A Puzzle about 'if', Update and Compositionality[1]

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Abstract. According to dynamic approaches to meaning, meanings are context change potentials: ways in which an assertion of a sentence affects the context or the common ground of a conversation. In this paper I will argue that it is not straightforward to square the idea that meanings are context change potentials with an adequate theory of the discourse dynamics of conditionals and the idea that meanings are compositional. As I will argue, there is a tension between providing a prima facie plausible theory of the discourse dynamics of indicative conditionals while holding on to a popular and widespread notion of what it is for meanings to be compositional. The tension disappears, if we reject the view that meanings are context change potentials. That said, I will close the paper by discussing what I take to be the two most promising routes defenders of a dynamic approach to meaning could take in order to resolve the tension.

Keywords: Theories of meaning, compositionality, dynamic semantics, conditionals.

1. Introduction

Discourse dynamics is the study of the context change potentials of sentences: how assertions of sentences of a given fragment of language affect the context of a conversation. In this paper I wish to investigate the discourse dynamics of simple indicative conditional (henceforth just *conditional*) sentences, such as

(1) If tweety is a bird, tweety is able to fly,

(2) It is not the case that if tweety is a bird, tweety is able to fly.

I will focus on conditional sentences that are *simple* in the sense that all of them can reasonably be modelled by a formal language generated by the following BNF:

$$\begin{aligned} &\phi := \alpha \mid (\alpha \to \alpha) \mid \neg(\alpha \to \alpha), \\ &\alpha := p \mid \neg\alpha \mid (\alpha \wedge \alpha), \end{aligned} \qquad (\mathcal{L}_s)$$

where p is supplied by a countable set of atomic sentences, $\mathtt{At}$.[2] Notice that $\mathcal{L}_s$ neither contains nested nor compound conditionals. For my purposes, focusing on sentences in this small fragment will suffice.

While the study of the discourse dynamics of conditionals is interesting in its own right, I am ultimately interested in a foundational question: the question of what meanings are. According to a popular view, our approach to meaning should be *dynamic*: *meanings are context change*

[1]I would like to thank John MacFarlane, Sven Neth, Adrian Ommundsen, Seth Yalcin as well as audiences at HU Berlin, ECAP11 and SuB28 for discussion and comments.

[2]As usual I will take $\to$, to model english 'if ... then ...' statements in the indicative mood and take $\wedge$ and $\neg$ to model conjunction and negation respectively. $\mathcal{B}$ will be used to refer to the non-conditional fragment of $\mathcal{L}_s$.

 In: Baumann, Geraldine, Daniel Gutzmann, Jonas Koopman, Kristina Liefke, Agata Renans, and Tatjana Scheffler (eds.) 2024. Proceedings of Sinn und Bedeutung 28. Bochum: Ruhr-University Bochum, 200-215.

potentials.[3] This is what we may call *the guiding slogan of dynamic semantics*.[4]

The goal of my paper is to convince the reader that it is not straightforward to square the guiding slogan with an adequate theory of the discourse dynamics of the simple conditional sentences in $\mathcal{L}_s$ while respecting the idea that meanings are compositional. As I will argue, there is a tension between providing a prima facie plausible theory of the discourse dynamics for $\mathcal{L}_s$ while holding on to a popular and widespread notion of what it is for meanings to be compositional. The tension disappears, as we will see, if we reject the guiding slogan of dynamic semantics. That said, I will close the paper by discussing what I take to be the two most promising routes defenders of dynamic approaches to meaning could take in order to respond to the tension.

Some caveats before we move on. First, I mentioned above that we will focus on a small conditional fragment $\mathcal{L}_s$. Hence, in the following it will be helpful to keep in mind that constraints on the discourse dynamics for simple conditionals we put forward should not be read as constraints that carry over to theories that aim to provide approaches to larger conditional fragments or fragments of language that, in addition to conditionals, contain modal or other operators apart from conjunction and negation.

Second, the literature on both the semantics and the discourse dynamics of indicative conditionals is vast. Accordingly it will be impossible to do justice to the wide range of puzzles and views surrounding natural language conditionals. However, I will address what I take to be the most pressing objections to the views I am interested in.

2. Discourse Dynamics

2.1. Discourse Dynamics in General

Let me start by saying more about what it is to provide a theory of the discourse dynamics for a fragment of language as well as what it is to provide a dynamic theory of meaning.

