in (35), the e-inference does not arise because the attitude holder is considered unreliable, and the conspiracy theorist's beliefs are likely incongruous with the speaker's beliefs. Since presuppositions typically cannot be canceled due to such pragmatic factors, the e-inference is not a presupposition.

- (35) The conspiracy theorist believes that the CIA finally stopped monitoring his thoughts.
 a. ✗ e-inference: *The CIA was monitoring the conspiracy theorist's thoughts*
 b. ✓ i-inference: *The conspiracy theorist believes that the CIA was monitoring his thoughts*

In addition, e-inferences can be canceled in contexts where the attitude holder holds a false belief (Geurts, 1998). For example, in (36), the *believe*-proposition is based on Barney's false belief and does not trigger an e-inference.

- (36) (Context: Barney believes that it was raining earlier today and it has stopped raining now. However, it never rained today.)
 Barney believes that it has stopped raining.
 a. ✗ e-inference: *It was raining*
 b. ✓ i-inference: *Barney believes that it was raining*

4.3.1. Response to Geurts' (1998) puzzles

This section addresses two puzzles raised by Geurts (1998) that challenge the i-inference first theories. The first puzzle concerns (37), where *know* presupposes that it is commonly believed that *x believes p*, and yet this presupposition does not imply the truth of *p*. The i-inference first theories would incorrectly predict the inference in (37b), given the presuppositional status of the inference in (37a).

- (37) Fred knows that John believes that someone came to the party.
 a. \rightsquigarrow *John believes that someone came to the party*

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- b. $\not\rightarrow$ *Someone came to the party*

In response to this challenge, Sudo (2014) assumes that *believe* undergoes pragmatic competition with *know* and argues that *believe* is felicitous only when the complement is contingent or false. According to the Principle of *Maximize Presupposition!* (Heim, 1991), when two propositions share the same assertive meaning but differ in their presuppositions, the proposition with the stronger presupposition is preferred in contexts where the presupposition is satisfied. For example, the proposition in (38) has a stronger presupposition than that in (37). In a context where someone indeed came to the party, (38) with the stronger presupposition is preferred, leaving (37) pragmatically odd.

- (38) Fred knows that John knows that someone came to the party.
a. \rightsquigarrow *John knows that someone came to the party*
b. \rightsquigarrow *Someone came to the party*

This competition between *believe* and *know* similarly explains why in (39) the e-inference does not arise from the presupposed i-inference. The Principle of *Maximize Presupposition!* predicts that in a context where someone indeed came to the party, the *know*-proposition in (40), which carries a stronger presupposition, is preferred over the *believe*-proposition in (39).

- (39) It is John who believes that someone came to the party.
a. \rightsquigarrow *Someone believes that someone came to the party*
b. $\not\rightarrow$ *Someone came to the party*
- (40) It is John who knows that someone came to the party.
a. \rightsquigarrow *Someone knows that someone came to the party*
b. \rightsquigarrow *Someone came to the party*

The second puzzle concerns why the i-inference, if it is a presupposition, is cancelable in *de re* contexts. An example of this is (41), where Barney does not conceptualize *x* as his sister, even though *x* is in fact his sister. The definite expression *his sister* receives a *de re* interpretation, picking out Barney's sister in the actual world and reflecting the way in which the speaker—but not Barney—identifies the referent of *x*.

- (41) (Context: Barney doesn't know that he has a sister *x*. He is somehow acquainted with *x* and believes that *x* is cute.)
Barney believes that his sister is cute. (adapted from Geurts 1998)
a. \checkmark e-inference: *Barney has a sister*
b. \times i-inference: *Barney believes he has a sister*

I provide two responses to this challenge. First, the judgment in (41) has been challenged. Nelson (2024) argues that the *believe*-proposition is actually infelicitous in a *de re* context. Based on the data in (42), Nelson observes that '[e]ven though Sally's brother is the happy guy in the corner, (42b) below doesn't seem to follow from the truth of (42a), as Sally does not conceptualize the person she sees in the corner as her brother.'

