A non-Boolean analysis of conjunction in additive numerals: A mereotopological approach[1]

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Abstract. In this paper, I investigate the underlying meaning of additive numerals such as *two hundred and three*. Cross-linguistically, additive numerals often involve a conjunction marker (e.g., Hurford, 1975; Ionin and Matushansky, 2018), which indicates a deep relationship between the expression of arithmetical addition and mereological sum formation. Inspired by the classical results in ontology of numbers showing that numbers are not primitives, but rather complex derived objects, and the ideas concerning the spatial metaphor for numbers by Nouwen (2016) and Matushansky and Zwarts (2017), I propose a mereotopological approach (Grimm 2012 et seq.) to the part-whole structure of additive numerals. I argue that in language numbers are conceptualized as vertically-oriented 1-dimensional maximally self-connected entities (type *e*) of a given height that can be fused by mereological sum formation. Given the nature of space they occupy and the adopted mereotopological framework, the result is a new greater maximally self-connected object that can be mapped onto its higher point. By utilizing ontological assumptions that were developed independently, the proposed systems allows to capture conjunction in additive numerals as non-Boolean 'and'.

Keywords: additive numerals, numbers, conjunction, mereology, mereotopology.

1. Introduction

In this paper, I explore the underlying meaning of additive numerals involving a conjunction marker such as *two hundred and three* (e.g., Hurford, 1975; Ionin and Matushansky, 2018). I argue that 'and' in additive numerals can be captured in non-Boolean terms as encoding mereological sum formation if we adopt a set of ontological assumptions including a view that numbers are conceptualized in language as vertically-oriented 1-dimensional entities (cf. Nouwen, 2016; Matushansky and Zwarts, 2017) and a mereotopological approach to part-whole structures (Grimm 2012 et seq.).

In ontology of numbers, it is standardly assumed that numbers are not primitive. Rather, they are assumed to be complex derived objects. For instance, Frege (1884) takes numbers to be higher ordered sets. Cantor (1895) proposes them to be sets of units (abstract objects deprived from all their individuating features beyond their being distinct from one another). Finally, Von Neumann's (1923) ordinals are complex set-theoretic objects. In each case, numbers are constructed from more primitive notions using set-formation.

At the same time, for at least half a century, a potential relationship between linguistic and arithmetical capacity has been seriously explored (see Hurford, 1975; Chomsky, 2008; Watanabe, 2017). For instance, proposals grounded in minimalism relate capacity for language and arithmetic by deriving arithmetic via recursive application of Merge to the minimal lexicon in-

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volving only one lexical item (Chomsky, 2008; Watanabe, 2017). Again, the result is a complex object constructed from more primitive notions.

Of course, in the context of the above, one should distinguish two independent questions. The first one concerns what numbers are, whereas the second is: how are numbers represented in language? I argue that adopting the proposed set of ontological assumptions concerning the linguistic representation of numbers allows us to understand the role of the conjunction marker in additive numerals in accordance with its semantic role when coordinating nominals.

It is well known that *and* is ambiguous between Boolean conjunction, illustrated in (1a), which is typically modeled in terms of set intersection, and non-Boolean conjunction, as in (1b), which is standardly captured as mereological sum formation (e.g., Hoeksema, 1988; Krifka, 1990; Lasersohn, 1995; Winter, 1996, 2001; Champollion, 2016; Schein, 2017; Schmitt, 2020).

(1) a. Mary went to the gym **and** Sue went to the office.
b. Mary **and** Sue met.

What is much less understood is the meaning of conjunction in additive numerals such as (2) where it encodes something that seems to be an entirely different operation, namely arithmetical addition. This gives rise to a three-way ambiguity adding even more intricacy to the puzzling behavior of *and*.

(2) two hundred **and** three

The proposals arguing to derive non-Boolean conjunction from the Boolean one or vice versa are well known. In this paper, I do not intend to contribute to the debate on whether and, if so, how to relate the two notions. Instead, I will attempt to contribute to our understanding of the meaning of conjunction in additive numerals and its relationship with its other meanings.

To the best of my knowledge, only Ionin and Matushansky (2006, 2018) took the phenomenon in (2) seriously and developed an account that attempts to capture the three-way ambiguity of *and*. Inspired by their work, I will attempt to answer the following question: is it possible to view linguistic expression of arithmetical addition in terms of (non-)Boolean conjunction? My answer is that yes, we can capture it as non-Boolean sum formation of 1-dimensional objects in 1-dimensional space. For this purpose, I will need to adopt two main assumptions both of which were developed in the field for independent reasons. First, I embrace a mereotopological approach to the notion of part-whole (Grimm 2012 et seq.), which assumes structured parthood involving topological relations holding between parts of a whole. The second assumption concerns linguistic representation of numbers as abstract vertically oriented vector-like objects in 1-dimensional space (see Nouwen, 2016; Matushansky and Zwarts, 2017). Combining these two assumptions allows for developing a non-Boolean analysis of *and* in additive numerals that will avoid certain problems with Ionin and Matushansky's proposal. This, in turn, has the desired consequence that there is no need to postulate a special meaning for arithmetical addition.

The paper is outlined as follows. In Section 2, I introduce the phenomenon of conjunction in additive numerals and discuss the previous analysis by Ionin and Matushansky (2018) and its problems with deriving the arithmetical meaning of additive numerals. In Section 3, I discuss several observations concerning the behavior of conjunction in additive numerals and the use of spatial language in relation to numbers. In Section 4, I introduce the assumptions adopted for the purpose of the analysis. They concern the interpretation of ‘and’ in additive numerals, syntactico-semantic structure of numerals, ontology of linguistic representation of numbers and mereotopological approach to part-whole structures. In Section 5, I spell out the details of the proposal based on these assumptions. The core idea is that within the adopted framework it is possible to capture the meaning of conjunction in additive numerals in standard non-Boolean terms as mereological sum formation if we take numbers to be underlyingly conceptualized as abstract integrated entities in 1-dimensional space. Section 6 concludes the paper.

2. Conjunction in additive numerals

Let us begin with a brief overview of the phenomenon of conjunction in additive numerals to be investigated in this paper.

2.1. Phenomenon

Cross-linguistically, addition in additive numerals can be expressed via juxtaposition, the comitative preposition ‘with’ and the vertical locative preposition ‘on’, but most often it is marked by a conjunction marker (e.g., Hurford, 1975, 1987; Greenberg, 1978; Ionin and Matushansky, 2018). For instance, German (3)–(4), Czech (5)–(6) and Farsi (7)–(8) all use the same marker in additive numerals and in coordinated subjects of sentences with collective predicates such as ‘meet’. This cross-linguistic tendency indicates a deep relationship between linguistic expression of arithmetical addition and mereological sum formation.

