A plea for a functional dependency approach to correlatives[1]

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Abstract. This paper compares two possible analyses of multi-head correlatives in Hindi; the canonical semantics of these structures proposed by Dayal (1996) and a comparable analysis by Gajewski (2008). While the first relies on the semantics of multiple-wh questions, the second builds on the semantics of wh- free relatives to derive the semantics of the Hindi multi-head correlative. Although the free relative approach seems to derive comparable truth conditions, I argue that this is only achievable through significant stipulations about the nature of pluralization and presupposition projection mechanisms. The required universal quantification over pair-lists which is central to the semantics of the multiple-wh correlative is best captured by a functional dependency between the wh-elements.. To that end, I propose an updated analysis of these structures based on the functional dependency approach of Dayal (1996) that aligns multiple headed correlatives with multiple wh-questions rather than free relatives.

Keywords: correlatives, free relatives, multiple-wh questions.

1. Introduction

Lipták (2009) defines a correlative as a left peripheral relative clause that is linked to a nominal correlate in the matrix through a demonstrative. From the surface structure, 4 characteristics broadly define the structure of a correlative:

(1) a. A left dislocated relative clause.
  b. A wh-element as a relative pronoun.
  c. A corresponding demonstrative in the main clause.
  d. The possibility of stacking relative pronouns in the relative clause.

However, capturing the semantics of this structure is far more tricky. The existing literature places correlatives somewhere on the spectrum between free relatives and conditionals in the broad landscape of quantificational structures (Grosu and Landman, 1998; Šimík, 2023). They are uncontroversially distinct from restrictive relative clauses of the form we are familiar from in English such as *I read the book that John wrote*, which are externally headed relative clauses that have an intersective meaning with the nominal head they modify. But they are also structurally similar to free relatives with a semantically vacuous external head, and interpretation of a multi-head correlative is often paraphrased as a conditional.

This paper teases apart the semantics of correlatives from that of free relatives. As the two existing approaches to correlatives presented here will illustrate, the two types of constructions do share a lot of similarities. However, a closer examination of the multi-head variant of correlatives shows that a unified semantics for the two might not be as appropriate. At the heart of this discussion is an examination of two possible ways to derive universal quantificational force in free relatives and multi-head correlatives. Both free relatives and correlatives show an alternation between definite and universal interpretations, but the comparison is not as straightforward. Wh-free relatives in English are constructions involving a single wh-DP and no plural

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marking which can refer to a single individual, or have a universal-like interpretation where it refers to multiple entities[2]. Jacobson (1995) accounts for this ambiguity in their meaning by using a pluralization operation in order to derive the apparent universal interpretation.

The two are comparable in that the simple correlative, which is structurally similar to a free relative also has a single definite interpretation. While the simple case unambiguously refers to a single individual, multi-head correlatives with multiple wh-elements in the main clause refer to multiple individuals that are universally quantified over, even though there is no overt plural morphology. A proposal by Gajewski (2008) attempts to derive universal quantification in multi-head correlatives via the same mechanisms as English wh-free relatives and provide a unified analysis for them. I show that it is not possible to derive a universal interpretation via the same mechanisms in both constructions and illustrate how it fails to make the correct predictions for multi-head correlatives. Finally, I argue for a functional dependency approach to the semantics of multi-head correlatives and offer a proposal along the lines of Dayal (1996). This line of reasoning aligns the semantics of correlatives more closely with that of multiple-wh questions rather than free relatives.

2. Defining Correlatives

Correlatives are not restrictive relative clauses or free relatives, or conditionals, although they have often been compared to all three. One of the most striking characteristics that distinguishes them from English-style relative clauses is the possibility of stacked relative pronouns in the relative clause. Consider the following classic examples of Hindi correlatives

(2) a. [jo larkii kharii hai] vo lambii hai.
[which girl stand is] that tall is.
The girl who is standing is tall.

b. [jis larkii-ne jis larke-ke saath khelaa] us-ne us-ko haraaya.
[which girl-ERG which boy-GEN with play] that-ERG that-ACC defeat.
Which girl played with which boy, she defeated him. (Dayal, 1996)

Grosu and Landman (1998) note, not all relative clauses have the same semantics as externally headed relative clauses. Crucially, unlike English restrictive relative clauses, correlatives do not have a set intersective meaning. Instead, the relative clause denotes a generalized quantifier which then takes the main clause as an argument. This, along with other properties of correlatives, such as their definite interpretation, and the ban on stacked relative clauses leads them to classify these structures as maximalizing relatives. Here, they form a unifying class of sorts with free relatives, which also do not have an intersective meaning and lack a head noun. Free relatives denote a maximal entity that can act as an argument to a predicate. The interpretation of the free relative is also affected by the presence of free choice items like the English *ever*.

(3) a. I met the girl who is standing at the back of the room. (RRC)

b. I met who(ever) is standing at the back of the room (FR)

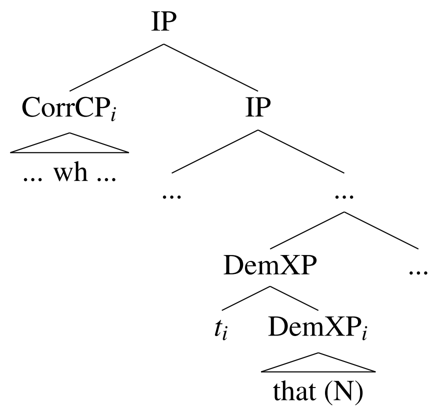
[2]In addition to a universal and definite interpretation Caponigro (2003, 2023) also reports the availability of free relatives that have an existential interpretation which have been attested Italian, Balto-Slavic, Semitic, and Mesoamerican languages

2.1. The syntax of correlatives

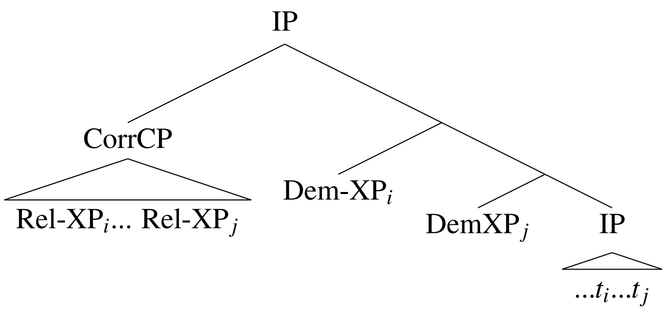
Bhatt (2003) proposes that simple correlatives in Hindi are base generated as adjuncts to the demonstrative (Dem-XP) in the main clause which then move to the left periphery. This is different from a post nominal relative clause that modifies the NP in the main clause and is extraposed to the right. Correlatives also allow multi-head correlatives; which are constructions where the relative clause contains multiple relative pronouns in the relative clause with corresponding demonstratives in the main clause. These multi-head counterparts are argued by Bhatt (2003) and Dayal (1995) to be base generated as adjuncts to the IP in the left peripheral position.