- (42) (Context: Sally is watching a person across the room laughing who happens to be her brother, although unbeknownst to Sally, and says to herself, "He [pointing at the laughing man] is happy.") (Nelson 2024)
a. Sally believes that the happy guy in the corner is happy.

- b. #Sally believes that her brother is happy.

Second, only i-inferences triggered by embedded *nominal* presupposition triggers are blocked in *de re* contexts. Nominal triggers include pronouns (*she*), definite descriptions (*the ghost*), and possessive constructions (*his sister*). In contrast, i-inferences resulting from other types of triggers, which are incompatible with a *de re* interpretation, are immune to such cancellation. For example, in (43), the *believe*-proposition contains an embedded cleft construction, which does not support a *de re* interpretation. The context features John's false belief: while John does not conceptualize the event *e* as a party-attending event, *e* is in fact a party-attending event involving some agent (not necessarily Andy). In this *de re* context, the *believe*-proposition is infelicitous.

- (43) (Context: There was a party and some attended, but John doesn't know this. He is somehow acquainted with an event *e* and believes that Andy completed the event *e*.)
#John believes that it is Andy who came to the party.

A similar infelicity is observed with the bookkeeping trigger *too*, which also renders the *believe*-proposition infelicitous in a *de re* context.

- (44) (Context: There was a party and someone other than Andy came to the party, but John doesn't know this. He is somehow acquainted with an event *e* and believes that someone other than Andy completed this event.)
#John believes that Andy came to the party too.

Taken together, these observations suggest that i-inferences may only be blocked when the embedded presupposition triggers are nominal and thus compatible with a *de re* reading. Other types of triggers, which do not allow *de re* interpretations, would render the *believe*-proposition infelicitous. Therefore, the observation in (41) does not undermine the presuppositional nature of i-inferences.

4.4. Prediction: asserted vs. presupposed i-inferences

The i-inference first theories make a key prediction: an e-inference can be pragmatically derived only when the i-inference is presupposed rather than asserted. This prediction is borne out empirically. When the i-inference is presupposed, as in (45), the e-inference arises. In contrast, when the i-inference is explicitly asserted, as in (46), the e-inference does not arise.

- (45) a. Assertion: John believes that it is Andy who came to the party.
b. Presupposition: *John believes that someone came to the party*
c. Implicature: *Someone came to the party*
- (46) a. Assertion: John believes that someone came to the party.
b. Implicature: *The speaker is not sure if someone came to the party*

The absence of the e-inference in (46) can be explained by the pragmatic competition between *believe* and *know*. According to the Principle of *Maximize Presupposition!* (Heim, 1991), when the embedded proposition is true, the *know*-proposition (47b), which carries a stronger presupposition, is preferred over the *believe*-proposition in (46a)/(47a). Consequently, (47a) implies that the embedded proposition is either contingent or false.

- (47) a. John believes that someone came to the party. ↗ *Someone came to the party*

- b. John knows that someone came to the party. \rightsquigarrow *Someone came to the party*

5. Explaining the puzzles

5.1. The context puzzle and the factive puzzle

To explain the puzzles, I combine the *believe*-based semantics of *regret* and *realize* with the i-inference first theories. The context puzzle is repeated in (48): emotive factives and verbs of discovery are infelicitous in simple-contradiction contexts but felicitous in false-belief contexts.

- (48) a. It is not raining, #but John regrets that it is raining.
b. Falsely believing that it is raining, John regrets that it is raining.

The *believe*-based semantics of *regret* and *realize* states that they presuppose a belief or disbelief relation with embedded factivity. Specifically, *x regrets p* presupposes that *x believes $\pi(p)$* , where π is a backgrounding operator that turns the asserted *p* into a presupposition of a tautology, \mathbf{T}_p . Following the i-inference first theories, I argue that the presupposition of *regret* in (49) presupposes the i-inference in (49a) and pragmatically implies the e-inference in (49b).

