Anaphoric potential of cumulative dependencies[1]

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Abstract. This paper discusses the anaphoric potential of non-quantificational plural arguments, inquiring whether cumulative readings introduce new *quantificational dependencies*. I show that (i) *quantificational subordination* against non-distributive readings is often quite degraded, but (ii) common knowledge inference sometimes improves its acceptability, and (iii) non-distributive plural anaphora against cumulative readings may induce a co-varying reading. This suggests that cumulative readings may indeed introduce new dependencies, but their availability is limited. I propose that non-distributive readings 'underspecify' dependencies, while distributive readings highlight specific dependencies, and its interaction with pronoun maximality blocks quantificational subordination against cumulative readings. I implement it with *State-based Dynamic Plural Logic* which keeps track of *quantificational alternatives*.

Keywords: anaphora, co-variation, cumulative readings, distributivity, dynamic plural logic, dynamic semantics, plurality, quantificational subordination, state-based semantics.

1. Introduction

A pronoun is often used to refer back to an entity that has been mentioned in the prior discourse, a phenomenon called *discourse anaphora*. Discourse anaphora is number sensitive: a singular pronoun may not refer back to entities that have been introduced with plural expressions.

(1) a. Tom^{u_1} wrote a paper^{u_2}. He$_{u_1}$ submitted it$_{u_2}$ to L&P.
  b. Tom^{u_1} wrote three papers^{u_2}. He$_{u_1}$ submitted {#it / them}$_{u_2}$ to L&P.

Singular indefinites under the scope of a quantifier may not antecedent singular pronouns.

(2) Every student^{u_1} wrote a paper^{u_2}. {#It / They}$_{u_2}$ is/are well written.

However, a singular indefinite under the scope of a quantifier may antecedent a singular pronoun if the pronoun is also under the scope of another quantifier. This phenomenon is called *quantificational subordination* (Karttunen, 1969: *et seq*) .

(3) a. Every student^{u_1} wrote a paper^{u_2}. b. They$_{u_1}$ **each** submitted it$_{u_2}$ to a journal.

Here, (3b) 'retrieves' the correspondence between students and papers, i.e. every student x submitted the paper x wrote. Such correspondences are called *quantificational dependencies*.

Now, a question arises: when are new quantificational dependencies introduced? And, when may a pronoun have access to quantificational dependencies stored in the context? I discuss

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these questions in light of non-distributive readings of non-quantificational plural arguments. Sentences with multiple plural arguments have several different readings as exemplified below.

(4) Three students^{u_1} read seven papers^{u_2}.
  a. **Collective reading**: a group of 3 students read 7 papers.
  b. **Distributive reading**: Each of the 3 students read 7 (possibly) different papers
  c. **Semi-distributive reading**: Each of the 3 students read the same 7 papers
  d. **Cumulative** (or **Co-distributive**) **reading**: each of the 3 students read at least one paper and each of the 7 papers are read by at least one student.
  e. **Paired-Cover reading**: there is a particular way to pair each of the 3 students with at least one paper and each of the 7 papers with at least one student, and each student read the paper that (s)he is paired with.

It turns out that the acceptability of quantificational subordination against cumulative readings is generally quite limited, but it sometimes improves in some contexts. Furthermore, co-varying readings are available with non-distributive plural anaphora. I propose that cumulative readings introduce new dependencies, but subordination fails because of the interaction between *under-specification of dependencies* and *pronoun maximality relative to quantificational alternatives*.

2. Technical background

I adopt a dynamic semantic approach, in which the meaning of a sentence updates the current discourse. *Discourse referents* (drefs) u_1, u_2,..., are addresses in which some values are stored, i.e. variables. *Information states* g, h,..., keep track of what entities have been mentioned at the discourse, i.e. variable assignments. Table 1 exemplifies information states and drefs.

	u_1	u_2	u_3	...
g	Alex	Beste	Chris	...

Table 1: Drefs and information states

Assignment extension updates an information state by adding a new value to u_n as defined in (5): g and h minimally differ in the new value on u_n.[2] An indefinite introduces a new value to an information state and a pronoun obtains its value directly from the current information state.

(5) $g[u_n]h = \forall u\,[u \neq u_n \rightarrow g(u) = h(u)]$

The role of quantificational dependencies has not been made clear yet. I adopt an enriched data structure to keep track of correspondences among values of drefs as well as their values. *Plural information states* (PIS) G, H, ..., are sets of information states (van den Berg, 1996; Nouwen, 2003; Brasoveanu, 2008: a.o.). A PIS can be given as a matrix as shown in Table 2.

Importantly, one may obtain plural individuals by summing up the values of a dref u across members of a PIS G even if each information state $g \in G$ assigns a singular value to u.[3]

[2] I assume total assignments and put aside the issues with them.
[3] I take plural individuals as sets of individuals and singular individuals as singleton sets of individuals.

G	u_1	u_2	u_3	u_4	...
g_1	Alex	David	Giorgio	Isla	...
g_2	Beste	Emile	Hannah	Isla	...
g_3	Chris	Fred	Hannah	Isla	...

Table 2: Plural information states

(6) $G(u) = \{g(u) : g \in G\}$

Dependencies are defined in (7b): u_m is dependent on u_n iff the values of u_m *co-vary* with the values of u_n. In Table 2, u_2 and u_3 are dependent on u_1, but u_4 is not.

(7) a. $G_{u_n=d} = \{g : g \in G \,\&\, g(u_n) = d\}$

b. In a plural information state G, u_m is *dependent* on u_n iff
$\exists d, e \in G(u_n)\,[G_{u_n=d}(u_m) \neq G_{u_n=e}(u_m)]$ (van den Berg, 1996)

The atomicity condition and the cardinality condition are defined 'globally', i.e. they check if the sum of the values of a dref under a PIS is an atom or comes with a certain cardinality.

(8) a. $\text{atom}(x) = \forall y\,[y \subseteq x \rightarrow y = x]$

b. $[\![\text{At}(u)]\!] = \{\langle G, H\rangle : G = H \,\&\, \text{atom}(H(u))\}$

c. $[\![\text{three}(u)]\!] = \{\langle G, H\rangle : G = H \,\&\, |(H(u))| = 3\}$

The dynamic distributivity operator δ (van den Berg, 1996: *et seq*) evaluates a formula ϕ with respect to subsets of a PIS. (9) evaluates ϕ with respect to $G_{u_n=d}$ and H_{u_d} for each $d \in G(u_n)$.

(9) $[\![\delta_{u_n}(\phi)]\!] = \{\langle G, H\rangle | G(u_n) = H(u_n) \& \forall d\,[d \in G(u_n) \rightarrow G_{u_n=d}[\![\phi]\!]H_{u_n=d})]\}$

This dynamic plural approach offers an analysis of quantificational subordination. Take the example (3) and a PIS illustrated in Table 3. Here, "each" introduces δ co-indexed with u_1. Accordingly, "it" picks up the value of u_2 with respect to $H_{u_1=d}$ for each d, i.e. h_1, h_2 and h_3, and its atomicity condition is evaluated against h_1, h_2 and h_3, instead of H as a whole.