Most of the notions we will be working with go back to Stalnaker (1999). According to Stalnaker, a conversation should be thought of as taking place on the background of a shared stock of information, the *context* or *common ground* of a conversation. To assert a sentences at a particular stage of the conversation is to contribute to that stock of information in a certain way. Such contributions may consist in information that is added to the context or help the discourse participants to coordinate on the context in some other way. A formal structure that helps to make this idea precise is what we may call a *model of conversation*.[5]

Definition 2.1 (Model of Conversation)**.** *A* ***model of conversation*** *for a fragment of language $\mathcal{L}$ is a pair $\langle C, \cdot[\cdot]\rangle$ where C is a set, the set of* ***contexts****, and $\cdot[\cdot]$ a function, the* ***update function****, that maps a context $c \in C$ and a sentence $\phi \in \mathcal{L}$ to a context $c[\phi] \in C$.*

[3] Such views were pioneered by Heim (1982) and Kamp (2013).

[4] This guiding slogan is found in many places (see Groenendijk and Stokhof, 1991; Veltman, 1996; Gillies, 2004: among many others).

[5] The notion is similar to what Bonnay and Westerståhl (2014) call an *abstract frame*. (Rothschild and Yalcin, 2016) and (Rothschild and Yalcin, 2017) discuss slightly more general structures.

The set of contexts corresponds to the context or the common ground at various possible stages of a conversation. The update function of such a model can be thought of as telling us how an assertion of a sentence at a context affects that context, how the conversation moves from one stage to another in light of an assertion. Note that the definition does not presuppose a particular notion of context. However, for the most part we will follow Stalnaker and think of contexts as sets of possible worlds: *ways the world might be in light of the information shared between the participants of the discourse*. Possible worlds, in turn, are taken to be functions from $\mathtt{At}$ to truth values in $\{1,0\}$. W will always denote a set of possible worlds and $\mathcal{P}(W)$ its power set. Let me also flag that, while it is natural to interpret such a structure as telling us about how the context of conversation evolves in light of assertions, this interpretation is not forced on us. Later on we will encounter a different, non-dynamic interpretation of an update function. For now, however, the way of looking at models of conversation just discussed will suffice.

The notion of a model of conversation puts us in a position to make the guiding slogan of dynamic semantics precise. Remember, according to the slogan, meanings are context change potentials. The context change potential of a sentence ϕ, in turn, is the way in which an assertion of ϕ would affect the context of conversation. Now, given a model of conversation, any sentences $\phi \in \mathcal{L}$ will induce a function from context to context $\cdot[\phi]$. Such a function can plausibly be take to model the context change potential of the sentences we are interested in. Accordingly, the following definition seems to adequately capture the guiding slogan.

Definition 2.2 (Dynamic approaches to meaning)**.** *According to a* ***dynamic approach to the meaning*** *of sentences in a fragment $\mathcal{L}$, meanings are modelled in terms of a recursively defined update function of a model of conversation for $\mathcal{L}$ which captures the context change potentials of the sentences in in the fragment.*

The term 'recursive' in the above definition should be read in a loose sense, as applying to any function that is defined by making use of the inductive structure of the fragment of language we are interested in. The notion is intended to capture the idea that on a dynamic approach to meaning an update function plays *two* roles. It is a model of the context change potentials of the fragment of language we are interested in *and* it describes its compositional structure. We will say more about what it is for a notion of meaning to be compositional later on, since typically our notion of compositionality is stronger than what we have so far. For now this more permissive way of capturing this notion will suffice.

Given this, let us contrast dynamic approaches to meaning to non-semantic approaches to discourse dynamics.

Definition 2.3 (Non-semantic approach to discourse dynamics)**.** *According to a* ***non-semantic approach to the discourse dynamics*** *for a fragment of language $\mathcal{L}$, a model of conversation for $\mathcal{L}$ is given in terms of a bridge principle that links a theory of meaning for $\mathcal{L}$, that does not provide the discourse dynamics of the fragment directly, to a theory about the context change potentials of the sentences in $\mathcal{L}$.*

Stalnaker's original proposal in (Stalnaker, 1999) is an example of such an non-semantic approach to discourse dynamics. His notion of update is determined via a bridge principle that links a theory of truth at an index (in the manner of Kaplan (1977)) to an appropriate model of

conversation. But note that the above definition does not require the input theory to be truth-conditional. What matters is that the system modeling the compositional theory of meaning does not deliver a theory of discourse dynamics directly. The latter is modelled via a bridge principle linking meanings to the update potentials of the sentences we are interested in.[6]

With these definitions in the back of our minds, let us turn to our first, concrete model of conversation for $\mathcal{L}_s$.