- (49) *x believes \mathbf{T}_p*
a. i-inference: *x believes p*
b. e-inference: *p* is true

Similarly, *x realizes p* presupposes that *x didn't previously believe $\pi(p)$* . The i-inference first theories then derive the i-inference (50a) as a presupposition and the e-inference (50b) as a pragmatic implicature.

- (50) *x didn't previously believe \mathbf{T}_p*
a. i-inference: *x believes p*
b. e-inference: *p* is true

Factivity is now derived as a pragmatic implicature, which I next show to be context-sensitive. As discussed in Section 4.4, the i-inference first theories predict that the e-inference is only derived when the i-inference is presupposed, but not when it is asserted. Crucially, in the simple-contradiction context, the i-inference is presupposed, while in the false-belief context, it is asserted.

In the simple-contradiction context in (51), the i-inference *John believes that it is raining* is presupposed by the presupposition of *regret*. Since the i-inference is not entailed by the first conjunct, it is inherited by the conjunction.

- (51) #It is not raining, but John regrets that it is raining.
 \rightsquigarrow *John believes that it is raining*

In contrast, in the false-belief context in (52), the i-inference *x believes p* is explicitly asserted. Although the i-inference is still presupposed by the presupposition of *regret*, it is entailed by the preceding gerund and does not project to the matrix level.

- (52) Falsely believing that it is raining, John regrets that it is raining.
 \rightarrow *John believes that it is raining*

The predictions are as follows. In a simple-contradiction context, the i-inference is presup-

Table 1. Explaining the context puzzle.

Context	I-inference (x believes p)	E-inference (p)	Contradicting $\neg p$
Simple-contradiction	presupposed	derived	Yes
False-belief	asserted	not derived	No

posed, and the e-inference is derived. Consequently, a contradiction arises between the e-inference, which implies the truth of the complement, and the preceding proposition, which denies the truth of the complement. In contrast, in a false-belief context, the i-inference is asserted, and the e-inference cannot be pragmatically derived. Therefore, no contradiction arises, which makes *regret* and *realize* felicitous in this context. The predictions are summarized in Table 1.⁷

The factive puzzle is repeated in (53): the factivity of *regret* and *realize* can be suspended in a false-belief context, in contrast to that of *know*.

(53) Falsely believing that it is raining, John regrets/#knows that it is raining.

I attribute this contrast to the factive semantics of *know*: it lexically presupposes the truth of its complement, regardless of the context. In both the simple-contradiction and false-belief contexts, the complement is false, which contradicts the factivity presupposition of *know*. Therefore, the factivity of *know* cannot be suspended in false-belief contexts.

5.2. Formalizations

I formally define the semantics of *regret* and *realize* in (54) and (55), respectively. Following the Hintakkan analysis of attitude predicates (Uegaki, 2023), I use $\text{DOX}_x^{w,t}$ to refer to the set of worlds compatible with what x considers to be the case in w at time t . In (54), NA is a two-place predicate that asserts that its first argument has a negative attitude toward its second argument.

$$(54) \quad \llbracket \text{regret} \rrbracket = \lambda w. \lambda p. \lambda x : \underline{\text{DOX}_x^w \subseteq \pi(p)}. \text{NA}_w(x, p)$$

$$(55) \quad \llbracket \text{realize} \rrbracket^t = \lambda w. \lambda p. \lambda x : \underline{\forall t' < t [\neg \text{DOX}_x^{w,t'} \subseteq \pi(p)]}. \text{DOX}_x^{w,t} \subseteq p$$

In (55), *realize* is evaluated against a time parameter t , which represents the time of realization. The time variable does not appear in the definition of the backgrounding operator π , as shown in (56) (the intensionalized version of (17)). This definition does not specify at which time the presupposition should be true.

$$(56) \quad \pi := \lambda p. \lambda w : p(w). \mathbf{T}$$

Empirically, for *realize*-propositions, the tenses of the matrix and subordinate clauses are independent. This is supported by corpus data from COCA, as seen in (57) through (59).