(3) a. Every student^{u_1} wrote a paper^{u_2}. b. They$_{u_1}$ **each**$_{\delta_{u_1}}$ submitted it$_{u_2}$ to a journal.

H	u_1	u_2	...
h_1	student$_1$	paper$_1$	...
h_2	student$_2$	paper$_2$	...
h_3	student$_3$	paper$_3$	...

Table 3: Student-paper correspondence in the discourse

3. Cumulative readings and dependencies

The previous literature in the dynamic plural approach agrees that δ introduces new dependencies, but disagrees on whether one may do so without δ. Brasoveanu (2008); Dotlačil (2013); Henderson (2014); Kuhn (2017: a.o.) allow new drefs to be dependent to old drefs as defined in (10a), and van den Berg (1996); Nouwen (2007); Law (2020: a.o.) do not as defined in (10b). I call (10a) *randomely dependent extension* and (10b) *dependency-free extension*.[4]

(10) a. **Randomly dependent extension**
$G[u]H \Leftrightarrow \forall g[g \in G \rightarrow \exists h[h \in H \& g[u]h]] \& \forall h[h \in H \rightarrow \exists g[g \in G \& g[u]h]]$

b. **Dependency-free extension**
$G[u]H \Leftrightarrow \exists D[H = \{h | \exists g \exists d[g[u]h \& \nu(u)(h) = d \& g \in G \& d \in D]\}]$

To see their difference, consider two PISs given in Table 4 and 5, which respectively exemplify PISs with dependencies and PISs without dependencies.

G	u_1
g_1	x_1
g_2	x_2

$\xrightarrow{G[u_2]H}$

H	u_1	u_2
h_1	x_1	y_1
h_2	x_2	y_2

Table 4: A context with dependencies

G	u_1
g_1	x_1
g_2	x_2

$\xrightarrow{G[u_2]H}$

H	u_1	u_2
h_1	x_1	y_1
h_2	x_1	y_2
h_3	x_2	y_1
h_4	x_2	y_2

Table 5: A context without dependencies

(10a) may produce both, but (10b) may only produce one exemplified in Table 5, i.e. only the former may introduce new dependencies without the δ operator. As a result, (10a) can describe cumulative readings with genuine quantificational dependencies while (10b) cannot. Table 6 and 7 respectively show possible output PISs that corresponds to a cumulative reading of (4).

H	u_1	u_2
h_1	student_1	paper_1
h_2	student_1	paper_2
h_3	student_2	paper_3
h_4	student_2	paper_4
h_5	student_2	paper_5
h_6	student_3	paper_6
h_7	student_3	paper_7

Table 6: Random dependency

H	u_1	u_2
h_1	student_1	paper_1
$\vdots$	$\vdots$	$\vdots$
h_7	student_1	paper_7
$\vdots$	$\vdots$	$\vdots$
h_{21}	student_3	paper_7

Table 7: Dependency-free

Accordingly, randomly dependent extension may describe a cumulative reading with the distributive evaluation of lexical relation as defined in (11). In this definition, evaluation of relations is fully faithful to the quantificational dependencies stored in the discourse. Thus, a cumulative reading arises as a direct consequence of cumulative dependencies.

(11) $[\![R(u_1,...,u_n)]\!] \Leftrightarrow \{\langle G,H\rangle : G = H \& \forall h \in H[\langle h(u_1),...,h(u_n)\rangle \in I(R)]\}$

[4]Elworthy (1995); Krifka (1996a) adopt different frameworks, but their predictions converge with (10a).

On the other hand, dependency-free extension cannot derive a cumulative reading with this distributive evaluation of relations. If one distributively evaluates a lexical relation against the PIS given in Table 7, it can only describe a semi-distributive reading. Thus, it has to be combined with an additional mechanism of cumulative predication. For example, (12) defines the cumulative evaluation of relations (Law, 2020: a.o.). In this definition, evaluation of relation is made independent of the quantificational dependencies stored in the discourse. Even if a PIS does not store any dependencies between two drefs, $*R$ expresses cumulative predication.[5,6]

(12) a. $[\![R(u_1,...,u_n)]\!] \Leftrightarrow \{\langle G,H\rangle : G = H \,\&\, \langle G(u_1),...,G(u_n)\rangle \in I(*R)\}$

b. (i) $R \subseteq *R$, (ii) if $\langle a_1,...,a_n\rangle \in *R$ and $\langle b_1,...,b_n\rangle \in *R$, then $\langle a_1+b_1,...,a_n+b_n\rangle \in *R$, and (iii) nothing else is in $*R$.

Now, these two options make opposite predictions for introduction of new dependencies under cumulative readings. All else being equal, if cumulative readings introduce new dependencies, they should be able to feed quantificational subordination and they should not, otherwise.

4. Quantificational dependencies with cumulative readings

In this section, I examine the predictions of randomly dependent extension and dependency-free extension. It turns out that the empirical picture seems more nuanced than these predictions. Only a few speakers accept the intended subordination reading (13b-ii).[7]

(13) a. Three^{u_1} students wrote seven papers^{u_2} (between them).

b. They$_{u_1}$ each submitted them$_{u_2}$.

  i. Each of the three students submitted the seven papers.

  ii. % Each of the three students submitted the papers they wrote

Furthermore, the acceptability of the intended anaphora varies depending on several factors. First, some speakers report that an example with a creation verb "write" in (13) is better than one with a non-creation verb "read" in (14). Not every speaker has found the contrast, though.

(14) a. Three^{u_1} students read seven papers^{u_2} (between them).

b. They$_{u_1}$ each wrote a review on them$_{u_2}$.

  i. Each of the three students wrote a review on the seven papers.

  ii. ?? Each of the three students wrote a review on the papers they read.

[5]One may also combine (10a) and (12). However, this combination predicts a discrepancy between dependencies relevant to anaphoric potential and dependencies relevant to evaluation of lexical relations.

[6]The combination of (10b) and (12) requires *mereological plurals*, i.e. D_e has to be closed under sum (union-formation in the assumption adopted in this paper), but the combination of (10a) and (11) does not.

[7]Note that a cumulative reading of (13a) would be true when one student wrote just one paper. Thus, one may argue that the degraded status would be due to violation of plurality requirement of "them." While it is surely relevant, Nakamura (2024) shows that plural pronouns also trigger a *partial plurality inference* (since Sauerland, 2003). See §5.4 for the data. Thus, quantificational subordination against cumulative dependencies are degraded not just because of violation of plurality requirement of "them."

Second, the intended anaphora becomes more accessible with common knowledge that helps one infer a functional correspondence. At this point, it seems that different speakers prefer different ways to make the correspondence salient.

