Van Benthem's problem, exhaustification, and distributivity[1]

Sam ALXATIB — *CUNY GC*
Natasha IVLIEVA — *HSE Moscow*

Abstract. We discuss the problem of deriving upper-bounded meanings of **few**, **fewer than**, and related expressions, in treatments where they are taken to denote predicates of individuals. In such analyses, the determiner-like uses of these expressions are derived by existentially-closing their predicate denotations, but this is known to give rise to problems (van Benthem, 1986). We show that the needed upper bound can be derived by applying an exhaustification operator above existential closure. Crucially, this exhaustification operator is insensitive to the distributive properties of the predicates in the sentence, an assumption that we see as consistent with recent work supporting the blindness view of implicatures (Fox and Hackl, 2006; Magri, 2009). We also discuss some similarities and differences between our analysis and Buccola and Spector's (2016) maximality-based approach.

Keywords: Modified numerals, exhaustification, distributivity, maximality.

1. Introduction

It is known that the expressions **many** and **few**, along with morphosyntactic derivatives like **fewer than 3**, show adjectival as well as determiner-like uses. Examples are shown in (1).

(1) a. The many/few people who smiled were smiling broadly
b. (The) people who smiled were many/few
c. Many/few people smiled

It is also known that, in the case of **many**, these multiple uses can be accommodated in a uniformly adjectival treatment, if it is complemented with an operation of Existential Closure (EC). The idea would be that **many** denotes a predicate of individuals, holding of those plural entities that reach a contextually-set cardinality, and in cases like (1c) where **many** behaves like a determiner, the truth conditions result from existentially closing the predicate.

But (as is also known), EC generates problematic results in the case of **few**. Suppose that, analogously to **many**, **few** denotes a predicate of (plural) individuals, holding of just those pluralities that fall below a certain cardinality threshold. If the determiner-like reading is to be derived from EC, the resulting truth conditions will require that some 'small' plurality exist that has whatever other properties appear in the given sentence. But these predicted conditions are inaccurate, because (i) they incorrectly require existence, and (ii) they fail to set a desired upper bound. The unwanted existence requirement comes from EC: under EC, and following standard ontological assumptions, sentences like (1c) where **few** behaves like a determiner cannot be true unless the given predicate(s) are verified by some *existing* plurality. But as is generally agreed in the literature, such sentences are intuitively true even if no individuals

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verify the predicate(s).[2] The second problem, the upper bound problem, is that the existence of small pluralities is compatible with the existence of larger ones; any large plurality of e.g. smiling people will have a small sub-plurality that satisfies the condition of fewness, which means that scenarios where a great number of people smiled will be predicted (incorrectly) to verify the truth conditions of **few people smiled**.

In this paper we will have little to say about the existence problem (problem (i) above). Our focus will be on the upper bound problem, known in current literature as van Benthem's problem (van Benthem 1986). We propose an exhaustification mechanism that correctly sets the needed upper bound, thus circumventing van Benthem's problem. The key detail in the proposal is that exhaustification applies without any sensitivity to the distributive/collective properties of the predicates appearing in the given construction. This assumption builds on recent work promoting the 'blindness' view of implicatures. We explain the details of our proposal in Section 2, and discuss some of its consequences in Section 3. In Section 4 we turn our attention to Buccola and Spector's (2016) discussion of these issues, and provide a brief (and at the moment inconclusive) comparison between one of their proposals and ours. In the remaining parts of the current section, we elaborate on the details of van Benthem's problem, taking note in particular of how it interacts with distributive predicates, and with non-distributive ones.

1.1. Van Benthem's problem and distributive/non-distributive predicates

To keep things simple we will assume that EC is the product of an unpronounced existential determiner, $\exists$, defined in (2).

(2) $[\![\exists]\!] = [\lambda P_{\langle e,t\rangle} \,.\, \lambda Q_{\langle e,t\rangle} \,.\, \exists x(P(x){=}Q(x){=}1)]$

We will also focus our attention on comparative phrases like **more than 4** and **fewer than 4**, instead of **many** and **few**, in order to sidestep the vagueness of the latter's semantics. To keep the presentation simple we assume that **more than 4** and **fewer then 4** denote predicates of individuals:[3]

(3) a. $[\![\textbf{more than 4}]\!] = [\lambda x_e \,.\, |x| > 4]$
b. $[\![\textbf{fewer than 4}]\!] = [\lambda x_e \,.\, |x| < 4]$

If predicates of type $\langle e,t\rangle$ can compose by Predicate Modification, then **more/fewer than 4 people** will denote another set/predicate of plural entities, namely entities of size greater (or less) than four that consist of people. Applying $\exists$ to these predicates will produce an existential quantifier that maps a given predicate Q to True iff Q holds of some plurality of people that contains more than (or less than) four atoms. The LFs and truth conditions for the two sentences are shown below.

[2]Though sentences like **few students smiled** license existence inferences by default, the inference is thought to be extra-semantic, given its disappearance in DE contexts, and cancellability in **in fact** continuations.

[3]As far as we can see, this assumption can be replaced with a more complex syntax/semantics for comparatives without affecting the tenability of the overall proposal. See Alxatib (2013), Chapter 7.

