How to bind into alternatives

Alexander SHILEN — *Johns Hopkins University*

Abstract. Pointwise abstraction has proven elusive to define, though language that requires the operation is commonplace. I show that modeling alternatives as sets of choice functions allows them to be bound into with a simple abstraction rule.

Keywords: binding, alternatives, choice functions

1. Introduction

Suppose we take the meaning of the question in (1) to be all of its possible answers.

(1) Who has some bad habits?

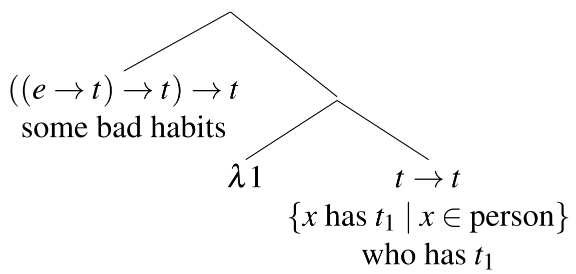
Suppose, also, that we interpret *some* as an existential quantifier. Then (1) asks if there is some bad habit that Louis has, some bad habit that Rhoda has, some bad habit that Percival has, and so on. When you ask this question, you are asking which if any of these claims is true.

Such an interpretation synthesizes the model of questions in Hamblin (1976) and the model of quantification in Heim and Kratzer (1998). Independently, these grammars are faithful to our intuitions about the meanings of questions and quantifiers. They are also compositional, in that they describe how *who* and *some* compose with their neighbors to determine the meaning of a phrase in which they occur.

We might therefore expect that together, Hamblin, Heim and Kratzer point towards a compositional model of (1). As Shan (2004) has observed, however, it is difficult to adapt H&K's notion of abstraction to Hamblin's pointwise setting.

To see why, first consider the tree in (2). As for Hamblin, *who* denotes a set of people, other lexical items are solitary members of singleton sets, and siblings compose via pointwise function application. As H&K would have it, the quantifier *some bad habits* has taken scope, depositing the trace t_1 and introducing the abstraction $\lambda 1$.

(2)



Here is the simpler half of the puzzle: how can *some bad habits* compose with *who has t_1* to yield the set of possible answers we have proposed? In order to be the pointwise argument of the quantifier, the complement must denote a set of predicates, something with the type $(e \to t) \to t$. H&K's abstraction rule, however, produces the expression in (4).

(3) $\{\lambda y. x \text{ has } y \mid x \in \text{person}\}$ $\qquad (e \to t) \to t$

(4) $\lambda y.\{x \text{ has } y \mid x \in \text{person}\}$ $\qquad e \to (t \to t)$

The difference between the two, plainly enough, is that in (4) abstraction is over the whole set of propositions, whereas in (3) abstraction is over each proposition in the set. What we need is a pointwise notion of abstraction.

This much we can achieve by fusing the two grammars in a particular fashion. Recall that to model abstraction, H&K rely on a map of variable assignments. The unambiguous denotation (5) for t_1 depends on a variable assignment g explicitly, as does the denotation for the phrase (6) in which it occurs.

(5) $[\![t_1]\!] = \lambda g.\{g(t_1)\}$ $\qquad i \to (e \to t)$

(6) $[\![\text{who has } t_1]\!] = \lambda g.\{x \text{ has } g(t_1) \mid x \in \text{person}\}$ $\qquad i \to (t \to t)$

When (6) is fed to H&K's abstraction rule, the location of its g parameter determines the location of the y parameter in the result (4). In order to move y, then, we may first move g, in an approach to alternative assignment functions introduced by Rooth (1992).

(7) $[\![t_1]\!] = \{\lambda g.g(t_1)\}$ $\qquad (i \to e) \to t$

Move g across the board, so that lexical entries with base types α have the uniform type $(i \to \alpha) \to t$. Then adapt application and abstraction to sets of assignment dependent elements, as opposed to assignment dependent sets.

(8) Pointwise Assignment Sensitive Application[1]

$$[\![a\ b]\!] : ((i \to \alpha \to \beta) \to t) \times ((i \to \alpha) \to t) \to (i \to \beta) \to t$$
$$[\![a\ b]\!] = \{\lambda g. f(g)(x(g)) \mid f \in [\![a]\!], x \in [\![b]\!]\}$$

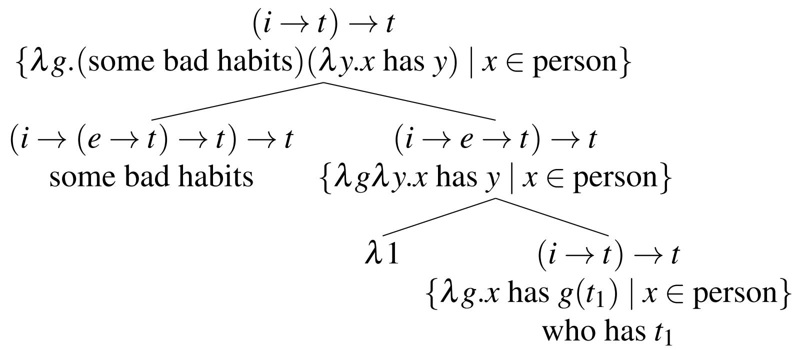
(9) Pointwise Assignment Sensitive Abstraction

$$[\![\lambda n\ a]\!] : n \times ((i \to \alpha) \to t) \to (i \to e \to \alpha) \to t$$
$$[\![\lambda n\ a]\!] = \{\lambda g \lambda x. y(g^{x/n}) \mid y \in [\![a]\!]\}$$

With these modes of composition we can derive a set of answers to (1) as in (10).

[1]Here the type constructor for functions $\to$ is right associative, so that $x \to y \to z$ means $x \to (y \to z)$. The type constructor for products $\times$ has higher precedence than $\to$, so that $x \times y \to z$ means $(x \times y) \to z$. I will sometimes omit parentheses to simplify the notation.

(10)



This takes us to the thornier half of the puzzle. While inverting alternatives and assignment dependence suffices to model sentences like (1), it's not obvious that the same grammar can handle sets of alternatives whose restrictive clauses are themselves assignment dependent. In (11), for example, *which of her marbles* denotes a set that varies with the value of *her*.

(11) Who lost which of her marbles?