(3) drei-**und**-zwanzig
three-and-twenty
‘twenty-three’ (German)

(4) Heidi **und** Alma haben sich getroffen.
Heidi and Alma AUX.PL REFL met
‘Heidi and Alma met.’ (German)

(5) tři-**a**-dvacet
three-and-twenty
‘twenty-three’ (Czech)

(6) Jana **a** Anna se potkal-y.
Jana and Anna REFL met-PL
‘Jana and Anna met.’ (Czech)

(7) bist **o** se
twenty and three
‘twenty-three’ (Farsi)

(8) Heidi **o** Alma ham-digar-o dide-and.
Heidi and Alma each-other-ACC seen-PL
‘Heidi and Alma have met.’ (Farsi)

Four decades of the research on part-whole structures revealed robust evidence for the relevance of pluralities across multiple ontological domains. In addition to the seminal work by Link (1983), who argued for pluralities of individuals, many proposals for mereologies of other ontological categories were put forward, including events (Bach, 1986), information states (Krifka, 1996), times (Artstein and Francez, 2003), roles (Wągiel, 2021a), propositions (Lahiri, 2002), questions (Beck and Sharvit, 2002), functions (Schmitt, 2019) and, most importantly from the perspective of this paper, degrees (Dotlačil and Nouwen, 2016).

It would then seem natural to treat conjunction in additive numerals on a par with other environments were it receives a non-Boolean interpretation as mereological sum formation $\sqcup$. However, applying straightforwardly a non-Boolean analysis of ‘and’ to (3) yields a wrong result, see (9) analogous to (10).

(9) $[\![\text{drei-und-zwanzig}]\!] = 3 \sqcup 20$

(10) $[\![\text{Heidi und Alma}]\!] = \textsc{h} \sqcup \textsc{a}$

The reason is that (9) refers to a plurality of two numbers instead of a single greater number, that is 23, which is the sum of the addends 3 and 20.

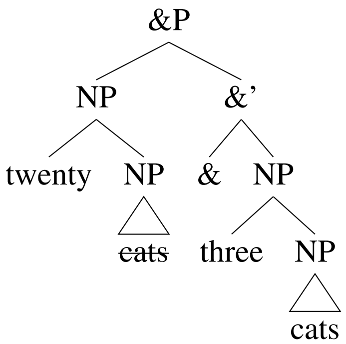
Nonetheless, I will argue that despite the incorrect result of the straightforward application of mereological sum formation, the interpretation of 'and' in examples such as (3) in non-Boolean terms is possible if we adopt a set of specific ontological assumptions concerning the nature and internal structure of numbers, as conceptualized in language. However, before we move to spelling out these assumptions, let us first discuss Ionin and Matushansky's account for conjunction in additive numerals.

2.2. Previous analysis

In their inspiring and influential account of complex numerals, Ionin and Matushansky (2006, 2018) argue that QPs involving additive numerals such as (11) have the syntactic structure in (12), where & represents conjunction, which can be expressed overtly, as in (2). On this analysis, *twenty* and *three* do not form a constituent in (12). Rather, the account postulates two full conjoined NPs with the first occurrence elided either via the RNR or phonological deletion.

(11) twenty-three cats

(12)



On the semantic side, Ionin and Matushansky propose the semantics for numerals as in (13), where the numeral is treated as a predicate modifier that provides the cardinality of a partition of a plural individual.[2] In (13), S is a partition Π of an individual x and the cardinality of S equals 3. For instance, (14) denotes a set of pluralities divisible into 3 non-overlapping individuals who are cats.

(13) $[\![\text{three}]\!] = \lambda P_{\langle e,t\rangle} \lambda x_e \exists S_{\langle e,t\rangle}[\Pi(S)(x) \wedge |S| = 3 \wedge \forall s \in S[P(s)]]$

(14) $[\![\text{three cats}]\!] = \lambda x_e \exists S_{\langle e,t\rangle}[\Pi(S)(x) \wedge |S| = 3 \wedge \forall s \in S[[\![\text{cat}]\!](s)]]$

Furthermore, Ionin and Matushansky (2018) observe that applying in (12) a standard type-lifted Boolean semantics of 'and', as given in (15), yields an incorrect result. The reason is that in (16) we end up with an individual that is simultaneously three cats and twenty cats, which is of course not a desirable result.

[2] A partition of a plurality x is a cover of x, i.e., a set of individuals such that the sum of all those individuals forms x. Moreover, a partition is a cover whose cells do not overlap (Gillon, 1987).

(15) $[\![\text{and}]\!] = \lambda P_{\langle e,t\rangle}\lambda Q_{\langle e,t\rangle}\lambda x_e[P(x) \wedge Q(x)]$

(16) $[\![\text{twenty-three cats}]\!] = [\![\text{and}]\!]([\![\text{three cats}]\!])([\![\text{twenty cats}]\!]) =$
$= \lambda x_e[[\![\text{three cats}]\!](x) \wedge [\![\text{twenty cats}]\!](x)]$

To solve this problem, Ionin and Matushansky (2018) develop an analysis based on the modified intersective theory of conjunction by Champollion (2016), which utilizes Existential Raising (ER), as defined in (17), combined with Minimization, which ensures minimal sets with no other entities (Winter, 2001). Application of this operation to each conjunct results in the denotation in (18).

(17) $[\![\text{ER}]\!] = \lambda P_{\langle \tau,t\rangle}\lambda Q_{\langle \tau,t\rangle}\exists x_\tau[x \in (P \cap Q)]$

(18) $[\![\text{twenty-three cats}]\!] = [\![\text{and}]\!]\big(\text{ER}([\![\text{three cats}]\!])\big)\big(\text{ER}([\![\text{twenty cats}]\!])\big) =$
$= \lambda P_{\langle e,t\rangle}\exists x_e\exists y_e[[\![\text{three cats}]\!](x) \wedge [\![\text{twenty cats}]\!](y) \wedge P(x) \wedge P(y)]$

Though the analysis in (18) solves the problem describes above, namely that an individual is simultaneously three cats and twenty cats, it still needs to assume a pragmatic requirement of non-overlap between the relevant sets in order to yield the desired truth conditions. Nevertheless, it is an important result, which shows that there is no need to postulate a special meaning of conjunction in QPs involving additive numerals, but rather its semantics can be captured in terms of the standard conjunction meaning.

This result, however, is restricted to the quantifying function of numerals when they are used as prenominal modifiers that count entities denoted by the modified NP. In the next section, I will discuss a problem with Ionin and Matushansky's approach in dealing with the arithmetic meaning of numerals.

2.3. Puzzle

Though most of the research on numerals has concerned their quantifying function in QPs such as *three cats*, there is a growing body of literature dedicated to the arithmetical meaning of numerals and its relationship with the quantifying meaning (e.g., Wiese, 2003; Bultinck, 2005; Moltmann, 2013, 2017; Rothstein, 2013, 2017; Wągiel and Caha, 2021). In (19), the numeral *three* does not count anything but rather it refers to an abstract mathematical object. It has been demonstrated that the arithmetical function of numerals has different properties than its quantifying use in (11).

(19) a. **Three** is an odd number.
b. Six divided by **three** equals two.

In order to capture the arithmetical meaning of numerals, as in (19), Ionin and Matushansky (2018) propose a silent nominalizing operation NOMNUM with the semantics in (20). When applied to a numerical predicate modifier, NOMNUM turns it into a singular term, which is an expression referring to a numeric value. For instance, for the numeral *three* it yields its corresponding cardinality, see (21).