(4) a. **[[∃ more than 4 people] smiled]**
b. $\exists x([\![\textbf{people}]\!](x)=1\ \&\ |x|>4\ \&\ [\![\textbf{smiled}]\!](x)=1)$

(5) a. **[[∃ fewer than 4 people] smiled]**
b. $\exists x([\![\textbf{people}]\!](x)=1\ \&\ |x|<4\ \&\ [\![\textbf{smiled}]\!](x)=1)$

As remarked above, the conditions in (5b) are less informative than desired: they hold even in situations where more than four people smiled, because if (say) five people smiled, there is guaranteed to be a smiling *sub*-group whose size is lower than four. In fact, if we look more closely we can see that the truth conditions of (5a) are identical to those that result when **4** is replaced with any other numeral. To see this, consider the minimally different **fewer than 5 people smiled**:

(6) a. **[[∃ fewer than 5 people] smiled]**
b. $\exists x([\![\textbf{people}]\!](x)=1\ \&\ |x|<5\ \&\ [\![\textbf{smiled}]\!](x)=1)$

If the conditions in (5b) hold, then there is a smiling plurality of size less than four, and therefore less than five, which means that (5a) entails (6a). Conversely, if (6b) holds, i.e. if there is a plurality of smilers of size less than five, then by the distributivity of the predicate **smiled** it follows that there is a smiling sub-plurality of size less than four. Therefore, (6a) entails (5a). This makes (5a) and (6a) equivalent, and the numeral in them semantically irrelevant.

At this point it is crucial to highlight the role of distributivity, of the predicate **smiled** in this case, in making (6)/(5) equivalent. In the case of non-distributive predicates like **lifted the piano (together)**, only one of the two entailments noted above will hold. Consider the LFs and truth conditions in (7) and (8).

(7) a. **[[∃ fewer than 4 people] lifted the piano]**
b. $\exists x([\![\textbf{people}]\!](x)=1\ \&\ |x|<4\ \&\ [\![\textbf{lifted the piano}]\!](x)=1)$

(8) a. **[[∃ fewer than 5 people] lifted the piano]**
b. $\exists x([\![\textbf{people}]\!](x)=1\ \&\ |x|<5\ \&\ [\![\textbf{lifted the piano}]\!](x)=1)$

(7b) says that there is a person-plurality of size less than four that lifted the piano. When this holds, the conditions in (8b) must also hold; if there is a piano-lifting plurality of size less than four, then that same plurality makes it true that there is one of size less than five. (7a) therefore entails (8a), in parallel to the entailment from (5a) to (6a). But the reverse entailment does not hold here; if there is a piano-lifting group of size less than five (=(8b)), it does not follow that there is one of size less than four (=(7b)): Suppose exactly four people lifted the piano together. The size of this group falls below five, but not below four, and there is no guarantee that some sub-group of the four lifters also lifted the piano, because the lifting is true of them collectively, not individually.

We therefore see that van Benthem's problem (the problem of the upper bound) causes numerals to be semantically redundant in the case of distributive predicates, though as we pointed out, this is not the case for non-distributive predicates. We will now explain what this redundancy means for attempts to derive the upper bound by pragmatic strengthening (or exhaustification): because (under distributivity) all alternatives of the form **fewer than n** are equivalent, there will be no way of selecting the right alternatives for exhaustification, and of using those alternatives to generate the desired upper bound at the correct **n**. We expand on this next. From this point on, we will write '$\exists$>**few** constructions' to refer to sentences where EC outscopes **few** and similar expressions. We also write 'distributive/non-distributive $\exists$>**few** constructions' to refer to $\exists$>**few** constructions that contain (or do not contain) distributive predicates.

1.2. Van Benthem's problem and pragmatic strengthening?

Our current truth conditions for distributive $\exists$>**few** constructions only require existence, and appear to make no use of the numeral modified by **fewer than**.[4] But what if the truth conditions were complemented with the negation of other, more informative alternatives? In (Neo-) Gricean pragmatics, a sentence *S* that is uttered in a certain context licenses not only the inferences that result from its literal meaning, but also the inference that alternative sentences which (i) were not uttered, (ii) would have been as relevant in that context, and (iii) are stronger than *S*, are false. It may seem that pragmatic principles along these lines can be used to derive the upper bound of $\exists$>**few** sentences, but as we now explain, pragmatic strengthening of distributive $\exists$>**few** LFs is either vacuous, or excessive.

The first of these two outcomes results if the only relevant alternatives to an $\exists$>**few** sentence are those where the numeral/degree is substituted for another. In the case of our example (5a), repeated below as (9a), this assumption limits the alternatives to those in (9b).

(9) a. **[[$\exists$ fewer than 4 people] smiled]**
  b. {**[[$\exists$ fewer than 3 people] smiled]**,
   [[$\exists$ fewer than 5 people] smiled],
   [[$\exists$ fewer than 6 people] smiled], $\cdots$}

The problem is that none of the alternatives in (9b) are stronger than (9a); they are all equivalent. This gives the strengthening mechanism nothing to negate (or 'exclude') in strengthening (9a), and leaves it vacuous.

Now, suppose we enrich our set of alternatives and add constructions of the form **[[n people] smiled]**, i.e. where **fewer than** is removed:

[4] We will return to the existence requirement in Section 4.