The Dem-XP proposal captures the correspondence between the relative pronoun and the demonstrative in the simple case, but it remains unclear how this plays out in the multi-head correlative. Since the relative clause is an IP adjunct in the multi-head correlative, this approach appeals to *'suitability'* for the adjunction operation. Essentially the relative clause with *n* relative pronouns can only combine with an *n*-ary predicate which is only possible if the number of demonstratives in the main clause matches the number of relative pronouns. Moreover, the demonstratives must undergo covert movement at LF within the matrix TP in order to generate a predicate of the correct semantic type. This results in a configuration as in (4b) I will be adopting these proposals for the syntax of the correlative in this paper.

(4) a. Simple correlatives

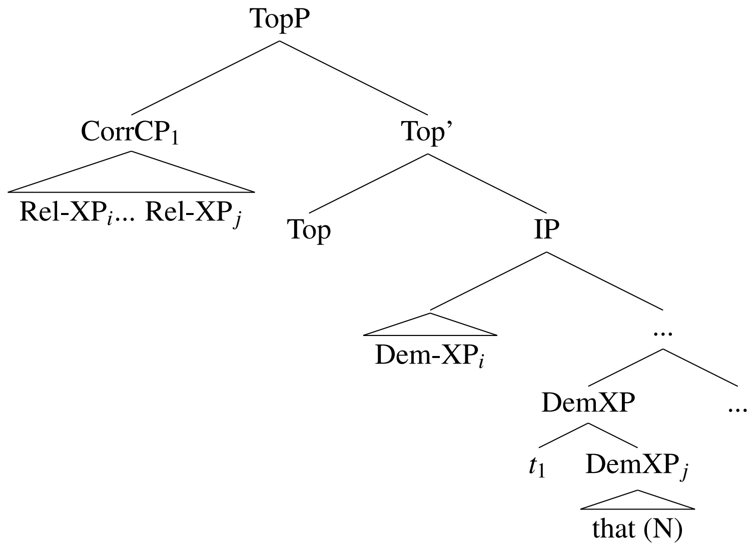


b. Multi-head correlatives



An alternative proposal for multi-head correlatives comes from Chierchia (2024) who suggests that the correlative in multi-head correlatives is based generated as an adjunct to the lower demonstrative and then moves to a topicalized position in the left periphery as in (5). Within the correlative, the relative pronouns undergo covert movement at LF to Spec CP.

(5)



3. Deriving Correlatives from questions

There are two existing approaches to the semantics of multi-head correlatives. The first is the seminal analysis from Dayal (1996), and the second is from Gajewski (2008). The first one draws on the semantics of simple and multiple wh-questions to propose an analysis of simple and multi-head correlatives. The second draws on the semantic operations that derive English wh-free relatives in Jacobson (1995) to argue that multi-head correlatives can essentially be treated as stacked free relatives, with the same operations being applied recursively. The two concur on the following aspects of the meaning of the correlative:

- **Universal Quantification**; single NP heads in a multiple correlative universally quantify over multiple ordered pairs.

- **Domain Exhaustivity**; There must be a pair in the set for each member of the thematically higher head.

- **Pointwise Uniqueness**; there can be no more than one pair for each member of the higher head.

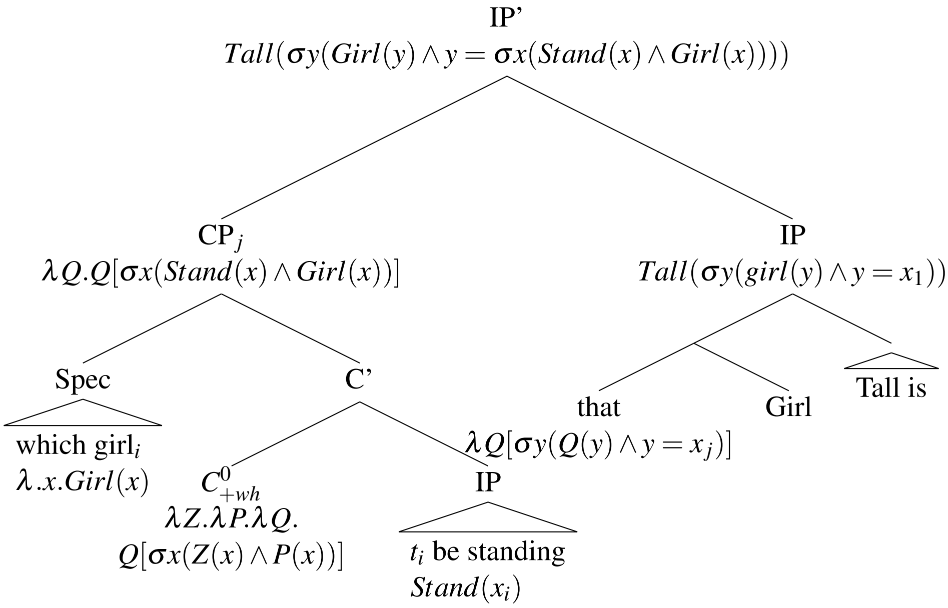
Both theories come with their own set of stipulations to account for these three characteristics and prima facie seem to derive equivalent truth conditions. In the third section I introduce a test context that pulls apart the difference in these two approaches to show that they are in fact not equivalent. A recursive approach, although appealing derives truth conditions that are far too weak. To that end, I motivate a return to an updated version of the functional dependency analysis of multi-head correlatives which I propose in the final section.

3.1. Correlatives according to Dayal (1996)

We know from Bhatt (2003) that Hindi has 4 relativization strategies. Setting aside the prenominal non-finite relative clauses, post-nominal relative clauses and right extraposed relative clauses in Hindi have been argued to behave like English-style restrictive relative clauses. Dayal (1995) notes that both these constructions have a set intersective meaning with the head noun that they modify. There are two features that set correlatives apart from typical restrictive relative clauses. One striking difference is the possibility of having multiple relative pronouns in the relative clause. Another is the absence of an overt NP in the main clause. Instead, the relative clause contains one or more internal heads, with corresponding semantically vacuous demonstratives in the main clause. According to Dayal (1996) the relative clause is a quantificational structure which takes the matrix clause as an argument. Like free relatives, correlatives involve a maximalization operation, but unlike free relatives which must have the semantic type of an argument, correlatives denote quantifiers.The correlative clause has the same structure and denotation as a wh-question. The C head of a correlative creates a quantifier like structure which takes first the relative clause and then the main clause as its arguments.

While Dayal (1995) assumes that the correlative is base generated as a IP adjunct, we can also derive this structure by following Bhatt (2003) and assuming that the relative clause is first base generated as an adjunct to the demonstration in the main clause and then undergoes movement to Spec TP. This would also explain the trace in the main clause that is co-indexed with the correlative. The semantic derivation proposed by Dayal would proceed in the same way.

(6)



The semantics of the correlative stems from the denotation of the C-head which creates a generalized quantifier; the set of properties that hold of a unique individual defined by the correlative, while the main clause denotes a property. The C-head is generalized to both atomic and plural entities, so the correlative can refer to a plurality if there is overt plural marking on the wh-DP. Crucially however, this plural interpretation is not available in the absence of plural marking on the DP in the relative clause. In the case of the simple correlative above, it could be straightforwardly translated to an English restrictive relative clause of the form *'the girl who is standing is tall'*.

