On the meaning and use of *okay* in spoken German[1]

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Abstract. We present an account of the meaning and use of *okay* in spoken German based on a conversation analytic study of the Berlin-Map-Task-Corpus, focusing on three uses: *pure acknowledgment*, *acceptance*, and *undecidedness marking*. Semantically, we propose that *okay* denotes a truth predicate. This allows for a uniform semantics: The different uses of *okay* can be derived from that meaning in connection with the meanings of prosodic contours which we take here to indicate stance marking. Pragmatically, we propose that the basic function of *okay* is to indicate uptake. For simplicity and lack of space, our proposal will be approximated in terms of a three-valued propositional multi-agent model, in which *okay* is taken to denote a constant function from formulas to the truth value *true*.

Keywords: *okay*, truth predicate, uptake, acceptance, undecidedness, speech acts

1. Introduction

The particle *okay* has barely received any attention in formal semantics and pragmatics. This is surprising, as it is omnipresent in our everyday lives, be that at conferences, doctor's visits, in court, but also at the breakfast table and in gossiping with friends. This is reflected in the large body of work in conversation analysis on *okay*, which does not just show that *okay* is used in many different settings, but also that *okay* is used for performing different kinds of actions. These range from such familiar uses as accepting a proposal, offer, and so on, to actions such as transitioning between tasks and topics, but also closings of interactions. Also, it can be found in different languages, where it is used similarly, if not even identically (for an overview and references see Betz et al., 2021).

In Krifka (2013), *okay* has been addressed shortly. *Okay* is said to react to the speech act as opposed to the proposition of an antecedent clause. Further, *okay* itself does not commit the *okay*-speaker to the propositional content of the speech act. For instance, if Alice asserts that *p* and Bob says *okay*, then Bob does not become committed to *p*. According to Krifka, Bob indicates compliance with Alice' assertion, which he considers to mean that Bob's *okay* indicates that Bob accepts integration of *p* into the common ground (see Krifka, 2013: 10–11). An overall similar proposal has been made in Krifka (2022).[2]

In Venant and Asher (2015), *okay* is used as a natural language example that illustrates acknowledgments. They discuss acknowledgments that only indicate that the *okay*-speaker has understood their interlocutor, and others that actually lead to grounding of the content of the speech act the *okay*-speaker is reacting to. The latter entails commitments by the *okay*-speaker

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[2]We lack space to discuss this in more detail, but the reader will see that our analysis of the meaning of *okay* differs substantially from Krifka (2013) and Krifka (2022).

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to the propositional content of the speech act they are reacting to. We will encounter examples of both types in this article.

While the literature is sparse, it is presumably a common place within the research community that *okay* contrasts with particles such as English *yes* or German *ja* in the context of polar interrogatives that receive a question interpretation (as opposed to say, a request or offer interpretation).[3] While saying *okay* in response to questions is judged off, replying *yes* or *ja* is totally acceptable.

Example 1. (*okay* in the context of polar interrogatives: analogous for German; ↓= final fall)
A: Is it raining? B: Yes↓./#Okay↓.
A: Shall we go to the Italian restaurant? B: Yes↓./Okay↓.
A: Can you pass me the salt? B: Yes↓./Okay↓.

The first question seeks information about the weather and while *yes* can be used to provide that sort of information, *okay* appears to be unable to do so. However, when a polar interrogative becomes interpretable as an offer or proposal (second case), or as a request or search for help (third case), *okay* becomes felicitous. These are speech acts which pose action-coordination problems and *okay* indicates compliance in such cases. The same goes for *yes* (again, this is alike for German).

We want to suggest that *okay* cannot be used to perform the speech act of answering information seeking questions, because *okay* conveys epistemic ignorance (see also Oloff (2019) for German, and Lindström (2018) for Swedish). Answering, however, seems to indicate knowledgability. On the other hand, if an interrogative can be interpreted as a proposal, offer, request or similarly, what can be taken at stake is whether the addressee is willing to undertake certain commitments. Thus, these interpretations do not (primarily) concern the knowledge of the addressee, but their current and future actions. In general, *okay* seems to be able to indicate acceptance rather than actual determining an action in these cases. The *okay*-speaker cannot be understood as claiming authority over the future actions of the interactants. Thus, the *okay*-speaker simply adopts the agenda proposed by their interlocutor. An in-depth study of *okay* in the context of interrogatives is planned for the future.

This article zooms in on three responsive *okay* uses, which we call *acceptance*, *pure acknowledgment*, and *undecidedness marking*, where the last kind of uses have not been discussed in the literature yet. We argue that *okay* semantically denotes a truth predicate and that pragmatically, *okay* is used to perform uptake. In combination with prosodic contours, different kinds of responses can be realized. For lack of space and for the sake of simplicity, we will approximate this idea in terms of a three-valued propositional multi-agent system.

2. Some uses of *okay* in spoken German

Our proposal is based on a large conversation analytic study of the *okay* tokens appearing in the video material of the L1-subcorpus (release 2.1) of the Berlin-Map-Task Corpus (BeMaTaC, Sauer (2013); on conversation analysis see for instance Schegloff (2007) and more generally Sidnell and Stivers (2013)). The map task corpus was interesting for our purposes, because the

[3]See also Krifka (2013) on *okay* in the context of questions.

setting constructs a joint project which requires coordination of knowledge. In a map task, we have a person with the full map, and a person who needs to copy the path from a dedicated starting point to a dedicated end point. The map contains pictures around which the path moves. In this particular map task experiment, each round has exactly one mismatch in pictures. Participant pairs perform the task twice with a role swap and a different map. The participants of the map task cannot see each other nor each other's sheet. This forces communication to be exclusively verbal and it creates an epistemic asymmetry. This provides a context in which words like *okay* are useful, because they can signal acknowledgments, acceptances, but they can also be used to manage the interaction via transition indication or indication of closings.[4]

The L1 sub-corpus provides around 66 minutes of video material. The recordings show a third-person perspective of the drawing person, which (in principle) allows to see what they are drawing, when they are drawing, etc. The video material contains 133 particle *okay* tokens. The tokens showcase many different uses of *okay*, ranging from transitional *okay*s, to acceptance marking *okay*s, undecidedness marking *okay*s, to auto-reflexive uses and uses that indicate the end of some activity. For lack of space, we will only illustrate transitional *okay*s.

Example 2. (Transitional *okay*, see BeMaTaC 2012-10-31-D, 00:00:00–00:00:15)
(The instructor and the instructee are seated and are waiting to start the map task. Then the door gets closed by the experimenters.)
Instructor: Okeh. Also du fängst auch unten links beim Startpunkt an. (*Okay, so you also begin in the bottom left corner at the starting point.*)
Instructee: Ja, den seh ich sowieso. (*Yes, I can see that anyway.*)

Here, the *okay* token is used to indicate a transition in activity. The closing of the door is an event that clearly indicates that the preparations for the map task have been completed and the participants in the sound proof booth can therefore start with the task. Since this is the agenda of the participants, this is inferrable from their common ground and it is clear that this is what is projected to happen. Indeed, participants work actively into that direction: The *okay* is being followed by the first instruction or a proposal for how to get started (a strategy proposal, so to speak). In terms of a truth predicate analysis of *okay*, the *okay*-speaker adds the fact that an event has ended to their representation of the situation, which formally is taken to be a set of formulas to which they add a new formula in a truth predicate. We propose that prosody is used to indicate that speakers have more to say, which thereby also indicates that more is to come. In all cases, prosody has been judged by ear. A detailed analysis is needed in the future.

In the following, we will further decontextualize the simplified corpus examples by referring to the interlocutors as *Alice* and *Bob* respectively, where the first turn in an example gets allocated to Alice. This is for convenience only.

2.1. Acceptance marking

The exchange in Ex.3 illustrates a typical case of acceptance marking:

Example 3. (Acceptance marking; BeMaTaC 2012-01-19-A; 00:00:18–00:00:21; ↓= final fall)
Alice: Hast du einen Stift? (*Do you habe a pen?*)

[4]On *okay* in joint projects see for instance Bangerter and Clark (2003).