(10) a. **[[∃ fewer than 4 people] smiled]**
b. {**[[∃ fewer than 3 people] smiled]**,
[[∃ fewer than 5 people] smiled],
[[∃ fewer than 6 people] smiled],
c. **[[∃ 2 people] smiled]**,
[[∃ 3 people] smiled],
[[∃ 4 people] smiled],
[[∃ 5 people] smiled], $\cdots$}

Each alternative in (10c) requires that there be an **n**-sized plurality of smiling people, and hence they each asymmetrically entail the weak existence truth conditions of (10a). It follows, by the simple pragmatic recipe described above, that an utterance of (10a) will semantically require existence, and via pragmatic strengthening will imply that no group of four or five smilers exists—so far correctly—but also that no group of three smilers exists, and no group of two smilers exists. By nullfying the semantic contribution of the numeral in the original utterance, the strengthening mechanism described above will not help recover the correct upper bound, because there will be no way to correctly identify its location.

In the next section, we will present a modified strengthening mechanism that bypasses van Benthem's problem. We will first introduce the mechanism's two key components, Innocent Exclusion and blindness, and show how they interact to produce the correct upper bound for distributive **∃>few** constructions. We will then discuss our predictions for non-distributive cases, and finally, compare our analysis to the analysis offered in Buccola and Spector (2016).

2. Reconsidering Pragmatic Strengthening

2.1. Innocent Excludability, Blindness, and Implicature Calculation

We will assume that the strengthened meaning of a sentence S is derived by applying an exhaustification operator Exh to the denotation of S (along the lines of Fox 2007a, following proposals by Groenendijk and Stokhof 1984; Krifka 1995; Landman 1998; van Rooy 2002). Exh may be viewed as a covert variant of **only**. Its semantics is defined below:

(11) $[\![\text{Exh}]\!]^{w}(A_{\langle st,t\rangle})(p_{\langle s,t\rangle}) = 1$ iff $p(w) = 1$ & $\forall q(q \in \text{EXCL}(p,A) \ \rightarrow \ q(w) = 0)$

The exhaustification operator takes a proposition p, its 'prejacent', and a set of alternatives A, and asserts that p is true and that all its excludable alternatives (from A) are false. The set of excludable alternatives $\text{EXCL}(p,A)$ is a subset of A. Different versions of Exh proposed in the literature differ with respect to how this set of excludable alternatives is defined. One possibility is to define $\text{EXCL}(p,A)$ as that subset of A that contains all (and only) propositions that are not entailed by p:

(12) $\text{EXCL}(p,A) = \{q : q \in A \ \& \ p \nvDash q\}$

But we will now show why (12) is problematic, and why (following Fox and others) we adopt the notion of 'Innocent Excludability' instead of it.

2.2. Innocent Excludability

It was argued in Sauerland (2004) that disjunctive constructions should have not only their conjunctive counterparts as alternatives, but also the disjuncts. This is based on cases like (13).

(13) John needs to talk to Mary or Sue.

(13) naturally implies that John does not need to talk to Mary specifically, nor to Sue. If this is to be derived as an implicature, then the mechanism that generates implicatures must have access to the alternatives seen in (14), where the disjunction is replaced with its disjuncts. The alternatives are shown together with the *conjunctive* alternative to (13), but in this example the conjunctive alternative will not play an important role.

(14)
  a. John needs to talk to Mary
  b. John needs to talk to Sue
  c. John needs to talk to Mary and Sue

Note that each of (14a) and (14b) is logically stronger than the original (13), which means that, if they are included in the set of alternatives, they will count as excludable by the definition in (12) and consequently be negated by the strengthening mechanism. Here, this seems to be a good result.[5] But in the case of unembedded disjunctions like (15), the same ingredients make strengthening contradictory.

(15) John saw Mary or Sue.

(16)
  a. John saw Mary.
  b. John saw Sue.
  c. John saw Mary and Sue.

Under the assumption that disjunction has its disjuncts among its alternatives, the application of Exh to (15) is predicted to assert (15) and negate both of (16a) and (16b) (in addition to negating (16c)). But this leads to the contradiction in (17):

(17) John saw Mary or Sue and he didn't see Mary and he didn't see Sue.

The conclusion from this result is that (12), our current definition of excludable alternatives, must be revised in a way that allows the alternatives in (14) to participate in strengthening (13), but blocks (16a–b) from participating in strengthening (15). The revision we adopt is Fox's (2007a), who defines excludability as 'innocent' excludability.

[5]Negating the conjunctive alternative in (14c) is vacuous here, since it follows from negating either of the disjuncts.