Suppose *who* introduces an abstraction that binds *her*. Then a plausible candidate for the denotation of the set of alternatives *which of her*$_1$ *marbles* is in (12).

(12) $\lambda g.\{x \mid x \in \text{marble of } g(\text{her}_1)\}$ $\qquad i \to (e \to t)$

But to construct (12), we have employed the original, problematic order for assignment dependence and alternatives, as we must close over the whole expression in order to bind the g in the set's restriction.[2]

Can we instead let *which* denote an unrestricted set and apply the compound predicate *of her*$_1$ *marbles* to each of its elements, as in (13)? We cannot, as this would yield the set of answers in (14), which includes such propositions as *Rhoda lost her moccasins, but they aren't marbles*, and *Rhoda lost marbles, but they aren't hers*. These are not appropriate answers to the question in (11).

(13) $\{\lambda g.x \text{ is marble of } g(\text{her}_1) \mid x \in D_e\}$ $\qquad (i \to e) \to t$

(14) $\{\lambda g.\, y \text{ lost } x \text{ and } x \text{ is marble of } g(\text{her}_1) \mid x \in D_e, y \in \text{person}\}$ $\qquad (i \to t) \to t$

The puzzle boils down to this: how can we make individual alternatives assignment dependent, and at the same time use that dependence to restrict membership in a set?

Faced with this tension – between assignment dependent sets and assignment dependent elements – Shan (2004) concludes that H&K's notion of variables and Hamblin's alternative semantics are incompatible. He offers a pointwise, variable free semantics in the spirit of Jacobson (1995) instead, and shows that it handles quantification into both kinds of expressions.

[2]Note that the difficulty is unrelated to the set-oriented notation I have adopted. When phrasing the set as its characteristic function in (1), the embedded g is similarly inaccessible.

(1) $\lambda y \exists x \in \text{marble of } g(\text{her}_1).x = y$ $\qquad e \to t$

Linguists after Shan have proceeded in different ways. Some have agreed that the tension is symptomatic of a fundamental incompatibility, and have resolved it – just like Shan – by forgoing one or the other of the two grammatical devices. Whereas Shan abandons assignment functions and retains pointwise composition, Charlow (2020) retains assignments and adopts, instead of pointwise composition, a variant of the intensional logic in Karttunen (1977).

Bumford (2022) offers a formalization of the devices' incompatibility. He assumes that the only way to model (11) while retaining each device is to define a shift from the denotation (12) into a corresponding denotation with the inverted type $(i \to e) \to t$. He motivates conditions that such a type shift should satisfy and shows that the conditions are incompatible with each other.

Kotek (2019) suggests that the recalcitrance of pointwise abstraction is a productive constraint in the grammar, and that it's responsible for intervention effects.

One exception to this spirit of argument is Novel and Romero (2010), who show that treating *which* as a definite description, as in Beck and Rullmann (1999), allows us to define precisely the kind of internally assignment dependent set – with the type $(i \to e) \to t$ – that we require instead of (12).

Here is the plan. In section 2 I will review Novel & Romero's proposal. In 3 I will present examples that Novel & Romero's grammar does not model, because they are not definite. In section 4 I will introduce a solution, due to Orin Percus, that preserves Novel & Romero's insight and can generalize to the rest of the data. To conclude, I will distill the solution into its essential components.

2. Novel & Romero

Novel and Romero (2010) begin with Rooth's $(i \to e) \to t$ type. They adapt Beck and Rullmann's (1999) treatment of wh-phrases as definite descriptions to construct a denotation for the set of marbles in (11) that varies according to the value assigned to the embedded pronoun even though the pronoun occurs in the nuclear scope, rather than the restrictive clause, of the set of alternatives.

Here's the idea. Suppose we apply each element of (15) to an assignment.

(15) $[\![\text{which of her}_1 \text{ marbles}]\!] = \{\lambda g \iota v.\, v \in \text{marble of } g(\text{her}_1) \wedge v = z \mid z \in D_e\} \quad (i \to e) \to t$

Some elements $z \in D_e$ are marbles of $g(\text{her}_1)$, but many are not. We can accommodate the failure of the definite description to refer for these choices of z by allowing ι to return some distinguished element – call it $\perp$. Now if *her*$_1$ *marble* consists exactly of $\{z_1, z_2\}$, then feeding an assignment to (15) will produce the set $\{z_1, z_2, \perp\}$. Adopt the convention whereby $\perp$ is excluded from sets of alternatives, and we have used a predicate that occurs in the nuclear scope of a set to define its membership conditions. Because we begin with the type $(i \to e) \to t$, it's straightforward to bind pronouns in the nuclear scope, and therefore straightforward to handle the configuration in sentences like (1) as well.

But it's possible to construct examples where bound pronouns restrict sets of alternatives that aren't introduced by definite determiners.

3. Some more data

In fact, it's possible to construct representative examples with any kind of wh-determiner.

We might expect, furthermore, any pair of phenomena that have been modeled with alternatives and assignments to be fertile ground for the puzzle. This is true in some cases, but not all. Work inspired by Kratzer and Shimoyama (2002), which treats indeterminacy writ large as alternative-denoting, requires pointwise abstraction in general. On the other hand, Rooth's (1992) model of focus operates on unrestricted domains, and so does not offer restrictive contexts in which to place bound material. Phenomena beyond quantifiers have been modeled with abstraction and assignment functions, such as pronominal anaphora (Büring, 2005) and so-called split intensionality (Keshet, 2010), and these elicit the puzzle when paired with any kind of alternative semantics that admits restrictive predicates for its sets.

3.1. Questions

While it's easiest to construct examples with the determiner *which*, which demands a restrictive complement, we can exemplify the puzzle with an arbitrary wh-question: place a quantifier in the nuclear scope of the determiner and a pronoun in a modifier.

(16) Who among her$_1$ peers does everyone$_1$ admire?

(17) Where in their$_1$ kitchen do [most cooks]$_1$ keep the salt?

(18) How, when they$_1$ travel so far, do [all of the geese]$_1$ migrate?

(19) Why, if they$_1$ are outraged, does [no one]$_1$ protest?

(20) What will everyone$_1$ do without their$_1$ power tools?

(21) When in their$_1$ golden years do [most people]$_1$ put their$_1$ feet up?