2.2. Heim's View

An early proposal for the discourse dynamics of indicative conditionals is found in Heim (2002). According to this view, we have the following model of conversation for $\mathcal{L}_s$.

Definition 2.4 (Heim's View)**.** *Heim's model is a pair* $\langle \mathcal{P}(W), \cdot[\cdot] \rangle$ *for some set of worlds* W *and for all contexts* $c \in \mathcal{P}(W)$*,* $p \in \mathtt{At}$ *and* $\alpha, \beta \in \mathcal{B}$*,* $\cdot[\cdot]$ *is recursively defined as follows:*

$$(p)\ \ c[p] = \{w \in c \mid w(p) = 1\}, \quad (\wedge)\ \ c[\alpha \wedge \beta] = c[\alpha][\beta],$$
$$(\neg)\ \ c[\neg\alpha] = c - c[\alpha], \quad (\rightarrow_h)\ \ c[\alpha \rightarrow \beta] = c - (c[\alpha] - c[\alpha][\beta]).$$

Note that this view satisfies our above definition of what it is for a theory of meaning to be dynamic. The theory is given in terms of a recursively defined model of conversation that is intended to capture the context change potentials of the sentences we are interested in. Let us check the view's predictions with respect to an example. Here is a vignette (slightly modified from Gillies, 2004) we may use to provide a context of conversation.

> **Crime at the Mansion:** A crime has been committed at the mansion. Ann and Bob are investigating the crime scene. It is common ground between Ann and Bob that the culprit acted alone and that there are three possible candidates. There is *the butler* (a member of the *house staff*), *the driver* and *the gardener* (both members of the *ground staff*).

Suppose Bob investigates the crime scene alone. Reporting back to Ann, he asserts:

(3) If a member of the ground staff did it, it was the driver.

A simplified but reasonable way of modeling the context of Ann and Bob's conversation is in terms of the set $\{w_b, w_d, w_g\}$, a set consisting of the three worlds in which the butler, the driver and the gardener are the sole culprits, respectively. Let g stand for 'a member of the ground staff did it' (a sentence true at w_g and w_d but false at w_b) and d for 'the driver is the culprit' (a sentence true at w_d only). Then, on Heim's view, we get

$$\{w_b, w_d, w_g\}[g \rightarrow d] = \{w_b, w_d\}.$$

This seems a reasonable prediction. Bob's assertion is predicted to be felicitous and informative. The butler is not ruled out as a candidate culprit for, after all, (3) does not seem to inform

[6] Dynamic approaches to meaning are sometimes contrasted to so-called *static* approaches. But there are different ways of making the notion precise. One way of contrasting dynamic and static approaches found in (Rothschild and Yalcin, 2016) and (Rothschild and Yalcin, 2017) is orthogonal to what I am interested in here. Hence, I will not use the notion in this paper.

us about the members of the house staff. However, Ann's assertion is predicted to rule out the gardener as a candidate culprit, a prediction that seems equally reasonable.

Unfortunately, Heim's view is faced with a well known objection (Gillies, 2004). It makes unwelcome predictions about negated conditionals. On her view, we have

$$\begin{aligned} c[\neg(\alpha \rightarrow \beta)] &= c - \big(c - (c[\alpha] - c[\alpha][\beta])\big), && \text{by } (\neg) + (\rightarrow_h) \\ &= c[\alpha] - c[\alpha][\beta], && \text{by set theory} \\ &= c[\alpha \wedge \neg\beta]. && \text{by } (\neg) + (\wedge) \end{aligned}$$

So an assertion of a negated conditional is predicted to convey both the antecedent and the negation of the consequent of the negated conditional asserted. To illustrate that this is an unwelcome consequence, consider (2) from above (repeated).

(2) It is not the case that if tweety is a bird, tweety is able to fly.

According to Heim's view (2) conveys both

(4) Tweety is a bird,

(5) Tweety is not able to fly.