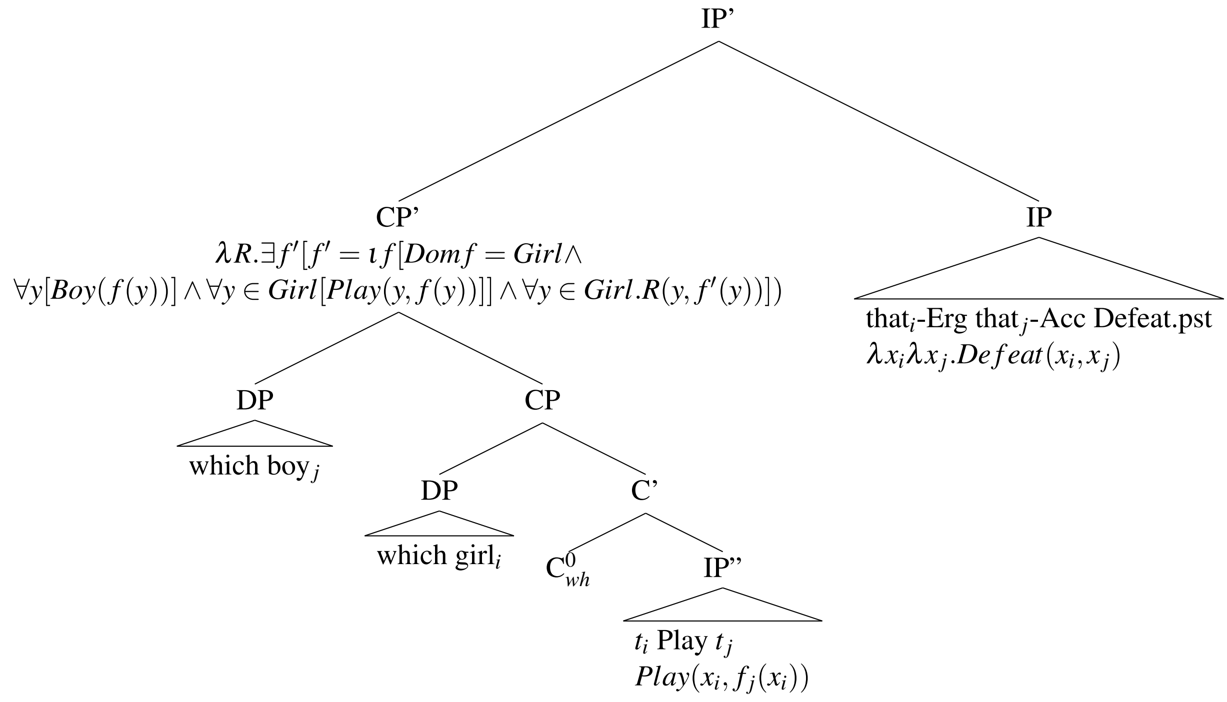
This analysis follows the Jacobson (1995) approach to free relatives by including an ι operator in the denotation of the C head. But the free relative denotes maximal individual while a simple correlative denotes a quantifier. Moreover, Hindi correlatives are sensitive to number marking on the DP, unlike free-relatives. Plural interpretations of correlatives are only available in the presence of overt plural morphology on the DP.

3.2. Multi-head correlatives

The universal quantificational force in multi-head correlatives in the absence of any overt plural morphology poses an interesting puzzle. The canonical example of a multi-head correlative in (2b) seems to refer to multiple girls, all of whom played with a unique boy, and each girl defeated that boy. By allowing multiple wh-DPs in the relative clause, the correlative shifts from referrring to a single unique individual to universally quantifying over pairs. In order to explain this shift, Dayal (1996) builds on her analysis of multiple wh-questions which can have a pair-list as a possible answer. The semantics of pair-lists impose two requirements on the answer — *domain exhaustivity*, and *uniqueness*. The set defined by the subject wh-DP must be exhaustified over; and each member of that set must be paired with a unique member of the set defined by the object wh-DP.

Multiple wh-questions display an asymmetry between the wh-DPs in the subject and object position, while the subject DP is exhaustified over, the object DP is not. The correlative must be defined for every element in the set denoted by the subject, but not in the object. Each individual in the subject DP must also be paired with just one individual in the object DP,i.e a one to many pairing is not available. These characteristics are derived by proposing a functional dependency between the subject and object wh-DPs.

(7)



(8) $[\![C^0_{+wh}]\!] = \lambda X.\lambda Y.\lambda Z.\lambda R.\exists f'[f' = \iota f[Dom f = Y \wedge \forall y[Z(f(y))] \wedge \forall y \in Y[X(y)(f)]] \wedge \forall y \in YR(y, f'(y))]$

(9) $[\![CP']\!] = [\![CP]\!]([\![DP]\!]) = ([\![C']\!]([\![DP]\!]))([\![DP]\!]) = ([\![C^0_{wh}]\!]([\![IP'']\!]))([\![DP]\!]))([\![DP]\!])$

a. $= (\lambda X.\lambda Y.\lambda Z.\lambda R.\exists f'[f' = \iota f[Dom f = Y \wedge \forall y[Z(f(y))] \wedge \forall y \in Y[X(y)(f)]] \wedge \forall y \in YR(y, f'(y))]$
$(\lambda x_i.\lambda f_j[Play(x_i, f_j(x_i))])([\![DP]\!]))([\![DP]\!])$

b. $= (\lambda Y.\lambda Z.\lambda R.\exists f'[f' = \iota f[Dom f = Y \wedge \forall y[Z(f(y))] \wedge \forall y \in Y.[Play(y, f(y))]] \wedge \forall y \in YR(y, f'(y))](Girl))(Boy)$

c. $= \lambda R.\exists f'[f' = \iota f[Dom f = Girl \wedge \forall y[Boy(f(y))] \wedge \forall y \in Girl[Play(y, f(y))]] \wedge \forall y \in Girl.R(y, f'(y))]])$

d. $[\![IP']\!] = [\![CP]\!]([\![IP]\!])$
$= \exists f'[f' = \iota f[Dom f = Girl \wedge \forall y[Boy(f(y))] \wedge \forall y \in Girl[Play(y, f(y))]] \wedge \forall y \in Girl.Defeat(y, f'(y))]])$

The semantics of the correlative is hard wired into the denotation in (8) which turns the correlative into a generalized quantifier (9c). In both the simple and multi-head cases, maximality is retained. Although both wh- DPs denote properties, one must assume that the two wh-DPs leave behind different types of traces. The subject wh-DP leaves an *e* type trace, just as in the simple correlative, but the object wh-DP leaves a functional trace of type $\langle e, e \rangle$. The functional trace left by the lower wh-DP is what encodes the functional dependency between the subject and object DP such that each element in the subject DP is mapped to a unique element in the object DP. This aspect of the analysis is not quite compositional since one DP must leave behind a complex trace even though both DPs denote properties.