Each of these examples has a reading in which the quantifier binds both the pronoun in the modifier and the trace it leaves behind. Some – (16), (17), (20), and (21) – have pair-list readings. At least (16) and (20) have functional readings. Let us set the pair-list and functional readings aside.

To derive the quantificational readings, assume that each quantifier takes wide scope. Here are sketches of the LFs of the nuclear scopes of the quantifiers post-extraction, before they compose with their quantifiers. In (25) I treat the conditional antecedent as a relativizer of the propositions that *why* quantifies over; in (26) *what* quantifies over events; in (27) *when* quantifies over a temporal domain. In each sketch I represent both the trace and the pronoun(s) with x, and I leave x free.

(22) $[\![$who among her$_1$ peers does t_1 admire$]\!] = \{x$ admires $y \mid y \in$ peer of $x\}$

(23) $[\![$where in their$_1$ kitchen does t_1 keep the salt$]\!] = \{x$ keeps the salt at $y \mid y \in x$'s kitchen$\}$

(24) $[\![$how, when they$_1$ travel so far, does t$_1$ navigate?$]\!] =$
$\{x$ navigates by y-ing $\mid y$ accommodates the distance x travels$\}$

(25) $[\![$why, if they$_1$ are outraged, does t$_1$ protest?$]\!] =$
$\{\lambda w.x$ protests in $w \wedge y(w) \mid y \in D_{s\to t}, y \subseteq (\lambda w.x$ is outraged in $w)\}$

(26) $[\![$what will t_1 do without their$_1$ power tools$]\!] =$
$\{x$ will $y \mid y \in D_v, x$ lacks power tools in $y\}$

(27) $[\![$when in their$_1$ golden years does t_1 put their$_1$ feet up$]\!] =$
$\{x$ puts x's feet up at $y \mid y \in D_T, y \subseteq x$'s golden years$\}$

Here is the observation. In order to derive sets of answers for each of these questions, we need the displaced determiner to compose pointwise with its complement. At the same time, we need the determiner to substitute values into its complement's restriction.

3.2. Choiciness

One line of work, beginning with Kratzer and Shimoyama (2002), casts free choice effects as the consequence of propositional alternatives introduced, as in *wh*-questions, by expanding sets of pointwise alternatives. K&S study indeterminate pronouns in Japanese and German *irgendein*, while further work extends their approach to English *any* (Menéndez-Benito, 2010), Spanish *algún* (Alonso-Ovalle and Menéndez-Benito, 2003), disjunction (Alonso-Ovalle, 2006), and particular species of conditionals (Alonso-Ovalle, 2009; Rawlins, 2013).

3.2.1. Indefinites

Indefinite determiners take complements that can embed bound pronouns, so it's unsurprising that when they're modeled as sets of alternatives, they elicit our problematic binding configuration. Consider the use of *any* in (28).

(28) Most lawyers owe any scruples they have to an ethics class.

Channeling K&S and Menendez-Benito, we might let *any scruples they possess* denote the set of scruples possessed by whoever *they* refers to. We can sketch the LF of the complement in (29).

(29) $[\![t_1$ owes any scruples they$_1$ have to an ethics class$]\!] =$
$\{x$ owes y to an ethics class $\mid y$ is scruple of $x\}$

Here x is free, again, in both a nuclear scope and a restrictive clause.

3.2.2. Unconditionals

Another application of Hamblin alternatives is to the analysis of unconditionals, as in Rawlins (2013), who treats (30) as a set of conditional antecedents with a single consequent.

(30) Whatever advice you get, you should seek a second opinion.

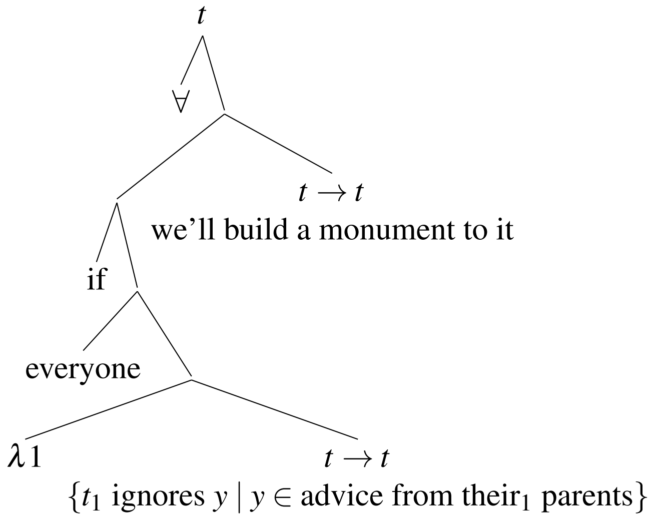
Rawlins models *whatever advice* as the subset of advice in D_e, and then allows it to compose pointwise with the predicate *you get*. This set of propositions composes pointwise with a conditional operator, which in turn composes with the singleton consequent, yielding a set of conditional propositions with varying antecedents and a fixed consequent: *if you get advice* a*, you should seek a second opinion; if you get advice* b*, you should seek a second opinion; if you get advice* c*,* etcetera. At the clause boundary, we close over the set, asserting that each of its elements is true.

Here's an unconditional example that elicits the puzzle.

(31) Whatever advice from their parents everyone ignores, we'll build a monument to it.

The sentence in (31) is about the kinds of advice-from-parents that are ignored by one and all: we'll build a monument to any such advice.[3] To model this reading, the quantifier in (31) must scope out of the relative clause, but below the operator that closes over the alternatives. We can sketch the LF in (32).

(32)



The complement of the quantifier *everyone* in this LF has a pronoun in both its nuclear and restrictive scope. Since both quantifier and complement are below the alternative closure operator, we need a pointwise definition of abstraction for them to compose.[4]

[3] To my ears, it's possible for a pronoun in the consequent of a similar sentence to vary with an antecedent-embedded quantifier.

(1) Whatever advice from their parents everyone ignores, they regret it later.

If this is the case, it's another kind of puzzle: *everyone* must take wide scope to bind *they*, and conditional antecedents are usually strong islands for universal quantifiers.

[4] As long as we're discussing scope configurations for unconditionals, here's an example worth noting for a different reason: treating *whatever* as a universal quantifier, with Rawlins, gives the wrong truth conditions.