But an assertion of (2) seems perfectly compatible with Tweety being a bat. Hence, intuitively, neither (4) nor (5) is conveyed. In a nutshell, the objection is that Heim's view is a dynamic version of the material conditional view, a view that is well known for its unwelcome predictions about negated conditionals.

2.3. The Dynamic Strict Conditional View

The problematic predictions of Heim's view about negated conditionals are typically taken to motivate a by now popular and widespread view in the dynamic semantics literature, the dynamic strict conditional view (Such a view is found in many places. For my purposes, the most relevant discussion is found in (Gillies, 2004) and Veltman (1985)).[7]

Definition 2.5 (The Dynamic Strict Conditional View)**.** *A dynamic strict model is a pair* $\langle \mathcal{P}(W), \cdot[\cdot] \rangle$ *where for any context* $c \in \mathcal{P}(W)$*,* $p \in \mathtt{At}$ *and* $\alpha, \beta \in \mathcal{B}$*,* $\cdot[\cdot]$ *is recursively defined as follows:*

$$\begin{array}{ll} (p)\ \ c[p] = \{w \in c \mid w(p) = 1\}, & (\wedge)\ \ c[\alpha \wedge \beta] = c[\alpha][\beta], \\ (\neg)\ \ c[\neg\alpha] = c - c[\alpha], & (\rightarrow_d)\ \ c[\alpha \rightarrow \beta] = \{w \in c \mid c[\alpha][\beta] = c[\alpha]\}. \end{array}$$

The view agrees with Heim's on how non-conditional sentences are treated. But it differs with respect to its entry for conditionals. The conditional operator is sometimes described as 'performing' a test, typically called the *Ramsey test*. According to this test we check whether the context at which the conditional is uttered has a certain global property: the property of being such that updating the context with the antecedent results in a context that accepts the consequent of the conditional. If the Ramsey test is passed, the context is left unchanged, and

[7] Many versions of the view agree about the context change potentials for $\mathcal{L}_s$ but differ on larger fragments of language such as (Willer, 2017), (Yalcin, 2012) and (Moss, 2018), to name a few.

the assertion is predicted to be felicitous at that context. If the Ramsey test is not passed, the context is 'crashed' to the empty set and the assertion is predicted to be infelicitous at that context.

This feature, together with Heim's entry for negation, leads to much more plausible predictions for negated conditionals.

$$\begin{aligned} c[\neg(\alpha \to \beta)] &= c - [\alpha \to \beta], && \text{by } (\neg) \\ &= c - \{w \in c \mid c[\alpha][\beta] = c[\alpha]\}, && \text{by } (\to)_d \\ &= \{w \in c \mid c[\alpha][\beta] \neq c[\alpha]\}. && \text{by set theory} \end{aligned}$$

For instance, an assertion of (2) can be thought of as a way for the speaker to point out that Tweety might be a bird that is not able to fly (a baby chick, for instance). So the effect it has on the common ground is not to add information to it, but rather to make sure it contains a certain possibility: in the case of (2) the possibility for Tweety being a bird that is not able to fly. Notice that since felicitous assertions of negated conditionals never change the original context, such conditionals do not share the unwelcome predictions of negated conditionals we observed for Heim's view.

Unfortunately, what we may take to be a feature of the dynamic strict conditional view with respect to negated conditionals is a bug with respect to assertions of plain conditionals. Consider the case from the previous section. We judged Ann's assertion of (3) as being felicitous and informative at the context in question. Heim's view leads to the reasonable prediction that the assertion will change the context to one which contains only the butler and the driver as candidate culprits. The strict conditional view, however, predicts

$$\{w_b, w_d, w_g\}[g \to d] = \varnothing.$$

For notice that our context does *not* pass the Ramsey test. It contains a world in which the anctecedent of our conditional is true but the consequent is false. Hence we have

$$\{w_b, w_d, w_g\}[g][d] = \{w_d\} \neq \{w_g, w_d\}\{w_b, w_d, w_g\}[g].$$

So Ann's assertion of (3) is predicted to be infelicitous. More precisely, the view predicts that Bob's assertion should be as infelicitous as any assertion that is either obviously false at the context or contradictory.

In light of this, choosing the dynamic strict conditional view over Heim's seems like an overreaction. We are trading in one inadequate prediction regarding negated conditionals for another inadequate prediction regarding plain conditionals.

