You can leave your dog outside, but do you have to?: An RSA approach[1]

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Abstract. A deontic possibility utterance can have a necessity interpretation in some contexts. For example, a café owner's utterance of "You can leave your dog outside." to a dog owner who is about to enter can be taken to mean that the dog owner *has to* leave their dog outside. This phenomenon, where a logically weaker item (possibility) implies a stronger item (necessity) on a scale, is not straightforwardly explained through standard theories of scalar implicature that predict that the assertion of a logically weaker item implies the negation of a stronger item. I attempt to account for this phenomenon of possibility-to-necessity inference using a theory of graded modality (Lassiter, 2011, 2017; Chung and Mascarenhas 2023) and the Rational Speech Act (RSA) framework (Frank and Goodman, 2012; Goodman and Stuhlmüller, 2013). I construct an RSA model where the pragmatic listener jointly infers about the state of the world and the value of the semantic variable as in Lassiter and Goodman (2013, 2017). The speaker utility includes social utility along epistemic utility, after Yoon et al. (2017). I consider two situations which differ in terms of whether explicit deontic necessity utterances are available to the speaker. The RSA model predicts a high probability of necessity interpretation in both. I further compare these results to the ones I obtain by controlling for components of the RSA model such as utterance cost and social utility, examining the effect that each has on the final result. The present study thus offers a way of systematically investigating a previously understudied phenomenon of deontic necessity inference arising from a deontic possibility utterance.

Keywords: possibility-to-necessity inference, graded modality, Rational Speech Act (RSA).

1. Introduction

Depending on the context, an utterance with a deontic possibility modal can have a necessity reading. (1) is a case in point, where the café owner's utterance is taken to mean that the listener has to leave their dog outside of the café.

(1) Context: A dog owner is walking their dog. They approach a café to get a cup of tea, but is unsure whether to leave the dog outside or to bring it inside. The café owner, seeing this, utters the following.
Café owner: You can leave your dog outside.

Modal necessity is usually considered logically stronger than modal possibility. Classical theories of scalar implicature (e.g., Horn's (1989) Q-principle) predicts that the assertion of a logically weaker item implies the negation of any stronger item. Example (1) is intriguing in that the opposite seems to be the case, i.e., the assertion of a weaker item implies a stronger

one. The question that naturally follows is how to systematically account for this phenomenon of possibility-to-necessity inference.

The present study aims to address this question by computationally modeling the above scenario. In Section 2, I introduce the two theoretical components that I use to do so: a theory of graded modality (Lassiter, 2011, 2017; Chung and Mascarenhas 2023) and the Rational Speech Act (RSA) framework (Frank and Goodman, 2012; Goodman and Stuhlmüller, 2013). Specifically, I adopt the truth condition for deontic necessity proposed in Chung and Mascarenhas (2023) and the truth condition for deontic possibility that follows from their formulation, which are defined in terms of whether the goodness of a proposition exceeds a contextually determined threshold value θ. Similar to Lassiter and Goodman's (2013, 2017) RSA model on adjective meanings dependent on a threshold value, I construct an RSA model of the listener in the scenario in (1) who infers about both the goodness of leaving their dog outside and the threshold value.

Section 3 provides the model predictions. By considering two situations which differ in terms of the set of alternative utterances that the speaker can choose from, I show how a possibility utterance can result in a necessity inference in languages with or without linguistic expressions that explicitly convey deontic necessity. By controlling for certain components of the RSA model, such as the utterance cost and social utility, I demonstrate the effect that they have on the listener's inference. The modeling results show that the listener's subjective prior beliefs about the state of the world, the cost involved in making an utterance, considerations of politeness, and the semantic denotation of utterances interact to enable a necessity interpretation of a possibility utterance. Section 4 identifies some desiderata for future studies and concludes.

2. Theoretical components

This section introduces the two theoretical components that I use to model the inference of a deontic necessity meaning from a deontic possibility utterance.

2.1. Graded modality

Lassiter (2011, 2017) puts forth a theory of graded modality based on probability calculus. Lassiter posits that we obtain the expected utility of a proposition by calculating the probability-weighted average of preferences. This expected utility can be understood as a measure function which takes a proposition as input and returns its goodness (i.e., how good the proposition is) in the form of a real number. A deontic modal statement is true if and only if the expected utility of the prejacent exceeds a contextually determined threshold value θ (Lassiter, 2011).