(1) Whatever he or she whispers, no dentist will persuade me to buy invisalign.
$\not\to$ there's no dentist x such that for everything x whispers, x will persuade me to buy invisalign.

The only scope that *no dentist* can take is widest, since it must bind both the pronoun in the antecedent and its trace in the consequent. But the wide scope gloss of (1) is verified if every dentist will persuade me to buy invisalign just once out of the many times they will whisper in my ear, and the sentence doesn't have that reading. Here is an appropriate gloss.

(2) there's no dentist x such that for <u>something</u> x whispers, x will persuade me to buy invisalign.

The accuracy of the gloss in (2) suggests that unconditionals have the same bivalence between universal and existential quantification that's a hallmark of other free choice items in downward entailing contexts (Ladusaw, 1980).

3.3. Assignment Sensitivity

Phenomena apart from quantification have been modeled with abstraction and assignment functions.

3.3.1. Nominal Anaphora

A variety of examples in the literature (e.g. in Déchaine and Wiltschko, 2002; Büring, 2005; Kratzer, 2009) demonstrate binding relationships – and therefore abstraction – among non-quantificational pronouns. I leave it to show that these occasions for abstraction surface in the restrictive clauses of alternative-denoting determiners.

3.3.2. Split Intensionality

Keshet (2010) uses movement and abstraction to distinguish separate worlds of evaluation in phrases that are ambiguous between de re and de dicto interpretations, in particular when movement is constrained by islands.

(33) Rhoda thinks my brother is my father.

With Keshet, we can derive an interpretation for (33) by interpreting one of the embedded DPs with respect to Rhoda's belief worlds and the other with respect to the actual world. We interpret *my brother* de re by scoping it out of its surface position and inserting a world abstraction $\lambda w'$. We fix *my brother* at the actual world w and allow *my father* to vary with w'.

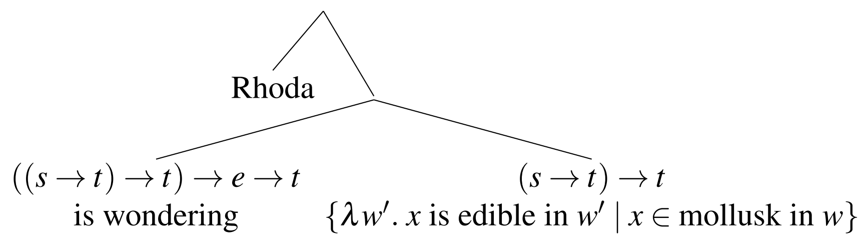
It turns out that abstraction over worlds provokes the same kind of tension for pointwise composition as abstraction over assignments. On the one hand, there are cases where each element of a set of alternatives needs to be parameterized with a world variable. On the other hand, we sometimes need a world parameter in the restriction of a set of world dependent alternatives.

To see why we need pointwise world parameters, consider (34). Suppose that Rhoda believes mistakenly that barnacles are mollusks, but that otherwise her beliefs about the animal kingdom are correct. She's wondering of all the mollusks, plus the (crustacean) barnacle, whether each is edible. Then there is a de dicto reading of (34) that is true and a de re reading that is false.

(34) Rhoda is wondering which mollusks are edible.

Deriving the de dicto reading is straightforward: we interpret the entire complement of *wonder* with respect to Rhoda's accessible belief worlds. To derive a set of de re mollusks for the false reading in the style of Keshet, we can move *which mollusks*, insert a world abstraction $\lambda w'$, and interpret *which mollusks* at the actual world w.

(35)



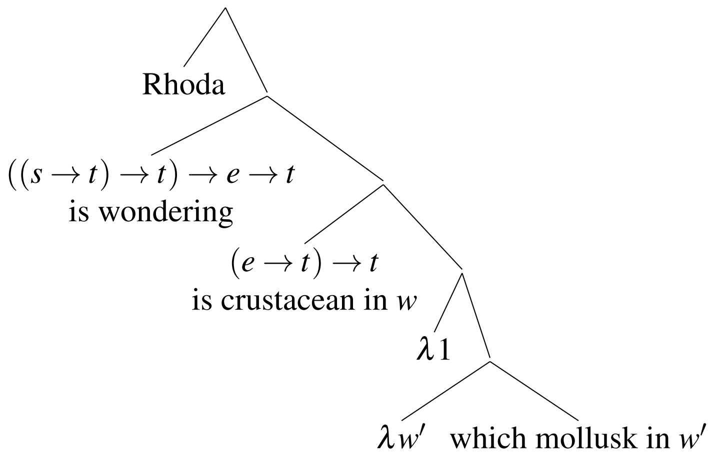
We still need a pointwise notion of intensional abstraction for (35) to compose, but as with the first half of the puzzle for assignment abstraction, there is a simple (and parallel) solution: we intensionalize individual alternatives rather than entire sets.

But now you tell Rhoda that one of the invertebrates she thinks is a mollusk is in fact a crustacean. You don't tell her which one, and she is left to ponder. She knows that nothing can be both a crustacean and a mollusk. We can describe her state of mind with (36).

(36) Rhoda is wondering which mollusk is a crustacean.

In this scenario, (36) is true just in case *crustacean* is de re and *which mollusk* is de dicto. If both predicates were de dicto, she would only be considering contradictions, as nothing is both a crustacean and a mollusk, and she knows it. In this case, we need to let *a crustacean* take scope, and then generate a set of mollusks with respect to Rhoda's belief worlds.

(37)



Critically, after we generate the de dicto mollusks, they have to compose pointwise with the rest of the clause.[5]

4. A solution

Recall the essential element of Novel & Romero's account: we use predicates in the nuclear scope of a set to define its membership conditions. It's possible to do this without relying on presupposition failure if we model alternative denoting determiners as sets of choice functions, adopting a suggestion by Orin Percus relayed in Charlow (2020).

4.1. Pointwise choice functions

Let $cf(\alpha)$ be the set of choice functions over subsets of a domain D_α. For non-assignment sensitive examples, we can model *which* as the set $cf(e)$.