- (57) *realized* + PAST/PRES/FUT:
- I finally realized that holding her **didn't** help her sleep.
 - I realized that these **are** incompatible statements.

⁷This analysis also accounts for the contrasts in (7) and (8). When the false belief is explicitly stated, the i-inference is asserted, and the e-inference cannot be derived. Therefore, no contradiction is predicted.

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- c. They want to attract Turkey into the conflict since they realized that they **are** stuck and **will** be defeated.

(58) *realizes* + PAST/PRES/FUT:

- a. He realizes that an injured zebra **got** on board from the ship during the storm.
 b. Many people say that she will come back after she realizes that she **made** a mistake.
 c. D'Antoni realizes that he **is** no longer in Phoenix or New York.
 d. He realizes that the name and logo **are** a concern to some Native Americans.
 e. The average person probably realizes that the rich **will** always be rich and control everything.

(59) *will realize* + PAST/PRES/FUT:

- a. The Nobel Committee will realize that we **came** up with this theory.
 b. Anyone who has read Gadaffi's little Green Book will realize that he **is** a thoughtful and enlightened leader.
 c. Maybe one day they will realize that they **accomplish** nothing constructive with this kind of thing.
 d. The businesses will realize that expenditures **will** exceed revenue.

These data suggest that the complements of *realize* are evaluated as true at the utterance time, rather than at the time of realization. To capture this, I adopt the tense semantics framework from von Stechow (2009), which assumes that propositions are of type $\langle s, \langle m, t \rangle \rangle$, where s represents the type of possible worlds and m the type of time points. I redefine π in (60): it now presupposes that the proposition it applies to is true at the utterance time. This ensures that the e-inference holds at the utterance time.

$$(60) \quad \pi := \lambda p. \lambda w. \lambda t : p(w)(t^*). \mathbf{T}$$

(where t^* represents the utterance time)

To illustrate, I derive the meaning of *Andy realized that John is a murderer*. The atomic proposition *John is a murderer* denotes the set of possible worlds in which John is a murderer at the utterance time t^* , as shown in (61).

$$(61) \quad \llbracket \text{John is a murderer} \rrbracket^{t^*} = \lambda w. \exists t = t^* [\text{murderer}_{w,t}(j)]$$

Embedding this proposition under *realize* yields the semantics in (62), which asserts that the realization event happened prior to t^* .

$$(62) \quad \llbracket \text{Andy realized that John is a murderer} \rrbracket^{t^*}$$

$$= \lambda w. \exists t < t^* [\text{DOX}_a^{w,t} \subseteq \lambda w. \lambda t'. \exists t' = t [\text{murderer}_{w,t'}(j)]]$$

defined if $\lambda w. \exists t < t^* [\forall t' < t [\neg \text{DOX}_a^{w,t'} \subseteq \pi(\lambda w. \lambda t''. \exists t'' = t' [\text{murderer}_{w,t''}(j)])]]$

Finally, applying the backgrounding operator to the embedded proposition yields the meaning in (63), which presupposes that John is a murderer at the utterance time t^* .

$$(63) \quad \pi(\lambda w. \lambda t''. \exists t'' = t' [\text{murderer}_{w,t''}(j)]) = \lambda w. \lambda t : \text{murderer}_{w,t^*}(j). \mathbf{T}$$

6. Conclusion

This paper explains why verbs of emotion and discovery, such as *regret* and *realize*, can embed false beliefs and have their factivity suspended. I propose a *believe*-based semantics for these predicates, incorporating a backgrounding operator that turns at-issue meanings into presuppositions. Adopting a theory of presupposition projection in attitude contexts, I derive the factivity of these predicates as context-sensitive pragmatic implicatures.

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