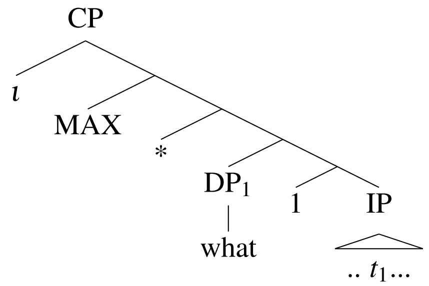
Moreover multi head correlatives have a matching requirement which states that relative pronouns must have corresponding demonstratives in the main clause. In the case of simple correlatives, the matching requirement follows naturally from the assumption that the relative clause is base generated as a Dem-adjunct and then moves to the left periphery. For the multi- head case, both Bhatt (2003) and Dayal (1995) assume that the relative clause is a TP-adjunct. But the TP-adjunct hypothesis does not explain the locality effects or the matching requirements in multi-head correlatives. The matiching requirement between the wh-DPs in the relative clause and the demonstratives in the main clause must be stipulated, or explained via a proposal like the one in Chierchia (2024).

4. Multi-head correlatives as multiple headed free relatives

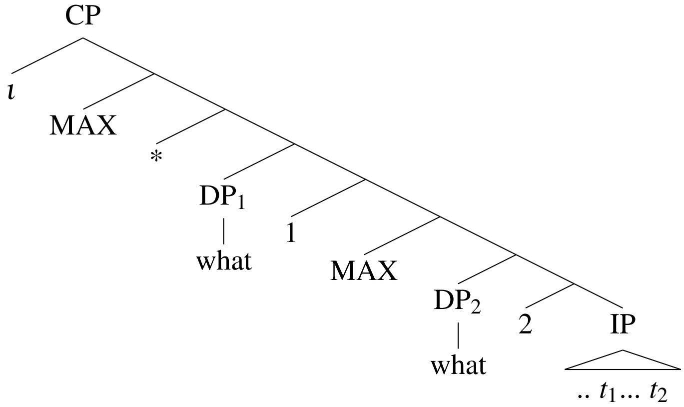
The alleged 'universal quantification' in the English free relative is a consequence of the free relative being defined as a maximal plural individual through a series of pluralization, maximalization, and ι shift operations. In order to get the desired result for multi-head correlatives, the same mechanisms must be adapted for ordered pairs rather than entities since the quantification in the multi-head case is over pairs defined by the subject and object wh-DP rather than entities. As we will see, although this process seems to involve components that are familiar from the free relative analysis, adapting them to be applicable to multiple wh-constructions like the correlative requires some major modifications. Moreover, it is not immediately apparent how these might be able to capture the domain exhaustivity and uniqueness requirements. The proposal by Gajewski (2008) discussed here argues that the universal quantificational force in multi-head correlatives can be derived through a recursive application of a maximalization operation over each wh-constituent followed by a pluralization and iota shift operation, just as in the case of the English wh-free relative shown in (10).

(10)

a. Simple free relative



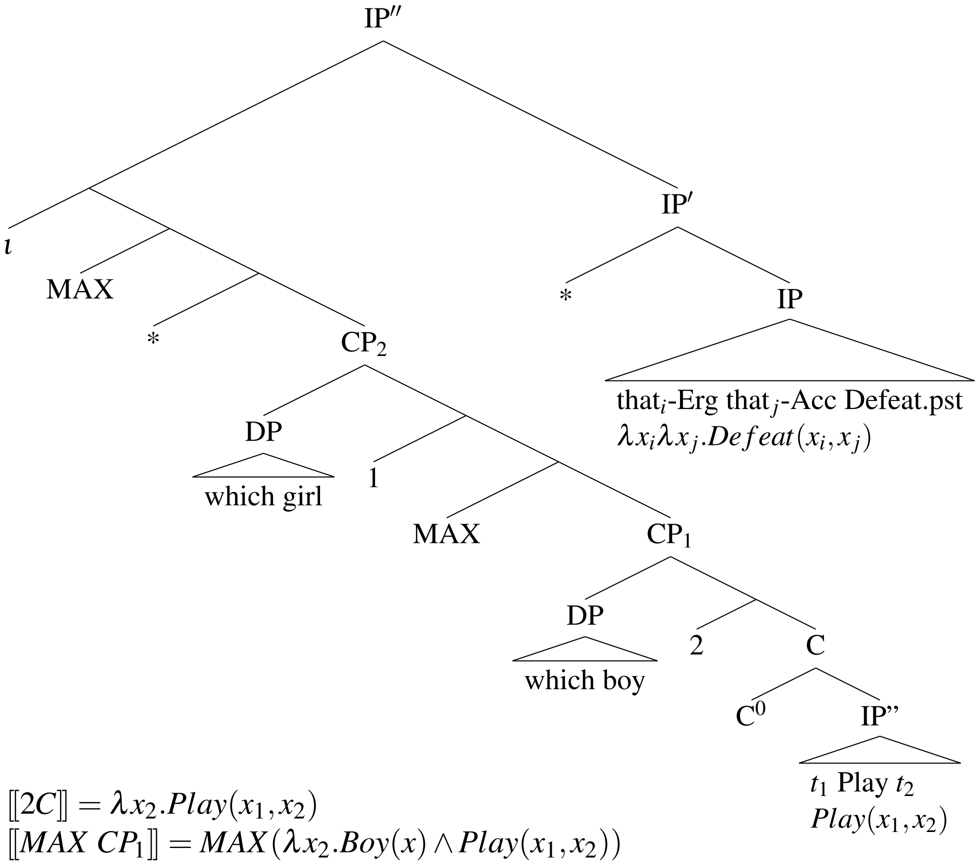
b. Multi-head free relative



Both approaches assume that the wh-DPs are properties, although this proposal treats both wh-DPs in assuming that both leave *e* type traces. This also has consequences for the wh-movement operations that generate the LF. Gajewski (2008) assumes a tucking in operation so that the subject-DP moves first and then the object-DP moves to a position below it to mirror the final word order we see in the correlative. Both wh-DPs leave behind *e* type traces. The tucking in of the object-DP is crucial for this analysis since without it, we would get the opposite result. Under the original Jacobson (1995) analysis, a wh-DP applies to the remainder of its clause, which denotes the maximal set of atomic and plural individuals, and turns its denotation into a singleton set containing the maximal individual of the input set. At this point, the ι shift applies to the singleton set to return its only member. The result is an *e* type clause. Under this proposal for multi-head correlatives, the two operations are kept separate. Crucially, the maximalization operation is applied in two places in the derivation before the pluralization and ι shift operations as seen in (11).

The number of MAX operators in the derivation depends on the number of wh-DPs in the clause. In this multi-head correlative with two wh-DPs, we see MAX being applied twice, once above each wh-DP. Unlike Dayal (1996), where the C-head creates the quantificational structure, here it is semantically vacuous. The first MAX operator applies above the object DP at CP_1. By returning a predicate that is true for some maximal individual, this first MAX operator captures the uniqueness requirement over the set of boys.