(15) a. Three participants^{u_1} interviewed seven elderly relatives^{u_2}.
b. They$_{u_1}$ each got informed consent from them$_{u_2}$ before talking them$_{u_2}$ through a questionnaire. (Robert Truswell, p.c.)

(16) a. Three MA students^{u_1} sent seven documents^{u_2} to the department this afternoon.
b. They$_{u_1}$ each intend to use them$_{u_2}$ to impress their$_{u_1}$ potential supervisors^{u_3}.

Thus, quantificational subordination with cumulative readings sounds better if common knowledge implies a unique mapping from the values of the subject to the values of the object.

One may suspect that it is because common knowledge reasoning makes a paired-cover reading more salient than a cumulative reading. To examine this possibility, I adopt the guided-reading-course-scenario in Haslinger (2021) with modification. Imagine that three students, Ann, Belle and Chris, took a guided-reading course in which seven papers listed. (17) is the target sentence. (17a), (17b) and (17c) respectively describe a semi-distributive scenario, a paired-cover scenario and a cumulative scenario. Contrary to the expectation, the informants I consulted with reject (17) under the pair-list scenario (17b) and the cumulative scenario (17c).

(17) Three students^{u_1} read seven papers^{u_2}. They$_{u_1}$ **each** wrote a review on them$_{u_2}$.
a. **Semi-distributive**: A, B and C all read all the seven papers. For assessment, they all wrote a review on the seven papers. → **felicitous**
b. **Paired-cover**: A is assigned papers 1-3, B is assigned papers 4-5 and C is assigned papers 6-7. They read all the papers assigned to them. For assessment, they all wrote a review on the papers that are assigned to them. → **infelicitous**
c. **Cumulative**: A, B and C are asked to read some of them. For assessment, they all wrote a review on the papers that they chose by themselves. In the end, three anonymized reviews are submitted. One reviews 1-3, another reviews 4-5 and the other reviews 6-7. It's not clear who read which, but all the three students wrote at least one review and all the seven papers are reviewed. → **infelicitous**

This suggests that both paired-cover readings and cumulative readings make subsequent quantificational subordination marginal. This is intuitively puzzling because the common knowledge inferences drawn in (15) and (16) seem to force a paired-cover reading, i.e. the context provides a particular way to pair individuals.

At the same time, there is a reason to believe that non-quantificational plural arguments indeed introduce new quantificational dependencies. Non-distributive plural anaphora may induce a co-varying readings as exemplified in (18b).[8]

[8]Nouwen (2003) wonders if (18b-ii) is a distinct reading or a sub-case of weak truth condition enabled with cumulative predication, pointing out that correspondences introduced with non-quantificational plural arguments may be permuted, unlike those introduced with δ.

(18) a. Three^{u_1} students wrote seven papers^{u_2} (between them).

b. They$_{u_1}$ submitted them$_{u_2}$ to a journal.

i. Each of the three students submitted the seven papers to a journal.

ii. Each of the three students submitted the papers they wrote to a journal.

Thus, the challenge is to derive the degraded status of quantificational subordination against dependencies introduced with non-quantificational plural arguments while (i) capturing its nuanced sensitivity to common knowledge reasoning, and (ii) deriving a co-varying reading with non-distributive plural anaphora. I will sketch a possible approach in the next section.[9,10]

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(1) a. [Tom, Dick and Harry]^{u_1} **each** wrote an article^{u_2}. They$_{u_1}$ (each) sent it$_{u_2}$ to L&P.

b. # To be precise, Tom sent the article Dick wrote, Dick sent the article Harry wrote and Harry sent the article Tom wrote. (Nouwen, 2003)

(2) a. [Tom, Dick and Harry]^{u_1} wrote three articles^{u_2}. They$_{u_1}$ sent them$_{u_2}$ to L&P.

b. To be precise, Tom sent the article Dick wrote, Dick sent the article Harry wrote and Harry sent the article Tom wrote. (Nouwen, 2003)

On this point, an anonymous reviewer of Homogeneity and Maximality Workshop 2 (HNM2) provides an interesting case of disambiguation. Here, the speaker denies her previous statement with "they$_{u_1}$ uploaded them$_{u_2}$ on EasyChair." because the dependencies between u_1 and u_2 were actually not preserved in this situation. This deniability with dependency permutation suggests that the co-varying interpretation is indeed a distinct reading.

(3) Three students^{u_1} wrote seven abstracts^{u_2}. Then, they$_{u_1}$ uploaded them$_{u_2}$ on EasyChair ... (5 minutes later) ... Ah, I was wrong, sorry! In fact, the 3 students did something weirder: they$_{u_1}$ uploaded each other's abstracts on EasyChair, not their own.

[9]In the earlier version of this work, I offered an analysis with a trivalent version of Dynamic Plural Logic, in which plural assignment extension may (re)assign *dummy value* ★ (cf. DeVries, 2016).

(1) $G[u]H \Leftrightarrow \exists D[H = \{h | \exists g \exists d[g[u]h \,\&\, h(u) = d \,\&\, g \in G \,\&\, d \in D \cup \{\bigstar\}]\}]$

In this approach, cumulative dependencies are semi-distributive dependencies in which the object dref stores ★ in some members of a PIS. It blocks quantificational subordination against cumulative dependencies with a trivalent definition of pronouns, i.e. a pronoun is defined iff its antecedent has a non-dummy value in each member of a PIS. Also, it emulates randomly dependent extension for bivalent expressions, i.e. some expressions 'ignore' ★ by treating it as a universal verifier, which does not contribute to their truth condition. However, this analysis neither explains why common knowledge sometimes improves quantificational subordination against cumulative dependencies nor derives co-varying readings of non-distributive plural anaphora.

[10]One may argue that plural anaphora with "each" is degraded simply because the same interpretation may be obtained without it. While this alternative is not implausible, it may not explain why common knowledge reasoning improves the acceptability of quantificational subordination against cumulative readings. Furthermore, it is not trivial to define "the same interpretation." For example, one can find a case in which the intended anaphora is still degraded even though its alternative without "each" cannot arrive at the same reading.

(1) Three^{u_1} students wrote seven papers^{u_2} (between them).

a. They$_{u_1}$ submitted them$_{u_2}$ to a different journal. → the internal reading available

b. They$_{u_1}$ each submitted them$_{u_2}$ to a different journal. → the internal reading **unavailable**

5. Quantificational alternatives and Maximality in State-based DPlL

I pursue an approach with randomly dependent extension that overcomes the challenge sketched above. Henceforth, I call sentences with multiple non-quantificational plural arguments *multi-plural sentences*.[11] The idea is that quantificational subordination fails because multi-plural sentences 'underspecify' dependencies, but the subsequent anaphora with a quantifier highlights specific dependencies. To see that multi-plural sentences underspecify dependencies, recall that randomly-dependent extension permits a wide range of PISs. The possible output PISs for (4) include one illustrated in Table 6, in which no pair of students read the same book.