Our formulation of Innocent Excludability (IE) is the following: given a proposition p and a set of propositions A, the IE-alternatives ($\textsc{excl}(p,A)$) are those that remain in A after the *non-innocent* subsets of A are removed from A. The non-innocent sets, in turn, are those whose 'set-negation' contradicts p, and that have no proper subsets whose set-negation contradicts p. The set negation of a set A is the conjunction of the negations of A's elements. These definitions are summarized below:

(18) $\textsc{excl}(p,A) = A - \bigcup\{B : B \subseteq A \text{ and } B \text{ is non-innocent w.r.t. } p\}$
  a. B is non-innocent w.r.t. p iff (i) $B^{\neg} \vDash \neg p$, and
  (ii) $\neg\exists B'(B' \subset B \;\&\; B'^{\neg} \vDash \neg p)$.
  b. $B^{\neg} = \bigwedge\{\neg q : q \in B\}$

This revised definition of $\textsc{excl}$ in (18) successfully distinguishes the case of (13) from (15). Let us represent the disjunction in (15) as $p \vee q$, and its alternatives as p, q, and $p \wedge q$:

(19) $A = \{p, q, p \wedge q\}$

Now, given the definition in (18), what subsets of A are non-innocent with respect to $p \vee q$? It is clear that negating all of the propositions in A will jointly contradict the disjunctive $p \vee q$, so A itself satisfies condition (i) in (18a). But A fails condition (ii) in (18b) because there is a proper subset of A whose set negation also contradicts $p \vee q$. This is the set $\{p,q\}$. And because $\{p,q\}$ does *not* have proper subsets whose set negations contradict $p \vee q$, it follows that $\{p,q\}$ is non-innocent with respect to $p \vee q$. So, the set of IE-alternatives in A, given $p \vee q$, is the result of subtracting the non-innocent set $\{p,q\}$ from A:

(20) $\textsc{excl}(p \vee q, A) = A - \{p,q\} = \{p \wedge q\}$

It follows that strengthening $p \vee q$ given the set of alternatives A will not be contradictory, and will generate the inference that the conjunctive $p \wedge q$ is false.

Consider now the case of (13), where disjunction is embedded under a universal modal. Here we may represent (13) itself as $\Box(p \vee q)$, and its alternatives as $\Box p$, $\Box q$, and $\Box(p \wedge q)$:

(21) $A' = \{\Box p, \Box q, \Box(p \wedge q)\}$

It should be clear that negating all of the alternatives in A' does not contradict $\Box(p \vee q)$; the negation will merely require that not all accessible worlds be p-worlds, and not all accessible worlds be q-worlds. This is consistent with $\Box(p \vee q)$, because the modal base may consist of a mix of worlds, some being p-worlds and others being q-worlds. There are therefore no non-innocent sets within A' given $\Box(p \vee q)$,

(22) $\textsc{excl}(\Box(p \vee q), A') = A' - \{\} = A'$

and from this it follows that exhaustifying (13), given the set of alternatives in A', will generate the inference that each element in A' is false:

(23) $\text{Exh}(A', \Box(p \vee q)) = \Box(p \vee q)\ \&\ \neg\Box p\ \&\ \neg\Box q\ \&\ \neg\Box(p \wedge q)$

This concludes our introduction of Innocent Excludability. We now turn to the second of our two key ingredients, blindness.

2.3. Blindness

We take 'blindness' to be a property of semantic operators or mechanisms. When we say that an operator $\boldsymbol{O}$ is blind to some informational content, we mean that $\boldsymbol{O}$ applies without any sensitivity to that content, that is, that $\boldsymbol{O}$ applies to representations (e.g. syntactic ones) where that information is absent. Though our discussion of blindness will remain somewhat vague in this paper, we see Gajewski (2002) as an important reference for how the notion may be made more precise (see specifically Gajewski's definition of 'logical skeleton').

We know of two proposals in the literature on implicatures that appeal to blindness. In one, Fox and Hackl (2006), it is argued that implicature calculation (and calculation of focus semantics) is blind to whether the predicates appearing in the given sentence utilize a discrete or dense scale of measurement. The claim is that implicatures are derived from representations where all measurement scales are assumed to be dense (we will explain the argument briefly below). If right, this means that implicature calculation (and calculation of focus semantics) is insensitive to content that on the surface appears to be *lexical*. As we will see later, our own proposal is similar to Fox and Hackl's in this respect. In another proposal, Magri (2009), implicature calculation is argued to be blind to contextual/world knowledge. We will review Magri's argument after we summarize Fox and Hackl's, but we want to make it clear that we will not talk in detail about how either of these two accounts might interact with our own. Our intention is to use the two proposals as precedent for the hypothesis that exhaustification is blind.[6]

2.3.1. Fox and Hackl (2006)

A central question in Fox and Hackl (F&H) is why sentences like (24) do not give rise to 'exact' implicatures.

(24) John read more than 3 books
$\not\Rightarrow$ $\neg$(John read more than 4 books)

Assuming that (24) has an alternative where the numeral **3** is replaced with **4**, we expect the latter to participate in exhaustifying the meaning of (24), since **John read more than 4 books** is stronger than (24). But then we expect (24) to imply that John read more than three but not more than four books, i.e. exactly four books.[7] In their discussion of the problem, Fox and Hackl point out that this prediction is specific to cases where the relevant scale is *discrete*; if

[6]Readers familiar with Fox and Hackl in particular will no doubt wonder how the density hypothesis interacts with our own exhaustification mechanism. This is by no means a trivial question, but we leave it for later work.

[7]Note that this predictions hinges on the assumption that comparatives have other comparatives as alternatives. One could add constructions of the form **exactly n . . .** , and we would no longer predict comparatives to give rise to these exact inferences. This possibility was in fact proposed in Spector (2005), but argued against in Fox (2007b).

the scale of degrees were dense, then there will be some alternative whose negation contradicts the utterance itself. Take (25) as an example.