(20) $\text{NOMNUM}(card)_{\langle\langle e,t\rangle,\langle e,t\rangle\rangle} = \iota x \forall P_{\langle e,t\rangle}\forall y[{*P}(y) \wedge |y| = x \rightarrow card(P)(y)]$

(21) $\text{NOMNUM}([\![\text{three}]\!]) = 3$

Of course, additive numerals also have the arithmetical meaning and can be used to refer to mathematical entities, as demonstrated in (22).

(22) a. **Twenty-three** is an odd number.
b. Forty-six divided by **twenty-three** equals two.

The problem is that on Ionin and Matushansky's view, additive numerals are not constituents, recall (12), and thus do not denote predicate modifiers required as the input of NOMNUM. Hence, the shift cannot be applied to the additive numeral as a whole, as used in the arithmetical environments in (22). Moreover, the role of conjunction in additive numerals used in their arithmetical function is left unexplained since in (22) it seems that conjunction should operate directly on numerical expressions. Therefore, this aspect of the meaning of the expressions in question needs to be further explored.

One might attempt to rescue Ionin and Matushansky's analysis by postulating a modified version of the nominalizing operation NOMNUM, which I will call here NOMNUM'. I will refrain from spelling out its exact semantics but the key difference is that NOMNUM' would take a structure with ellipsis as its input. Specifically, one could assume that a sentence such as (22a) in fact has the structure in (23), where the numerals *twenty* and *three* modify a very vague semantically unspecified nominal predicate such as *thing*, which then gets elided. Hence, NOMNUM' would take a vague plurality as its input and yield a number corresponding to its cardinality as an output.

(23) [NOMNUM' [twenty ~~things~~ and three ~~things~~]] AUX an odd number

The solution suggested above would solve the problem with the inability of Ionin and Matushansky's original proposal to derive arithmetical meanings of additive numerals. However, there are at least three serious challenges it faces.

2.4. Challenges

It is well known that in general there are several problems with Ionin and Matushansky's approach to numerals (e.g., He, 2015; He et al., 2017; Tatsumi, 2021). However, in this paper I limit my focus to three specific challenges that a potential attempt to rescue their system in the context of deriving the arithmetical meaning of additive numerals, as suggested in the (23), would face.

The first problem with the solution suggested above is that given the syntactic structure in (23), we would expect additive numerals in their arithmetical function to pattern in terms of number agreement with nominal conjunctions and show plural agreement, as in (24). This is, however, contrary to fact, as witnessed in (25). Notice that though (25c) is not ungrammatical, it requires the plural nominal in predicate position and it forces the distributive interpretation, i.e., the number 200 and the number 3 are a natural number each. Assuming the underlying structure in (23), the behavior in (25) is unexpected.

(24) a. *The two guards and the three forwards **is** a good team.
b. The two guards and the three forwards **are** a good team.

(25) a. Two hundred and three **is** a natural number.
b. *Two hundred and three **are** a natural number.

c. Two hundred and three **are** natural **numbers**. DISTRIBUTIVE

The second challenge concerns the distribution of predicates denoting arithmetical properties and relations. As observed by Rothstein (2013, 2017), QPs with numerals modifying semantically trivial nominals such as *thing* behave unlike bare numerals and still pattern with other QPs involving numerals modifying NPs in that they are incompatible with arithmetical predicates. This problem, of course, extends also to additive numerals, which is demonstrated by the contrast between (26) and (27).

(26) a. Forty-six divided by **twenty-three** equals two.
b. **Twenty-three** is a natural number.

(27) a. #Forty-six things divided by **twenty-three things** equals two things.
b. #**Twenty-three things** are a natural number.
c. ***Twenty-three things** is a natural number.

The third problem to be discussed in this paper concerns the behavior of classifier phrases in certain classifier languages with additive numerals. Specifically, colloquial Farsi has both overt conjunction markers in additive numerals and obligatory numeral classifiers.[3] The example in (28) shows that in constructions with numerals the classifier cannot be omitted and *o* in (29) is a conjunction marker, recall also (7).

(28) se ***(tâ)** ketâb
three CLF book
'three books' (Colloquial Farsi)

(29) bist **o** se
twenty and three
'twenty-three' (Colloquial Farsi)

Given the syntactic structure suggested in (23), we would expect two occurrences of the classifier in constructions with additive numerals. However, contrary to this expectation we observe that only one occurrence of the classifier is allowed, as demonstrated in (30). Specifically, the numeral-internal occurrence of the classifier after *bist* 'twenty' is illicit, whereas the classifier has to appear between the additive numeral and the modified NP.

(30) bist **(*tâ)** o se ***(tâ)** ketâb
twenty CLF and three CLF book
'twenty-three books' (Colloquial Farsi)

Importantly, the fact that in (30) the classifier does not appear after the first constituent of the additive numeral cannot be explained by postulating the ellipsis of the classifier phrase (or whatever syntactic label one would assign to the classifier+noun complex) along the lines of (31). The reason is that the ellipsis of the classifier and noun to the exclusion of the numeral is impossible in other relevant environments in which the classifier is obligatorily present in the absence of the noun. These include, e.g., fragment answers, as illustrated in (32).

(31) [$_{\&P}$ [$_{ClP}$ 'twenty' [$_{ClP}$ ~~CLF 'book'~~]] 'and' [$_{ClP}$ 'three' [$_{ClP}$ CLF 'book']]]

[3]All Colloquial Farsi examples discussed in this paper are due to Fereshteh Modarresi (p.c.). I would like to sincerely thank her for sharing and discussing the data with me.

(32) Q: How many cats sleep?
  a. Se ***(tâ)**.
  three CLF
  'Three.'
  b. Bist (*tâ) o se ***(tâ)**.
  twenty CLF and three CLF
  'Twenty-three.' (Colloquial Farsi)

Moreover, no classifier can appear in arithmetical uses of additive numerals, see (33), which shows that an analysis in the spirit of (23) is inadequate.

(33) a. bist (*tâ) o se addad-e fard e
  twenty CLF and three number-LNK odd is
  'Twenty-three is an odd number.'
  b. #bist (*tâ) o se **tâ** addad-e fard e
  twenty CLF and three CLF number-LNK odd is
  Intended: 'Twenty-three is an odd number.' (Colloquial Farsi)

Based on the challenges discussed above, I conclude that an approach in the spirit of Ionin and Matushansky (2018) suggested in (23) is problematic and does not seem to offer a solution to the puzzle concerning the meaning of conjunction in arithmetical uses of numerals. Before I spell out my own proposal, let us first discuss two sets of observations regarding conjunction in additive numerals.

3. Observations

In this section, I will discuss several observations concerning properties of conjunction in additive numerals and the use of spatial expressions as numeral modifiers. While the observations regarding conjunction are (to the best of my knowledge) novel, most of the facts regarding numeral modifiers are well known in the linguistic literature. However, my contribution is that I suggest to view these facts as indicating a particular way in which numbers are conceptualized in natural language, namely as abstract vertically oriented objects. And this conceptualization will help us to understand the role of conjunction in additive numerals.