(11)



(12) a. $[\![2C]\!] = \lambda x_2.Play(x_1, x_2)$
b. $[\![MAX\ CP_1]\!] = MAX(\lambda x_2.Boy(x) \wedge Play(x_1, x_2))$
$= \lambda x_2.x_2$ is a unique boy $\wedge\, Play(x_1, x_2)$

By separating MAX and ι which perform essentially the same function, Gajewski (2008) requires a special MAX operator (13a) that maps a predicate of type $\langle e,t \rangle$ to a predicate that is true of exactly one individual. This is a departure from a typical maximality operator that takes a predicate as an argument and returns the maximal individual for which that predicate is true. So the denotation of (12b) is spelled out as in (13b)

(13) a. $[\![MAX]\!] = \lambda f_{e,t} : \exists x[f(x) = 1 \wedge \forall y[f(y) = 1 \rightarrow y \preceq x]].\lambda z.z = \sigma x[f(x) = 1 \wedge \forall y[f(y) = 1 \rightarrow y \preceq x]$
b. $[\![MAX\ CP_1]\!] = \lambda z.z = \sigma x[Boy(x) \wedge Play(x_1, x) = 1 \rightarrow \forall y[Boy(y) \wedge Play(x_1, y) = 1 \rightarrow y \preceq x]]$

At this stage in (13b), we are still left with an $\langle e,t \rangle$ type predicate. This first MAX operator seems to successfully capture the pointwise uniqueness requirement between the subject and object Wh-DP. Next, the trace left by the subject wh-DP is abstracted over, resulting in a two place relation which then combines with the predicate denoted by the subject wh-DP via predicate modification. The result is a set of ordered pairs of girls and unique boys that they played with in CP_2 as shown in (14b). But we aren't done yet. The challenge now is to end up with the semantics of a maximal plural individual in the correlative.

4.1. Pluralization for a pair-list interpretation in correlatives

Since we are concerned with 2-place relations rather than properties or sets, the extension to plurals requires some extra machinery. In order to accurately describe the process, consider the following scenario in which it is felicitious to utter the multi-head correlative. *There is a tennis match between girls and boys and 3 girls g1,g2,g3 each played against just one boy b1, b2, b3 respectively* At this stage CP_2 (14b) denotes a set of ordered girl-boy pairs such that each girl played with one unique boy which can be defined as in (14c) .

(14) a. $[\![1CP_1]\!] = (\lambda x_1.\lambda z.z = \sigma x[Boy(x) \wedge Play(x_1,x) = 1 \rightarrow \forall y[Boy(y) \wedge Play(x_1,y) = 1 \rightarrow y \preceq x]])$

b. $[\![CP_2]\!] = \lambda x.\lambda z.Girl(x) \wedge z = \sigma y[Boy(y) \wedge Play(x,y) = 1 \rightarrow \forall z[Boy(z) \wedge Play(x,z) = 1 \rightarrow z \preceq y]]$

c. $[\![CP_2]\!] = \left\{ \begin{matrix} < g1, b1 > \\ < g2, b2 > \\ < g3, b3 > \end{matrix} \right\}$

Now in order to derive the universal quantificational force that we see in English wh- free relatives, the correlative must refer to a maximal plural individual. Obtaining this with multi-head correlatives not straightforward. In the case of an English free relative, this process involved a straightforward application of the pluralizing * operator which when applied to the wh-DP returns a set of maximal plural entities.

(15) $*P = \lambda X.\lambda x.X \subseteq P \wedge x = \bigcup X$

This is not applicable in our multi-head correlative which involves a set of ordered pairs. At this stage Gajewski (2008) resorts to a special pluralizing operator with the denotation in (16a). When applied to CP2, we are left with the denotation in (16b) which is of type $<< e, < e, t >>, t >$. There are two important points to note about this particular * operator since this is crucial to deriving the domain exhaustivity and the universal quantification in the correlative.

(16) a. $[\![*]\!] = \lambda R_{<e,<e,t>>}.\lambda S_{<e,<e,t>>}.S \subseteq R \wedge S \neq \emptyset$

b. $[\![*CP_2]\!] = \lambda S_{<e,<e,t>>}.S \subseteq \{< x, z >: Girl(x) \wedge z = \sigma y[Boy(y) \wedge Play(x,y) = 1 \rightarrow \forall z[Boy(z) \wedge Play(x,z) = 1 \rightarrow z \preceq y]]\} \wedge S \neq \emptyset$

First, note is that such an operator is clearly not motivated by any overt plural morphology in the multi-headed correlative which strikingly exhibits only singular morphology on each DP. While free relatives do display an ambiguity between a plural and definite singular interpretation due to the number neutrality of the wh-word, simple correlatives do not, even if the multi-head correlative can refer to multiple entities. Second, the * operator from the English free relative proposal must be type lifted to apply to relations instead of sets of individuals. It results in a set of sets of ordered pairs as shown in (16b). While it would be reasonable to use a pluralization operator for two place predicates, this is not the typical * operator. Furthermore, although it is never specified in the denotation in (16a), this * operator also encodes that presupposition that it is total over the domain of individuals defined by the subject DP ($\boldsymbol{\lambda S.\lambda S'} : \mathbf{Tot}(\mathbf{U},\mathbf{S})\mathbf{S'} \subseteq \mathbf{S}$). This presupposition is crucial for the universal quantification interpretation.

In the literature, * has been generalized to relations in order to capture cumulative readings in cases like *the children's teachers* or *the students' parents* This pluralization operator over two place predicates denoted by ** is defined by Krifka (1986) as closure of a set under sum formation. So **R is defined as the smallest superset closed under mereological sum formation.

(17) $**R(x)(y) = 1 iff R(x)(y) = 1 or \exists x_1, x_2, y_1, y_2 : x_1 \oplus x_2 = x and y_1 \oplus y_2 = y and **R(x_1)(y_1) = 1 and **R(x_2)(y_2) = 1$ (Krifka, 1986)

As Gajewski (2008) notes, the canonical ** operator in (17) would break the pairing between each girl and the unique boy who she defeats due to the cumulative interpretation it gives, thus deriving the incorrect truth conditions for the correlative. To that end, he proposes the novel * operator in (16a). This plural operator performs a very specific operation quite different from the canonical ** operator. In this case, * takes the set of ordered pairs in (14c) and returns a set of set of ordered pairs rather than the superset containing its mereological sum. This pluralized set would be of the form in (18). This set contains the maximal set of ordered pairs as well as sets containing its atomic parts.

(18) $$[\![*CP_2]\!] = \left\{ \begin{matrix} \left\{ \begin{matrix} <g1,b1> \\ <g2,b2> \\ <g3,b3> \end{matrix} \right\} & \left\{ \begin{matrix} <g1,b1> \\ <g2,b2> \end{matrix} \right\} & \left\{ \begin{matrix} <g1,b1> \\ <g3,b3> \end{matrix} \right\} & \left\{ \begin{matrix} <g2,b2> \\ <g3,b3> \end{matrix} \right\} \\ \{<g1,b1>\} & \{<g2,b2>\} & \{<g3,b3>\} & \end{matrix} \right\}$$

As we can see from (18) this special * operator maintains the unique boy-girl pairing rather than their sum. The multi-head correlative is potentially the only configuration which might require such a non-standard semantics of pluralization involving the pluralization of individual pairs rather than the mereological sum. This operator creates plural individuals corresponding to pairs and sets of pairs which is unprecedented. A novel type of plural individual needs to be introduced in our ontology for the treatment of multi-head correlatives.