The dynamic strict conditional view can, as we will see shortly, also be criticised for how it handles the context change potentials of negated conditionals, and I will comment on one such objection later on. However, assuming we agree that the dynamic strict conditional view makes superior predictions about negated conditionals while Heim's view does better with respect to plain conditionals, it seems that we should be aiming for a view that combines the benefits and rids us of the drawbacks of both views.

3. A Conflict with Compositionality

3.1. Constraints

Given the above discussion, let us put down some constraints a more adequate theory of the discourse dynamics for $\mathcal{L}_s$ should satisfy. Let us call any model of conversation that satisfies the constraints an *unorthodox model*:

Definition 3.1 (An unorthodox view)**.** *Let $\cdot[\cdot]$ be an update function that makes the same predictions as Heim's for atomic sentences, negation and conjunction. An **unorthodox model** for $\mathcal{L}_s$ is a pair $\langle \mathcal{P}(W), \cdot[\cdot]^* \rangle$ such that for all contexts $c \in \mathcal{P}(W)$, $p \in \mathtt{At}$ and $\alpha, \beta \in \mathcal{B}$, $\cdot[\cdot]^*$ satisfies the following constraints.*

***Conservativity**: $c[\alpha]^* = c[\alpha]$,*

***Materiality**: $c[\alpha \to \beta]^* = c - (c[\alpha] - c[\alpha][\beta])$,*

***Strict Negation**: $c[\neg(\alpha \to \beta)]^* = \{w \in c \mid c[\alpha][\beta] \neq c[\alpha]\}$.*

In light of the Conservativity constraint, an unorthodox model agrees with both Heim's and the dynamic strict conditional view on how to treat non-conditional sentences. In light of the Materiality constraint, such a model shares the predictions of Heim's view with respect to plain conditionals but, in light of the Strict Negation constraint, it sides with the dynamic strict conditional view for negated conditionals. Accordingly, an unorthodox model would combine the benefits and rid us of the drawbacks of both of the views discussed above.

Before we proceed, let me make two general remarks about the Materiality and the Strict Negation constraint. First, as mentioned above, the Materiality constraint tells us to treat assertions of (plain) indicative conditionals essentially as assertions of the material conditional, a view that has a bad reputation in the literature on the *semantics* of indicative conditionals. Hence, we may worry that well known objections to the view carry over to any view about the discourse dynamics of $\mathcal{L}_s$ that accepts Materiality. In response to this worry, note first that the arguably strongest objection against the view is related to its predictions about the interaction of conditionals and negation. That worry is circumvented by any unorthodox model, since negated conditionals are treated in terms of Strict Negation. Second, note that the constraint is formulated with respect to a small fragment of language and is not intended as a constraint that holds for fragments larger than $\mathcal{L}_s$. Hence, objections involving operators other than conditionals, negation and conjunction do not apply out of the box. Finally, let me highlight that semantic objections straightforwardly apply only if we adopt a dynamic approach to meaning. But this is a supposition we can reject (something we will come back to later).[8]

Let us turn to the Strict Negation constraint. One, general objection the dynamic strict conditional view faces is that felicitous assertions (assertions that do not crash the context) of simple conditional sentences are never informative. This objection is circumvented for plain conditionals, if an unorthodox model is assumed (remember, Heim's view predicts assertions

[8]It is interesting to note that recent work in bayesian epistemology has led to views according to which bayesian updating on conditional information should be spelled out in terms of updating on a the corresponding material conditional (see Eva et al., 2020; Günther, 2018). While these are not views about discourse dynamics, I take them to provide, at least, indirect evidence in favour of the materiality constraint.

of conditionals to be informative in appropriate contexts). However, it still holds for unorthodox models when focusing on simple, negated conditionals. In response to this worry, let me highlight that I do not think that the aim of asserting a sentence must be to contribute information to the context of conversation. Some assertions may just be ways of making sure that the context stays the way it is. That said, let me mention one unorthodox view that treats negated conditionals as informative in appropriate contexts.