3.1. Conjunction

The first observation is that conjunction in additive numerals poses a challenge for a conjunction-reduction analysis (e.g., Gleitman, 1965; Ross, 1967). The key data are provided in (34), where the truth-conditions of (34a) differ form those of (34b)–(34c). While the former is true, the latter are not. This shows that conjunction in additive numerals cannot be reduced to disguised instances of sentential conjunction. In this way, conjunction in additive numerals patterns with nominal conjunction with collective predicates. Consequently, the standard arguments for a non-Boolean treatment based on sum formation apply.

(34) a. Tři-**a**-dvacet je liché číslo.
  three-and-twenty is odd number
  'Twenty-three is an odd number.'
  b. Tři **a** dvacet jsou lichá čísla.
  three and twenty are odd numbers

'Three and twenty are odd numbers.'
c. Tři je liché číslo **a** dvacet je liché číslo.
three is odd number and twenty is odd number
'Three is an odd number and twenty is an odd number.' (Czech)

Second, cross-linguistically one can frequently encounter two types of conjunction, which are marked differently, namely regular (non-distributive) conjunction and distributive conjunction (e.g., Szabolcsi, 2015; Mitrović and Sauerland, 2016; Flor et al., 2017). It seems, however, that only regular conjunctions work in additive numerals. For instance, the data in (35) show the distinction between the two types in Czech (Dočekal et al., 2022). The contrast in (36) demonstrates that the distributive 'and' is illicit in additive numerals.

(35) a. Kluk **a** dívka rozbili dvě okna.
boy and girl broke two windows
'A boy and a girl broke two windows.'
b. Kluk **i** dívka rozbili dvě okna.
boy $\text{and}_{\text{DISTR}}$ girl broke two windows
'A boy and a girl each broke two windows.' (Czech)

(36) a. tři-**a**-dvacet
three-and-twenty
'twenty-three'
b. *tři-**i**-dvacet
three-$\text{and}_{\text{DISTR}}$-twenty
Intended: 'twenty-three' (Czech)

These two observations suggest that conjunction in additive numerals relates to sum formation.

3.2. Spatial metaphor for number

The second set of observations concerns the spatial metaphor in relation to numerals (Lakoff and Johnson, 1980; Corver and Zwarts, 2006; Nouwen, 2016). When talking about numbers, we often use linguistic expressions that invoke vertical orientation. This is illustrated by the examples in (37).

(37) a. One million is a { **high / big / large** } number.
b. Ten is { **higher / bigger / larger** } than nine.
c. John found { **over / under** } 50 typos in the manuscript.
d. Jasper is allowed to invite **up to** 10 children to his party.
e. { **growth / rise / fall** } in a number
f. a number { **jumped / dropped** }

For instance, in (37a)–(37b) the vertically oriented adjective *high* and the adjectives *big* and *large*, which can be interpreted this way, are used to express properties of numbers concerning their relative ordering both in positive and comparative contexts. Moreover, it is well known that only vertically oriented spatial prepositions can be used as numeral modifiers, see (37c)–(37d) (Corver and Zwarts, 2006). Finally, we use expressions of upward/downward movement to express the increase/decrease in values, as depicted in (37e)–(37f). All of these examples indicate that the size of a number is linguistically represented as height.

According to Lakoff and Johnson's metaphor, humans conceptualize quantity as a pile of stuff. In simple words, the bigger quantity the higher the pile. Though this metaphor seems to be restricted to quantifying uses of numerals, it seems plausible that a conceptualization along these lines does reflect the way humans think about quantity. More importantly, however, the data in (37) show that the vertically oriented notion of size in connection with numbers is linguistically relevant across different kinds of natural language expressions. Yet, so far only a few attempts have been made to try to connect the empirical pattern with this general concept and to test whether it makes sense compositionally. The two notable exceptions are the proposals by Nouwen (2016) and Matushansky and Zwarts (2017).

To summarize this part of the paper, I conclude in accordance with the previous literature that there is a cross-linguistic tendency to use conjunction markers inside additive numerals. This suggests a deep relationship between sum-formation and linguistic expression of arithmetical addition. The previous analysis of conjunction in numerals by Ionin and Matushansky (2006, 2018) faces several problems and does not offer a promising account to capture the arithmetical meaning of additive numerals. In the following sections, I will propose an alternative account that will build on the empirically supported idea that in natural language numbers are conceptualized as vertical objects of a certain height.

4. Assumptions

Let us begin with spelling out four assumptions that are crucial for my proposal. They concern the interpretation of 'and' in additive numerals, the syntactico-semantic structure of numerals, the mereotopological approach to part-whole structures and the ontology of linguistic representations of numbers.

4.1. Interpretation of 'and'

First of all, I assume that 'and' is analyzed in terms of non-Boolean conjunction as the mereological sum formation $\sqcup$. Importantly, however, I assume that 'and' in additive numerals does not show semantic effects indicating idempotence. This assumption is also supported by the behavior of nominal conjunction in sentences such as (38), which are not felicitous when 'and' conjoins co-indexed expressions with the same reference.

(38) #Yesterday at 20:13 pm, Amy met **Kim**$_i$ and { **Kim**$_i$ / **Kim**$_i$ and Tim }.

This effect is most probably pragmatic in nature but if so, then I assume that it is grammaticalized in additive numerals.

4.2. Syntactico-semantic structure of numerals

The second assumption concerns the internal structure of numerals. In particular, I adopt the nanosyntactic approach to the form and meaning of numerals by Wągiel and Caha (2020, 2021), who argue that numerals spell out complex syntactico-semantic structures consisting of various configurations of primitive meaning components. The approach is grounded in Nanosyntax (Starke 2009; Caha 2009 et seq.), whose key feature is late insertion, which means that lexical entries are not tailor-made to express a specific meaning, but rather their semantics depends on their syntactic environment. In this model, syntax combines abstract semantic features into complex structures that are mapped to their pronunciation via language-specific lexical entries.

The mapping is ensured by a language-invariant spellout procedure.

Wągiel and Caha's analysis postulates three universal semantic features that make up the syntactico-semantic structure of numerals. The first one is $\textsc{Scale}_m$, defined in (39), which is taken to denote a closed interval, i.e., a set of numbers from 0 to the lexically encoded upper bound. For instance, for the numeral 'five' it would be the number 5.

(39) a. $[\![\textsc{Scale}_m]\!]_{\langle n,t\rangle} = \lambda n_n[0 \leq n \leq m]$
b. $[\![\textsc{Scale}_5]\!] = [0,5]$

The second component is the invariant syntactic head $\textsc{Num}$ with the semantics in (40). It introduces the maximization operator $\textsc{max}$, which yields the biggest number in the interval, and thus forges a name of a number concept (type *n*).

(40) a. $[\![\textsc{Num}]\!]_{\langle\langle n,t\rangle,n\rangle} = \lambda P_{\langle n,t\rangle}[\textsc{max}(P)]$
b. $[\![\textsc{Num}]\!]([\![\textsc{Scale}_5]\!]) = 5$

Finally, the $\textsc{Cl}$ head in (41) introduces a classifier semantics by shifting a number to a counting device that allows for numeric quantification over referents denoted by the modified NP.[4]

(41) a. $[\![\textsc{Cl}]\!]_{\langle n,\langle e,t\rangle\rangle} = \lambda n_n \lambda x_e[\#(x) = n]$
b. $[\![\textsc{Cl}]\!]([\![\textsc{Num}]\!]([\![\textsc{Scale}_5]\!])) = \lambda x_e[\#(x) = 5]$

Consequently, the quantifying function of the numeral 'five' is represented by the structure in (42a), whereas its arithmetical meaning is represented by (42b).