(25) John is taller than 6 ft.

Let us assume that (25) requires that John reach a degree of height d above six feet. Unlike the integer scale, the scale of height is intuitively dense, meaning that for any two degrees of height d_1 and d_2 there is a degree of height d_3 in between them. If (25) requires that John reach some height d above six feet, then by the density of the height scale there must exist some degree of height d' above six feet and below John's actual height d. Consider now the alternative in (26).

(26) John is taller than d'

Since d' is above 6 feet, it follows that (26) is stronger than (25), which means that in strengthening (25) the algorithm should negate (26). But negating (26) means that John is at most d'-tall, which makes him shorter than d, i.e. shorter than his assumed actual height. The consequence is that, no matter how tall John is assumed to be, there is always a degree of height just below his own (and above six feet) that can form a stronger comparative sentence than (25), and whose negation will contradict the semantic content of (25).[8] The broader consequence of this is that implicatures of comparatives are predicted to generate contradictions if the relevant scale of degrees is dense. Yet the absence of implicatures in comparatives appears to hold regardless of whether the scale is discrete, as in the case of (24), or dense, as in the case of (25). Given this, Fox and Hackl propose that the level of representation at which implicatures are calculated (as well as focus etc.) is one where all scales are dense. This is their Universal Density of Measurement hypothesis.

The proposal we borrow from Fox and Hackl is that implicature calculation is in some sense blind to information provided by neighboring lexical material. That is, implicature calculation seems to proceed as if some abstract representations replaced the elements that make up the given sentence. By masking the properties of the elements they stand in for, e.g. the discreteness of the counting scale in (24), the abstraction renders implicature calculation insensitive to information that would otherwise change its outcome.

2.3.2. Magri (2009)

Another empirical argument in favor of the blindness view was presented in Magri (2009).[9] Magri claims that the notion of entailment relevant for implicature calculation is *logical*, rather than contextual. His argument is based on the oddness of sentences like (27):

(27) Some Italians come from a warm country.

The argument is this. (27) is odd because the use of **some** triggers a scalar implicature that the alternative with the universal quantifier, **all Italians come from a warm country**, is false.

[8]See Gajewski (2009) for a discussion of how this interacts with innocent excludability.
[9]See also Magri (2011); Schlenker (2012); and Magri (to appear) for discussion.

This leads to the strengthened proposition that *some but not all* Italians come from a warm country, which contradicts our knowledge that all Italians come from the same country. But as Magri points out, this explanation works only if we assume that this piece of world knowledge is unavailable to implicature calculation. While the alternative **all Italians come from a warm country** is logically stronger than (27), the two sentences are *contextually* equivalent; knowing that all Italians come from the same country, if some Italians come from a warm country, then it must be the case that all of them do (and vice versa). If this information is factored into the implicature-calculating mechanism, the **all**-alternative would not be negated, because it is equivalent to (27), and the oddness would no longer be predicted. If, on the other hand, the mechanism was blind to world-knowledge, and was sensitive only to logical information, then exhaustification would apply, and produce the detected oddness of (27).[10]

In what follows, we will attempt to exhaustify our $\exists$>**few** structure again, but with the blindness hypothesis and Innocent Excludability in mind. We will show that if the distributivity of the given predicate is hidden from the exhaustification mechanism—in parallel to how scale discreteness is hidden, in Fox and Hackl's proposal—we predict that the upper bound be placed correctly, thus overcoming the challenge of van Benthem's problem.

2.4. Exhaustification revisited

Consider once again the $\exists$>**few** LF in (28),

(28) **[[$\exists$ fewer than 4 people] smiled]**

And consider the alternatives below:

(29) a. {**$\exists$ fewer than 2 people smiled**,
$\exists$ fewer than 3 people smiled,
b. **$\exists$ fewer than 5 people smiled**,
$\exists$ fewer than 6 people smiled, $\cdots$
c. **$\exists$ 2 people smiled**,
$\exists$ 3 people smiled,
d. **$\exists$ 4 people smiled**,
$\exists$ 5 people smiled,
$\exists$ 6 people smiled, $\cdots$}

As we saw in Section 1.2, the distributivity of the verb **smiled** makes each of the alternatives in (29a–b) equivalent to (28), so negating any of them individually will contradict (28). This means that each alternative in (29a–b) makes its own *singleton* non-innocent set, since by definition, a set is non-innocent if its set-negation contradicts the utterance, and if it has no proper subset whose set-negation contradicts the utterance. The alternatives in (29a–b) are therefore not innocently excludable.

[10] As Magri himself notes, the success of his analysis requires that (27) be *obligatorily* exhaustified, since the oddness of the sentence would otherwise not be predicted. The reader is referred to the original paper for details.