Chung and Mascarenhas (2023) adopt Lassiter's theory of graded modality and propose a unified truth condition for modal necessity, applicable for both deontic and epistemic modal flavors:

(2) $[\![must\ \varphi]\!]^{\theta,w} = (E_w[\mu_{EVAL}|\varphi] > \theta) \wedge \forall\psi \in Alt(\varphi): (E_w[\mu_{EVAL}|\psi] \leq \theta)$

$E_w[\mu_{EVAL}|\varphi]$ represents the expected utility, or the goodness, of prejacent φ at world w. θ is a contextually determined threshold, and $Alt(\varphi)$ is the set of alternatives of φ. Assuming a deontic modal flavor, (2) reads as 'MUST(φ) is true iff φ is the only good-enough choice among the available options.' This is the truth condition for deontic MUST(φ) that I utilize in the present study.

I further assume the truth condition in (3) for deontic CAN(φ):

(3) $\llbracket can\ \varphi \rrbracket^{\theta,w} = (E_w[\mu_{EVAL}|\varphi] > \theta)$

(3) reads as 'CAN(φ) is true iff φ is a good-enough choice'. Chung and Mascarenhas (2023) do not explicitly provide this formulation, but (3) naturally follows from (2) under the assumption that (i) possibility modals and necessity modals are duals, (ii) $Alt(\varphi)$ is a singleton set of $\neg\varphi$, and (iii) the goodness of φ and $\neg\varphi$ are inversely proportionate. The derivation of (3) from (2) is given in the Appendix. (2) and (3) capture our intuition about deontic necessity and possibility, as well as the logical entailment relation that whenever MUST(φ) is true, CAN(φ) is also true.

Chung and Mascarenhas (2023) provide support from natural language evidence for the truth condition in (2). In Korean, deontic modal statements are expressed by combining conditionals and evaluative predicates. The compositional semantics of such conditional evaluatives involves comparing the expected utility of propositions to threshold values. (4) below, an example of a Korean deontic necessity statement, showcases this:

(4)   Example of Korean deontic necessity statement
      a.   *Kangaji-nun bakk-ey   tu-si-Ø-**eya**   **toy**-eyo.*
           Puppy-TOP   outside-DAT leave-HON-PRES-**only.if EVAL**-DECL
           'You have to leave your dog outside.'
           Lit. 'Only if you leave your dog outside, it suffices.'
      b. $\llbracket only_{-(e)ya} \rrbracket^w \Theta(\llbracket if\ \varphi,\ then\ EVAL \rrbracket^w)$
           $= E_w[\mu_{EVAL}|\varphi] > \theta \wedge \forall\psi \in Alt(\varphi): (E_w[\mu_{EVAL}|\psi] \leq \theta)$

(4a), the Korean counterpart of the English sentence "You have to leave your dog outside", involves the particle *-eya*, which means 'only if', and the evaluative predicate *toy*. Combining these with the prejacent, we arrive at the truth condition in (4b), which is identical to the truth condition in (2).

Korean deontic possibility statements provide similar support for the truth condition in (3), as shown below:

(5)   Example of Korean deontic possibility statement
      a.   *Kangaji-nun bakk-ey   tu-si-Ø-**myen**   **toy**-eyo.*
           Puppy-TOP   outside- DAT leave- HON-PRES-**if**   **EVAL**-DECL
           'You can leave your dog outside.'
           Lit. 'If you leave your dog outside, it suffices.'

b. $\Theta(\llbracket if\ \varphi,\ then\ EVAL \rrbracket^w) = E_w[\mu_{EVAL}|\varphi] > \theta$

(5a) is the Korean counterpart of the English sentence "You can leave your dog outside", with the particle *-myen*, which means 'if', and the evaluative predicate *toy*. Its truth condition is given in (5b), and this is identical to the truth condition for deontic possibility given in (3).

This section has identified the truth conditions for deontic necessity and possibility that I employ in the present study based on Lassiter (2011, 2017) and Chung and Mascarenhas (2023), providing evidence from Korean as empirical support. The following section turns to the RSA framework that I use to model how a listener makes pragmatic inferences.