(42) a. [$\textsc{Cl}$ [$\textsc{Num}$ $\textsc{Scale}_5$]]
b. [$\textsc{Num}$ $\textsc{Scale}_5$]

Crucially, these structures can be mapped onto morphology in various ways and the system allows for capturing systematic ambiguities and cross-linguistic variation in a principled way (for technical details concerning implementation, see Wągiel and Caha, 2020, 2021).

4.3. Ontology of linguistic representation of numbers

The third assumption concerns the way in which numbers are conceptualized in language. It is grossly inspired by the proposals by Nouwen (2016) and especially Matushansky and Zwarts (2017) and embraces a view that numbers are represented as abstract vertically oriented entities (type *e*) that have a part-whole structure.

The starting point is that following Zwarts and Winter (2000), I assume that in addition to the basic domain D_e there is also the ontological domain D_p of spatial points (type *p*), which is endowed with the appropriate geometrical structure that enables points to be related to each other in terms of distance and direction. Though the full spatial domain is 3-dimensional, it also encompasses 2- and 1-dimensional subspaces, e.g., horizontal planes and vertical lines. These subspaces maintain the fundamental geometrical structuring of distance and direction.

Building on the ideas by Nouwen (2016) and Matushansky and Zwarts (2017), I assume that in grammar the number line is conceptualized as vertically oriented 1-dimensional space that

[4]This is a simplified version of Wągiel and Caha's proposal since they assume that the $\textsc{Cl}$ shifts a number to a predicate-modifier semantics. However, these details are irrelevant for the current paper.

has a non-arbitrary starting point, which we can represent as 0 and which corresponds to the intuitive notion of GROUND. As suggested by Nouwen, gravity might play an essential role in the special nature of the vertical axis since it provides an absolute notion of direction to the vertical axis. Be it as it may, I take the concept of the number line to be half-open and I also assume that the space in question is dense in the sense that it has no gaps between the points relative to the relevant level of granularity (see Fox and Hackl, 2006).

Furthermore, I embrace an absolute view of space, which assumes that it exists irrespective of the existence of entities that occupy it (see Hoefer et al. 2021 for an overview and discussion). Thus, it can be conceptualized as an abstract container of sorts. Any entity that is located in space corresponds to a set of spatial points, which constitute its location in space. Consequently, in line with Matushansky and Zwarts, I assume that the function LOC (type $\langle e, \langle p,t \rangle \rangle$) maps an object to the set of points it occupies in space. Crucially, locations of distinct objects cannot overlap at a given moment, i.e., two entities cannot simultaneously occupying the same location. However, they can be, e.g., stacked, i.e., one can be placed on top of another.

Given the above, it is now possible to reinterpret intervals corresponding to SCALEs discussed in the previous section, recall (39), as abstract vector-like entities, which are abstract objects of type *e* located in 1-dimensional space. Such objects have a specific height and an integrated part-whole structure, i.e., do not involve gaps. I stipulate that there is no determined location that these entities occupy but their bottom is located at 0 (ground) as a default. When the LOC operation is applied to the meaning of a SCALE, it yields a set of points this entity occupies in space, see (43).

(43) $\text{LOC}_{\langle e,\langle p,t\rangle\rangle}(\llbracket \text{SCALE}_5 \rrbracket_e) \Rightarrow$ a set of points $\llbracket \text{SCALE}_5 \rrbracket$ occupies

In addition, the maximization operation MAX, recall (40), can be now reinterpreted as a function that takes a set of points and yields the highest point in that set, i.e., the point that is located farthest from 0 (ground), see (44).

(44) $\text{MAX}_{\langle\langle p,t\rangle,p\rangle}\big(\text{LOC}_{\langle e,\langle p,t\rangle\rangle}(\llbracket \text{SCALE}_5 \rrbracket_e)\big) \Rightarrow$ the highest point in (43)

In the spirit of Matushansky and Zwarts (2017), I take that point to intuitively correspond to the notion of a degree or number since the relevant 1-dimensional structure shares properties with scale structures, and thus the spatial points are ordered in a similar way as degrees or numbers. In other words, the consequent application of the operations LOC and MAX to the meaning of a SCALE yields a result that can be viewed as a number.

4.4. Mereotopological approach to part-whole structures

The final assumption concerns the nature of the part-whole structures encoded in SCALEs. Specifically, I assume that these part-whole structures are conceptualized in a way that is best captured in mereotopological terms.

Mereotopology is a theory of parts and wholes that extends standard mereology with topological notions. It was introduced to natural-language semantics by Grimm (2012) and since then a growing body of the literature has emerged that applies the theory to explain, e.g., singulatives (Grimm, 2012; Wągiel and Shlikhutka, 2023; Kagan, 2024), atomizers such *grain* (Scontras, 2014), notional mass nouns (Lima, 2014), partitive constructions (Wągiel, 2021b, 2022b), classifier constructions (Schvarcz and Wohlmuth, 2021), object mass nouns (Sutton and Filip, 2021), spatial collectives (Wągiel, 2021a), adjectives such as *whole* (Igel, 2021), multipliers like *double* (Wągiel, 2022a), clustered plurals (Gréa, 2023) and multiplicatives (Wągiel, 2023).

In mereotopology, the standard mereological notion of PARTHOOD ($\sqsubseteq$) is augmented with the topological relation of CONNECTEDNESS (C), which is reflexive and symmetric (Casati and Varzi, 1999). C is implied by OVERLAP (O) and incorporated into the system in a way that ensures its interactions with $\sqsubseteq$ (Varzi, 2007). As a result, one can distinguish between purely mereological structures, which are based exclusively on $\sqsubseteq$, and mereotopological structures, which involve $\sqsubseteq$ and C. Consequently, in contrast to classical mereology, mereotopology provides means to model various arrangements of parts within a whole, and thus various types of spatial configurations of entities.

Mereotopological notions allow for capturing ontological distinctions between different spatial configurations of entities. A particularly useful ontological category concerns integrated wholes (Casati and Varzi 1999; Grimm 2012; Wągiel 2021b; see also Moltmann 1997). This notion plays a significant role in human cognition and is based on the intuition that an integrated entity forms a cohesive whole not divisible into separated parts. In mereotopology, this intuition corresponds to the notion of SELF-CONNECTED (SC) defined in (45) (Casati and Varzi, 1999: 57), which states that an entity is self-connected if any two parts that form the whole of that entity are connected to each other.