What about the alternatives in (29c–d)? As we showed in Section 1.2, negating these together does not contradict the truth conditions of (28), so they are all predicted to be innocently excludable. It follows that exhaustifying (28) with respect to the alternatives in (29) will produce an excessively low upper bound:

(30) $\text{Exh}((29),(28))=1$ iff $\exists x(\llbracket\textbf{people}\rrbracket(x)=1\ \&\ |x|<4\ \&\ \llbracket\textbf{smiled}\rrbracket(x)=1)\ \&$
$\neg\exists x(\llbracket\textbf{people}\rrbracket(x)=1\ \&\ |x|\geq 2\ \&\ \llbracket\textbf{smiled}\rrbracket(x)=1)\ \&$
$\neg\exists x(\llbracket\textbf{people}\rrbracket(x)=1\ \&\ |x|\geq 3\ \&\ \llbracket\textbf{smiled}\rrbracket(x)=1)\ \&\cdots$

But recall also that the equivalence of (29a–b) and (28) does not hold when **smiled** is replaced with a collective predicate. In those cases, the alternatives with the lower numeral asymmetrically entail those with the higher numeral. If there exists a piano-lifting group of size less than four, then there exists a piano-lifting group of size less than five, namely the same one, but the reverse does not hold: if there is a lifting group of size less than five, it does not follow that there is one of size less than four.

Now suppose that, by blindness, the exhaustification mechanism were insensitive to the lexical properties of the verb. Then the grouping of alternatives into non-innocent sets will change, and consequently change the contents of the (innocently) excludable set. Let us repeat (29) and (30), but abstract away from the NP/VP:

(31) **[[∃ fewer than 4 NP] VP]**

(32) a. {**∃ fewer than 2 NP VP**,
∃ fewer than 3 NP VP,
b. **∃ fewer than 5 NP VP**,
∃ fewer than 6 NP VP, ⋯
c. **∃ 2 NP VP**,
∃ 3 NP VP,
d. **∃ 4 NP VP**,
∃ 5 NP VP,
∃ 6 NP VP, ⋯}

We can immediately see that the alternatives in (32b) are non-innocent, because each of them is individually weaker than (31), and therefore comprises a singleton non-innocent set of its own. The alternatives in (32a) are stronger than (31), but they are not innocently excludable because they form non-innocent sets with alternatives in (32c). As an example, take the first alternative in (32a) and the first alternative in (32c). Negating the former amounts to saying that there are no groups (of NPs that VP) of size less than 2, which means that the size of VPing NP groups is 2 or greater. This contradicts the negation of the first alternative in (32c), which says that there are no groups of VPing NPs of size 2 or more. Note that, by themselves, each of these alternatives can be negated consistently with (31), so on their own, they do not form non-innocent sets with respect to (31).

More generally, then, for any numeral (or degree) d lower than 4, there is a non-innocent set that contains the alternatives **[ꓱ fewer than *d* NP VP]** together with the alternative **[ꓱ *d* NP VP]**. Any set containing just these two sentence is non-innocent, because its set negation is contradictory and therefore inconsistent with (31).

What about the alternatives in (32d)? It may appear that each of these can be paired with an alternative from (32b) into a non-innocent set. But in fact, these sets have non-innocent proper subsets: remember that each alternative in (32b) individually forms a singleton non-innocent set, so pairing it with an alternative from (32d) will not produce a non-innocent set.

It follows, then, that the excludable alternatives from (32) are the ones in (32d), and negating these produces the desired upper bound.

(33) $\text{Exh}((32),(31))\!=\!1$ iff $\exists x([\![\mathbf{NP}]\!](x)\!=\!1 \;\&\; |x|\!<\!4 \;\&\; [\![\mathbf{VP}]\!](x)\!=\!1)$ &
$\neg\exists x([\![\mathbf{NP}]\!](x)\!=\!1 \;\&\; |x|\!\geq\!4 \;\&\; [\![\mathbf{VP}]\!](x)\!=\!1)$ &
$\neg\exists x([\![\mathbf{NP}]\!](x)\!=\!1 \;\&\; |x|\!\geq\!5 \;\&\; [\![\mathbf{VP}]\!](x)\!=\!1)$ & $\cdots$

3. Discussion and consequences

In a nutshell, our proposal derives the upper bound of ꓱ>**few** constructions not from their literal semantics, but from an exhaustification mechanism that negates whatever is excludable from the set of alternatives. In Section 2 we showed that, in order for the exhaustification mechanism to produce the correct upper bound, the entailment relations it 'sees' between the alternatives must be independent of whether the predicates in those alternatives are distributive or collective. We pointed to findings in the literature on implicatures, notably Fox and Hackl (2006) and Magri (2009), that suggest that exhaustification is blind to lexical/contextual information.

But the proposal brings many questions with it. First, it is known that implicatures are often cancellable, so if an implicature-generating mechanism is responsible for the upper bound in the sentences that concern us, why is it that the upper bound is intuitively obligatory?

Our answer to this is that, in the case of distributive predicates, lack of exhaustification produces underinformative truth conditions. We make this precise and borrow Buccola and Spector's Pragmatic Economy Constraint:

(34) **Pragmatic economy constraint (Buccola and Spector, 2016)**:
An LF ϕ containing a numeral n is infelicitous if, for some m distinct from n, ϕ is truth-conditionally equivalent to $\phi[n \to m]$ (the result of substituting m for n in ϕ).