2.2. RSA framework

Assuming the truth conditions for deontic modality in the previous section, I used the RSA framework (Frank and Goodman, 2012; Goodman and Stuhlmüller, 2013) to model how a listener capable of pragmatic reasoning would infer about the state of the world and the value of semantic variables. A standard assumption in an RSA model is that a pragmatically competent listener (L_1) reasons about the speaker's (S_1) choice of utterance, which is in turn based on reasoning about a hypothetical listener who only considers literal meaning (L_0). Following Lassiter and Goodman (2013, 2017), I assume that the free threshold variable θ in the truth condition passes through the speaker to L_1, and L_1 jointly infers about the goodness of the prejacent and the value of θ.

2.2.1. Literal listener L_0

L_0 is a hypothetical agent who only considers the literal meaning of an utterance. L_0 has subjective beliefs about the world prior to a conversation, represented with probability distributions. Upon hearing an utterance u, L_0 conditions their beliefs on the truth of u, as formulated in (6) below:

(6) Literal listener L_0
   a. $P_{L_0}(A|u, \theta) = P_{L_0}(A|\llbracket u \rrbracket^\theta = 1)$
   b. $P_{L_0}(A|u = can\ \varphi,\ \theta = 0.7)$
       $= P_{L_0}(A|\llbracket u \rrbracket^\theta = 1)$
       $= P_{L_0}(A|A > 0.7)$

In (6), A represents the goodness of the prejacent φ of a deontic modal utterance u. As established in Section 2.1 above, the truth of a deontic modal utterance depends on the threshold variable θ. (6a) states that, given an utterance u and a threshold value θ, the literal listener L_0 conditions their prior belief on the utterance being true. (6b) lays out what this will look like when $u = $ CAN(φ) and $\theta = 0.7$. The literal listener L_0 assigns zero probability to worlds where the goodness of the prejacent φ does not exceed 0.7, and renormalizes the probability of the rest so that the sum of probabilities stays as 1.

2.2.2. Speaker S_1

The speaker S_1 chooses their utterance from a set of alternative utterances based on speaker utility. After Yoon et al. (2016) and Yoon et al. (2017), I assume that speaker utility comprises of social, as well as epistemic, utility, defined as follows:

(7) Speaker utility
$$\mathbb{U}_{S_1}(u; A, \theta) = \beta \cdot \mathbb{U}_{epistemic}(u; A, \theta) + (1 - \beta) \cdot \mathbb{U}_{social}(u; A, \theta) - Cost(u)$$
$$= \beta \cdot \{\log\left(P_{L_0}(A|u, \theta)\right)\} + (1 - \beta) \cdot \mathrm{E}_{P_{L_0}(\theta, A|u)}[Val(\theta, A)] - Cost(u)$$

Epistemic utility increases with the probability that the hypothetical literal listener L_0 correctly figures out about the state of the world. Social utility represents politeness, and is defined in terms of the speaker's subjective preference on the state of the world. Here, I assume that the speaker has motivation to indicate a lower threshold value than actuality, in order to avoid being too imposing on the listener, while there is no motivation for or against signaling that the goodness of the prejacent is a certain value. Hence, I posit a value function $Val(\theta, A)$ which is inversely proportionate to θ and is a constant with respect to A. By calculating the average of $Val(\theta, A)$ weighted by $P_{L_0}(\theta, A|u)$, I am able to derive the social utility of a certain utterance u.

The proportion that epistemic and social utility each occupy in computing the overall utility is represented by the parameter β. In the present study, I set the value of β to 0.5, but in principle the β parameter can also be subject to the pragmatic listener's inference. Finally, I assumed that all utterances have an equal cost of 2, except for the null utterance, or silence, which I assumed to have zero cost.

The probability that a speaker chooses an utterance u increases with its utility. Following Lassiter and Goodman (2013, 2017), I assume that S_1 uses a soft-max choice rule:

(8) Speaker S_1
$$P_{S_1}(u|A, \theta) \propto exp(\alpha \times \mathbb{U}_{S_1}(u; A, \theta))$$

The parameter $\alpha \geq 0$ represents how heavily the speaker's choice relies on speaker utility: if $\alpha = \infty$, S_1 would always choose the utterance with the highest speaker utility, and if $\alpha = 0$, there is equal probability that S_1 would choose each utterance, regardless of the speaker utility. In the present study, I set the value of α to 2.