(45) $\textsc{sc}(x) \stackrel{\text{def}}{=} \forall y \forall z[\forall w(\textsc{o}(w,x) \leftrightarrow (\textsc{o}(w,y) \vee \textsc{o}(w,z))) \to \textsc{c}(y,z)]$

However, (45) is too weak to capture the notion of an integrated whole since it should also ensure maximality. The reason is that among multiple SC entities within a part-whole structure, e.g., contiguous proper parts of a line segment, only the biggest such entity is the integrated whole, i.e, the entire line segment. Furthermore, it is useful to relativize integrity to a property, rather than to define it in absolute terms. In mereotopology, these intuitions are captured by the notion of MAXIMALLY SELF-CONNECTED (MSC) defined in (46) (Casati and Varzi, 1999: 60, adapted).[5] An entity is maximally self-connected relative to a property if (i) every part of the entity is connected to (overlaps) the whole (SC) and (ii) anything else which has the same property, is self-connected, and overlaps it is once again part of it (maximality).

(46) $\textsc{msc}(P)(x) \stackrel{\text{def}}{=} P(x) \wedge \textsc{sc}(x) \wedge \forall y[P(y) \wedge \textsc{sc}(y) \wedge \textsc{o}(y,x) \to y \sqsubseteq x]$

The mereotopological framework described above allows us to discern the ontological category of an integrated whole, which has been argued to be a crucial notion for individuation (Moltmann, 1997; Grimm, 2012; Wągiel, 2021b). I assume that the notion is applicable not only to concrete physical individuals, but also to abstract entities and that MSC entities are conceptualized as independent objects in their own right.

In the following section, I will spell out an account of the meaning of conjunction in the arithmetical function of additive numerals that is an alternative to the problematic proposal by Ionin and Matushansky (2018). This account is based on the combination of a mereotopological approach (Grimm 2012 et seq.) with the treatment of numerals as vertically oriented vector-

[5]I ignore here the distinction between an entity and its interior, and thus the corresponding notion of STRONGLY SELF-CONNECTED, which is often taken to play a role in modeling integrated wholes (for details, see Casati and Varzi, 1999: 59–60).

like objects in 1-dimensional space inspired by the work of Nouwen (2016) and especially Matushansky and Zwarts (2017).

5. Proposal

Given the set of assumptions spelled out in the previous section, it is now time to put the pieces together in order to see how they allow us to explain the behavior of conjunction in additive numerals. For this purpose, let us focus on the semantic representation of the number 25 referred to, e.g., by the Czech example in (47). In a nutshell, under the proposed analysis, the arithmetical function of (47) gets the interpretation in (48), where the mereological sum of the entities denoted by $\textsc{Scale}_5$ and $\textsc{Scale}_{20}$ is first mapped by the LOC operation onto the set of points corresponding to its location in abstract 1-dimensional space and then the MAX operation yields the highest point in that set.

(47) pět-a-dvacet
five-and-twenty
'twenty-five' (Czech)

(48) $[\![\text{pět-a-dvacet}]\!] = \textsc{max}\big(\textsc{loc}([\![\textsc{Scale}_5]\!] \sqcup [\![\textsc{Scale}_{20}]\!])\big)$

5.1. Derivation

To explain the details and how their interplay yields the desired result, let us demonstrate the step-by-step derivation of the semantics in (48). For presentation purposes, each step of the proposal will be visualized graphically in the figures in (51)–(55) below.[6] Specifically, I propose that the arithmetical meaning of (47) is constructed in the following way.

Step 1: We start with the representation of the number line, which in the postulated approach is taken to be vertically-oriented 1-dimensional space. This space is dense in the sense that there are no gaps between points (see Fox and Hackl, 2006) and half-open, which means that it has a non-arbitrary starting point, represented as 0, and no endpoint. In other words, it has the lowest point (corresponding to the notion of GROUND) and no highest point and there are no gaps between the points. The number line is represented graphically in (51).

Step 2: The $[\![\textsc{Scale}_5]\!]$ component denotes a vertically-oriented 1-dimensional MSC entity, i.e., an object of type *e*, which is 'of height 5'. By default, it occupies the space beginning at the lowest point, which is 0. This is visualized in the figure in (52).

Step 3: $[\![\textsc{Scale}_{20}]\!]$ denotes another 1-dimensional MSC entity, this time 'of height 20'. When 'and' sums up $[\![\textsc{Scale}_5]\!]$ and $[\![\textsc{Scale}_{20}]\!]$, they are distinct objects, and thus in accordance with the adopted assumptions, they cannot overlap. Instead, $[\![\textsc{Scale}_{20}]\!]$ occupies the space immediately above $[\![\textsc{Scale}_5]\!]$, which has already been located in (52). In other words, it is stacked on top of it, as illustrated in (53).

Step 4: But now, due to the nature of the assumed dense 1-dimensional space and the internal structure of the two entities corresponding to $[\![\textsc{Scale}_5]\!]$ and $[\![\textsc{Scale}_{20}]\!]$ in (53), the MSC property holds also of the sum of $[\![\textsc{Scale}_5]\!] \sqcup [\![\textsc{Scale}_{20}]\!]$. As a result, they are not considered two independent objects, and this gives rise to a new MSC entity, i.e., $[\![\textsc{Scale}_{25}]\!]$ 'of height 25',

[6]Notice that the grid in the figures is provided only for the visualization purposes.

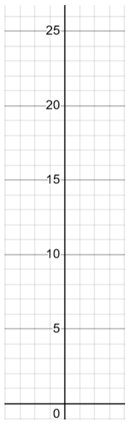
which is conceptualized as a singular integrated object in (54).

Step 5: Finally, LOC shifts $[\![\textsc{Scale}_{25}]\!]$ to the set of points it occupies and MAX yields the highest point in that set. The application of MAX is represented by the blue circle in (55). Due to the properties of the assumed 1-dimensional space, the result corresponds to what I argue to be the linguistic concept of the number 25. This object can be then shifted to a counting device semantics via the CL head, recall (41), which would account for the quantifying meaning of the numeral. Due to space limitations, the exact details are left unexplored here; however, the main point is that the discrete entity denoted by (48) and represented in (55) can be subject to further operations, which would account for other functions of the numeral.

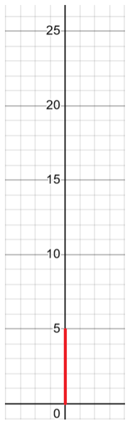
(49) $\textsc{loc}_{\langle e,\langle p,t\rangle\rangle}([\![\textsc{Scale}_5]\!]_e) \Rightarrow$ a set of points $[\![\textsc{Scale}_5]\!]$ occupies

(50) $\textsc{max}_{\langle\langle p,t\rangle,p\rangle}\big(\textsc{loc}_{\langle e,\langle p,t\rangle\rangle}([\![\textsc{Scale}_5]\!]_e)\big) \Rightarrow$ the highest point in that set

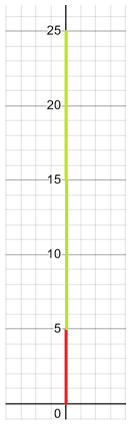
(51) Step 1



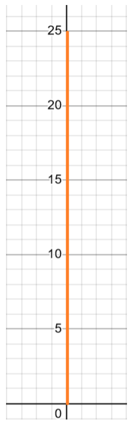
(52) Step 2



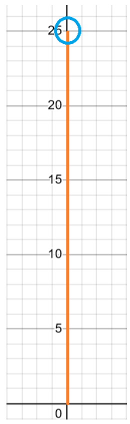
(53) Step 3



(54) Step 4



(55) Step 5



Before I conclude, let us briefly discuss one interesting consequence of the proposal.