Bob: Ja. (*Yes.*)
Alice: Okeh↓. Also du musst jetzt vom Startpunkt . . . (*Okay. So, you have to move now from the starting point . . .*)

Alice asks Bob whether he has a pen. Without a pen, Bob cannot copy the path. Bob answers affirmatively. Then, Alice responds using an *okay* token with a final falling intonation. This is followed by a turn that implements an instruction for Bob to now draw a line form the starting point to some point not specified in the example.[5]

Alice's question publicly conveys that she is ignorant about Bob's pen-ownership (cf. Heritage, 2012). Thus, this is not common ground among the two. It is common ground among them that Bob can perform the map task only if he has a pen. This also means that it is common ground that Bob can only perform drawing instructions if he has a pen. Now, from Alice's instruction, we can infer that she assumes that Bob has a pen. Thus, the instruction in connection with Bob's answer gives us that both consider this true. Further, this can become common ground among the two, for both can infer that the other can reason that way. This is because both know that only under that assumption are instructions of that kind reasonable (the instruction is meant to be carried out now, not at a later point). Therefore, the instruction indicates that Bob's pen-ownership can be considered common ground at this point. Importantly, the instruction is presented as a conclusion from the information available to Alice at the point of instruction giving (the import of *also*). Therefore, that Bob has a pen is assumed by Alice prior to the instruction. Her question suggests that she did not assume that he has a pen prior to the instruction (in that case, she could just have made the instruction). Her assuming so is best seen a consequence of Bob's answer. But to get from his answer to assuming that he has a pen, she must do more than acknowledging his claim, she has to align with it: she has to consider it true that he has a pen. Prior to the *okay*, we cannot infer so because of her question and lack of display of uptake. After the *okay* token, it is already assumed as a premise for the instruction making. Thus, the *okay* token is the only option left for situating this inference such that it is publicly accessible. As a consequence, we get that this *okay* can be used to make Bob's pen-ownership common ground, because it publicly indicates a change in knowledge.

2.2. Undecidedness marking

Undecidedness markings look similar to acceptance markings in terms of position, however, they are non-aligning actions, yet they acknowledge what others have said:

Example 4. (Undecidedness marking; BeMaTaC 2011-12-14-A, 00:01:36–00:02:05; ↑= final rise)
Alice: Du hast keine Nägel? (*You don't have nails?*)
Bob: Mhmh. (*Uhuh.*)
Alice: Okeh↑? (*Okay?*)
Bob: Ich habe einen Schornsteinfeger. (*I've got a chimney sweeper.*)
Alice: Okeh↓. Dann sind das aber nicht so ganz gleiche Bilder. (*Okay. But then the pictures aren't exactly the same.*)

[5]The end point of the line is inessential to the analysis so we cut off the turn at that point.

The displayed sequence is preceded by an instruction of Alice which turns out to be problematic. The instruction asks Bob to draw the line to a picture that is not part of Bob's sheet. Bob makes overt his confusion by initiating repair, which gets responded to by Alice asking whether he doesn't see the picture in question. Bob states that he has no such picture. Then, the above situation occurs.

Given the context, Alice's question is readily understood as a confirmation seeking question. This check is confirmed by Bob (*Mhmh.*) and responded to by Alice using an *okay* token, the item of interest here. The token has final rising intonation and occurs in the same kind of position as the token from Ex.3. This case differs though. Naturally, Alice assumes that they have matching pictures (they did not know of the mismatches prior to this encounter). That assumption and Bob's information do not go together. If one is true, the other must be false. While the *okay* token indicates acknowledgment of Bob's answer and understanding of it, we cannot infer from it that Alice treats Bob's answer true, nor can we infer that she treats it false. What she said up to that point does not decide between these options and this includes the contribution of the *okay* turn. Bob's subsequent turn allows for an interpretation where the intonation is interpreted as indicating trouble: *What to do now? How to continue?* His turn can be taken to contribute to this issue. That scenario does not entail though that Alice takes Bob to say the truth necessarily (she may still think that maybe he has overseen the picture, or something else). In any case, it seems clear that his claim of having a chimney sweeper picture is best understood as an alternative picture to the pictures of nails Alice was referring to. It is the response to that turn which allows us to infer that Alice' issue of inconsistent information has been resolved. The second *okay* token in the example is clearly an acceptance marking *okay*, which can be argued for again via the subsequent conclusion drawing. After this *okay* token Alice publicly concludes that they must have different pictures in at least a few cases. It is only here that we can indeed infer that she considers Bob's answer true and her initial assumption false. Because the first *okay* token does not allow for an inference to a classical truth value, we call this kind of *okay* an *undecidedness marking okay*.

2.3. Acknowledging without accepting

A case along the lines of Ex.5 has been suggested by an anonymous reviewer of the original abstract, and it goes in the direction of acknowledgments discussed in Venant and Asher (2015), where we do not find alignment. We are unaware of such cases in the BeMaTaC corpus.

Example 5. (Pure acknowledgment)
Alice: Sag mir was du über Charlie weißt. (*Tell me what you know about Charlie.*)
Bob: Charlie kommt aus Hamburg. (*Charlie is from Hamburg.*)
Alice: Okeh↓, das ist falsch. Charlie kommt aus Homburg. (*Okay, that's wrong. Charlie is from Homburg.*)

This case contrasts with the acceptance marking case. Here, Alice asks Bob to tell her what he knows about Charlie. Bob starts by saying that Charlie is from Hamburg. Alice responds to this by using an *okay*-token with a final falling intonation (at least that is the premise of the constructed case). Given the general idea that *okay* denotes a truth predicate, this may be taken to mean that Alice considers it true that Charlie is from Hamburg. If we assume so, we will have to conclude that Alice is inconsistent. Semi-formally, this means that we have *True*(p) and *False*(p), which is a contradiction. However, we are not forced to this logical form of the interaction. Bob saying that Charlie is from Hamburg can be taken to introduce a commitment by him to the truth of Charlie being from Hamburg. If we represent this by the formula $C_B p$, we now get *True*$(C_B p)$ for Alice' *okay*-response which is consistent with *False*(p). In fact, this line of thought mirrors that of Venant and Asher (2015) and Krifka (2013), just in terms of truth predicates as opposed to commitments.

Contrasting this case and the acceptance marking case suggests that the final fall introduces an implicature: unless it is indicated otherwise, an acknowledging *okay* seems to be interpreted as also aligning. This reading is indeed available if we leave aside any opposing material:

Example 6. (Acknowledgment without denial)
Alice: Sag mir was du über Charlie weißt. (*Tell me what you know about Charlie.*)
Bob: Charlie kommt aus Hamburg. (*Charlie is from Hamburg.*)
Alice: Okeh↓. (*Okay.*)

We can interpret Alice as accepting what Bob says, but we are not forced to. We may be doubtful about Alice's own stance on this, for instance, if this were some kind of interrogation. With Venant and Asher (2015) we can say that Alice's discourse move is ambiguous.

Overall, cases like Ex.5 seem to differ from acceptance marking *okay*s (Ex.3) only in the inferences they sanction. We want to stress that this is a constructed example and it is thus indispensable to study natural occurring cases which also allow for prosodic studies. We suspect such cases to occur frequently in argumentative settings, where it can be relevant to acknowledge without aligning.

3. The meaning of intonational contours

Our proposal assumes a division of labor between *okay*'s semantics and the meaning of final intonational contours in the derivation of illocutionary force. In particular, we have proposed that final rising contour has a meaning that indicates that the speaker cannot provide a classical truth value judgment for the propositional content of the locution. The final fall has instead been taken to contribute an implicature that as long as it can be assumed to be applicable will lead to alignment.

The literature on the meaning of prosodic contours in the context of declaratives made similar observations for both German and English.[6] With regard to final rises, a common idea is that they do not add commitments (for German, see for instance Truckenbrodt (2006): 271–272; for English see for instance Rudin (2022): 358–359, Goodhue (2021): 960–964) The commitment proposal for final rises translates quite neatly to our truth value proposal. From the lack of commitment, we cannot infer a commitment to the contrary (such requires additional assumptions). Thus, the speaker cannot be taken to be committed to the proposition being true, nor to it being false. This is essentially what we call undecidedness.