2.2.3. Pragmatic listener L_1

L_1, a pragmatically competent listener, takes into account S_1's choice of utterance and their subjective prior beliefs about the world to make a Bayesian inference on the joint estimate of A and θ. This is formulated in (9) below:

(9) Pragmatic listener L_1

$$P_{L_1}(A, \theta | u) \propto P_{S_1}(u | A, \theta) \cdot P_{L_1}(A, \theta)$$
$$\propto P_{S_1}(u | A, \theta) \cdot P_{L_1}(A) \cdot P_{L_1}(\theta)$$

The second line holds under the assumption that the prior distributions of A and θ are independent (Lassiter and Goodman, 2013).

Finally, the marginal posterior distribution of A and θ can each be derived from the joint distribution in (9) by integrating out the other variable:

(10) Pragmatic listener's marginal distribution of A and θ
    a. $P_{L_1}(A | u) = \int P_{L_1}(A, \theta | u) \, d\theta$
   b. $P_{L_1}(\theta | u) = \int P_{L_1}(A, \theta | u) \, dA$

The marginal posterior distribution of A is derived by integrating the joint distribution in (9) on θ, as is laid out in (10a). Similarly, the marginal posterior distribution of θ is derived by integrating the joint distribution in (9) on A, as shown in (10b).

This section has laid out the details of the RSA model that I constructed to model the way a pragmatically competent listener infers about the state of the world upon hearing a deontic modal utterance. In the following Section 3, I present the model predictions.

3. Model predictions

This section presents the predictions that the RSA model yields regarding the pragmatic listener's inferences upon hearing the utterance "You can leave your dog outside" in the scenario in (1). Throughout this section, φ represents the proposition 'You leave your dog outside', which is the prejacent of the deontic possibility statement "You can leave your dog outside".

I assumed the prior distributions for the goodness of φ and the threshold value θ as in Figure 1 The listener's prior beliefs on the goodness of φ and the threshold value θ.
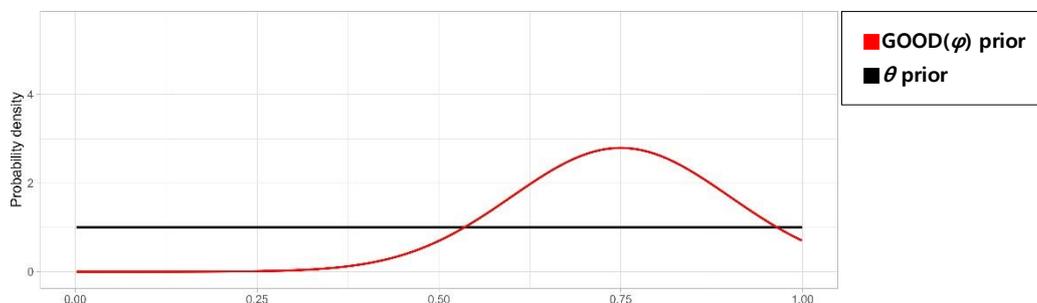
Figure 1 The listener's prior beliefs on the goodness of φ and the threshold value θ

The red curve represents the listener's prior belief on how good the proposition φ is. It has more weight in the higher values than in the lower values to reflect the general knowledge that it is usually more acceptable to leave one's dog outside of an establishment than to bring it

inside. The flat black line represents the listener's prior belief on what the threshold value θ might be. It has uniform distribution over the support $(0, 1)$, assuming that the listener has no prior information on what the speaker's internal threshold is. The prior distributions of the goodness of φ and of θ are assumed to be statistically independent.

I considered two situations which differ in terms of the set of alternative utterances that the speaker can choose from. Situation 1 is where the speaker chooses their utterance among {Ø, CAN(φ), CAN($\neg\varphi$)}, where Ø is the null utterance, or silence. Situation 2 is where the speaker chooses among {Ø, CAN(φ), CAN($\neg\varphi$), MUST(φ), MUST($\neg\varphi$)}. Each of these situations is presented in Section 3.1 and 3.2 respectively.

In both situations, I assumed that the listener considers φ and $\neg\varphi$ as alternatives when evaluating the truth value of modal statements. In other words, the set $Alt(\varphi)$ in the truth condition in (2) is the singleton set {$\neg\varphi$}. Operating with a more fine-grained set of alternatives (e.g., $Alt(\varphi)$ = {'You hire a dogsitter', 'You leave your dog at the dog park', …}) would yield a better approximation of reality, a task I relegate to future research.