(4) Three students^{u_1} read seven papers^{u_2}.

However, cumulative readings tolerate cases in which some students read the same books. The context may also include such PISs exemplified in Table 8. Semi-distributive dependencies can be taken as an extreme case, in which all the students read all the books as shown in Table 9.

H	u_1	u_2
h_1	student$_1$	book$_1$
h_2	student$_1$	book$_2$
h_3	student$_1$	book$_3$
h_4	student$_2$	book$_2$
h_5	student$_2$	book$_3$
h_6	student$_2$	book$_4$
h_7	student$_3$	book$_5$
h_8	student$_3$	book$_6$
h_9	student$_3$	book$_7$

Table 8: Cumulative dependency

H	u_1	u_2
h_1	student$_1$	book$_1$
$\vdots$	$\vdots$	$\vdots$
h_8	student$_2$	book$_1$
$\vdots$	$\vdots$	$\vdots$
h_{15}	student$_3$	book$_1$
$\vdots$	$\vdots$	$\vdots$
h_{21}	student$_3$	book$_7$

Table 9: Semi-distributive dependency

Thus, randomly-dependent extension allows the same formula to express a semi-distributive reading, paired-cover readings and a cumulative reading. In this sense, a multi-plural sentence underspecifies dependencies, i.e. it does not highlight any particular dependencies. I propose that this prevents subordination against dependencies introduced with a multi-plural sentence.

I build a positive proposal in the rest of this section. §5.1 introduces a *state-based* version of *Dynamic Plural Logic (DPlL)*, and propose that pronouns perform maximization relative to *quantificational alternatives*. §5.2 shows how it accounts for the core data, §5.3 discusses the effect of common knowledge, and §5.4 discusses its consequence for pronoun number.

[11] As far as I know, this term comes from Haslinger (2021).

5.1. Maximization relative to quantificational alternatives

I introduce *State-based DPlL* to express the relevant notion of 'underspecified dependencies'.[12] First, I define *quantificational alternatives* as sets of *possibilities*, i.e. pairs of a possible world and a PIS.[13] I use s as a variable for quantificational alternatives. Then, I take a context to be a set of quantificational alternatives, and a formula denotes a function from an input context to an output context (Heim, 1982; Groenendijk et al., 1995). Note that c is a *downward closed set* of quantificational alternatives s, i.e. if $s \in c[\phi]$, for any $s' \subset s$, $s' \in c[\phi]$. In this setting, evaluation of lexical relations, cardinality conditions, and (non-)atomicity conditions are defined in (19).[14]

(19) a. $c[R(u_1,...,u_n)] = \{s : s \in c \,\&\, \forall\langle w,G\rangle \in s\, \forall g \in G\, [\langle g(u_1),...,g(u_n)\rangle \in I_w(R)]\}$
b. $c[\text{three}(u)] = \{s : s \in c \,\&\, \forall\langle w,G\rangle \in s\, [|(G(u))| = 3]\}$
c. $c[\text{At}(u)] = \{s : s \in c \,\&\, \forall\langle w,G\rangle \in s\, [\text{atom}(G(u))]\}$
d. $c[\text{Non-At}(u)] = \{s : s \in c \,\&\, \forall\langle w,G\rangle \in s\, [\neg\text{atom}(G(u))]\}$

I also introduce the sequencing operator ';' to signify dynamic conjunction.

(20) $c[\phi;\psi] = c[\phi][\psi]$

Now, I refine plural extension in State-based DPlL as given in (21).

(21) $c[u] = \{s' : \exists s \in c\, [\forall\langle w,G\rangle \in s\, \exists\langle w',H\rangle \in s'\, [G[u]H] \,\&\, \forall\langle w',H\rangle \in s'\, \exists\langle w,G\rangle \in s\, [G[u]H]$ $\&\, \forall\langle w,I\rangle \in s'\, \forall\langle w',J\rangle \in s'\, [I(u) = J(u)]]\}$

(21) is a straightforward refinement in the current setting except the last conjunct. This ensures that every possibility in a quantificational alternative s agrees on the global value of u, i.e. c is partitioned based on the value of u. Due to randomly dependent extension, every quantificational alternative in $c[u]$ covers all the possible patterns of dependencies. In other words, if ϕ is non-distributive, each quantificational alternative in $c[\phi]$ covers a semi-distributive reading and all the possible paired-cover readings, and those quantificational alternatives are distinguished just in terms of the global values of drefs that occur in ϕ. In this sense, dependencies introduced with plural extension is underspecified at the level of quantificational alternatives.

Next, I refine the δ operator in this state-based system as given in (22).

(22) $c[\delta_{u_n}(\phi)] = \{s' : \exists s \in c\, [\forall\langle w',H\rangle \in s'\, \exists\langle w,G\rangle \in s\, [w = w' \,\&\, G(u_n) = H(u_n) \,\&\, \forall d \in H(u_n)$ $\exists s'' \in c[\phi]\, \forall\langle w'',K\rangle \in s'\, [\langle w,G_{u_n=d}\rangle \in s \,\&\, \langle w',H_{u_n=d}\rangle \in s'' \,\&\, \langle w'',K_{u_n=d}\rangle \in s'']]]\}$

(22) is a straightforward refinements of δ in this setting except an additional universal quantification over $\langle w'',K\rangle \in s'$. This ensures that for any two possibilities $\langle w,I\rangle$ and $\langle w',J\rangle$ in s', their

[12] The idea that plural predication involves some kind of underspecification is not new (Schwarzschild, 1996; Krifka, 1996b; Malamud, 2012; Bar-Lev, 2019; Križ and Spector, 2021; Haslinger, 2021: a.o.).

[13] This idea can be taken as a successor of the notion expressed with this term in Brasoveanu (2011, 2013).

[14] However, see §5.4 for a reason to regard the non-atomicity condition as a pragmatic inference. As this is not directly relevant to the main point of the discussion, I assume that plurals hard-wire the non-atomicity condition.

subset possibilities with respect to each value d of u_n belong to the same quantificational alternative in $c[\phi]$, i.e. $I, J \in s$ iff $I(u_n) = J(u_n)$ and $\forall d \in J(u_n) \exists s' \in c[\phi] [\langle w, I\rangle \in s' \,\&\, \langle w', J\rangle \in s']$. It interacts with $c[u]$ in an important way. Recall that $c[u]$ distinguishes quantificational alternatives based on the global value of u. If it is evaluated under the scope of δ, $c[u]$ distinguishes quantificational alternatives based on 'local' values of u under each subset of PIS $H_{u_n=d}$. Crucially, the above mentioned universal quantification over $\langle w'', K\rangle$ in (22) ensures that two possibilities belong to the same quantificational alternative iff their subset possibilities belong to the same quantificational alternative in $c[\phi]$ with respect to each d in u_n.

To see this, consider three possibilities illustrated in Table 10.