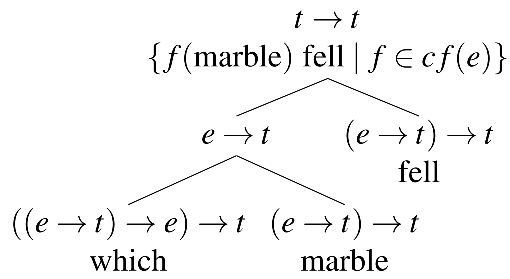
[5] Here is a challenge: come up with an example that has world dependence in the nuclear and restrictive scopes of a set of alternatives simultaneously, as with the assignment dependence in many of our other examples. As it stands, (34) and (36) force a hamblinesque split intensionalist to say something about each kind of pointwise world dependence, but they could be different things, at the expense of generality. A dual example forces a generalization.

(38) $cf(\alpha) = \{f \mid f \in D_{(\alpha\to t)\to\alpha}, \forall p \in D_{\alpha\to t}.\ p(f(p))\}$ $\qquad ((\alpha \to t) \to \alpha) \to t$

(39) $[\![\text{which}]\!] = cf(e)$ $\qquad ((e \to t) \to e) \to t$

Applied to a simple example such as *which marble fell?*, this denotation derives simple answers.

(40)



The denotation for *which* can be used to compose other wh-words.

(41) $[\![\text{who}]\!] = [\![\text{which person}]\!] = \{f(\text{person}) \mid f \in cf(e)\}$ $\qquad e \to t$

(42) $[\![\text{where}]\!] = [\![\text{which place}]\!] = \{f(\text{place}) \mid f \in cf(e)\}$ $\qquad e \to t$

Applied to simple examples, these denotations also derive simple answers.

(43) $[\![\text{who left?}]\!] = \{f(\text{person})\text{ left} \mid f \in cf(e)\}$ $\qquad t \to t$

(44) $[\![\text{where did you go?}]\!] = \{\text{you went to } f(\text{place}) \mid f \in cf(e)\}$ $\qquad t \to t$

To accommodate abstraction, it's enough to make *which* trivially assignment sensitive.

(45) $[\![\text{which}]\!] = \{\lambda g. f \mid f \in cf(e)\}$ $\qquad (i \to (e \to t) \to e) \to t$

If we adopt the definitions for assignment sensitive application and abstraction in (8) and (9), (45) immediately yields the correct answerhood conditions for our original, problematic example, neglecting plurality.

(46) $[\![\text{Who lost which of her marbles?}]\!] = \{\lambda g. f'(\text{person})\text{ lost } f(f'(\text{person})\text{'s marble}) \mid f, f' \in cf(e)\}$

In English: to ask *who lost which of her marbles* is to ask for which choices of someone x and x's marbles y it's true that x lost y.

Choice functions are not a novel device in models of indeterminacy. They have been applied to the interpretation of both wh-items (Engdahl, 1985; Reinhart, 1998; Romero, 1999) and indefinites (Reinhart, 1997; Kratzer, 1998; Matthewson, 1998). But in general, these authors invoke choice functions to derive the scopal behavior of wh-items and indefinites without committing to movement. So for them, choice functions are another method for achieving some of the behavior that a pointwise grammar achieves. I don't know of any work that combines the two devices in this fashion.

4.2. The Binder Roof Constraint

Charlow (2020) entertains this approach to the same binding problem when it surfaces amid an alternative semantics for exceptionally scoping indefinites, but he doesn't pursue it. To see why he doesn't, first consider a classic exceptionally scoping indefinite.

(47) If a relative of mine dies, I'll inherit a house.

This sentence has a reading where the quantifier remains in the antecedent: if any relative of mine dies, I'll inherit a house. It also has a reading where the quantifier appears to take scope over the conditional. On this reading, there is a relationship between specific relatives and inheritance: there is relative x such that if x dies, I will inherit a house.

Modeling *a relative of mine* as a set of pointwise composing relatives gives the DP a kind of scope over the conditional, which derives the second reading without committing the grammar to extraction out of the antecedent, which is otherwise a scope island. That's the basic motivation for applying an alternative semantics.

Exceptionally scoping indefinites can embed bound pronouns in their restrictive clauses, as in (48), so to pursue an alternative semantics for (47), it's necessary to define pointwise abstraction.

(48) If a brother of his_1 dies, some priest_1 will inherit a flock.
$\leadsto$ there's a priest x and a brother of x's y such that if y dies, x will inherit a flock.

The binding configuration for the gloss in (48), when modeled with an alternative semantics, is another example of the puzzle we've been discussing. And modeling *a* as a set of choice functions derives the correct reading for (48).

The issue, for Charlow, is that a pointwise semantics for indefinites derives unattested truth conditions for sentences like (49), which has no wide scope reading for the indefinite.

(49) Nobody_1 submitted a paper she_1 wrote.

Now, this seems to me to demonstrate that an alternative semantics is unsuitable for modeling exceptionally scoping indefinites, rather than that the approach to binding is misled. Since I'm not concerned with the relative merits of an alternative semantics for exceptionally scoping indefinites, but with the tractibility of pointwise abstraction, I will set the Binder Roof Constraint aside.

4.3. Stacking modifiers

What about our examples from 3.1, in which determiners other than *which* combine with restrictive modifiers? Recall one example.

(50) Who among her_1 peers does everyone_1 admire? (16)

Here we need to further restrict the set of people in *who* with the predicate *among her peers*, but we've already expended our choice functions by applying them to *person* in the lexical entry for *who*. What's more, modifiers with embedded pronouns can be stacked.

(51) Who among her$_1$ peers, other than herself$_1$, does everyone$_1$ admire?

The basic relationship between an exceptive like *other than herself* and a determiner it modifies is one of domain subtraction (c.f. von Fintel, 1993 and references therein). A rough gloss of (51) narrows the domain of *who* twice, once for each modifier, as in (52).

(52) (everyone)($\{t_1$ admires $x \mid x \in$ person, $x \in$ her$_1$ peers, $x \neq$ herself$_1\}$)

To accommodate these kinds of examples, we might introduce a slightly higher typed alternative for *which* and its cousins. Call this the transitive version of the entry in (45).

(53) $[\![\text{which}]\!] = \{\lambda g \lambda p \lambda q. f(p \cap q) \mid f \in cf(e)\}$ $\quad (i \rightarrow (e \rightarrow t) \rightarrow (e \rightarrow t) \rightarrow e) \rightarrow t$

This will work for determiners whose domains are restricted by two predicates. For an arbitrarily tall stack of restrictions, we might appeal to predicate modification and allow the stack to compose into a single coordinated predicate. Thus the entry in (53) might be rich enough to model any kind of restrictive pointwise modification of a determiner.