3.1. Situation 1: speaker chooses from {Ø, CAN(φ), CAN($\neg\varphi$)}

Figure 2 *L1's* inference on the goodness of φ, the goodness of $\neg\varphi$, and threshold θ in Situation 1 summarizes the model predictions for Situation 1, where the set of alternative utterances is {Ø, CAN(φ), CAN($\neg\varphi$)}, and the speaker has chosen to utter CAN(φ).
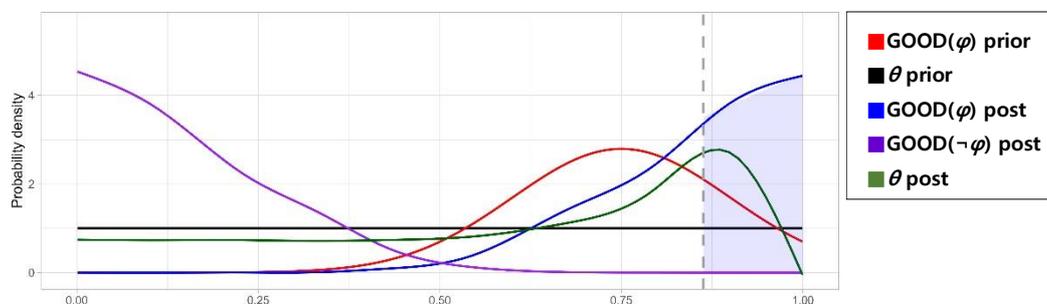
Figure 2 L_1's inference on the goodness of φ, the goodness of $\neg\varphi$, and threshold θ in Situation 1

After the pragmatic reasoning process, the listener assigns even higher probability to the higher values of the goodness of φ, as represented by the blue curve. Conversely, the posterior distribution of the goodness of $\neg\varphi$, represented by the purple curve, has more weight in the lower values. The posterior distribution of θ is shifted to the right compared to its prior distribution, as evidenced by the green curve. In sum, L_1 infers that it is likely that the goodness of φ is very high and the goodness of $\neg\varphi$ is very low, and the threshold θ is likely to be high. Combining these, it is highly likely that the goodness of φ exceeds the threshold while the goodness of $\neg\varphi$ does not, i.e., φ is the only good-enough option among the alternatives. Thus, L_1 would assign a high probability to a necessity interpretation of the utterance "You can leave your dog outside".

The first row of Table 1 shows how this is borne out. Having computed the posterior distribution of θ, L_1 may choose as an estimate of θ its mode, i.e., the value at which it attains its maximum. In Situation 1, the value is 0.8630 (vertical dotted line in Figure 2). The probability that the goodness of φ exceeds this value is 0.5581, and the probability that the goodness of $\neg\varphi$ exceeds this value is 1.464×10^{-6} (area under the blue and the purple curves, respectively, on the right of the dotted line in Figure 2). Given these probabilities, and further assuming $\text{CAN}(\varphi)\vee\text{CAN}(\neg\varphi)$ is true[2], the probability that φ is allowed but not required is nearly 0, while the probability that φ is required is nearly 1.

Cost	θ est.	$P(\text{GOOD}(\varphi)>\theta)$	$P(\text{GOOD}(\neg\varphi)>\theta)$	$P(\text{CAN}(\varphi)=1 \wedge \text{MUST}(\varphi)=0)$ (given $\text{CAN}(\varphi\vee\neg\varphi)$)	$P(\text{MUST}(\varphi)=1)$ (given $\text{CAN}(\varphi\vee\neg\varphi)$)
O	0.8630	0.5581	1.464×10^{-6} (nearly 0)	$\leq 2.623\times10^{-6}$ (nearly 0)	≥ 0.9999
X	0.5005	0.9892	0.0122	≤ 0.0123	≥ 0.9877

Table 1 Probability that different modal utterances are true, taking the mode of θ as its estimate, in Situation 1.

Utterance cost contributes to the high probability of a necessity interpretation. The second row of Table 1 represents the predictions of a model where utterance cost is not included, and identical to the one discussed so far in all other respects. When utterance cost is not taken into account, the probability that φ is required is greater than or equal to 0.9877, which is a smaller probability than when utterance cost is part of the model.