Once the * operator generates this plural entity that is comprised of multiple girl-boy pairs, the derivation proceeds in the following way; A second MAX operator must extract the maximal plural individual which in this case is the set containing the maximal set of ordered pairs in (19b). Following this, an ι shift operation shifts the type of the relative clause from a set containing the maximal set of ordered pairs , to the maximal set of ordered pairs in (20b).

(19) a. $MAX(*CP) = \lambda S.S = \{<x,y>: \lambda x_1.\lambda x_2.Girl(x_1) \wedge x_2 \text{ is a unique boy} \wedge Play(x_1,x_2)\} \wedge S \neq \emptyset$

b. $$[\![MAX*CP_2]\!] = \left\{ \left\{ \begin{matrix} <g1,b1> \\ <g2,b2> \\ <g3,b3> \end{matrix} \right\} \right\}$$

(20) a. $\iota(MAX(*CP)) = \iota S = \{<x,y>: \lambda x_1.\lambda x_2.Girl(x_1) \wedge x_2 \text{ is a unique boy} \wedge Play(x_1,x_2)\} \wedge S \neq \emptyset$

b. $$[\![\iota MAX*CP_2]\!] = \left\{ \begin{matrix} <g1,b1> \\ <g2,b2> \\ <g3,b3> \end{matrix} \right\}$$

Finally, the * operator is also applied to the set of ordered pairs denoted by the main clause, which take the correlative as an argument. This defines a subset relation between the two clauses which stated that the set of ordered pairs defined by the correlative is a subset of the set of ordered pairs defined by the main clause as in (21c).

(21) a. $[\![IP']\!] = *[\lambda x.\lambda y.Beat(x,y)] = \lambda S.S \subseteq \{<x,y>: Beat(x,y)\} \wedge S \neq \emptyset$

b. $[\![*IP']\!]([\![\iota CP']\!]) = \lambda S.S \subseteq \{<x,y>: Beat(x,y)\} \wedge S \neq \emptyset(\iota S = \{<x,y>: \lambda x_1.\lambda x_2.Girl(x_1) \wedge x_2 \text{ is a unique boy} \wedge Play(x_1,x_2)\} \wedge S \neq \emptyset)$

c. $(\iota S = \{<x,y>: \lambda x_1.\lambda x_2.Girl(x_1) \wedge x_2 \text{ is a unique boy} \wedge Play(x_1,x_2)\} \wedge S \neq \emptyset) \subseteq \{<x,y>: Beat(x,y)\} \wedge S \neq \emptyset$

True iff for every pair of a girl x and a unique boy y that x played, x beat y.

The context in which the multi-head correlative was uttered is one in which there was a tennis tournament between boys and girls, where each girl *g1,g2,g3* each played against just one boy *b1, b2, b3* respectively. So we are left with 3 ordered pairs that sastisfy the correlative and are a subset of the set of ordered pairs defined by the main clause.

4.2. The problem of presupposition projection

Although the denotation in (21c) seem to be the right truth conditions for the multi-head correlative, a simple application of the same MAX and ι operators as the free relative does not actually result in the domain exhaustivity and universal quantification that are integral to the semantics of the multi-head correlative. In order to illustrate this, let us compare the denotations of the set of ordered pairs defined by CP_2 with the final denotation of the correlative. We see that CP_2 in (14c) and the set of ordered pairs denoted by the correlative in (20b) are equivalent. This raises the question of the need for 3 additional operators to return to exactly the same result.

At CP_2 in (14c), the derivation does not yet account for domain exhaustivity or universal quantification which are meant to be presuppositions under this approach. These two points are meant to be captured by the * operator, the second MAX operator and the final ι shift. However, it is unclear how these operators manage to achieve this when considering a context where domain exhaustivity and universal quantification are naturally satisfied. To illustrate the problem, consider a model U as in Figure 1 which illustrates the context in (22)

(22) *Context: There is a tennis tournament between girls and boys with 5 girls* $\{g_1,g_2,g_3,g_4,g_5\}$ *and 3 boys* $\{b_1,b_2,b_3\}$. *3 girls* $\{g_1,g_2,g_3\}$ *played 2 matches each and lost, one girl* $\{g_4\}$ *did not play at all, and one girl* $\{g_5\}$ *played against one boy* $\{b_1\}$ *and defeated him.*

The context in (22) is not one in which the multi-head correlative can be felicitously uttered. Not all of the girls played. Most of the girls played against more than one boy, and only one girl played against one boy and defeated him. A set of ordered pairs as defined in (14b) and repeated below would only be true of one ordered pair $< g5,b1 >$ in this model and false or undefined for the rest.

(23) a. $[\![CP_2]\!] = \lambda x.\lambda z.Girl(x) \wedge z = \sigma y[Boy(y) \wedge Play(x,y) = 1 \rightarrow \forall z[Boy(z) \wedge Play(x,z) = 1 \rightarrow z \preceq y]]$

b. $[\![CP_2]\!] = \{< g5,b1 >\}$

c. $[\![\iota MAX * CP_2]\!] = \{< g5,b1 >\}$

Applying the *, MAX and ι operators in such a context would only return this same set consisting of the one ordered pair that verifies the correlative. While this clearly derives the wrong truth conditions for the multi-head correlative, this is precisely how one would expect a higher order free relative to behave. We know from Jacobson (1995) that English wh-free relatives do not truly exhibit universal quantificational force. A free relative with a single wh-DP refers to both singular and plural maximal entities, with the reference to maximal plural entities giving rise to an alleged universal interpretation.

The proposal in Gajewski (2008) accomplishes exactly the same result, with the stacked free relative referring to ordered pairs, and not individuals. In the felicitous conditions, the free relative is defined and true for the maximal set of ordered pairs which includes all the girls who played with a unique boy. The presupposition from the lower MAX operator filters out all the girls who played with multiple boys. In the infelicitous context, this would be all the entities in the set of girls and the set would only include the singleton pair $\langle g_5, b_1 \rangle$ which is then pluralized by the higher * operator and then selected by the MAX and ι operators.

This is clearly insufficient for a correlative which has much stronger truth conditions. Gajewski (2008) must introduce certain stipulations to rectify this. After the * operator pluralizes the set of ordered pairs defined by CP_2, a second MAX operator must extract the maximal plural individual which in this case is the set containing the maximal set of ordered pairs. This is a crucial point in the derivation because this is where the universal quantificational force, and domain exhaustivity of the correlative is derived, and it relies on the following stipulations about the * operator. **1)** The * operator has the denotation in (16a) and is not upward closed under sum formation. **2)** The uniqueness presupposition from the lower Max projects universally over the domain of the subject DP, i.e girls.

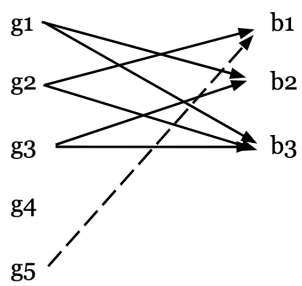


Figure 1:

This is because an existence presupposition is far too weak and would satisfy the correlative in a context like (22). Gajewski (2008) assumes that the special * operator behaves like a universal quantifier to allow the lower presupposition to project universally over the domain of the DP. This is based on the argument that the presupposition of a relative clause projects universally over the domain of the restrictor in quantificational structures. In constructions of the form $Q.(R)\lambda x.A(x)_{p(x)}$, the presupposition $p(x)$ holds for every individual in the domain of quantification i.e $\forall x.[R(x) \rightarrow p(x)]$. So sentences like *Every/No tennis player brings both of his rackets to a match*, presuppose that every tennis player has two rackets.