Definition 3.2 (An unorthodox Boethian view)**.** *Let* $\cdot[\cdot]$ *be an update function that makes the same predictions as Heim's for atomic sentences, negation and conjunction. An* ***unorthodox Boethian*** *model for* $\mathcal{L}_s$ *is a pair* $\langle \mathcal{P}(W), \cdot[\cdot]^* \rangle$ *such that for all contexts* $c \in \mathcal{P}(W)$*,* $p \in \mathtt{At}$ *and* $\alpha, \beta \in \mathcal{B}$*,* $\cdot[\cdot]^*$ *satisfies the following constraints.*

Conservativity*:* $c[\alpha]^* = c[\alpha]$*,*

Materiality*:* $c[\alpha \to \beta]^* = c - (c[\alpha] - c[\alpha][\beta])$*,*

Boethian Negation*:* $c[\neg(\alpha \to \beta)]^* = c[\alpha \to \neg\beta]^*$*.*

Boethian Negation is a dynamic version of a claim sometimes referred to as Boethius' Theses: the claim that negated conditionals reduce to a corresponding plain conditionals with a negated consequent.[9] To illustrate, consider an assertion of

(6) It is not the case that if a member of the ground staff did it, it is the driver.

With respect to the context of our above vignette, the view predicts:

$$
\begin{aligned}
\{w_b, w_d, w_g\}[\neg(g \to d)]^* &= \{w_b, w_d, w_g\}[g \to \neg d]^*, && \text{by Boethian Negation} \\
&= \{w_b, w_d, w_g\} && \text{by Materiality} \\
&\quad - (\{w_b, w_d, w_g\}[g] - \{w_b, w_d, w_g\}[g][\neg d]), \\
&= \{w_b, w_d\}. && \text{by Conservativity}
\end{aligned}
$$

Hence, an assertion of (6) is predicted to be informative in this context. Whether this prediction is empirically adequate (or superior to Strict Negation) is a question I wish to leave open here. The key arguments presented below do not depend on whether an unorthodox or an unorthodox Boethian view is chosen.

3.2. Compositionality Conflicts

In light of the previous sections, it seems we should either opt for an unorthodox or an unorthodox Boethian model of conversation rather than Heim's or the dynamic strict conditional view. What are the prospects of providing such models in light of what we have called the guiding slogan of dynamic semantics? Let us turn to this question in this section.

According to the guiding slogan, meanings are context change potentials. As argued above, this entails that our notion of update plays *both* the role of modeling the context change potentials of the fragment we are interested in as well as the compositional mechanisms governing the meanings of that fragment. Above we thought of 'compositional' as 'recursively defined' in a loose

[9] (Wansing, 2023) contains a helpful discussion of Boethius' Thesis, albeit in a non-dynamic setting. See also (Santorio, 2022), for a more recent discussion. More on Santorio's view below.

sense. However, typically theories of meaning are taken to satisfy a stronger compositionality constraint.

To formulate the constraint let us adopt some notation. Let $\mathcal{L}$ be an arbitrary fragment of language and ϕ, ψ and χ sentences of that fragment. Then $\chi^{\phi/\psi}$ denotes that sentence in $\mathcal{L}$ that is just like χ except that any occurence of ϕ is replaced by ψ. Given this, the compositionality constraint I have in mind goes as follows:

Definition 3.3 (Compositionality (General))**.** *A theory of meaning for a fragment of language $\mathcal{L}$ is compositional, just in case for all $\phi, \psi, \chi \in \mathcal{L}$ our theory satisfies*

$$\textit{if } \textit{meaning}(\phi) = \textit{meaning}(\psi), \textit{ then } \textit{meaning}(\chi) = \textit{meaning}(\chi^{\phi/\psi}).$$

That is, if ϕ and ψ have the same meaning according to the theory, so do χ and $\chi^{\phi/\psi}$.

The constraint is motivated by a fairly standard approach to semantic theorizing. Typically, if a sameness in meaning for two sentences is postulated, this can be tested for by looking at what is called the *embedding behaviour* of the sentences in question. That is, we expect no difference in meaning to occur if we look at the sentences in embedded environments. The other way around, if two sentences differ in meaning at an embedded context, our principle tells us that they must differ in meaning when occurring unembedded.

While all of this is well known, let us illustrate the principle by looking at a classic argument from embedding behaviour, the argument for why Moore paradoxical sentences are not contradictions.