Because the prior distribution of the goodness of φ has more weight in the higher values, there is already substantial probability that the listener would decide it is better to leave their dog outside, even if the speaker opts for the null utterance (silence). In the current setting, the null utterance involves zero utterance cost, while uttering $\text{CAN}(\varphi)$ has the utterance cost of 2. The fact that the speaker put in the effort to utter $\text{CAN}(\varphi)$, incurring extra costs, when the null utterance was also an option, implies that $\text{CAN}(\varphi)$ is highly informative, signaling not only possibility but also necessity.

Situation 1 can be understood as a model of a language that does not have lexical items that directly express deontic necessity. Nez Perce, for example, has a possibility modal *o'qa* which can be used flexibly for the necessity meaning in upward-entailing environments (Deal 2011). Deal (2011) demonstrates that *o'qa* is truth-conditionally a possibility modal by examining its truth conditions in downward-entailing environments. In upward-entailing environments, however, depending on the context, *o'qa* can be interpreted as a necessity modal. It may be

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[2] In Kratzer's (1981) analysis of modality, $\text{CAN}(\varphi)\vee\text{CAN}(\neg\varphi)$ is always true for any proposition φ. The truth condition for deontic possibility given in (3), which I assume for the present study, does not guarantee this: in principle, both the goodness of φ and the goodness of $\neg\varphi$ could be below the threshold value. Because it would be logical to assume that at least one of φ and $\neg\varphi$ is possible, for now I tackle this issue by conditioning the probabilities on the truth of $\text{CAN}(\varphi)\vee\text{CAN}(\neg\varphi)$. Whether the truth condition in (3) can be modified to ensure that $\text{CAN}(\varphi)\vee\text{CAN}(\neg\varphi)$ is a tautology is a subject for further investigation. I thank an anonymous reviewer for SuB29 for their comment on this matter.

argued that a pragmatic inference process like above contributes to the necessity reading of scaleless possibility modals such as *o'qa*. In Section 3.1, I have presented the model predictions for Situation 1, where the speaker chooses their utterance from the set $\{\emptyset, \text{CAN}(\varphi), \text{CAN}(\neg\varphi)\}$. The modeling results show that, after the pragmatic inference process, L_1 would assign a high probability to φ being the only good-enough option, which is the truth condition for deontic necessity I assumed in (2). Utterance cost is a main factor in this inference, and Situation 1 can be understood as a model of a language with explicit possibility, but not necessity, modals, such as Nez Perce. In Section 3.2 below, I present the model predictions for Situation 2.

3.2. Situation 2: speaker chooses from $\{\emptyset, \text{CAN}(\varphi), \text{CAN}(\neg\varphi), \text{MUST}(\varphi), \text{MUST}(\neg\varphi)\}$

Figure 3 summarizes the model predictions for Situation 2, where the set of alternative utterances is $\{\emptyset, \text{CAN}(\varphi), \text{CAN}(\neg\varphi), \text{MUST}(\varphi), \text{MUST}(\neg\varphi)\}$, and the speaker has chosen to utter $\text{CAN}(\varphi)$.
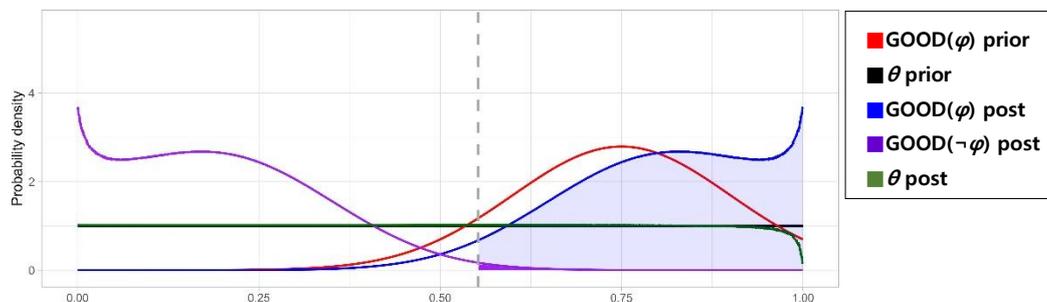
Figure 3 L_1's inference on the goodness of φ, the goodness of $\neg\varphi$, and threshold θ in Situation 2

The inference happens in generally the same direction as in Situation 1, if less dramatically. Similar to Situation 1, the posterior distribution of the goodness of φ has shifted rightward compared to its prior distribution, as is evidenced by the blue curve. The posterior distribution of the goodness of $\neg\varphi$, on the other hand, has shifted leftward, as shown by the purple curve. The posterior distribution of the threshold θ has similar weight through the lower to mid-high values, with less weight in the highest values. Together, it is still highly probable that φ is the only good-enough option, though less so compared to Situation 1.