5.2. Additive numerals vs. coordinated numerals

The proposed system allows us to capture the truth-conditional difference between additive numerals and coordinated numerals, recall (34), repeated here as (56).

(56) a. Tři-**a**-dvacet je liché číslo.
three-and-twenty is odd number
'Twenty-three is an odd number.'

b. Tři **a** dvacet jsou lichá čísla.
three and twenty are odd numbers
'Three and twenty are odd numbers.' (Czech)

The contrast can be straightforwardly captured as the distinction between the representation in (57) and in (58), respectively.

(57) $[\![\text{tři-a-dvacet}]\!] = \textsc{max}\big(\textsc{loc}([\![\textsc{Scale}_3]\!] \sqcup [\![\textsc{Scale}_{20}]\!])\big)$

(58) $[\![\text{tři a dvacet}]\!] = \textsc{max}\big(\textsc{loc}([\![\textsc{Scale}_3]\!])\big) \sqcup \textsc{max}\big(\textsc{loc}([\![\textsc{Scale}_{20}]\!])\big)$

In (57), the LOC and MAX operators are applied to the sum of the two entities denoted by the SCALE element. On the other hand, in (58) the two operations in question apply to each of the SCALE components and the result is summed up by the mereological sum formation $\sqcup$. In effect, while (57) denotes the highest point of the MSC entity arisen from the fusion of $[\![\text{SCALE}_3]\!]$ and $[\![\text{SCALE}_{20}]\!]$, (58) denotes a plurality of points that represent two different numbers.

6. Conclusion

In this paper, I examined the underlying meaning of additive numerals such as *two hundred **and** three* and argued that it is not necessary to postulate that conjunction encodes the operation of arithmetical addition, which would add another meaning to the well-known ambiguity of 'and'. Instead, I demonstrated that a non-Boolean analysis of conjunction in such numerals is possible if one adopts specific assumptions concerning the ontology of numbers. Specifically, I argued that in natural language numbers are represented as vertically oriented vector-like entities in 1-dimensional space. Furthermore, I argued that these objects have a part-whole structure that is best represented in mereotopological terms. Based on these claims, I proposed an analysis that has an advantage of capturing the meaning of 'and' in additive numerals used in the arithmetical function, which has not been accounted for until now.

References

Artstein, R. and N. Francez (2003). Plural times and temporal modification. *Amsterdam Colloquium 14*, 63–68.

Bach, E. (1986). The algebra of events. *Linguistics and Philosophy 9*(1), 5–16.

Beck, S. and Y. Sharvit (2002). Pluralities of questions. *Journal of Semantics 19*(2), 105–157.

Bultinck, B. (2005). *Numerous Meanings: The Meaning of English Cardinals and the Legacy of Paul Grice*. Amsterdam: Elsevier.

Caha, P. (2009). *The Nanosyntax of Case*. Ph. D. thesis, University of Tromsø.

Cantor, G. (1895). Beiträge zur Begründung der transfiniten Mengenlehre. *Mathematische Annalen 46*(4), 481–512.

Casati, R. and A. C. Varzi (1999). *Parts and Places: The Structures of Spatial Representation*. Cambridge, MA: MIT Press.

Champollion, L. (2016). Ten men and women got married today: Noun coordination and the intersective theory of conjunction. *Journal of Semantics 33*(3), 561–622.

Chomsky, N. (2008). On phases. In R. Freidin, C. P. Otero, and M. L. Zubizarreta (Eds.), *Foundational Issues in Linguistic Theory: Essays in Honor of Jean-Roger Vergnaud*, pp. 133–166. Cambridge, MA: MIT Press.

Corver, N. and J. Zwarts (2006). Prepositional numerals. *Lingua 116*(6), 811–836.

Dotlačil, J. and R. Nouwen (2016). The comparative and degree pluralities. *Natural Language Semantics 24*(1), 45–78.

Dočekal, M., N. Haslinger, E. Rosina, M. Roszkowski, I. Šafratová, V. Schmitt, M. Wągiel, and V. Wurm (2022). Cumulative readings of distributive conjunctions: Evidence from Czech and German. In D. Gutzmann and S. Repp (Eds.), *Proceedings of Sinn und Bedeutung 26*, pp. 239–257. Cologne: University of Cologne.

Flor, E., N. Haslinger, H. Koopman, E. Rosina, M. Roszkowski, and V. Schmitt (2017). Cross-linguistic evidence for a non-distributive lexical meaning of conjunction. In A. Cremers,

T. van Gessel, and F. Roelofsen (Eds.), *Proceedings of the 21th Amsterdam Colloquium*, pp. 255–264. Amsterdam: ILLC.

Fox, D. and M. Hackl (2006). The universal density of measurement. *Linguistics and Philosophy 29*, 537–586.

Frege, G. (1884). *Die Grundlagen der Arithmetik*. Breslau: Wilhelm Koebner.

Gillon, B. S. (1987). The readings of plural noun phrases in English. *Linguistics and Philosophy 10*(2), 199–219.

Gleitman, L. R. (1965). Coordinating conjunctions in English. *Language 41*(2), 260–293.

Greenberg, J. H. (1978). Generalizations about numeral systems. In J. H. Greenberg (Ed.), *Universals of Human Language*, Volume 3, pp. 249–295. Stanford, CA: Stanford University Press.

Grimm, S. (2012). *Number and Individuation*. Ph. D. thesis, Stanford University, California.

Gréa, P. (2023). Quelques in French: A clustered plural. *Journal of Semantics 40*(2-3), 427–482.

He, C. (2015). Complex numerals in Mandarin Chinese are constituents. *Lingua 164*, 189–214.

He, C., O.-S. Her, X. Hu, and W. Zhu (2017). Overt coordination in additive numerals of minority languages in south china. *Syntax 20*(3), 292–316.

Hoefer, C., N. Huggett, and J. Read (2021). Absolute and Relational Space and Motion: Classical Theories. In E. N. Zalta and U. Nodelman (Eds.), *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University.

Hoeksema, J. (1988). The semantics of non-Boolean 'and'. *Journal of Semantics 6*(1), 19–40.

Hurford, J. R. (1975). *The Linguistic Theory of Numerals*. Cambridge: Cambridge University Press.

Hurford, J. R. (1987). *Language and Number: The Emergence of a Cognitive System*. Oxford: Blackwell.

Igel, M. (2021). Part-whole-modifiers and the *-operator. In L. Martí, H. Pearson, and Y. Sudo (Eds.), *Proceedings of Sinn und Bedeutung 25*, pp. 430–447. London: University College London and Queen Mary University of London.

Ionin, T. and O. Matushansky (2006). The composition of complex cardinals. *Journal of Semantics 23*(4), 315–360.

Ionin, T. and O. Matushansky (2018). *Cardinals: The Syntax and Semantics of Cardinal-Containing Expressions*. Cambridge, MA: MIT Press.