[6]We note that this literature is more sophisticated than the presentation here. For instances, different kinds of rises are distinguished. Such is not the case here. This should be addressed in the future.

Final falls have been seen in different lighting. Both Truckenbrodt and Rudin take final falls to indicate commitment. Thus, a declarative sentence such as *It is raining* with a final falling contour will be understood as committing the speaker to the propositional content of the sentence (see Truckenbrodt (2006): 271–272 and Rudin (2022): 358–359, respectively). Goodhue sees final falls as unmarked defaults that indicate speakers have not indicated lack of commitment. Thus, in saying *It is raining*, the speaker is taken to not have indicated lack of commitment and thus by pragmatics is considered to actually undertake a commitment (see Goodhue, 2021: 960–964).[7]

Our proposal is thus alike to Goodhue's semantics for final rises and falls in that rises are taken to indicate lack of a definite truth value judgment and the final fall is taken to implicate a definite truth value judgment, which in the responsive *okay* uses discussed earlier leads to alignment.

4. *okay* as a truth predicate and marker of uptake

In this section we state our proposal for the semantics of *okay*. This is argued for on two grounds. First, it is consistent with our earlier examples and allows for a uniform semantics of *okay* in the three cases. Differences are allocated to prosody. Second, other similar particles are reasonably taken to be about matters of truth. Thus, the truth predicate idea seems a reasonable general account for these kinds of particles. What are truth predicates though?

4.1. Truth predicates

Truth predicates, say T, are predicate symbols of (usually) a first-order language, which are read as *is true*. Thus, they apply to terms. For instance, we can say "2+2=4 is true", which can be rendered by $T(\ulcorner 2+2=4 \urcorner)$. Here $\ulcorner 2+2=4 \urcorner$ is a term denoting the closed formula $2+2=4$. Typically, $\ulcorner 2+2=4 \urcorner$ denotes the Gödel number of the formula $2+2=4$ (so, there is a coding device assumed which derives names for formulas). A basic desideratum for truth predicates is that they satisfy instances of the truth scheme $T(\ulcorner \phi \urcorner) \leftrightarrow \phi$ (see Tarski (1944)). For further details on truth predicates see Leitgeb (2007).

4.2. Truth and uptake as the common elements

In Ex.3, 4, and 5, we found that the *okay*-speaker is taking up what the interlocutor says. This can be formally expressed in a very succinct way: The *okay*-speaker adds facts to their knowledge base, a set of formulas.[8] In a way, this is a simple model of memory, where formulas are stored as well as information about their truth values. Adding facts means adding a formula that is considered true. In a propositional setting this means that the assignment for the database

[7]Other commitment-based accounts of the meaning of prosodic contours along the same or similar lines as Truckenbrodt (2006), Rudin (2022), and Goodhue (2021) have been made in, for instance, Gunlogson (2003), Farkas and Roelofsen (2019), and Jeon (2018), and many others exist.

[8]The notion is familiar in computer science and artificial intelligence in the context of databases and knowledge representation. The commitment stores of table models can be thought of in that way.

must be such that the new fact is true under the assignment. In the first order case, any models of the database must make the newly added fact true. With truth predicates, this can be directly encoded in the language, $T(\ulcorner\phi\urcorner)$.

More concretely, let us assume Alice asserts p. Let us represent this using the propositional letter q. Assume now that Bob reacts by saying *okay*. Our claim is that *okay* is used to add q as a fact to Bob's representation of what is going on, i. e., he updates his knowledge base with the formula $T(\ulcorner q\urcorner)$. Put simply, Bob believes that Alice asserted p and this is expressed by saying *okay*. In doing so, Bob has taken up what Alice is doing (an assertion) and what the content of that doing is (p). Hence, we can analyze uptake in terms of knowledge base updates utilizing truth predicates. Similar ideas on uptake have been proposed in connection with commitments, where to understand is taken to mean to have a commitment to your interlocutor having a commitment to a proposition (see for instance Schlöder et al., 2019).

The differences between the three uses can either be related to the meaning of the (final) intonational contour, which gives us the split fall versus rise, or that difference can be related to the availability of non-monotonic inferences (implicatures), which gives us the difference of pure acknowledgments versus acceptance markings. The meaning of the final fall could then, roughly at least, be stated as an integrity constraint in the context of logic programming. The idea is that when $T(\ulcorner q\urcorner)$ is satisfied, and it is not the case that cooperativity is doubtful, then Bob should adopt Alice's stance on p, thus, we call for an update with $T(\ulcorner p\urcorner)$ for Bob's knowledge base. If cooperativity is doubtful, this inference is no longer licensed, and the update call $T(\ulcorner p\urcorner)$ will not happen.[9] Final rises could be taken to denote a special predicate U which is true of a term $\ulcorner\phi\urcorner$ if neither T nor F are true of it relative to a database. We can call U an *undecidedness predicate*; F reads *is false*. The exact details are for another occasion.

4.3. Okay and other particles

The following is intended not as a firm result of conducted research, but initial glimpses of potential future work. German possesses particles such as *ja*, *genau*, and *richtig*, which are evidently about matters of truth:

Example 7. (Response particles indicating truth value judgments)
Alice: Charlie hatte heute seine Verteidigung. (*Today was Charlie's thesis defense.*)
Bob: Ja↓./ Richtig↓./ Genau↓./ Okeh↓. (*Yes./ Right./ Exactly./ Okay.*)

We have Alice asserting that Charlie's thesis defense was today. From that, we can infer (non-monotonically) that Alice considers this true. Likewise, any response option displayed for Bob does the same. When Bob responds using *ja*, he will be taken to consider it true that Charlie's thesis defense was today. The same goes in case of *richtig* or *genau*. From that viewpoint, it makes sense to have such expressions denoting truth predicates. We observe that an *okay* response allows for similar inferences (if Bob is cooperative). Thus, it makes sense to group them together and treat them uniformly.

Of course, there are differences among these expressions. When Bob responds by saying *okay*

[9] On logic programming see Doets (1994). On integrity constraints in this dynamic sense, see Kowalski (1995). See also van Lambalgen and Hamm (2005) for application in formal semantics.

instead of *ja*, *richtig*, or *genau*, he may only perform an acknowledgment move. Such is not possible when saying *ja*, *richtig*, or *genau* with final falling intonation. With the latter expressions, he cannot be considered as non-aligning with Alice, even in non-cooperative settings like an argument. This consequence may be due to another factor though. All expressions considered here convey an epistemic stance in assertoric contexts. For instance, when Bob responds by saying *genau*, he will not be understood as being ignorant, instead, he is not just aligning, but confirming. The same is true of *richtig* and *ja*. This stance may provide to further entailments from the resulting logical form, because Bob conveys to know already what Alice is telling him, thus, he must consider the propositional content of her locution true already. However, *okay* does not convey knowledgability (see also Oloff (2019) on German *okay*, and Lindström (2018) on Swedish *okay* in this regard). This is clear from Ex.3, where *okay* is used to integrate information. It does not convey that Alice was knowledgeable (such would be incoherent in fact). As a consequence, we may say that all these expressions are devices for indicating uptake (they all do so in fact).[10] But *ja*, *richtig*, and *genau* have meanings that add to that, while *okay*'s meaning is too weak to do so. Implicatures may strengthen its meaning to an acceptance marker though.

The general idea then is that all these expressions denote truth predicates (which allows to perform uptake explicitly). They differ in the epistemic stances they can convey, which also contributes to differences in possible illocutionary force. The differences allow for fine-tuning of epistemic claims conveyed by epistemic stances, a matter that is known to be of high interactional significance (see for instance Heritage (2013) on the interactional significance of knowledge and knowledge claims).

5. A propositional discourse model

We present a propositional multi-agent model as an approximation of the idea that *okay* semantically denotes a truth predicate and is used to indicate uptake. In that model, *okay* is taken to denote a constant function mapping propositional letters to the truth value 1 from the set of truth values $\mathbb{T}_3 := \{u, 0, 1\}$.