The first row of Table 2 shows how this is borne out. The posterior distribution of θ attains maximum at 0.5526 (vertical dotted line in Figure 3). Taking this value as the estimate of θ, the probability that the goodness of φ exceeds θ is 0.9707, while the probability that the goodness of $\neg\varphi$ exceeds θ is 0.0089 (area under the blue and the purple curves, respectively, on the right of the dotted line in Figure 3). Given these probabilities and assuming $\text{CAN}(\varphi) \vee \text{CAN}(\neg\varphi)$ is true, the probability that φ is allowed but not required is less than or equal to 0.0092, while the probability that φ is required is greater than or equal to 0.9908. The latter probability is quite high, while still smaller than the minimum probability that L_1 assigns to φ being required in Situation 1, which is 0.9999.

Social utility	θ est.	P(GOOD(φ)>θ)	P(GOOD($\neg\varphi$)>θ)	P(CAN(φ)=1∧MUST(φ)=0) (given CAN(φ∨$\neg\varphi$))	P(MUST(φ)=1) (given CAN(φ∨$\neg\varphi$))
O	0.5526	0.9707	0.0089	≤ 0.0092	≥ 0.9908
X	0.2564	0.9935	0.4371	≤ 0.4399	≥ 0.5601

Table 2 Probability that different modal utterances are true, taking the mode[3] of θ as its estimate, in Situation 2.

Such a diminishment compared to Situation 1 is due to the availability of MUST(φ): L_1 infers that, if the speaker had wanted to strongly signal necessity, they would have chosen MUST(φ). But the speaker instead chose CAN(φ), which is logically weaker than MUST(φ). Hence, L_1 assigns a smaller probability to the necessity interpretation compared to Situation 1, where MUST(φ) is not part of the set of alternative utterances.

Social utility, on the other hand, is a factor that boosts the probability of a necessity interpretation. As was described in Section 2.2.2, social utility is defined in such a way as to discourage signaling a high threshold value. The truth conditions for necessity and possibility are defined in such a way that a necessity utterance would, on average, signal a higher threshold than a possibility utterance[4]. Thus, social utility puts a damper against asserting MUST(φ), even though it is more informative. L_1 infers that the speaker might have asserted CAN(φ) for the sake of politeness although MUST(φ) is also true. This kind of inference results in a higher probability for a necessity interpretation of a possibility utterance.

This is evidenced by the second row of Table 2, which presents the predictions of a model where social utility is not included, all other aspects being equal to the model so far discussed. When only epistemic utility, and not social utility, is considered, the probability that L_1 assigns to a necessity interpretation is greater than or equal to 0.5601, a smaller value than when social utility is considered along epistemic utility. This shows that social utility plays the role of promoting a necessity interpretation of a possibility utterance in the current model.

In Section 3.2, I have presented the model predictions for Situation 2, where the speaker chooses their utterance from the set {Ø, CAN(φ), CAN($\neg\varphi$), MUST(φ), MUST($\neg\varphi$)}. Similar to Situation 1, the model predicts that L_1 would assign a high probability to φ being the only good-enough option, i.e., a necessity interpretation. Because of the scalar reasoning involving CAN(φ) and MUST(φ), we observe a diminishment in the probability that L_1 assigns to the necessity interpretation compared to Situation 1. Social utility works to offset the effect of such epistemic reasoning, as it discourages the assertion of MUST(φ) even if it is true. All in all, the modeling results show how a listener can infer a necessity meaning from a possibility utterance in languages such as English which have separate lexical items for encoding modal possibility and necessity.

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[3] When there were clearly two modes, I calculated their average to obtain the estimated value of θ.
[4] As an illustration, suppose that the goodness of φ is 0.6, and the goodness of $\neg\varphi$ is 0.4, where $Alt(\varphi) = \{\neg\varphi\}$. By the definitions in (2) and (3), MUST(φ) = 1 would signal 0.4≤θ<0.6, whereas CAN(φ) = 1 signals θ<0.6. The assertion of MUST(φ) thus introduces a lower bound for the threshold value which the assertion of CAN(φ) does not, resulting in a higher expectation of the threshold value inferred from the utterance.