Kagan, O. (2024). The slavic suffix -in/-yn as partition shifter. *Natural Language Semantics 32*(1), 35–63.

Krifka, M. (1990). Boolean and non-Boolean 'and'. In L. Kálmán and L. Polos (Eds.), *Papers from the Second Symposium on Logic and Language*, pp. 161–188. Budapest: Akadémiai Kiadó.

Krifka, M. (1996). Parametrized sum individuals for plural reference and partitive quantification. *Linguistics and Philosophy 19*, 555–598.

Lahiri, U. (2002). *Questions and answers in embedded contexts*. Oxford: Oxford University Press.

Lakoff, G. and M. Johnson (1980). *Metaphors We Live By*. Chicago: University of Chicago Press.

Lasersohn, P. (1995). *Plurality, Conjunction and Events*. Boston: Kluwer Academic Publishers.

Lima, S. (2014). All notional mass nouns are count nouns in Yudja. In T. Snider, S. D'Antonio, and M. Weigand (Eds.), *Proceedings of Semantics and Linguistic Theory 24*, pp. 534–554. Ithaca, NY: CLC Publications.

Link, G. (1983). The logical analysis of plural and mass nouns: A lattice–theoretical approach. In R. Bäuerle, C. Schwarze, and A. von Stechow (Eds.), *Meaning, Use, and Interpretation of Language*, pp. 302–323. Berlin: Mouton de Gruyter.

Matushansky, O. and J. Zwarts (2017). Making space for measures. In A. Lamont and K. Tetzloff (Eds.), *NELS 47: Proceedings of the Forty-Seventh Annual Meeting of the North East Linguistic Society*, pp. 261–274. Amherst, MA: GLSA.

Mitrović, M. and U. Sauerland (2016). Two conjunctions are better than one. *Acta Linguistica Hungarica 63*, 471–494.

Moltmann, F. (1997). *Parts and Wholes in Semantics*. New York, NY: Oxford University Press.

Moltmann, F. (2013). Reference to numbers in natural language. *Philosophical Studies 162*(3), 499–536.

Moltmann, F. (2017). Number words as number names. *Linguistics and Philosophy 40*(4), 331–345.

Nouwen, R. (2016). Making sense of the spatial metaphor for number in natural language. Unpublished manuscript. Available online on LingBuzz.

Ross, J. R. (1967). *Constraints on variables in syntax*. Ph. D. thesis, Massachusetts Institute of Technology.

Rothstein, S. (2013). A Fregean semantics for number words. In M. Aloni, M. Franke, and F. Roelofsen (Eds.), *Proceedings of the 19th Amsterdam Colloquium*, pp. 179–186. Amsterdam: ILLC.

Rothstein, S. (2017). *Semantics for Counting and Measuring*. Cambridge: Cambridge University Press.

Schein, B. (2017). *'And': Conjunction Reduction Redux*. Cambridge, MA: MIT Press.

Schmitt, V. (2019). Pluralities across categories and plural projection. *Semantics and Pragmatics 12*(17), 1–49.

Schmitt, V. (2020). Boolean and non-Boolean conjunction. In D. Gutzmann, L. Matthewson, C. Meier, H. Rullmann, and T. E. Zimmermann (Eds.), *The Wiley Blackwell Companion to Semantics*, pp. 1–32. Hoboken, NJ: Wiley-Blackwell.

Schvarcz, B. and K. Wohlmuth (2021). An MSSC-approach to Hungarian classifiers. *Acta Linguistica Academica 68*(3), 257–273.

Scontras, G. (2014). *The Semantics of Measurement*. Ph. D. thesis, Harvard University, Cambridge, MA.

Starke, M. (2009). Nanosyntax: A short primer to a new approach to language. *Nordlyd 36*(1), 1–6.

Sutton, P. R. and H. Filip (2021). The count/mass distinction for granular nouns. In H. Filip (Ed.), *Countability in natural language*, pp. 252–291. Cambridge: Cambridge University Press.

Szabolcsi, A. (2015). What do quantifier particles do? *Linguistics and Philosophy 38*, 159–204.

Tatsumi, Y. (2021). The architecture of complex cardinals in relation to numeral classifiers. In M. Dočekal and M. Wągiel (Eds.), *Formal approaches to number in Slavic and beyond*, pp. 323–346. Berlin: Language Science Press.

Varzi, A. C. (2007). Spatial reasoning and ontology: Parts, wholes, and locations. In M. Aiello, I. E. Pratt-Hartmann, and J. van Benthem (Eds.), *Handbook of Spatial Logics*, pp. 945–1038.

Berlin: Springer.

Von Neumann, J. (1923). Zur Einführung der transfiniten Zahlen. *Acta Litterarum ac Scientiarum Regiae Universitatis Hungaricae Francisco-Josephinae, sectio scientiarum mathematicarum 1*, 199–208.

Watanabe, A. (2017). Natural language and set-theoretic conception of natural number. *Acta Linguistica Academica 64*(1), 125–151.

Wiese, H. (2003). *Numbers, Language, and the Human Mind*. Cambridge: Cambridge University Press.

Winter, Y. (1996). A unified semantic treatment of singular NP coordination. *Linguistics and Philosophy 19*(4), 337–391.

Winter, Y. (2001). *Flexibility Principles in Boolean Semantics: The Interpretation of Coordination, Plurality, and Scope in Natural Language*. Cambridge, MA: MIT Press.

Wągiel, M. (2021a). Slavic derived collective nouns as spatial and social clusters. In M. Dočekal and M. Wągiel (Eds.), *Formal Approaches to Number in Slavic and Beyond*, pp. 175–205. Berlin: Language Science Press.

Wągiel, M. (2021b). *Subatomic Quantification*. Berlin: Language Science Press.

Wągiel, M. (2022a). Quantifying over hidden (parts of) events. *Glossa: A Journal of General Linguistics 7*(1), 1–26.

Wągiel, M. (2022b). A typology of proportional quantifiers: Evidence from Polish partitives. *Acta Linguistica Academica 69*(1), 128–159.

Wągiel, M. (2023). Acts, occasions and multiplicatives: A mereotopological account. In J. Kim, B. Öney, Y. Zhang, and F. L. Zhao (Eds.), *Proceedings of Semantics and Linguistic Theory 33*, pp. 276–297. Washington, DC: Linguistic Society of America.

Wągiel, M. and P. Caha (2020). Universal semantic features and the typology of cardinal numerals. *Catalan Journal of Linguistics 19*, 199–229.

Wągiel, M. and P. Caha (2021). Complex simplex numerals. *Acta Linguistica Academica 68*(4), 470–515.

Wągiel, M. and N. Shlikhutka (2023). Clustering and declustering things: The meaning of collective and singulative morphology in Ukrainian. In J. Kim, B. Öney, Y. Zhang, and F. L. Zhao (Eds.), *Proceedings of Semantics and Linguistic Theory 33*, pp. 194–214. Washington, DC: Linguistic Society of America.

Zwarts, J. and Y. Winter (2000). Vector space semantics: A model-theoretic analysis of locative prepositions. *Journal of Logic, Language and Information 9*, 169–211.