5.1. Truth value judgments and interpretation of truth values in Strong Kleene Logic

An appropriate choice for approximating our truth predicate proposal in a propositional setting is Strong Kleene Logic (Kleene, 2000), though we will only utilize the interpretation of truth values underlying the logic at this point. Formally, we will assume a countably infinite set of propositional letters Φ, no connectives will be used.[11] We will use functions $v_{a_j}^{a_i,t}$ that map formulas $\phi \in \Phi$ to a set of truth values, $\mathbb{T}_3 = \{u, 0, 1\}$, where a_i, a_j are agents and t is a point in time. The functions $v_{a_j}^{a_i,t}$ will be finite.

Strong Kleene Logic is interesting for our purposes because the truth value u, conceptually at

[10]Not every utterance indicates uptake, and thus the indication of uptake is non-trivial. Repairs concerning uptake are non-uptake indicating.

[11]Extensions of the model using Strong Kleene Logic proper are possible. The chosen size is for definiteness only; finite sets work just as well and in applications they suffice.

least, is not considered a proper truth value (Kleene (2000); see also Feferman (1984)). It is used to encode that no classical truth value could be found given some procedure. Thus, truth value judgments are seen as values of partial functions, and u is used to represent them as total functions.

In conversing, agents are faced with decision making such as whether to align or disalign with others, but also communicating whether they are knowledgeable as in when they get asked a question. Much of this can be represented in terms of truth value judgments as in the case of assertions. There, we can say that to assert means to judge true (at least such is made public; agents can lie). To not know can be taken to not be able to judge true or false. In fact, these points are reflected in our undecidedness marking case (Ex.4): While the first *okay* indicates that the speaker has not been able to resolve the issue, and thus has not found a classical solution, the second *okay* makes clear that the issue has been resolved and a classical solution has been found. In that connection, we note that because u is not a truth value in the sense of 1 and 0, formulas that are judged u can be judged 1 or 0 subsequently as a monotone update of the model. This property is of more general importance for conversing. In case of ignorance, we do not perceive others as revising a definitive point of view on the world, but instead as reaching such a point of view, which is a monotone update. Overall, these properties make Strong Kleene Logic a reasonable choice for current purposes.

5.2. Agents, contexts, and speech acts

We provide a simple yet for current purposes sufficient multi-agent system for modeling speech acts as context changing functions.

We will fix a set of propositional letters $\Phi := \{p_n : n \in \mathbb{N}\}$ as well as an index set $\mathscr{A} = \{a_1, \ldots, a_n\}$, the set of agents. Further, we let time range over the natural numbers, thus $\mathscr{T} := \mathbb{N}$. The addition of time is for convenience at this point. For $a_i, a_j \in \mathscr{A}$, $i, j \in \{1, \ldots, n\}$, $t \in \mathscr{T}$, and $X \subset \Phi$ a finite subset of formulas, we define *judgment functions* $v_{a_j}^{a_i,t} : X \to \mathbb{T}_3$. Judgment functions allow us to represent perspectives. For instance, if Alice (a) asserts at t that ϕ, we take this to add a judgment $(\phi, 1)$ to Alice's judgment function $v_a^{a,t}$. If Alice asserts at t that ϕ is false, we would instead add $(\phi, 0)$ to $v_a^{a,t}$. And if Alice would publicly express her ignorance about p at t, we would represent this by adding (ϕ, u) to $v_a^{a,t}$. On the other hand, if Bob (b) asserts at t that q and Alice uptakes Bob doing so, we can represent this by adding $(q, 1)$ to $v_b^{a,t}$, where this function represents Alice's perspective on Bob's judgments. We make this idea formally precise by defining *agent contexts*, where an agent context is a tuple of judgment functions $C_{a_i,t} := (v_{a_1}^{a_i,t}, \ldots, v_{a_i}^{a_i,t}, \ldots, v_{a_n}^{a_i,t})$ for $a_i \in \mathscr{A}$. Conceptually, we distinguish judgments to 1 and 0 from such that assign u as value. The former are properly thought of as judgments, for they deliver a result for a computation, whereas the latter in fact are not really judgments; assignment of u reflects that no judgment could be made. This is the import of Strong Kleene Logic's conceptual interpretation of $0, 1$, and u.

Given a set of agents $\mathscr{A}$, we define contexts $\mathscr{C}_t$ at t as tuples of agent contexts $(C_{a_1,t}, \ldots, C_{a_n,t})$. While agent contexts are local representations of a conversation (they encode an agent's total perspective at t), a context $\mathscr{C}_t$ is really the global representation of the conversation at t: it contains all information about each agent's judgments as well as what they think others have judged at and up to t. A context here is really an abstraction of the analyst that allows to judge what is indeed happening, while it is not a perspective conversational agent's can necessarily obtain (Alice may have false beliefs about Bob's beliefs).

We use contexts for modeling conversations by modeling changes within agent contexts, where changes are induced by illocutionary acts, which we call here for simplicity speech acts. Speech acts are functions from a context $\mathscr{C}_t$, an agent a_i, and a formula ϕ to a context $\mathscr{C}_{t+1}$. Thus, they also induce changes in time: no events, no flow of time. Depending on the speech act, different effects occur. For instance, if agent a_i asserts ϕ at t, we want that the agent context of a_i at t changes in such a way that $v_{a_i}^{a_i,t}$ is updated or revised to $v_{a_i}^{a_i,t} \cup \{(\phi,1)\}$. We will therefore deal now with updates and revisions of judgment functions $v_{a_i}^{a_i,t}$.

Given the interpretation of truth values in Strong Kleene Logic different kinds of updates are intelligible. One kind of update consists in adding the result of a judgment of a proposition not considered so far:

Definition 1. (+-update of judgment functions) Let $\phi \in \Phi$ be a formula, $v_{a_j}^{a_i,t}$ be a judgment function, and $\tau \in \mathbb{T}_3$ be a truth value. We set:

$$v_{a_j}^{a_i,t} + (\phi,\tau) := \begin{cases} v_{a_j}^{a_i,t+1} := v_{a_j}^{a_i,t} \cup \{(\phi,\tau)\} & \text{if } \neg\exists\tau' \in \mathbb{T}_3.(\phi,\tau') \in v_{a_j}^{a_i,t} \\ \text{undefined} & \text{else} \end{cases}$$

Example 8. Consider the function $v_{a_i}^{a_i,t} := \{(p,1),(q,0)\}$. The formula r has not been judged explicitly yet. Thus, we can perform the update $v_{a_i}^{a_i,t} + (r,1)$, which results in the function $v_{a_i}^{a_i,t+1} = \{(p,1),(q,0),(r,1)\}$. We cannot perform updates with p, because of $(p,1) \in v_{a_i}^{a_i,t}$.

If a judgment procedure has not yielded u as result, we may later update u to a classical result:

Definition 2. ($\uparrow$-update of judgment functions) Let $\phi \in \Phi$ be a formula, $v_{a_j}^{a_i,t}$ be a judgment function. We set

$$v_{a_j}^{a_i,t} \uparrow (\phi,\tau) := \begin{cases} v_{a_j}^{a_i,t+1} := (v_{a_j}^{a_i,t} \setminus \{(\phi,u)\}) \cup \{(\phi,\tau)\} & \text{if } (\phi,u) \in v_{a_j}^{a_i,t}, \tau \in \{0,1\} \subset \mathbb{T}_3 \\ \text{undefined} & \text{else} \end{cases}$$

Example 9. Consider $v_{a_i}^{a_i,t} := \{(p,1),(q,1),(r,u)\}$. We can perform $v_{a_i}^{a_i,t} \uparrow (r,1) = v_{a_i}^{a_i,t+1} = \{(p,1),(q,1),(r,1)\}$, and $v_{a_i}^{a_i,t} \uparrow (r,0) = v_{a_i}^{a_i,t+1} = \{(p,1),(q,1),(r,0)\}$, but not $v_{a_i}^{a_i,t} \uparrow (p,\tau), \tau \in \{0,1\}$, since $(p,u) \notin v_{a_j}^{a_i,t}$.