$\langle w,$

G	u_1	u_2
g_1	d_1	e_1
g_2	d_2	e_1
g_3	d_2	e_2

$\rangle$

$\langle w',$

H	u_1	u_2
h_1	d_1	e_1
h_2	d_1	e_2
h_3	d_2	e_2

$\rangle$

$\langle w'',$

K	u_1	u_2
k_1	d_1	e_1
k_2	d_1	e_2
k_3	d_2	e_1
k_3	d_2	e_2

$\rangle$

Table 10: Two possibilities with different dependencies

These possibilities agree in the global values of u_1 and u_2, but store different patterns of dependencies. First, suppose that $[u_2]$ is not evaluated under the scope of δ_{u_1}. In this case, all the three possibilities belong to the same quantificational alternative because $G(u_2) = H(u_2) = K(u_2)$. Second, suppose that $[u_2]$ is evaluated under the scope of δ_{u_1}. In this case, the partition of the context relies on local values of u_2. If one looks at the case in which u_1 has the value d_1, then $G_{u_1=d_1} = \{e_1\}$ and $H_{u_1=d_1} = K_{u_1=d_1} = \{e_1, e_2\}$. Therefore, there is $s_1 \in c[u_2]$ such that $\langle w, G_{u_1=d_1}\rangle \in s_1$ and there is $s_2 \in c[u_2]$ such that $\langle w', H_{u_1=d_1}\rangle \in s_2$ and $\langle w'', K_{u_1=d_1}\rangle \in s_2$. If one looks at the case in which u_1 has the value d_2, then $G_{u_1=d_2} = K_{u_1=d_2} = \{e_1, e_2\}$ and $H_{u_1=d_2} = \{e_2\}$. Therefore, there is $s_3 \in c[u_2]$ such that $\langle w, G_{u_1=d_2}\rangle \in s_3$ and $\langle w'', K_{u_1=d_2}\rangle \in s_3$, and there is $s_4 \in c[u_2]$ such that $\langle w', H_{u_1=d_2}\rangle \in s_4$. As a result, none of these possibilities belong to the same quantificational alternative. If one focuses on the case in which u_1 has the value d_1, $\langle w', H_{u_1=d_1}\rangle$ and $\langle w'', K_{u_1=d_1}\rangle$ belong to the same quantificational alternative s_1. However, they differ when u_1 has the value d_2 because $\langle w', H_{u_1=d_2}\rangle$ and $\langle w'', K_{u_1=d_2}\rangle$ belong to different quantificational alternatives s_3 and s_4. Similarly, if one focuses on the case in which u_1 has the value d_2, $\langle w, G_{u_1=d_2}\rangle$ and $\langle w'', K_{u_1=d_2}\rangle$ belong to the same quantificational alternative s_3. However, they differ when u_1 has the value d_1 because $\langle w', H_{u_1=d_1}\rangle$ and $\langle w'', K_{u_1=d_1}\rangle$ belong to different quantificational alternatives s_1 and s_2. Hence, each of these three possibilities belongs to a different quantificational alternative if $[u_2]$ is evaluated under the scope of δ_{u_1}.

In general, if plural extension $[u_m]$ is evaluated under the scope of δ_{u_n}, quantificational alternatives are distinguished based the value of u_m under each particular value d of u_n. In other words, quantificational alternatives are distinguished by the precise dependency between u_n and u_m established in each possibility (recall the definition (7b).) Hence, if ϕ is distributive, each quantificational alternative in $c[\phi]$ covers a specific pattern of dependencies and each dependency pattern is expressed with a different quantificational alternative. In this sense, δ removes underspecification in quantificational alternatives. Note that if ϕ does not introduce new values to any dref, $c[\delta_{u_n}(\phi)]$ just preserves partition given in c because ϕ itself does not partition c.

In this state-based setting, I propose that pronouns require maximality *relative to* quantificational alternatives as defined in (23), which plays the central role in my analysis.[15]

(23) $c[\textsc{max}(u_n)] =$
$\{s'' : \exists s \in c \forall s' \in c\, [s \not\subset s' \rightarrow s'' \subseteq s \,\&\, \forall \langle w, G\rangle \in s \forall \langle w', H\rangle \in s''\, [G(u_n) \subseteq H(u_n)]\}$

It requires that each quantificational alternative in $c[\textsc{max}(u_n)]$ only contains the possibilities that assign the maximal value to u_n among the possibilities that belong to the same partition in c. Consider a quantificational alternative $s \in c$ without any $s' \in c$ such that $s \subset s'$. Then, for each such s, $c[\textsc{max}(u_n)]$ maintains the possibilities that assign the maximal value on u_n and discard the other possibilities from s. Crucially, (23) is 'blind' to the worlds in c and thus it does not distinguish a semi-distributive reading, paired-cover readings and a cumulative reading. If (23) occurs outside the scope of δ, it maximizes the global value of u_n. If it occurs inside the scope of δ co-indexed with u_m, it maximizes the local value of u_n for each value d in u_m.

(23) is sensitive to whether an antecedent sentence comes with δ or without δ. In the former case, δ_{u_m} in the antecedent sentence has already offered specific quantificational alternatives. Thus, (23) under the scope of δ_{u_m} just requires that those alternatives store the maximal value on u_n for each particular value d in u_m. In the latter case, the antecedent sentence does not come with δ, i.e. each quantificational alternative covers all the possible dependencies. This is the direct consequence of randomly-dependent extension. Now, since (i) each quantificational alternative of a multi-plural sentence contains semi-distributive dependencies, and (ii) $\textsc{max}(u_n)$ maximizes u_n blindly to worlds in each quantificational alternative, if (23) is evaluated under the scope of δ_{u_m}, each quantificational alternative maintains the semi-distributive dependency and discard other dependency patterns. Accordingly, if $\textsc{max}(u_n)$ is evaluated under δ and against dependencies introduced with a multi-plural sentence, the output context is only compatible with a semi-distributive reading. This is my short answer to the puzzle described in §4.[16]

5.2. Interaction between cumulative dependencies and maximisation

In this section, I demonstrate how the proposed analysis works in each paritular case. I start with cases of quantificational subordination with a quantifier. Consider (3).

(3) a. Every student^{u_1} wrote a paper^{u_2}. b. They$_{u_1}$ **each** submitted it$_{u_2}$ to a journal.

Quantifiers introduce a maximal value to its dref and I adopt a different maximisation operation defined in (24) (cf. Brasoveanu, 2008; Dotlačil and Roelofsen, 2021; Roelofsen and Dotlačil, 2023). I take $\textsc{max}^{u_n}(\phi)$ as an abbreviation of $u_n; \textsc{max}_{u_n}(\phi)$. While (23) is blind to worlds and takes no description, (24) is world-sensitive and takes a description ϕ.

[15] It is crucial that s in (23) is the maximal quantificational alternative, i.e. *alternative* in Inquisitive Semantics (Ciardelli et al., 2018). Otherwise, it is trivially satisfied in each s' that is a singleton set of possibilities.