4. Conclusion

The present study started with the observation that a deontic possibility utterance can have a necessity reading in certain contexts, as exemplified in (1). I adopted truth conditions for deontic necessity and possibility that are based on a graded notion of modality (Lassiter 2011, 2017; Chung and Mascarenhas 2023). I then modeled the pragmatic inferences that a listener makes within the RSA framework (Frank and Goodman 2012; Goodman and Stuhlmüller 2013). In this model, the pragmatic listener jointly infers about the state of the world and the value of the semantic variable as in Lassiter and Goodman (2013, 2017), and the speaker utility includes social utility along epistemic utility, after Yoon et al. (2017). I considered two situations which differ in terms of whether explicit deontic necessity utterances are available to the speaker as an option. The model predicts a high probability of necessity interpretation in both. I further compared the model predictions when certain components of the RSA model (e.g., utterance cost, social utility) are absent or present, examining the effect that each component has on the overall result.

Future studies could expand upon the present work by trying out different values for the various parameters involved in the model. For example, one could examine how the model predictions differ when assumptions about the listener's prior beliefs change, or if epistemic utility and social utility take up different proportions in computing the speaker utility. Experimental and/or field work could provide data that serves as a standard against which to evaluate the current model, and help determine which set of parameters lead to the best results. It would also be interesting to observe the way that advancement in theoretical issues, such as determining the set of alternative propositions considered when computing the truth of modal necessity, interacts with the RSA model to yield new predictions.

The present study has endeavored to offer a way of systematically investigating a previously understudied phenomenon of deontic necessity inference arising from a deontic possibility utterance. It has looked into how the listener's prior beliefs, the semantic denotation of utterances, and reasoning about politeness and the effort it takes to say something each plays a role to derive the possibility-to-necessity inference. In doing so, the present study will hopefully serve as a step in furthering our understanding of modal meanings and inference in general.

5. Appendix

Deriving deontic possibility from Chung and Mascarenhas's (2023) deontic necessity
Assumptions:
(i) Possibility modals and necessity modals are duals
(ii) $Alt(\varphi)$ is the singleton set $\{\neg\varphi\}$
(iii) The goodness of φ and the goodness of $\neg\varphi$ are inversely proportionate, and sum up to some constant C

Derivation
a. $E_w[\mu_{EVAL}|\varphi] = -E_w[\mu_{EVAL}|\neg\varphi] + C$ (from Assumption (iii))
b. $[\![can\ \varphi]\!]^w = [\![\neg must\ \neg\varphi]\!]^w$

$$= (\mathrm{E}_w[\mu_{EVAL}|\neg\varphi] \leq \theta) \vee (\mathrm{E}_w[\mu_{EVAL}|\varphi] > \theta)$$
$$= (C - \mathrm{E}_w[\mu_{EVAL}|\varphi] \leq \theta) \vee (\mathrm{E}_w[\mu_{EVAL}|\varphi] > \theta)$$
$$= (\mathrm{E}_w[\mu_{EVAL}|\varphi] \geq C - \theta) \vee (\mathrm{E}_w[\mu_{EVAL}|\varphi] > \theta)$$
$$= \mathrm{E}_w[\mu_{EVAL}|\varphi] > \min(\theta, C - \theta)$$
$$= \mathrm{E}_w[\mu_{EVAL}|\varphi] > \tau, \quad \text{where } \tau = \min(\theta, C - \theta)$$

Changing the name of the variable τ in the last line to θ, we obtain the truth condition in (3).

RSA model and parameters

Literal listener L_0

L_0 prior distributions: $A \sim \mathrm{N}_{(0,1)}(0.75, 0.15^2)$ (truncated normal distribution)

$\qquad \theta \sim \mathrm{U}(0, 1)$

Speaker S_1

Value function: $Val(\theta, A) = 5(1-\theta)$

Social utility: $\mathrm{E}_{P_{L_0}(\theta,A|u)}[Val(\theta,A)] = \int_0^1 \int_0^1 Val(\theta,A) \cdot P_{L_0}(\theta,A|u) \; d\theta \; dA$

$\qquad$ where $P_{L_0}(\theta,A|u) = P_{L_0}(\theta|A,u) \cdot P_{L_0}(A|u)$

$$= P_{L_0}(\theta|[\![u]\!]^A = 1) \cdot \int_0^1 P_{L_0}(A|\theta,u) \cdot P_{L_0}(\theta) \; d\theta$$

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