Besides updates, conversations contain revisions. For instance, assume Bob learns from Alice about an event he was ill-informed about. Bob will then adjust his judgments to fit the new information. A very general and naive revision function consists in keeping everything as is besides the item that is to be revised:

Definition 3. (Revision *RE*) Let $\phi \in \Phi$ be a formula, $v_{a_j}^{a_i,t}$ be a judgment function, and $\tau \in \mathbb{T}_3$ be a truth value. Then:

$$v^{a_i,t}RE(\phi,\tau) := \begin{cases} v_{a_j}^{a_i,t+1} := (v_{a_j}^{a_i,t} \setminus \{(\phi,\tau')\}) \cup \{(\phi,\tau)\} & \text{if } \tau' \in \{0,1\}, (\phi,\tau') \in v_{a_j}^{a_i,t}, \tau \neq \tau' \\ \text{undefined} & \text{else} \end{cases}$$

We have defined revisions as operations altering truth values 1 or 0 to any other value in $\mathbb{T}_3$. This reflects the conceptual point that u is not a proper truth value. Revisions are non-monotonic changes of proper judgments, thus changes with respect to 1 and 0.

To obtain simpler definitions of speech acts we will define an operation $\sqcup$ that selects among $\uparrow, +$, and *RE* or else only increments time (the final option in the definition):

Definition 4. ($\sqcup$-update of judgment functions) Let $\phi \in \Phi$ be a formula, let $v_{a_j}^{a_i,t}$ be a judgment function, and let $\tau \in \mathbb{T}_3$ be a truth value. We define:

$$v_{a_j}^{a_i,t} \sqcup (\phi, \tau) := \begin{cases} v_{a_j}^{a_i,t} \uparrow (\phi, 1) & \text{if } \tau \in \{0,1\}, (\phi, u) \in v_{a_j}^{a_i,t} \\ v_{a_j}^{a_i,t} + (\phi, \tau) & \text{if } \neg\exists\tau' \in \mathbb{T}_3((\phi,\tau') \in v_i) \\ v_{a_j}^{a_i,t} RE(\phi,\tau) & \text{if } (\phi,\tau') \in v_{a_j}^{a_i,t}, \tau' \in \{1,0\}, \tau \neq \tau' \\ v_{a_j}^{a_i,t+1} \text{ with } v_{a_j}^{a_i,t+1} = v_{a_j}^{a_i,t} & \text{if } \exists\tau' \in \mathbb{T}_3(\tau = \tau', (\phi,\tau') \in v_{a_j}^{a_i,t}) \end{cases}$$

The last option which makes $\sqcup$ into an identity function that increments time (the elements of the set stay the same, but the index t increments by 1) is in place to account for redoings. A simple case is when an agent repeats an assertion again at another occasion. In the current system, this is but a redoing, thus time increments, because there is an event, but the event is redundant and thus the effect will not distinguish the state at t and the state at $t+1$.

As before, we will assume a fixed set of propositional letters Φ and a fixed index set $\mathscr{A}$. We define a positive assertion operation (*ASSERT-P*) and a negative one (*ASSERT-N*) since we lack negation in our object language:

Definition 5. (Assertion) Let $\mathscr{C}_t = (C_{a_1,t}, \dots, C_{a_i,t}, \dots, C_{a_n,t})$, $a_i \in \mathscr{A}$, and $\phi \in \Phi$ be given. $ASSERT\text{-}P(\mathscr{C}_t, a_i, \phi) := \mathscr{C}_{t+1}$ with $\mathscr{C}_{t+1} = (C_{a_1,t+1}, \dots, C_{a_i,t+1}, \dots, C_{a_n,t+1})$, $C_{\ell,t+1} = C_{\ell,t}$ for $\ell \in \mathscr{A} \setminus \{a_i\}$, and $C_{a_i,t+1} = (v_{a_1}^{a_i,t+1}, \dots, v_{a_i}^{a_i,t} \sqcup (\phi,1), \dots, v_{a_n}^{a_i,t+1})$ with $v_\ell^{a_i,t+1} = v_\ell^{a_i,t}$ for $\ell \in \mathscr{A} \setminus \{a_i\}$.

$ASSERT\text{-}N(\mathscr{C}_t, a_i, \phi)$ is defined alike, but we have the pair $(\phi, 0)$ instead of $(\phi, 1)$.

Example 10. Assume Alice (a) tells Bob (b) about the Elden Ring lore.[12] She asserts at t that Queen Marika and Radagon are the same person (p). Assuming that we are in the context $\mathscr{C}_t$ with $C_{a,t} = (v_a^{a,t} = \emptyset, v_b^{a,t} = \emptyset)$ and $C_{b,t} = (v_a^{b,t} = \emptyset, v_b^{b,t} = \emptyset)$ the assertion results in $\mathscr{C}_{t+1}$ with $C_{a,t+1} = (v_a^{a,t+1} \sqcup (\phi,1), v_b^{a,t+1})$, $v_b^{a,t+1} = v_b^{a,t}$, where $v_a^{a,t+1} \sqcup (\phi,1) = v_a^{a,t+1} + (\phi,1) = \{(\phi,1)\}$. We have $C_{b,t+1} = C_{b,t}$.

Next, we define an operation, *COPY*, which serves to register what others did. It is an integral part of our account of uptake and the pragmatics of *okay*:

Definition 6. (Copy) Let $\mathscr{C}_t = (C_{a_1,t}, \dots, C_{a_i,t}, \dots, C_{a_n,t})$, $a_i \in \mathscr{A}$, and $\phi \in \Phi$ be given.

$$COPY(\mathscr{C}_t, a_i, \phi) := \begin{cases} \mathscr{C}_{t+1} & \text{if } \exists x \in \mathscr{A} \exists \tau \in \mathbb{T}_3.(\phi,\tau) \in v_x^{x,t} \\ \text{undefined} & \text{else} \end{cases}$$

where $\mathscr{C}_{t+1} := (C_{a_1,t+1}, \dots, C_{a_i,t+1}, \dots, C_{a_n,t+1})$, $C_{\ell,t+1} = C_{\ell,t}$ for $\ell \in \mathscr{A} \setminus \{a_i\}$, and $C_{a_i,t+1} := (v_{a_1}^{a_i,t+1}, \dots, v_x^{a_i,t+1}, \dots, v_{a_n}^{a_i,t+1})$ with $v_\ell^{a_i,t+1} = v_\ell^{a_i,t}$ for $\ell \in \mathscr{A} \setminus \{x\}$; $v_x^{a_i,t+1} := v_x^{a_i,t} \sqcup (\phi,\tau)$

Thus, this operation copies a judgment from another agent. Note that this operation is factual. Thus, our agents will not have false representations. This is just a convenient choice at this point.

Example 11. Returning to Alice and Bob, let us assume that Bob heard and understood what Alice said. In that case, we may assume that Bob adjusted his representation of what Alice is

[12] Elden Ring is an action role-playing video game developed by FromSoftware.

committing to and is believing, by adding $(p,1)$ to Alice' judgment function. Thus, we have an update of the context $\mathscr{C}_{t+1} = (C_{a,t+1}, C_{b,t+1})$ to $\mathscr{C}_{t+2} = (C_{a,t+2}, C_{b,t+2})$ with $C_{a,t+2} = C_{a,t+1}$, and $C_{b,t+2} = (v_a^{b,t+2} \sqcup (p,1) = \{(p,1)\}, v_b^{b,t+2})$ with $v_b^{b,t+2} = v_b^{b,t+1}$. Later, we will say that Alice and Bob have secured uptake of p, because they have shared interpretations of Alice's judgment of p (see Def.15).

Assertions can be reacted to in different ways. They may be accepted:

Definition 7. (Acceptance) Let $\mathscr{C}_t = (C_{a_1,t}, \ldots, C_{a_i,t}, \ldots, C_{a_n,t})$, $a_i \in \mathscr{A}$, and $\phi \in \Phi$ be given.

$$\mathit{ACCEPT}(\mathscr{C}_t, a_i, \phi) := \begin{cases} \mathscr{C}_{t+1} & \text{if } \exists x \in \mathscr{A} \exists \tau \in \{0,1\} \subset \mathbb{T}_3.(\phi,\tau) \in v_x^{x,t}, (\phi,\tau) \in v_x^{a_i,t} \\ \text{undefined} & \text{else} \end{cases}$$

where $\mathscr{C}_{t+1} := (C_{a_1,t+1}, \ldots, C_{a_i,t+1}, \ldots, C_{a_n,t+1})$, $C_{a_i,t+1} := (v_{a_1}^{a_i,t+1}, \ldots, v_{a_i}^{a_i,t+1}, \ldots, v_{a_n}^{a_i,t+1})$ with $v_\ell^{a_i,t+1} = v_\ell^{a_i,t}$ for $\ell \in \mathscr{A} \setminus \{a_i\}$ and $C_{\ell,t+1} = C_{\ell,t}$ for $\ell \in \mathscr{A} \setminus \{a_i\}$; $v_{a_i}^{a_i,t+1} := v_{a_i}^{a_i,t} \sqcup (\phi,\tau)$.