[16] As one may notice, State-based DPlL resembles *Dynamic Plural Inquisitive Semantics* in Dotlačil and Roelofsen (2021); Roelofsen and Dotlačil (2023) in the sense that both are state-based dynamic plural semantics. However, they crucially differ in the sense that the partitions in c in State-based DPlL represent specifications of dependencies while the partitions in c in Dynamic Plural Inquisitive Semantics represent resolution conditions.

(24) a. $c[\textsc{max}^{u_n}(\phi)] = c[u_n][\textsc{max}_{u_n}(\phi)]$

b. $c[\textsc{max}_{u_n}(\phi)] = \{s : s \in c[\phi] \,\&\, \forall\langle w,H\rangle \in s\, \forall\langle w',G\rangle \in \cup c[\phi]\, [w = w' \rightarrow G(u_n) \subseteq H(u_n)]\}$

(3a) and (3b) are respectively translated as (25). I assume that the translation of pronouns comes with the (non-)atomicity condition and the maximization condition (23).[17,18]

(25) a. $[\textsc{max}^{u_1}(\delta_{u_1}(\text{At}(u_1);\text{student}(u_1))); \delta_{u_1}(u_2;\text{At}(u_2);\text{paper}(u_2);\text{wrote}(u_1)(u_2))]$

b. $[\text{Non-At}(u_1); \textsc{max}(u_1); \delta_{u_1}(\text{At}(u_2); \textsc{max}(u_2); u_3;\text{journal}(u_3);\text{submit}(u_1)(u_2)(u_3))]$

Table 11 and 12 respectively exemplify typical PISs in the output context of (3a) and (3b).

⟨w,

G	u_1	u_2
g_1	student$_1$	paper$_1$
g_2	student$_2$	paper$_2$
g_3	student$_3$	paper$_3$

⟩

Table 11: An output possibility in (3a)

⟨w,

H	u_1	u_2	u_3
h_1	student$_1$	paper$_1$	journal$_1$
h_2	student$_2$	paper$_2$	journal$_2$
h_3	student$_3$	paper$_3$	journal$_3$

⟩

Table 12: An output possibility in (3b)

First, $\textsc{max}(u_1)$ is evaluated outside the scope of δ_{u_1} and requires that each quantificational alternative only maintains possibilities $\langle w,H\rangle$ and discards other possibilities $\langle w',K\rangle$ such that $K(u_1) \subset H(u_1)$. In this case, $\textsc{max}^{u_1}(\text{At}(u_1);\text{student}(u_1))$ has already maximized the value of u_1, i.e. u_1 stores the maximal set of students in each s, and thus $\textsc{max}(u_1)$ does not discard any possibility from each quantificational alternative. Second, $\textsc{max}(u_2)$ is evaluated inside the scope of δ_{u_1}, interacting with $\text{At}(u_1)$. For each $s \in c$, it requires that u_2 stores an atomic value for each $H_{u_1=d}$ and it is maximal among any $K_{u_1=d}$ such that $\langle w,K\rangle \in s$. Since it requires the maximal singular value, it is amount to the uniqueness requirement relative to each subset of $H_{u_1=d}$. In this case, the quantificational alternatives in c are distinguished based on dependency patterns because of δ_{u_1} in (25a). Thus, $\textsc{max}(u_2)$ does not discard any possibilities from each quantificational alternatives, and thus does not block quantificational subordination in this case.

Next, consider cases of co-varying interpretations of plural anaphora without quantifiers.

(18) a. Three^{u_1} students wrote seven papers^{u_2} (between them).

b. They$_{u_1}$ submitted them$_{u_2}$ to a journal^{u_3}.

These sentences are translated as (26), and Table 13 and 14 exemplify possible PISs in the output context of (18a) and (18b) which support a cumulative reading.

(26) a. $[u_1;\text{three}(u_1);\text{students}(u_1); u_2;\text{seven}(u_2);\text{papers}(u_2);\text{wrote}(u_1)(u_2))]$

b. $[\textsc{max}(u_1); \textsc{max}(u_2); u_3;\text{journal}(u_3);\text{submit}(u_1)(u_2)(u_3)]$

[17]The translation of "every" essentially follows van den Berg (1996); Brasoveanu (2008), but I omit some details that are not relevant to the main point of this paper.

[18]I put aside sub-clausal compositionality, but one may easily make this system compositional in the style of Dotlačil and Roelofsen (2021); Roelofsen and Dotlačil (2023), which follow Muskens (1996); Brasoveanu (2008).

$\langle w,$

G	u_1	u_2
g_1	student$_1$	paper$_1$
g_2	student$_1$	paper$_2$
g_3	student$_1$	paper$_3$
g_4	student$_2$	paper$_2$
g_5	student$_2$	paper$_3$
g_6	student$_2$	paper$_4$
g_7	student$_3$	paper$_5$
g_8	student$_3$	paper$_6$
g_9	student$_3$	paper$_7$

$\rangle$

Table 13: An output PIS for (18a)

$\langle w,$

H	u_1	u_2	u_3
h_1	student$_1$	paper$_1$	journal$_1$
h_2	student$_1$	paper$_2$	journal$_1$
h_3	student$_1$	paper$_3$	journal$_1$
h_4	student$_2$	paper$_2$	journal$_1$
h_5	student$_2$	paper$_3$	journal$_1$
h_6	student$_2$	paper$_4$	journal$_1$
h_7	student$_3$	paper$_5$	journal$_1$
h_8	student$_3$	paper$_6$	journal$_1$
h_9	student$_3$	paper$_7$	journal$_1$

$\rangle$

Table 14: An output PIS for (18b)

Since neither (18a) nor (18b) introduce δ, the quantificational alternatives in c are distinguished with the global values of u_1 and u_2, and each alternative covers all the possible patterns of dependencies. As (26) does not involve any occurrence of δ, both occurrences of MAX are non-distributively evaluated. Importantly, pronoun maximization $\text{MAX}(u_n)$ outside the scope of δ only concerns the global value of u_n relative to possible worlds and quantificational alternatives, i.e. the sum of student$1-3$ and the sum of paper$_{1-7}$ in this particular case. Since a semi-distributive reading, paired-cover readings and a cumulative readings are indistinguishable with the global values of argument drefs alone, non-distributive pronoun maximization does not rule out any of these possible readings. As a result, the resultant c maintains quantificational alternatives each of which exhaust a semi-distributive reading and all the possible paired-cover readings. Thus, (18b) does not block a co-varying reading of plural pronouns.[19]

It can also handle cases in which the antecedent sentence involves a quantifier but the pronoun sentence does not. Take (27) as an example. It also has a reading that each of the students submitted the paper which (s)he wrote.

(27) a. Every^{u_1} student wrote a paper^{u_2}. b. They$_{u_1}$ submitted them$_{u_2}$ to a journal^{u_3}.