Very well. I think it's worth asking whether the approach can be generalized. Specifically, can we decompose *which* into a lemma and an independent choice functional morpheme?

5. Decomposing the solution

Shan (2002) made the observation that many grammatical devices can be phrased as monads. Informally, for the application at hand, a monad is a device for applying a sequence of environment-dependent functions to an argument, where the environment may change with each application. Assignment dependence is one such environment, and the assignment sensitive interpretation function from H&K implicitly uses the *reader* monad. Nondeterminacy is another, and the pointwise interpretation function from Hamblin makes implicit use of the *powerset* monad.

Importantly, monads can themselves be composed, and the order in which they're composed is significant.

What I want to suggest is that composing the reader and powerset monads in a particular order yields a monad that's useful for modeling a pointwise semantics. Not coincidentally, this monad is an inversion of the monad that Charlow (2020) shows is implicit in Karttunen's semantics for questions. If Charlow's is the Karttunen monad, this is the Hamblin monad. Interestingly, to define this monad in the first place, I have found choice functions essential.

Whereas Charlow embeds the powerset within the reader, we will embed the reader within the powerset. The objects of Charlow's monad – call it I_s – are the assignment sensitive sets that we departed from in order to define pointwise abstraction; they have the type $i \rightarrow (\alpha \rightarrow t)$. The objects of our monad – let's call it S_i – will have the type $(i \rightarrow \alpha) \rightarrow t$.

Let me define I_s and demonstrate how it handles our binding problem. Then I will suggest a definition for S_i and briefly explore how it might do the same.

5.1. Defining I_s

To define I_s, we will first define the reader and powerset monads I and S. Formally, I and S are functors m equipped with transformations μ that flatten nested instances of m. So to define I and S, we will first define their underlying functors. These consist of unit functions η, which equip arbitrary values with functorial structure, and map functions $>$, which apply functions to values embedded in that structure. Using $>$ to map a function over either of these functors transforms the embedded values and preserves the surrounding structure.

(54) Reader Functor

$$\eta_i(x) = \lambda g.x \qquad \alpha \to i \to \alpha$$
$$f >_i m = f \circ m \qquad (\alpha \to \beta) \times (i \to \alpha) \to (i \to \beta)$$

(55) Powerset Functor

$$\eta_s(x) = \{x\} \qquad \alpha \to \alpha \to t$$
$$f >_s m = \{f(x) \mid x \in m\} \qquad (\alpha \to \beta) \times (\alpha \to t) \to (\beta \to t)$$

To construct the corresponding monads we carry along η and define flattening functions μ. We will also define type constructors – functions from types to types – that abbreviate the notation and serve as names for the monads.

(56) Reader Monad

$$I\alpha = i \to \alpha$$
$$\eta_i(x) = \lambda g.x \qquad \alpha \to I\alpha$$
$$\mu_i(m) = \lambda g.m(g)(g) \qquad I(I\alpha) \to I\alpha$$

(57) Powerset Monad

$$S\alpha = \alpha \to t$$
$$\eta_s(x) = \{x\} \qquad \alpha \to S\alpha$$
$$\mu_s(m) = \{y \mid x \in m, y \in x\} \qquad S(S\alpha) \to S\alpha$$

To compose the two, first compose their underlying functors.

(58) Reader Functor for Powersets

$$\eta_{is}(x) = \lambda g.\{x\} \qquad \alpha \to i \to \alpha \to t$$
$$f >_{is} m = \lambda g.\{f(x) \mid x \in m(g)\} \qquad (\alpha \to \beta) \times (i \to \alpha \to t) \to (i \to \beta \to t)$$

The composite monad inherits the unit η of the composite functor and defines a new μ.

(59) Reader Monad for Powersets

$$I_s\alpha = I(S\alpha)$$
$$\eta_{is}(x) = \lambda g.\{x\} \qquad \alpha \to I_s\alpha$$
$$\mu_{is}(m) = \lambda g.\{y \mid x \in m(g), y \in x(g)\} \qquad I_s(I_s\alpha) \to I_s\alpha$$

To derive a denotation for our running example we can let *which* denote a curried version of $>_{is}$ with arguments flipped, α fixed to e, and β fixed to $I_s t$.

(60) $[\![\text{which}]\!] = \lambda m \lambda f \lambda g.\{f(x) \mid x \in m(g)\}$ $\qquad I_s e \to (e \to I_s t) \to I_s(I_s t)$

As above, we can use $[\![\text{which}]\!]$ to define $[\![\text{who}]\!]$.

(61) $[\![\text{person}]\!] = \lambda g \lambda x.\text{person}(x)$ $\qquad I_s e$

(62) $[\![\text{who}]\!] = [\![\text{which person}]\!] : (e \to I_s t) \to I_s(I_s t)$
$[\![\text{who}]\!] = [\![\text{which person}]\!] = \lambda f \lambda g.\{y \mid x \in [\![\text{person}]\!](g), y \in f(x)(g)\}$

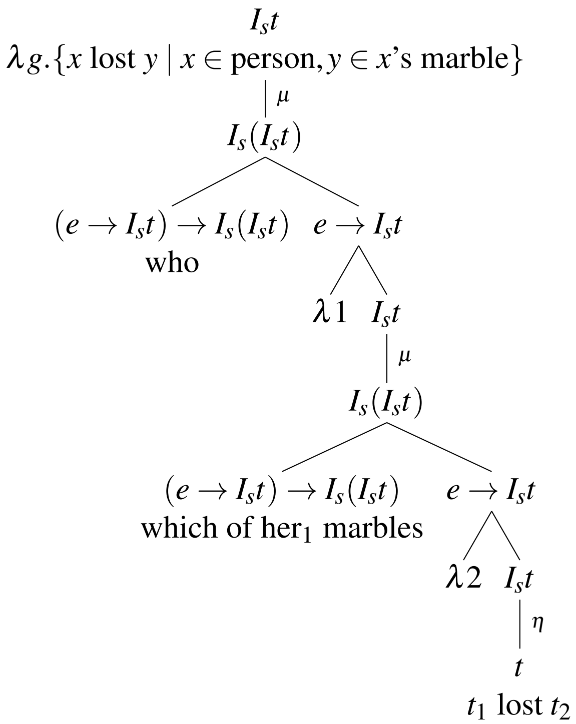
Similarly, we can compose *which* with *her marbles*.