To accept means to align with the judgment of others. This motivates restricting τ to either 0 or 1, because u is not a proper truth value. We note that the definition requires that someone made a judgment, thus agents cannot mistakenly accept jugdments without there having been a judgment that can be accepted. This reflects the anaphoric nature of accepting, but also incorporates a factual element, which is again a convenient choice.

Another response type available in the context of assertions is that of a (strong) rejection. Here, the speaker takes an opposing point of view:

Definition 8. (Rejection) Let $\mathscr{C}_t = (C_{a_1,t}, \ldots, C_{a_i,t}, \ldots, C_{a_n,t})$, $a_i \in \mathscr{A}$, and $\phi \in \Phi$ be given.

$$\mathit{REJECT}(\mathscr{C}_t, a_i, \phi) := \begin{cases} \mathscr{C}_{t+1} & \text{if } \exists x \in \mathscr{A} \exists \tau \in \{0,1\} \subset \mathbb{T}_3.(\phi,\tau) \in v_x^{x,t}, (\phi,\tau) \in v_x^{a_i,t} \\ \text{undefined} & \text{else} \end{cases}$$

where $\mathscr{C}_{t+1} := (C_{a_1,t+1}, \ldots, C_{a_i,t+1}, \ldots, C_{a_n,t+1})$, $C_{a_i,t+1} := (v_{a_1}^{a_i,t+1}, \ldots, v_{a_i}^{a_i,t+1}, \ldots, v_{a_n}^{a_i,t+1})$ with $v_\ell^{a_i,t+1} = v_\ell^{a_i,t}$ for $\ell \in \mathscr{A} \setminus \{a_i\}$ and $C_{\ell,t+1} = C_{\ell,t}$ for $\ell \in \mathscr{A} \setminus \{a_i\}$; $v_{a_i}^{a_i,t} := v^{a_i,t} + a_i \sqcup (\phi, 1-\tau)$

Similar comments apply here as with acceptance. The reader should note that the rejection operation emulates the semantics of classical negation via $1-\tau$ and the restriction $\tau \in \{0,1\}$.

Assertions may also simply be acknowledged:

Definition 9. (Acknowledgment) Let $\mathscr{C}_t = (C_{a_1,t}, \ldots, C_{a_i,t}, \ldots, C_{a_n}, t)$, $a_i \in \mathscr{A}$, and $\phi \in \Phi$ be given. $\mathit{ACKNOWLEDGE}(\mathscr{C}_t, i, \phi) := \mathit{COPY}(\mathscr{C}_t, i, \phi)$

Thus acknowledgments are defined in terms of the copy operation. Equating acknowledgments with registering of others' judgments is not a conceptual claim though. Acknowledgments are illocutionary acts, whereas registering is a cognitive task or result thereof. However, given that we do not model locutionary acts here, we cannot distinguish these phenomena in our model (unless we introduce primitive propositional formulas whose intended meaning can be provided in first-order or higher-order logic). Acknowledgments, *qua* illocution, could be made dependent on the occurrence of an appropriate locution. In that way, we could make acknowledgments indeed a publicly accessible action, and registrations (the operation represented by our *COPY*-operation) would be distinguishable as a cognitive process, which may not find a public display.

A simple representation for polar interrogatives can be provided:

Definition 10. (Questions) Let $\mathscr{C}_t = (C_{a_1,t}, \ldots, C_{a_i,t}, \ldots, C_{a_n,t})$, $a_i \in \mathscr{A}$, and $\phi \in \Phi$ be given. *QUESTION*$(\mathscr{C}_t, a_i, \phi) := \mathscr{C}_{t+1}$ where $\mathscr{C}_{t+1} = (C_{a_1,t+1}, \ldots, C_{a_i,t+1}, \ldots, C_{a_n,t+1})$ with $C_{\ell,t+1} = C_{\ell,t}$ for $\ell \in \mathscr{A} \setminus \{a_i\}$, and $C_{a_i,t+1} = (v_{a_1}^{a_i,t+1}, \ldots, v_{a_i}^{a_i,t} \sqcup (\phi, u), \ldots, v_{a_n}^{a_i,t+1})$ with $v_\ell^{a_i,t+1} = v_\ell^{a_i,t}$ for $\ell \in \mathscr{A} \setminus \{a_i\}$.

The idea is that polar interrogatives express that there is an open issue regarding the truth value of a formula ϕ. This can be represented by taking ϕ to be undecided. This comes close to notions of inquisitiveness, where a state neither entails ϕ nor $\neg\phi$.

We define new operations, *ANSWER-P* and *ANSWER-N*, which are used to perform positive and negative answers respectively. Similar comments apply as with assertions:

Definition 11. (Answers) Let $\mathscr{C}_t = (C_{a_1,t}, \ldots, C_{a_i,t}, \ldots, C_{a_n,t})$, $a_i \in \mathscr{A}$, and $\phi \in \Phi$ be given.

$$\textit{ANSWER-P}(\mathscr{C}_t, a_i, \phi) := \begin{cases} \mathscr{C}_{t+1} & \text{if } \exists x \in \mathscr{A}.(\phi, u) \in v_x^{x,t}.(\phi, u) \in v_x^{a_i,t} \\ \text{undefined} & \text{else} \end{cases}$$

where $\mathscr{C}_{t+1} := (C_{a_1,t+1}, \ldots, C_{a_i,t+1}, \ldots, C_{a_n,t+1})$, $C_{a_i,t+1} := (v_{a_1}^{a_i,t+1}, \ldots, v_{a_i}^{a_i,t+1}, \ldots, v_{a_n}^{a_i,t+1})$ with $v_\ell^{a_i,t+1} = v_\ell^{a_i,t}$ for $\ell \in \mathscr{A} \setminus \{a_i\}$ and $C_{\ell,t+1} = C_{\ell,t}$ for $\ell \in \mathscr{A} \setminus \{a_i\}$; $v_{a_i}^{a_i,t+1} := v_{a_i}^{a_i,t} \sqcup (\phi, 1)$.

ANSWER-N$(\mathscr{C}_t, a_i, \phi)$ is defined similarly, but we have the pair $(\phi, 0)$ instead of $(\phi, 1)$.

Example 12. (Question-answer sequence) Continuing with Alice and Bob, assume Bob asks Alice a question about the Elden Ring lore, say, whether Queen Rennala is a demi-god (q). Thus, given the context $\mathscr{C}_{t+2} = (C_{a,t+2}, C_{b,t+2})$ with $C_{a,t+2} = (v_a^{a,t+2} = \{(p,1)\}, v_b^{a,t+2} = \emptyset)$ and $C_{b,t+2} = (v_a^{b,t+2} = \{(p,1)\}, v_b^{b,t+2} = \emptyset)$, Bob's question results in the new context $\mathscr{C}_{t+3} = (C_{a,t+3}, C_{b,t+3})$ with $C_{a,t+3} = C_{a,t+2}$ and $C_{b,t+3} = (v_a^{b,t+3} = \{(p,1)\}, v_b^{b,t+3} \sqcup (q,u) = \{(q,u)\})$. Assuming that Alice was attentive and understood what Bob said, we get the context $\mathscr{C}_{t+4} = (C_{a,t+4}, C_{b,t+4})$ with $C_{a,t+4} = (v_a^{a,t+4} = \{(p,1)\}, v_b^{a,t+4} = \{(q,u)\})$ and $C_{b,t+4} = C_{b,t+3}$. Alice can now proceed to answer Bob's question. She answers negatively, yielding the context $\mathscr{C}_{t+5} = (C_{a,t+5}, C_{b,t+5})$ with $C_{a,t+5} = (v_a^{a,t+5} = \{(p,1)\} \sqcup (q,0) = \{(p,1),(q,0)\}, v_b^{a,t+5} = \{(q,u)\})$ and $C_{b,t+5} = C_{b,t+4}$. If Bob takes up on this, he may accept this answer which leads him to judge q false as well. This would lead to making common ground that q is false (see Def.16).