These sentences are translated as (28).

(28) a. $[\text{MAX}^{u_1}(\delta_{u_1}(\text{At}(u_1);\text{student}(u_1)));\delta_{u_1}(u_2;\text{At}(u_2);\text{paper}(u_2);\text{wrote}(u_1)(u_2))]$
b. $[\text{MAX}(u_1);\text{MAX}(u_2);u_3;\text{journal}(u_3);\text{submit}(u_1)(u_2)(u_3))]$

Table 11 and Table 12 can be reused as illustrations of typical PISs in the output context of (27a) and (27b). In this case, the occurrence of δ_{u_1} in (28a) expands c so that each quantificational alternative corresponds to a particular type of dependencies. Then, $\text{MAX}(u_1)$ and $\text{MAX}(u_2)$ are both evaluated against this set of quantificational alternatives. Since they are not under the scope of δ, however, they simply narrow down each s to the possibility that assigns the globally maximal value to u_1 and u_2. Since each s contains a particular PIS due to δ_{u_1}, it does not discard any possibility. Thus, just like (26b), (28) does not block a co-varying reading.

[19]To derive a permuted reading given in Footnote 4, one may assume that a plural pronoun may take the global value of its antecedent as a mereological sum, cf Footnote 3.

Now, I finally turn to cases of quantificational subordination against cumulative readings.

(13) a. Three^{u_1} students wrote seven papers^{u_2} (between them).
b. They$_{u_1}$ each submitted them$_{u_2}$ to a journal.

These sentences are translated as (29).

(29) a. $[u_1;\text{three}(u_1);\text{students}(u_1);u_2;\text{seven}(u_2);\text{papers}(u_2);\text{wrote}(u_1)(u_2))]$
b. $[\text{MAX}(u_1);\delta_{u_1}(\text{MAX}(u_2);u_3;\text{journal}(u_3);\text{submit}(u_1)(u_2)(u_3))]$

Table 13 and Table 14 can be reused to illustrate possible PISs in the output context of (13a) and (13b). The first occurrence $\text{MAX}(u_1)$ does not cause a problem because it is in the exactly the same environment as the one in (26). The question is which value the second occurrence $\text{MAX}(u_2)$ picks. Now, the additional layer of information with quantificational alternatives becomes crucial. Since (29a) does not involve any δ, each quantificational alternative in c contains possibilities that cover a semi-distributive reading and all the possible paired-cover readings. This is not problematic if $\text{MAX}(u_2)$ is not evaluated under the scope of δ, as we have just seen in the discussion on (18b). However, this time, it is evaluated under the scope of δ_{u_1}. In this environment, in each s, $\text{MAX}(u_2)$ only maintains the possibilities that assign the maximal value on u_2 for $K_{u_1=d}$ for each d. Since a possibility with a semi-distributive dependency assigns the globally maximal value of u_2 with respect to each value of u_1, this possibility is always the one that meets this maximization requirement in each s. As a result, each quantificational alternative in the resultant c only maintains possibilities with a semi-distributive dependency. Thus, the proposed maximization blocks quantificational subordination in this case. Importantly, this explains the degraded status of quantificational subordination against cumulative dependencies while deriving co-varying readings of non-distributive plural anaphora. The remaining question is why common knowledge sometimes improves quantificational subordination against cumulative dependencies. The next section addresses this question in light of State-based DPlL.

5.3. Common knowledge, expectation and adjustment of the context

I propose that common knowledge inference may improve the status of quantificational subordination against cumulative dependencies because it may 'adjust' the context prior to maximization by discarding possibilities with semi-distributive dependencies from c.

First of all, the lexical meaning of creation verbs and common knowledge reasoning contribute to identification of correspondences. For example, the common knowledge on writing tells that there is a unique author (or a unique group of co-authors) for each product of writing. This may disambiguate a multi-plural sentence. Consider a toy example (30).

(30) Two students^{u_1} wrote two papers^{u_2}.

Suppose $D = \{a_1, a_2, p_1, p_2\}$ and consider three worlds w_1, w_2 and w_3 given in (31).

(31) a. $I_{w_1}(R) = \{\langle a_1, p_1\rangle, \langle a_2, p_2\rangle\}$ (paired-cover reading)

b. $I_{w_2}(R) = \{\langle a_1, p_1\rangle, \langle a_1, p_2\rangle, \langle a_2, p_2\rangle\}$ (paired-cover reading with an overlap)
c. $I_{w_3}(R) = \{\langle a_1, p_1\rangle, \langle a_1, p_2\rangle, \langle a_2, p_1\rangle, \langle a_1, p_2\rangle\}$ (semi-distributive reading)

While $I_{w_1}(R)$ is consistent with the common knowledge because any distinct paper is associated with a unique author, $I_{w_2}(R)$ and $I_{w_3}(R)$ are not. Now, consider the PISs that are respectively associated with w_2 and w_3 as illustrated in Table 15 and Table 16.

K	u_1	u_2
k_1	a_1	b_1
k_2	a_1	b_2
k_3	a_2	b_2

Table 15: The PIS corresponding to (31b)

K'	u_1	u_2
k'_1	a_1	b_1
k'_2	a_1	b_2
k'_3	a_2	b_1
k'_4	a_2	b_2

Table 16: The PIS corresponding to (31c)

Suppose that one may discard the possibilities $\langle w_2, K\rangle$ and $\langle w_3, K'\rangle$ from the output context of (30) due to contextual inconsistency. Then, consider that (32) is uttered after (30).[20]

(32) They$_{u_1}$ each submitted them$_{u_2}$.

As discussed in §5.2, MAX(u_2) in this environment only maintains the possibilities with semi-distributive dependencies. However, by supposition, such possibilities have already been discarded from the context. As a result, the possibilities with the maximal value of u_2 have to be chosen from paired-cover dependencies compatible with the common knowledge of writing. This means that (32) now has a reading in which each student submitted papers that (s)he wrote, which is a subordination reading. Thus, if common knowledge reasoning narrows down the input context 'prior' to maximization, quantificational subordination against cumulative dependencies becomes available. In this sense, the individual variation may point to variation in common knowledge sensitivity, i.e. it hinges on whether an individual takes common knowledge inference as contextual entailment and 'accommodates' the input context along with it.

This consideration might also apply to comprehenders' expectation for future discourse (Rohde, 2008: a.o.). For example, Krifka (1996a) reports that the subordination reading is clearer if the two cardinal modifiers match.

(33) a. Three students^{u_1} wrote three articles^{u_2}.
b. They$_{u_1}$ each sent them$_{u_2}$ to L&P. (Krifka, 1996a)

Also, it seems that quantificational subordination against cumulative readings becomes more acceptable for some speakers if the object numeral is divisible by the subject numeral.[21]

(34) a. Two students^{u_1} wrote four papers^{u_2}.