Gajewski (2008) argues that the uniqueness presupposition in (14b) poses a similar case. The presupposition that there is a unique boy is in the form of restrictive relative clause modifying the restrictor of a quantifier. So (14b) is of the form $Q.[\lambda x.A(x)_{p(x)}]N$ and the presupposition project universally, i.e $\forall x.[N(x) \rightarrow p(x)]$. This means that CP_2 is defined only if every girl played a unique boy. However, while this view on presupposition projection is borne out in the case of universal quantifiers, it has been brought into question for existentials, as well as definite descriptions [3]. Sentences like *Some tennis player who brought both of his rackets to a match broke them* do not necessarily require that every tennis player has two rackets. Without this, it is not possible to state that every girl played a unique boy, or that every girl even played at all. Moreover, there is nothing in the denotation of * in (16a) that suggests that it resembles a universal quantifier. While $\forall$ defines a subset relation between the restrictor and the nuclear scope, the special * operator in this case defines a superset relation between the two.

The puzzle of presupposition projection at this stage of the derivation continues as this presupposition must project past the * operator. This is the quantifier-like element which determines the projection. Once this presupposition is defined totally over the domain of girls, each set

[3]See discussion in Fox (2013) about the nature of universal projection in quantificational structures.

of ordered pairs defined in $*CP_2$ contains a set of girl-boy pairs for every girl in the domain who played with a unique boy. A second modified MAX operator returns the singleton set containing the maximal set of ordered pairs. In a final step, the iota shift operation extracts this maximal set of girl-boy pairs such that every girl in the domain played with a unique boy (24b).

(24) a. $MAX(*CP) = \lambda S.S = \{<x,y>: \lambda x_1.\lambda x_2.Girl(x_1) \wedge x_2$ is a unique boy $\wedge Play(x_1,x_2)\} \wedge S \neq \emptyset$

b. $\iota(MAX(*CP)) = \iota S = \{<x,y>: \lambda x_1.\lambda x_2.Girl(x_1) \wedge x_2$ is a unique boy $\wedge Play(x_1,x_2)\} \wedge S \neq \emptyset$

The rest of the derivation proceeds as before, but with this crucial stipulation that the presupposition is total over the domain of girls. This ensures the universal quantification and domain exhaustivity such that the subset relation defined by the correlative is true for every girl in the context (22) and gives us the desired truth conditions.

To recapitulate, this approach involves two separate MAX operators over each wh-DP , with the lower MAX generating a uniqueness presupposition over the set of boys. This results in the set of girl-boy pairs being defined only for those girls who played with just one boy. Next, * operator type applies to result in a set of sets of ordered pairs. We must assume that this quantifier-like operator allows the presupposition to project universally over the domain of the subject-DP so that the correlative is defined totally over the domain of girls. A second type-lifted MAX operator extracts singleton set containing the maximal set of ordered pairs, and the final ι shift outputs a set of ordered pairs. The pluralized relation defined by the main clause takes the correlative as an argument and defines a subset relation between the correlative and the matrix clause such that the maximal set defined by the correlative is a subset of the set defined by the matrix clause. For our multi-head correlative in (2b), this means that the maximal set of ordered pairs of girls who played a unique boy is a subset of the set of girls who defeated boys.

5. Returning to functional dependency

It is certainly true that both analyses derive the correct truth conditions in the original context for the multi-head correlative in (25) , which is a case that satisfies all the felicity requirement of a multi-head correlative. This context illustrates that domain exhaustivity applies to the set denoted by the subject-DP as this is a situation where all the girls will play against some boy, not all the boys will get a chance to play.

However, this equivalence between the two approaches can only be achieved with 2 very construction specific assumptions about the denotation of pluralization operators in multi-head correlatives, as well as the nature of presupposition projection in these constructions. In contrast, the proposal in Dayal (1995) largely rests on the nature of functional traces in multiple-wh questions as well as quantificational questions. Moreover, the uniqueness requirement is part of the denotation of C-head, as is the domain exhaustivity and the universal quantification. This rules out the possibility of the multi-head correlative being felicitous in a context like (26) without any further stipulations.

(25) *Context: There is a tennis tournament between girls and boys. There are three girls interested in playing against boys* $\{g_1,g_2,g_3\}$ *but there are four boys interested in playing* $\{b_1,b_2,b_3,b_4\}$ (*adapted from Dayal (1995)*)

a. ✓[jis larkii-ne jis larke-ke saath khelaa] us-ne us-ko haraaya.
[which girl-ERG which boy-GEN with play] that-ERG that-ACC defeat.
Which girl played with which boy, she defeated him.

(26) *Context: There is a tennis tournament between girls and boys with 5 girls* $\{g_1, g_2, g_3, g_4, g_5\}$ *and 3 boys* $\{b_1, b_2, b_3\}$*. 3 girls* $\{g_1, g_2, g_3\}$ *played 2 matches each and lost, one girl* $\{g_4\}$ *did not play at all, and one girl* $\{g_5\}$ *played against one boy* $\{b_1\}$ *and defeated him.*

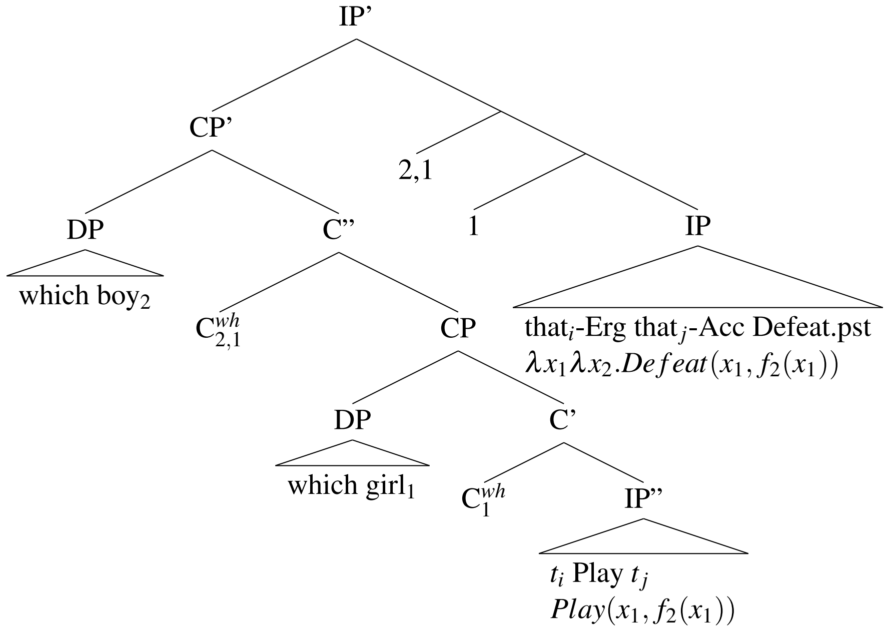
a. ✗[jis larkii-ne jis larke-ke saath khelaa] us-ne us-ko haraaya.
[which girl-ERG which boy-GEN with play] that-ERG that-ACC defeat.
Which girl played with which boy, she defeated him.