5.3. *okay* and intonational contours

We proposed that *okay* is combined with prosody to perform different kinds of speech acts. Particularly, we claimed that when *okay* combines with a final rise, it is used to perform undecidedness marking. The final rise is used to convey speaker's stance, in this case that they cannot assign a classical value to the formula in question. Thus, a final rise conveys a sense of inquisitiveness. The *okay* token, we proposed, is used to perform an acknowledgment of what the prior speaker said. In this way, *okay* is used to perform or indicate uptake:

Definition 12. (*okay* meaning) Let $\mathscr{C}_t = (C_{a_1,t}, \ldots, C_{a_n,t})$ and $\phi \in \Phi$ be given.

$$\textit{OKAY}(\mathscr{C}_t, \phi) := \begin{cases} (\phi, \tau) & \text{if } \exists x \in \mathscr{A} \exists \tau \in \mathbb{T}_3.(\phi, \tau) \in v_x^{x,t} \\ \text{undefined} & \text{else} \end{cases}$$

The *if*-clause is used to reflect the anaphoric nature of *okay*, though we want to note that *okay* is applicable in contexts where an agent has made a private inference, because we do not ask for a different agent in the test (this also applies with the other definitions). Such is especially the case with *okay*s that are used to indicate the (approaching) ends of tasks, transitional *okay*s, but also auto-reflexive *okay*s which are used in individual thought processes within multi-agent settings.[13] Our definition allows for such cases by not having constraints on agents.

We define the meaning of the final rise as a constant function mapping formulas to u:

Definition 13. (Final rise meaning) Let $\mathscr{C}_t = (C_{a_1,t}, \ldots, C_{a_n,t})$ and $\phi \in \Phi$ be given.
$FR(\mathscr{C}_t, \phi) := (\phi, u)$

Final rising *okay*s can be represented as a primitive composition of the meanings of *okay* and the final rise, which together implement an illocutionary act: $OKAY(\mathscr{C}_t, \phi) + FR(\mathscr{C}_t, \phi) := QUESTION(COPY(\mathscr{C}_t, a_i, \phi), a_i, \phi)$.

We treat $+$ as a primitive symbol here. The intended meaning is that it combines the two meaning contributions which then feeds into illocutionary force ascription and results in the expression on the right side of the identity symbol. The *okay* as an anaphoric expression picks up on what has been said (or thought) and registers this contribution, i. e., *okay* is used to register a judgment, commitment, whatever you have. On the other hand, the final rise adds an ignorance stance with respect to what has been said, here a formula ϕ. For the future a formal account of illocutionary force ascription is desirable (ideally in a type-free system so as to utilize truth predicates; on type-free systems see Feferman (1984)).

Our discussion of final falls suggests that they can be taken to contribute an implicature. Now, for the sake of simplicity, we will not attempt to add implicatures to the current propositional model. Instead, we will assume that only in the cooperative cases does the final fall make a meaning contribution. Therefore, we will assume that only for such cases exists a lexical entry and in non-cooperative cases, the fall is simply meaningless, i.e., non-cooperative final falls do not provide a meaning contribution.

Definition 14. (Final fall meaning) Let $\mathscr{C}_t = (C_{a_1,t}, \ldots, C_{a_n,t})$ and $\phi \in \Phi$ be given.

$$FF_{coop}(\mathscr{C}_t, \phi) := \begin{cases} (\phi, \tau) & \text{if } \exists x \in \mathscr{A} \exists \tau \in \{0,1\} \subset \mathbb{T}_3.(\phi, \tau) \in v_x^{x,t} \\ \text{undefined} & \text{else} \end{cases}$$

Our definition treats final falls as anaphoric. In fact, it differs only slightly from the semantics of *okay*. We do so, because we wish to model it as a stance aligning device. This is a responsive action, thus it needs a relatum.

The acceptance marking reading of *okay* can then be rendered by $OKAY(\phi) + FF_{coop}(\mathscr{C}_t, \phi) := ACCEPT(COPY(\mathscr{C}_t, a_i, \phi), a_i, \phi)$, with a_i the speaker. Here the *okay* token is used to uptake and provide a context for an aligning stance. Thus, the illocutionary contribution of *okay* needs to apply first. And the purely acknowledging use of *okay* boils down to $OKAY(\mathscr{C}_t, \phi) := COPY(\mathscr{C}_t, a_i, \phi)$, with a_i the speaker.

[13] Two constructed examples using English (they work like that in German too): Bob carries a box to the car. Reaching the car and while putting the box down, Bob says: “Okay.” (⇒ Completion of task *okay*) Bob: “Does this say that I have to pay them $100?” Alice: “Let me see . . . Okay. It says you have to pay $100, but . . . ” (⇒ Auto-reflexive *okay*)

Given the above, we can now model our undecidedness marking example Ex.4:

Example 13. (Undecidedness marking) We assume for simplicity an initially empty context $\mathscr{C}_0 = (C_{a,0}, C_{b,0})$ with $C_{a,0} = (v_a^{a,0} = \emptyset, v_a^{b,0} = \emptyset), C_{b,0} = (v_a^{b,0} = \emptyset, v_b^{b,0} = \emptyset)$. Alice asks Bob a question, which yields $\mathscr{C}_1$ with $C_{a,1} = (v_a^{a,1} = \{(p,u)\}, v_a^{b,1} = \emptyset)$ and $C_{b,1} = (v_a^{b,1} = \emptyset, v_b^{b,1} = \emptyset)$. Bob uptakes on Alice' question and answers it subsequently. This gives us $\mathscr{C}_2$ in which uptake occurs, with $C_{a,2} = C_{a,1}$ and $C_{b,2} = (v_a^{b,2} = \{(p,u)\}, v_b^{b,2} = \emptyset)$. The *COPY*-operation provides the context for negatively answering, which is $\mathscr{C}_3$ with $C_{a,3} = C_{a,2}$ and $C_{b,3} = (v_a^{b,3} = \{(p,u)\}, v_b^{b,3} = \{(p,0)\})$. Alice responds to Bob using an undecidedness marking *okay* resulting in the context $\mathscr{C}_5$ resulting from a *COPY*-operation and a *QUESTION*-operation, with $C_{a,5} = (v_a^{a,5} = \{(p,u)\}, v_a^{b,5} = \{(p,0)\})$ and $C_{b,5} = C_b^4 = C_b^3$. We note that Alice re-asks her question on this analysis. Then, Bob makes an assertion, which gets accepted by Alice in next position. This gives us first the context $\mathscr{C}_6$ with $C_{a,6} = (v_a^{a,6} = \{(p,u)\}, v_a^{b,6} = \{(p,0)\})$ and $C_{b,6} = (v_a^{b,6} = \{(p,u)\}, v_b^{b,6} = \{(p,0)\})$ in which Bob uptakes Alice' response. Next, $\mathscr{C}_7$ results from Bob's assertion with $C_{a,7} = (v_a^{a,7} = \{(p,u)\}, v_a^{b,7} = \{(p,0)\})$ and $C_{b,7} = (v_a^{b,7} = \{(p,u)\}, v_b^{b,7} = \{(p,0),(q,1)\})$, followed by Alice' uptake and acceptance yielding $\mathscr{C}_9$ with $C_{a,9} = (v_a^{a,9} = \{(p,u),(q,1)\}, v_a^{b,9} = \{(p,0),(q,1)\})$ and $C_{b,9} = (v_a^{b,9} = \{(p,0)\}, v_b^{b,9} = \{(p,0),(q,1)\})$. Finally, we have Alice resolving the issue p by aligning with Bob, yielding $\mathscr{C}_{10}$ with $C_{a,10} = (v_a^{a,10} = \{(p,0),(q,1)\}, v_a^{b,10} = \{(p,0),(q,1)\})$ and $C_{b,10} = C_{b,9}$. This alignment is made public via an assertion (here a negative assertion).