(63) $[\![\text{her}_1 \text{ marbles}]\!] = \lambda g \lambda x.x \text{ is } g(\text{her}_1) \text{ marble}$ $\qquad I_s e$

(64) $[\![\text{which of her}_1 \text{ marbles}]\!] : (e \to I_s t) \to I_s(I_s t)$
$[\![\text{which of her}_1 \text{ marbles}]\!] = \lambda f \lambda g.\{y \mid x \in [\![\text{her}_1 \text{ marble}]\!](g), y \in f(x)(g)\}$

To put these DPs to work, we give each one scope. Once they compose with their complement, μ flattens the result into a propositional type.[6]

(65) $[\![\text{Who lost which of her marbles?}]\!] =$



[6]I have broken Charlow's $\gg\!=$ into μ and $>$, in the following way. For any monad M, the formula in (1) gives back $\gg\!=$.

(1) $f \gg\!= m = \mu_M(f >_M m)$ $\qquad (\alpha \to M\beta) \times M\alpha \to M\beta$

A nearly equivalent approach is to assign *which* the denotation $\gg\!=$ directly, with currying and argument order parallel to (60). I use μ on its own to facilitate comparison with what follows.

We derive an assignment sensitive set of answers, one for each pair of a person and her marbles, which is correct.

5.2. Defining S_i
But what if we invert the monad? First invert the underlying functor.

(66) Powerset Functor for Readers

$$\eta_{si}(x) = \{\lambda g.x\} \qquad \alpha \to (i \to \alpha) \to t$$
$$f \gt_{si} m = \{f \circ x \mid x \in m\} \qquad (\alpha \to \beta) \times ((i \to \alpha) \to t) \to (i \to \beta) \to t$$

Then define μ.

(67) Powerset Monad for Readers

$$S_i\alpha = (i \to \alpha) \to t$$
$$\eta_{si}(x) = \{\lambda g.x\} \qquad \alpha \to S_i\alpha$$
$$\mu_{si}(m) = \{\lambda g.(f \circ x)(g)(g) \mid x \in m, f \in cf(i \to \alpha)\} \qquad S_i(S_i\alpha) \to S_i\alpha$$

Notice that we quantify over choice functions in order to flatten the nested functors. I have not been able to construct a μ that does the right thing *without* using choice functions. You may confirm that this is a challenge; (Charlow, 2014: p. 151) suggests that it is impossible, with choice functions or without.

Notice, also, that our choice functions are of a different type than above: the set is $cf(i \to \alpha)$, rather than $cf(\alpha)$. It is possible to give a μ with the correct shape over $cf(\alpha)$ as follows.

(68) $\mu'_{si}(m) = \{\lambda g.f(\{y(g) \mid y \in x(g)\}) \mid x \in m, f \in cf(\alpha)\} \qquad S_i(S_i\alpha) \to S_i\alpha$

But it turns out that μ'_{si} violates the naturality condition, one of a trio of laws any μ must satisfy in order to behave predictably.[7]

Is μ_{si} natural? I have not proven that it is, but the higher type of its choice functions relieves it of the counter examples that disqualify μ'_{si}. It's worth observing, as well, that the functions in $cf(i \to \alpha)$ bear a resemblance to skolemized choice functions, which Kratzer (1998), Chierchia (2001), and Mirrazi (2024) use to model exceptionally scoping indefinites, and which Chierchia (1992) uses to model pair-list readings for question-embedded quantfiers. In fact, uncurrying each $f \in cf(i \to \alpha)$ yields an f with the type $((i \to \alpha) \to t) \times i \to \alpha$, which, if you squint at it, is a choice function with a skolem parameter i.

[7]Here are the laws, for any $m : M$, $\eta : \alpha \to M\alpha$, $\mu : M(M\alpha) \to M\alpha$, $h : \alpha \to \beta$, and $\gt$ uncurried as map.

(1) $(map(h) \circ \mu)(m) = (\mu \circ map(map(h)))(m)$ Naturality

(2) $(\mu \circ map(\mu))(m) = (\mu \circ \mu)(m)$ Associativity

(3) $\mu \circ \eta = id = \mu \circ map(\eta)$ Identity

5.3. Using S_i

Suppose μ behaves predictably. How is it useful? We can let *which* denote the function such that $\mu \circ [\![\text{which}]\!] = \{\lambda g. f \mid f \in cf(i \to e)\}$.[8]

(69) $[\![\text{which}]\!] = \{\lambda g \lambda p \lambda q. (p \circ q)(g)\}$ $\quad S_i((e \to t) \to S_i e)$

(70) $[\![\text{who}]\!] = [\![\text{which person}]\!] = \{\lambda g \lambda q. (\text{person} \circ q)(g)\}$ $\quad S_i(S_i e)$

(71) $[\![\text{which of her}_1 \text{ marbles}]\!] = \{\lambda g \lambda q. ((g(\text{her}_1) \text{ marble}) \circ q)(g)\}$ $\quad S_i(S_i e)$

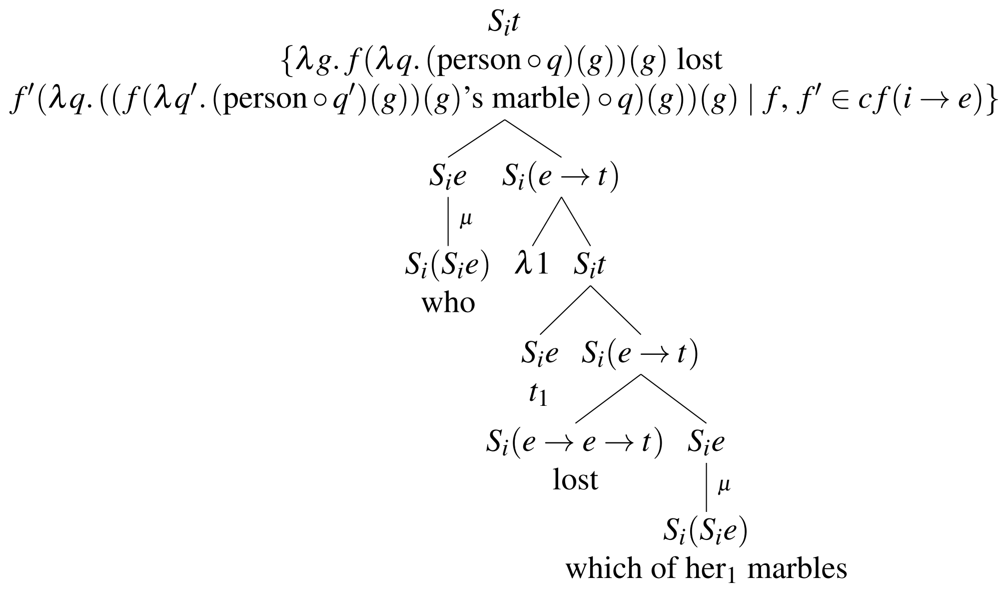
Once the determiner composes with a restriction, μ flattens it into a lower type.