[20] I take (32) as a toy example, and ignore the competition between "it" and "them" for an expository sake.
[21] I thank to an anonymous reviewer of Homogeneity and Maximality workshop 2 (HNM2) for this example. Note that the reviewer's original point is that small numbers improve the acceptability, though.

b. They$_{u_1}$ each sent them$_{u_2}$ to L&P. (an anonymous reviewer for HnM workshop)

There is no *prima facie* reason to think that a specific choice of two cardinals triggers common knowledge inference. However, if a multi-plural sentence involves two matching numerals or the combination of a numeral and its multiple, a comprehender may think that a future continuation of the discourse may retrieve a non-trivial dependency expressed with this multi-plural sentence. If such an expectation may lead to adjustment of the context, discarding possibilities with semi-distributive dependencies prior to maximization, then it may have the same effect as common knowledge inference. Much more work is necessary to assess this conjecture and it is left for the future work. However, the point is that the individual variation may indicate individual variation in sensitivity to common knowledge and expectation, and the proposed analysis is flexible enough to incorporate these factors by letting them adjust the input context for sentences with pronouns so that some possibilities are discarded prior to maximization.

5.4. Remarks on pronoun number

In this section, I discuss the effect of pronoun number on maximization. First, I have shown that maximization on singular pronouns leads to the uniqueness requirement. One may wonder if it predicts that quantificational subordination only has a *strong reading*, i.e. (3) is true iff every student submitted **all** the papers (s)he wrote. This seems to be a good prediction for (3).[22]

(3) a. Every student^{u_1} wrote a paper^{u_2}. b. They$_{u_1}$ **each** submitted it$_{u_2}$ to a journal.

However, (35) seems to allow a weak reading: it may still be judged true when a customer had more than one credit card and used one of these credit cards to pay the check.

(35) a. Every customer^{u_1} had a credit card^{u_2}. b. They$_{u_1}$ each used it$_{u_2}$ to pay the check.

This is not necessarily a problem, though. Even if a singular indefinite introduces an atomic value to a dref, it is still compatible with the *at least one* reading. For example, consider (36) in regard to a PIS H and a world w. The value of u_1 under H is atomic, but the (dynamic) truth condition of (36) is still compatible with I_w(dog) and I_w(own) (cf. Sudo, 2023).

(36) Ann owns a dog^{u_1}.
a. $H(u_1) = d_1$ b. $I_w(\text{dog}) = \{d_1, d_2, d_3\}$ c. $I_w(\text{own}) = \{\langle \text{Ann}, d_1\rangle, \langle \text{Ann}, d_2\rangle, \langle \text{Bill}, d_3\rangle\}$

Thus, one may just assume that in any possible output PIS of (35), there is at most one value in u_2 for each distinct value of u_1. As $\textsc{max}(u_n)$ is blind to the information stored in possible worlds, the result of maximization is still compatible with possibilities that support a weak reading.[23]

Second, Nakamura (2024) shows that *partial plurality inferences* (since Sauerland, 2003) are also observed with plural pronouns as exemplified in (37): "them" only requires that its value is plural with respect to at least one of the values of u_1.

[22] See also Nouwen (2003) for the related discussion.

[23] One may combine the proposed analysis with a pragmatic approach to the weak/strong ambiguity, e.g., a *homogeneity approach* (Chatain, 2018; Champollion et al., 2019).

(37) Scenario: There are ten PhD students in this department. This semester, seven of them wrote exactly one paper, while the other three students wrote more than one paper. They all submitted their papers to a journal.

a. Every PhD student^{u_1} wrote (some) papers^{u_2} in this semester.

b. They$_{u_1}$ each submitted {#it / them}$_{u_2}$ to a journal.

This suggests that the non-atomicity inference arises due to pragmatic competition (Sauerland, 2003; Sauerland et al., 2005: a.o.), and Nakamura (2024) proposes a DPlL analysis to derive this presuppositional inference, applying Sudo's (2023) *dynamic scalar implicature* approach. The analysis in this paper is compatible with it. Recall that δ splits c based on dependency patterns. For example, consider d_1, e_1, e_2 such that $I_{u_1=d_1}(u_2) = e_1$ and $J_{u_1=d_1}(u_2) = \{e_1, e_2\}$. As the value of u_2 is introduced under the scope of δ_{u_1}, such I and J are members of different quantificational alternatives, i.e. the local value of u_2 is different in $I_{u_1=d_1}$ and $J_{u_1=d_1}$. As a result, maximization relative to quantificational alternative does not discard possibilities with I because J belongs to a different quantificational alternative. Thus, those PISs which support pronominal partial plurality inferences survive through the update with pronoun maximization.

6. Conclusion

There is a good reason to believe that cumulative readings introduce new dependencies, but it raises a puzzle of why quantificational subordination against them is marginal. I offered a solution with *State-based DPlL* equipped with pronoun maximality relative to *quantificational alternatives*: it discards dependencies other than semi-distributive dependencies only if the antecedent sentence is non-distributive, and a pronoun is evaluated distributively. This derives co-varying readings of non-distributive plural anaphora while blocking quantificational subordination against cumulative readings. Then, I argued that common knowledge may improve it by discarding possibilities with semi-distributive dependencies from the context prior to maximization, and conjectured that expectations for discourse continuation may play the same role.

Some issues are left for future work. First, cumulative readings with quantificational or definite objects also do not seem to feed quantificational subordination.

(38) a. Three^{u_1} students read {seven papers/the (seven) papers/all the papers/every paper}^{u_2}.

b. ?? They$_{u_1}$ each wrote a review on them$_{u_2}$.

No speaker has reported a contrast among (38) so far, i.e. they sound equally bad. Although more work is necessary, the same analysis may work for them. Also, one speaker reported a subtle difference between definite plurals and universal quantifiers in an example with "write."

(39) It's surprising that this many papers have been written in this research group this year.

a. Actually, three^{u_1} students wrote the (seven) papers^{u_2}. ?They$_{u_1}$ each submitted them$_{u_2}$ to L&P.

b. Actually, three^{u_1} students wrote {all the papers / every paper}^{u_2}. ??They$_{u_1}$ each submitted them$_{u_2}$ to L&P.

Second, factors other than common knowledge and expectation may affect the acceptability of quantificational subordination against cumulative readings. For example, signaling the speaker's ignorance about dependencies improves the felicity for some but not all speakers. It might also be due to the pair-list interpretation of "who read which paper."

(40) Three students^{u_1} read seven papers^{u_2}. ??(I don't know who read which paper, but) they$_{u_1}$ each wrote a review on them$_{u_2}$.

Also, an overt description sometimes improves the acceptability of subordination.

(41) Three PhD applicants^{u_1} provided seven supporting documents^{u_2}. They$_{u_1}$ each use {??them / the documents}$_{u_2}$ to convince their$_{u_1}$ potential supervisors.^{u_3}

There could be yet other relevant factors, and investigation on them is left for future research.

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