We will look more closely at Buccola and Spector's proposal in the next section. At the moment we can simply point out that (34) is used by them to get around a similar problem to ours: why is exhaustification (or, on their account, maximality) obligatory? The effect of (34) is to require that numerals make a truth conditional difference. As we saw in Section 1.2, unexhaustified ꓱ>**few** constructions have the same truth conditions regardless of the numeral that appears in them, and because of this, they are ruled out by the economy constraint in (34). On the other hand, exhaustifying $\exists>$**few** constructions (blindly) retrieves the semantic import of the numeral, and thus satisfies (34).

But now we face a second question. If exhaustification is made obligatory because of a filter against uninformative numerals, we predict that exhaustification be *optional* in *non-distributive* $\exists>$**few** sentences. The reason is simply that, as we saw in Section 1.2, changing the numeral in non-distributive $\exists>$**few** constructions *does* change the truth conditions, so there is no reason yet to favor either of the exhaustified or unexhaustified parses of (35):

(35) Fewer than 4 people lifted the piano (together).

Interestingly, sentences like (35) do not intuitively place an upper bound in the same way as distributive $\exists>$**few** examples. As Buccola and Spector point out, (35) requires only that *some* group of lifters reach a size below four; the sentence is still true if the piano was lifted by some other group of size five, six, etc. This reported judgment fits the prediction of the current proposal, which permits unexhaustified parses of (35) given that the numeral in it is non-trivial. But importantly, the proposal as it stands also allows exhaustified parses of non-distributive $\exists>$**few** structures. So we currently predict (35) to also have upper-bounded readings. We leave this issue to future work.

Another important problem that we have set aside so far is the existence prediction of EC. The predicted truth conditions of $\exists>$**few** constructions, whether exhaustified or not, require that some *existing* plurality verify the predicates appearing in the sentence. We do not have a solution to this problem yet, but we will say more about it after we discuss Buccola and Spector's account, to which we now turn.

4. Maximality: Buccola and Spector (2016)

We must make it clear that our review of Buccola and Spector (B&S) is by no means representative of the many ideas they discuss. Our attention will be restricted to their syntactic maximality account (SMax) of modified numerals. Once we go over the basics of the account, we offer a brief comparison between it and our own proposal. As announced in the introduction, the comparison will be inconclusive, but it will highlight an advantage of B&S that has to do with the existence inference discussed earlier.

4.1. Syntactic Maximality (SMax)

The ingredients of B&S's SMax account are the following. First, expressions like **fewer than 4** denote generalized quantifiers (GQs) over degrees (type $\langle dt, t\rangle$) rather than predicates of individuals. Second, nodes that denote degrees (type d) can undergo two type-shifting operations, which we represent syntactically. One, ISCARD, takes a degree d and returns a predicate of (plural) individuals. The predicate holds of individuals whose size equals d. The other, ISMAX, takes a degree d and returns a GQ over degrees. The GQ holds of a set of degrees provided that its maximal element is d. The definitions are shown below:

(36) a. $[\![\textbf{fewer than 4}]\!] = [\lambda D_{\langle d,t\rangle} \,.\, \exists d(d{<}4 \;\&\; D(d){=}1)]$
b. $[\![\textsc{IsCard}]\!](d) = [\lambda x_e \,.\, |x|{=}d]$
c. $[\![\textsc{IsMax}]\!](d) = [\lambda D_{\langle d,t\rangle} \,.\, \max(D){=}d]$

On B&S's account, a sentence like (37) can have two possible LFs. In one LF, the phrase **fewer than 4** undergoes QR and binds a trace of type d in its base position. The trace is shifted by ISCARD into a predicate of individuals, and the result is composed with other predicates ultimately closed by an EC operation.

(37) **Fewer than 4 people smiled**

(38) **[fewer than 4] [λd [[∃ [ISCARD(d) people]] smiled]]**

With the dislocated modified numeral scoping above EC, the resulting truth conditions require that some degree d below 4 exist, and that some smiling plurality be of size d. These are the same (uninformative) truth conditions as those predicted on our account (without exhaustification):

(39) $[\![(38)]\!]{=}1$ iff $\exists d(d{<}4 \;\&\; \exists x(|x|{=}d \;\&\; [\![\textbf{people}]\!](x){=}1 \;\&\; [\![\textbf{smiled}]\!](x){=}1))$
$={1}$ iff $\exists x(|x|{<}4 \;\&\; [\![\textbf{people}]\!](x){=}1 \;\&\; [\![\textbf{smiled}]\!](x){=}1)$

In another LF of (37) the phrase **fewer than 4** is moved further, and binds a trace that undergoes shifting by ISMAX.

(40) **[fewer than 4] [$\lambda d'$ [ISMAX(d')] λd [[∃ [ISCARD(d) people]] smiled]]**

The truth conditions of this LF require that, for some degree d below 4, the maximal size of existing smiling groups equal d. This is the same as saying that the maximal size of existing smiling groups fall below 4, which matches the intuited upper-bounded reading of the sentence.