(72) $\mu([\![\text{who}]\!]) = \{\lambda g. f(\lambda q. (\text{person} \circ q)(g))(g) \mid f \in cf(i \to e)\}$ $\quad S_i e$

(73) $\mu([\![\text{which of her}_1 \text{ marbles}]\!]) : S_i e$
$\mu([\![\text{which of her}_1 \text{ marbles}]\!])$
$= \{\lambda g. f(\lambda q. ((g(\text{her}_1) \text{ marble}) \circ q)(g))(g) \mid f \in cf(i \to e)\}$

The resulting derivation for our running example is not quite as legible as with the other approaches, but the answerhood conditions are correct.

(74) $[\![\text{Who lost which of her marbles?}]\!]$ =



In English: to ask *who lost which of her marbles?* is, given some assignment g, to ask for which two choices of functions a and b from g into e it's true that $a(g)$ lost $b(g)$, with the conditions that $a(g)$ return a person and $b(g)$ return a marble of $a(g)$.

[8] If we bake pointwise composition into the lexical entry, this is again the map function $>$ of the underlying functor.

References

Alonso-Ovalle, L. (2006). Disjunction in alternative semantics. ph.d. thesis, university of massachusetts, amherst.

Alonso-Ovalle, L. (2009). Counterfactuals, correlatives, and disjunction. *Linguistics and Philosophy 32*(2), 207–244.

Alonso-Ovalle, L. and P. Menéndez-Benito (2003). Some epistemic indefinites.

Beck, S. and H. Rullmann (1999). A flexible approach to exhaustivity in questions. *Natural Language Semantics 7*(3), 249–298.

Bumford, D. (2022, 09). Composition under distributive natural transformations: Or, when predicate abstraction is impossible. *Journal of Logic, Language and Information 31*, 1–21.

Büring, D. (2005). *Binding Theory*. Cambridge Textbooks in Linguistics. Cambridge University Press.

Charlow, S. (2014). *On the semantics of exceptional scope*. NYU.

Charlow, S. (2020). The scope of alternatives: Indefiniteness and islands. *Linguistics and Philosophy 43*(4), 427–472.

Chierchia (2001). *A puzzle about indefinites* (Editd by Cecchetto, Chierchia, and Guasti ed.)., pp. 51–89.

Chierchia, G. (1992). 1 993. questions with quantifiers. *Natural Language Semantics 1*(1), 81–234.

Déchaine, R.-M. and M. Wiltschko (2002, 07). Decomposing pronouns. *Linguistic Inquiry 33*(3), 409–442.

Engdahl, E. (1985). Constituent questions: The syntax and semantics of questions with special reference to swedish.

Hamblin, C. L. (1976). Questions in montague english.

Heim, I. and A. Kratzer (1998). *Semantics in Generative Grammar*. Malden, MA: Blackwell.

Jacobson, P. (1995). On the quantificational force of english free relatives.

Karttunen, L. (1977, 01). Syntax and semantics of questions. *Linguistics and Philosophy 1*, 3–44.

Keshet, E. (2010). Split intensionality: A new scope theory of de re and de dicto. *Linguistics and Philosophy 33*(4), 251–283.

Kotek, H. (2019). *Composing Questions*. Massachusetts Institute of Technology, Department of Linguistics and Philosophy.

Kratzer, A. (1998). *Scope or Pseudoscope? Are there Wide-Scope Indefinites?*, pp. 163–196. Dordrecht: Springer Netherlands.

Kratzer, A. (2009, 04). Making a pronoun: Fake indexicals as windows into the properties of pronouns. *Linguistic Inquiry 40*(2), 187–237.

Kratzer, A. and J. Shimoyama (2002). Indeterminate pronouns: The view from Japanese. In Y. Otsu (Ed.), *The Third Tokyo Conference on Psycholinguistics*, pp. 1–25.

Ladusaw, W. A. (1980). Polarity sensitivity as inherent scope relations.

Matthewson, L. (1998). On the interpretation of wide-scope indefinites. *Natural Language Semantics 7*(1), 79–134.

Menéndez-Benito, P. (2010). On universal free choice items. *Natural Language Semantics 18*(1), 33–64.

Mirrazi, Z. (2024, 12). Indefinites in negated intensional contexts. *Semantics and Pragmatics 17*.

Novel, M. and M. Romero (2010, 01). Movement, variables and hamblin alternatives.
Rawlins, K. (2013). (un)conditionals. *Natural Language Semantics 21*(2), 111–178.
Reinhart, T. (1997). Quantifier scope: How labor is divided between qr and choice functions. *Linguistics and Philosophy 20*(4), 335–397.
Reinhart, T. (1998). Wh-in-situ in the framework of the minimalist program. *Natural Language Semantics 6*(1), 29–56.
Romero, M. (1999). Intensional choice functions for which phrases. *Semantics and Linguistic Theory*.
Rooth, M. (1992). A theory of focus interpretation. *Natural Language Semantics 1*(1), 75–116.
Shan, C.-c. (2002). Monads for natural language semantics. *ArXiv cs.CL/0205026*.
Shan, C.-c. (2004, 09). Binding alongside hamblin alternatives calls for variable-free semantics. *Semantics and Linguistic Theory 14*.
von Fintel, K. (1993). Exceptive constructions. *Natural Language Semantics 1*(2), 123–148.