5.1. An updated version of functional dependency

From the previous section, we can safely conclude that despite their apparent similarities there are different mechanisms at play in free relatives and correlatives, so perhaps a unified analysis of the two is not a viable option, and a functional dependency approach is the best way to analyse multi-head correlatives. In this section I provide an updated version of the original functional dependency approach in Dayal (1995) that is more in line with some of the recent literature on the semantics of questions, and decomposes the complex denotation of the C-head that generates the quantificational structure of the correlative.

This proposal retains the functional dependency between the subject and object wh-DP. However, instead of properties, these wh-elements are existential quantifiers. The subject DP is an existential quantifier over the domain of girls while the object DP is a quantifier over choice functions. As in the original approach, one leaves behind an *e* type trace while the other leaves a functional trace of type $\langle e, e \rangle$. Furthermore, rather than a single complex C-head, I adopt a split-Comp approach along the lines of Chierchia (2023). While the lower C-head in (28a) abstracts over the trace left by the subject and encodes universal quantification over the domain of the subject, the higher C-head in (28b) abstracts over the functional trace and generates the quantificational structure of the correlative.

(27)



The denotation of C_1 incorporates the Montague BE operator. It abstracts over the index on the trace of subject wh-word, and takes 2 arguments, the subject DP, and the IP.

(28) a. $[\![C_1]\!] = \lambda Q.\lambda \mathscr{P}.\forall y[BE(\mathscr{P})(y) \rightarrow Q(y)]$
b. $[\![C_2]\!] = \lambda P_f.\lambda \mathscr{P}.\lambda R.\mathscr{P}[\lambda f'[P_f(f') \wedge \forall z \in Dom(f') \rightarrow R(z, f')]]$
c. $[\![CP]\!] = [\![C_1]\!]([\![IP'']\!])([\![DP]\!])$
d. $[\![CP]\!] = \lambda Q.\lambda \mathscr{P}.\forall y[BE(\mathscr{P})(y) \rightarrow Q(y)(\lambda x_1.Play(x_1, f_2(x_1)))](\lambda P.\exists x[Girl(x) \wedge P(x)])$
e. $= \forall y[(\lambda \mathscr{P}\lambda w.[\mathscr{P}(\lambda z(z = w))])(\lambda P.\exists x[Girl(x) \wedge P(x)])(y) \rightarrow Play(y, f_2(y))]$
f. $= \forall y[(\lambda w.[\lambda P.\exists x[Girl(x) \wedge P(x)](\lambda z(z = w))])(y) \rightarrow Play(y, f_2(y))]$
g. $= \forall y(\lambda w.[\exists x[Girl(x) \wedge \lambda z(z = w))](x)](y) \rightarrow Play(y, f_2(y))$
h. $= \forall y.Girl(y) \rightarrow Play(y, f_2(y))$

The second Comp head C_2 abstracts over the functional trace of the object wh-phrase.

(29) a. $[\![CP']\!] = [\![C_2]\!]([\![CP'']\!])([\![DP_2]\!])$
b. $[\![C_2]\!] = \lambda P_f.\lambda \Pi.\lambda R.\Pi[\lambda f'[f' = \iota f.[P_f(f) \wedge \forall z \in Dom(f) \rightarrow R(z, f)]]]$
c. $[\![CP']\!] = \lambda P_f.\lambda \Pi.\lambda R.\Pi[\lambda f'[f' = \iota f.[P_f(f) \wedge \forall z \in Dom(f) \rightarrow R(z, f)]]]$ $(\lambda f_2(\forall y.Girl(y) \rightarrow Play(y, f_2(y)))(\lambda P.\exists f.[P(f)])$
d. $= \lambda \Pi.\lambda R.\Pi[\lambda f'[f' = \iota f.[(\lambda f_2([\forall y.Girl(y) \rightarrow Play(y, f_2(y))]))(f) \wedge \forall z \in Dom(f) \rightarrow R(z, f)]]](\lambda P.\exists f''.[P(f'')])$
e. $= \lambda \Pi.\lambda R.\Pi[\lambda f'[f' = \iota f.[[\forall y.Girl(y) \rightarrow Play(y, f(y))] \wedge \forall z \in Dom(f) \rightarrow R(z, f)]]]$ $(\lambda \mathbf{P}.\exists \mathbf{f}''.[\mathbf{P}(\mathbf{f}'')])$
f. $= \lambda R.\exists \mathbf{f}''.[\mathbf{f}'' = \iota \mathbf{f}.[[\forall y.Girl(y) \rightarrow Play(y, f(y))] \wedge \forall z \in Dom(f) \rightarrow R(z, f)]]$

Turning to the main clause, I assume that the demonstratives corresponding to the relative pronouns also encode similar e and $\langle e, e\rangle$ type traces that are abstracted over.

(30) a. $[\![IP']\!] = \lambda R.\exists \mathbf{f}''.[\mathbf{f}'' = \iota \mathbf{f}.[[\forall y.Girl(y) \rightarrow Play(y, f(y))] \wedge \forall z \in Dom(f) \rightarrow R(z, f)]]$ $(\lambda f_j.\lambda x_i.Defeat(x_i, f_j))$
b. $= \exists \mathbf{f}''.[\mathbf{f}'' = \iota \mathbf{f}.[[\forall y.Girl(y) \rightarrow Play(y, f(y))] \wedge \forall z \in Dom(f) \rightarrow Defeat(z, f)]]$

6. Conclusion

We are left with the truth conditions in (30b) which states that the correlative is true if there is some unique function defined by the correlative also verifies the main clause. As in the original formulation, it is total over the domain of the subject and is predicted to be infelicitous in (22). This modified proposal for functional dependency is equally compatible with the original proposal for simple correlatives in Dayal (1995). An advantage of this approach is that it makes the correct predictions for an unambiguous single definite interpretation for the simple correlative, while still accounting for the universal interpretation in the multi-head case without postulating a covert pluralization operator.

In this paper, I have examined the semantics of multi-head correlatives and compared two possible analyses; one by Dayal (1995) and another Gajewski (2008). The first, which builds on the semantics of multiple wh-questions, and the second which builds on the semantics of free relatives. While the two may seem equivalent at first glance, I have illustrated how the free relative approach derives much weaker truth conditions without some major stipulations about the nature of pluralization and presupposition projection in this construction. Covert pluralization in the absence of plural morphology followed by a maximalization operation might derive the desired definite-universal alternation in English style wh-free relatives, but it results in the wrong truth conditions for Hindi multiple-wh correlatives. Encoding a functional dependency between the subject and object wh-DPs in this construction, just as in multiple wh-questions seems to be the correct way to deriving universal quantification over pair-lists in multiple-headed correlatives.

This approach still needs to assume three distinct ∃-heads; the C-head in the simple case has to be different from both C-head in the multi-head correlative, even if they both have a maximalization semantics. But this also gives us the correct predictions for simple correlatives. There is still much to be done in terms of decomposing the semantics of correlatives, and their interactions with elements such as free choice items and quantificational adverbs which I leave for future work.

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