(41) $[\![(40)]\!]{=}1$ iff $\exists d(d{<}4 \;\&\; \max\{d' : \exists x(|x|{=}d' \;\&\; [\![\textbf{ppl}]\!](x){=}1 \;\&\; [\![\textbf{smiled}]\!](x){=}1)\}{=}d)$
$={1}$ iff $\max\{d : \exists x(|x|{=}d \;\&\; [\![\textbf{ppl}]\!](x){=}1 \;\&\; [\![\textbf{smiled}]\!](x){=}1)\}{<}4$

The choice between LFs (38) and (40), for sentence (37), is based on the Pragmatic Economy Constraint (PEC) that we cited earlier. Recall that the constraint blocks LFs that contain uninformative numerals. In (38), the numeral is uninformative because of the distributivity of **smiled** (see discussion in Section 1.2), but in (40), changing the numeral clearly changes the truth conditions. By the PEC, then, only the latter LF can be associated with (37), which means that upper-bounded readings are obligatory when distributive predicates are used. In the case of non-distributive predicates, however, 'non-maximal' LFs incur no violation of the PEC, because their truth conditions depend crucially on the numeral. It follows that sentences like (42) can be interpreted without an upper bound—LF (43)—or with an upper bound—LF (45).

(42) **Fewer than 4 people lifted the piano (together)**

(43) **[fewer than 4] [λd [[ꟻ [ISCARD(d) people]] lifted the piano]]**

(44) $[\![(43)]\!]=1$ iff $\exists d(d<4$ & $\exists x(|x|=d$ & $[\![\textbf{people}]\!](x)=1$ & $[\![\textbf{smiled}]\!](x)=1))$
$=1$ iff $\exists x(|x|<4$ & $[\![\textbf{people}]\!](x)=1$ & $[\![\textbf{lifted...}]\!](x)=1)$

(45) **[fewer than 4] [$\lambda d'$ [ISMAX(d')] λd [[ꟻ [ISCARD(d) people]] lifted...]]**

(46) $[\![(45)]\!]=1$ iff $\exists d(d<4$ & $\max\{d' : \exists x(|x|=d'$ & $[\![\textbf{ppl}]\!](x)=1$ & $[\![\textbf{lifted...}]\!](x)=1)\}=d)$
$=1$ iff $\max\{d : \exists x(|x|=d$ & $[\![\textbf{ppl}]\!](x)=1$ & $[\![\textbf{lifted...}]\!](x)=1)\}<4$

4.2. A comparison: Exhaustification or maximality?

As we said earlier, we cannot comprehensively compare our proposal to that of B&S. Nevertheless, we can take note of some features that the two accounts have in common, and other features that distinguish them. We leave a more detailed assessment to future investigation.

One common ingredient to B&S and the current proposal is the reliance on the PEC, and the resulting prediction that, with non-distributive predicates, upper-bounded readings are *permitted but not obligatory*. In our account, this is because unexhaustified parses are not blocked by the PEC in non-distributive ꟻ>**few** LFs; in B&S, it is because the PEC permits both maximal as well as non-maximal LFs. Whether or not this prediction fits the facts remains to be seen, but it appears that in this respect the two accounts are similar.[11] The proposals are also alike in deriving the upper bound from sources that are external to the modified numeral itself. In our account, the source of the upper bound is the (blind) exhaustification operator, while in B&S it is the ISMAX shift operation.

But there are differences. Consider first the existence inference mentioned in Section 3. On our proposal the inference results from existentially closing the predicate **fewer than 4**, but importantly, the inference remains even when the LF is (blindly) exhaustified. We therefore predict that upper-bounded readings still require existence, and that sentences like (37) be *false* in situations where no one smiled. We see this as a disadvantage of our account, and an advantage of B&S's, where applying ISMAX above ISCARD effectively removes the requirement of existence.[12]

There is, however, a possible amendment to exhaustification that deserves further research, which involves adding null individuals to the domain of entities. At the moment we cannot present a concrete version of this idea, and leave it to future development (see Landman 2004, and also B&S's maximal-informativity proposals in their Section 8). If such an analysis can be formulated, it may come with different predictions from B&S's SMax proposal, which es-

[11] B&S discuss cases where collective predicates take upper-bounded readings, citing personal communication with Philippe Schlenker. See their Section 7.

[12] It must be noted that removing the existence inference in B&S depends on defining max so that it returns the degree 0 when its input is the empty set. In situations where no one smiled, there are no degrees in the set $\{d : \exists x(|x|=d$ & $[\![\textbf{ppl}]\!](x)=1$ & $[\![\textbf{smiled}]\!](x)=1)\}$, and hence no maximal degree that can inform the semantics of ISMAX. To get around this, it must be explicitly stated that $\max\{\}=0$.

sentially correlates non-existence with upper-boundedness, since the two come together from applying ISMAX.

5. Conclusions

We discussed a problem that arises with the interpretation of expressions **few** and **fewer than 3** in certain environments. The problem can be described as follows. When combined with distributive predicates, those expressions intuitively impose an upper bound. However, assuming the adjectival semantics for **few** and deriving its quantificational meaning through EC, sentences with **few** are predicted to have uninformative truth conditions. We showed also that the needed upper bound cannot result from standard pragmatic strengthening mechanisms.

We proposed a modified strengthening mechanism that can derive the correct upper bound. The key properties of the proposed mechanism are (i) its insensitivity to information about distributivity, and (ii) innocent exclusion (Fox, 2007a). Our proposal was compared to Buccola and Spector's maximality-based account. As we pointed out, the comparison is incomplete and requires further investigation.

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