Ex.3 is rendered this way:

Example 14. (Acceptance marking) We assume again an initially empty context $\mathscr{C}_0$ with $C_{a,0} = (v_a^{a,0} = \emptyset, v_b^{a,0} = \emptyset)$ and $C_{b,0} = (v_a^{b,0} = \emptyset, v_b^{b,0} = \emptyset)$. Alice asks Bob a question yielding the context $\mathscr{C}_1$ with $C_{a,1} = (v_a^{a,1} = \{(p,u)\}, v_b^{a,1} = \emptyset)$ and $C_{b,1} = (v_a^{b,1} = \emptyset, v_b^{b,1} = \emptyset)$. After uptake, Bob answers affirmatively, yielding the context $\mathscr{C}_3$ with $C_{a,3} = C_{a,2} = C_{a,1}$ and $C_{3,b} = (v_a^{b,3} = \{(p,u)\}, v_b^{b,3} = \{(p,1)\})$. Then, Alice uses an acceptance marking *okay* resulting in the context $\mathscr{C}_5$ with $C_{a,5} = (v_a^{a,5} = \{(p,1)\}, v_b^{a,5} = \{(p,1)\})$ and $C_{b,5} = (v_a^{b,5} = \{(p,u)\}, v_b^{b,5} = \{(p,1)\})$.

This contrasts with the formal modeling of Ex.5:

Example 15. (Pure acknowledgment) We assume again an initially empty context $\mathscr{C}_0$ with $C_{a,0} = (v_a^{a,0} = \emptyset, v_b^{a,0} = \emptyset)$ and $C_{b,0} = (v_a^{b,0} = \emptyset, v_b^{b,0} = \emptyset)$. Alice instructs Bob to tell her every-thing about Charlie resulting in the context $\mathscr{C}_1$ with $C_{a,1} = (v_a^{a,1} = \{(p,1)\}, v_b^{a,1} = \emptyset)$ and $C_{b,1} = (v_a^{b,1} = \emptyset, v_b^{b,1} = \emptyset)$.[14] After uptake and acceptance, Bob makes an assertion, yielding the context $\mathscr{C}_4$ with $C_{a,4} = C_{a,3} = C_{a,2} = C_{a,1}$ and $C_{b,4} = (v_a^{b,4} = \{(p,1)\}, v_b^{b,4} = \{(p,1),(q,1)\})$. Alice acknowledges Bob's assertion using an *okay* token resulting in the context $\mathscr{C}_5$ with $C_{a,5} = (v_a^{a,5} = \{(p,1)\}, v_b^{a,5} = \{(p,1),(q,1)\})$ and $C_{b,5} = C_{b,4}$. Then, Alice rejects Bob's answer resulting in $\mathscr{C}_6$ with $C_{6,a} = (v_a^{a,6} = \{(p,1),(q,0)\}, v_b^{a,6} = \{(p,1),(q,1)\})$ and $C_{b,6} = C_{b,5}$.

Finally, a polar interrogative case for illustration purposes. We proposed that *okay* tokens cannot be used to answer questions. To some extend this is represented in our formal rendering of such cases.

[14]Note that we simplify instructions to assertions here.

Example 16. (*okay* in context of polar interrogative) Assuming an initially empty context $\mathscr{C}_0$ with $C_{a,0} = (v_a^{a,0} = \emptyset, v_b^{a,0} = \emptyset)$ and $C_{b,0} = (v_a^{b,0} = \emptyset, v_b^{b,0} = \emptyset)$, assume that Alice asks a question, yielding the context $\mathscr{C}_1$ with $C_{1,a} = (v_a^{a,1} = \{(p,u)\}, v_b^{a,1} = \emptyset)$ and $C_{b,1} = C_{b,0}$. Further, assume Bob uptakes on Alice' question using an *okay*-token. We have then $\mathscr{C}_2$ with $C_{a,2} = C_{a,1}$ and $C_{b,2} = (v_a^{b,2} = \{(p,u)\}, v_b^{b,2} = \emptyset)$. If Bob uses an *okay* token with final rise, then this additionally leads to ignorance stance marking. In that case, we get $\mathscr{C}_3$ with $C_{a,3} = C_{a,2}$ and $C_{b,3} = (v_a^{b,3} = \{(p,u)\}, v_b^{b,3} = \{(p,u)\})$. However, if Bob uses an *okay* token with final falling intonation in a cooperative setting, then this cannot be comprehended as implementing an acceptance marking *okay*, because the presupposition of the final fall is not satisfied. A proper judgment is required. And in a non-cooperative setting, *okay* with final falls only implement acknowledgment. Thus, on this analysis, *okay* with final falls cannot be used to provide an answer, because they cannot be used to express judgments on their own. This offers an alternative explanation to the suggestion made in the introduction.

5.4. Uptake and Common Ground

We finish off with uptake and common ground. We take both to be states of a multi-agent system here, thus properties of a context $\mathscr{C}_t$:

Definition 15. (Uptake) Let $\mathscr{C}_t = (C_{a_1,t}, \ldots, C_{a_n,t})$ be a context, $\phi \in \Phi$ be a formula, and $G \subseteq \mathscr{A}$ be a group of agents. We say that ϕ has been taken up in $\mathscr{C}_t$ among agents of G if and only if $\exists x \in G \exists \tau \in \mathbb{T}_3 \forall y \in G.(\phi, \tau) \in v_x^{y,t}$.

Thus we have that a formula ϕ has been taken up in a group of agents if and only if someone in the group has judged ϕ or expressed ignorance regarding ϕ and everyone else in the group has registered this. This is a fair approximation of uptake.

It is easy to see that on our analysis, *okay* is an expression that can lead to uptake by means of implementing acknowledgments (see for instance Ex.11).

Definition 16. (Common Ground) Let $\mathscr{C}_t = (C_{a_1,t}, \ldots, C_{a_n,t})$ be a context, $\phi \in \Phi$ be a formula, and $G \subseteq \mathscr{A}$ be a group of agents. We say that ϕ is common ground in $\mathscr{C}_t$ among agents of G if and only if $\exists \tau \in \{0,1\} \subset \mathbb{T}_3 \forall x,y \in G((\phi,\tau) \in v_x^{x,t} \wedge ((\phi,\tau) \in v_x^{y,t}))$.

We can see that common ground presupposes uptake, which means that we require shared understanding of what others have done (represented here in terms of truth value judgments). However, we do not have that if a formula is taken up, that such is common ground. We also observe that only acceptance uses of *okay* can lead to common ground. Pure acknowledgment uses do not add a judgment of the *okay*-speaker that can lead to grounding, and undecidedness markings indicate that the *okay*-speaker is not in a position currently to provide a classical judgment. Consequently, our formal representations do behave correctly with respect to our empirical findings.

6. Conclusion

This article contributes a first substantial study of the meaning and use of *okay* in formal semantics and pragmatics. Other studies have considered *okay* either as an aside, or as just a linguistic example of another more general phenomenon. Besides, they have operated on extremely small samples. The current study is informed by a large conversation analytic study of *okay* in spoken German and zoomed in on three specific uses. We proposed that *okay* denotes a truth predicate, which provides a precise account of the semantics of *okay*. Pragmatically, we said, *okay* is essentially used to mark uptake by implementing acknowledgments. For ease of exposition and lack of space, we have spelled out these ideas in a propositional model based on the interpretation of truth values in Strong Kleene Logic. Clearly, the idea of *okay* denoting a truth predicate can be generalized. Particles such as *ja* (*yes*), *genau* (*exactly*), and *richtig* (*right*) can be analyzed in the same way. Similarly, expressions such as *nein* (*no*) may be taken to denote falsity predicates (however, see Incurvati and Schlöder (2017)). On such a position, differences between expressions have to be motivated differently. One possibility is to say that, for instance, *okay* and *ja* have different epistemic profiles: while *okay* indicates ignorance, *ja* indicates knowledgability. A thorough discussion of this idea has to await the future and should be done minimally within a first-order account. The same goes for the mentioned yet not discussed uses of *okay*.

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