(7)	(ϕ)	It is raining but I do not believe it is raining,
(8)	(ψ)	I believe it is raining and I do not believe it is raining,Ê
(9)	(χ)	Suppose it is raining but I do not believe it is raining,
(10)	$(\chi^{\phi/\psi})$*Boethian*	Suppose I believe it is raining and I do not believe it is raining.

(7) and (8) sound equally infelicitous. But (9) seems felicitous while (10) is not. Hence, there is a difference in meaning between (10) and (9). So, by our principle, (7) and (8) must differ in meaning as well. Note that this argument in favour of a difference in meaning between (7) and (8) would not be valid without the just mentioned compositionality constraint.

What does the principle tell us about the guiding slogan? Since according to the guiding slogan meanings are context change potentials, and context change potentials of sentences are modelled with the help of an update function of a model of conversation, it seems reasonable to require a corresponding model of conversation to be compositional in the following sense (see also Rothschild and Yalcin, 2016: for a discussion of such a constraint in a dynamic setting):

Definition 3.4 (Compositionality (Dynamic))**.** *A model of conversation for a fragment of language $\mathcal{L}$ is compositional just in case for all $\phi, \psi, \chi \in \mathcal{L}$, its update function $\cdot[\cdot]$ satisfies*

$$\textit{if } \cdot[\phi] = \cdot[\psi], \textit{ then } \cdot[\chi] = \cdot[\chi^{\phi/\psi}].$$

Now, both Heim's view (Def. 2.4) as well as the dynamic strict conditional view (Def. 2.5), satisfy this compositionality constraint. However, given our above discussion, it should not come as a surprise that this compositionality constraint is in conflict with choosing an unorthodox or

an unorthodx Boethian model of conversation. To be precise, let us call a model of conversation *non-trivial*, if it can handle cases like the Crime at the Mansion example discussed above.[10] We then have the following proposition.

Proposition 3.1. *No unorthodox nor an unorthodox Boethian model of conversation that is non-trivial is compositional in the sense of Def. 3.4.*[11]

Hence, providing an unorthodox (Boethian) model of conversation for $\mathcal{L}_s$ is in tension with a popular and widespread notion of what it is for meanings to be compositional.

In the next section we will look at potential ways to resolve the tension. We first give the outlines of a radical solution, a solution that gives up on dynamic approaches to meaning by choosing a non-semantic approach to discourse dynamics. We close the paper by looking at the two most promising ways for fans of the guiding slogan to respond to the conflict.

4. Escaping the Conflict

4.1. Rejecting the Guiding Slogan

As mentioned above (Def 2.3), there are non-semantic approaches to discourse dynamics which distinguish between providing a theory of meaning that tells us about the compositional mechanisms governing the fragment of language we are interested in, and a theory of the discourse dynamics, telling us about the context change potentials of the fragment. On such a view theories of meaning are linked to a models of conversation capturing the discourse dynamics of the fragment of language via some bridge principle. But meanings do not provide a theory of the context change potentials directly. Distinguishing between meanings and context change

[10] Even more precisely, let us call a model of conversation non-trivial if we have a set of contexts $\mathcal{P}(W)$ based on a set W containing at least three possible worlds w, w' and w'' such that there are sentences α and β with α true at w', w'' and β true at w'' only.

[11] The proof basically mirrors the example we discussed above. But, for the sake of completeness, here are the details. Let W contain at least three worlds w and w' and w''. Then there is a context $c \in \mathcal{P}(W)$ such that $c := \{w, w', w''\}$. Now pick a sentence α that is true at both w' and w'' (and false at w) as well as a sentence β that is true at w'' only. Let us consider the conditional $\alpha \rightarrow \beta$. By Conservativity and Materiality we have for any context c,

$$c[\alpha \rightarrow \beta]^* = c[\neg(\alpha \wedge \neg\beta)]^*.$$

Moreover, by Conservativity we have $\{w, w', w''\}[\neg\neg(\alpha \wedge \neg\beta)] = \{w'\}$.
Now, Strict Negation entails

$$\{w, w', w''\}[\neg(\alpha \rightarrow \beta)]^* = \varnothing$$

and Boethian Negation entails

$$\begin{aligned}\{w, w', w''\}[\neg(\alpha \rightarrow \beta)]^* &= \{w, w', w''\}[\alpha \rightarrow \neg\beta]^* \\ &= \{w, w'\}\end{aligned}$$

Thus, in light of Materiality and Conservativity $\alpha \rightarrow \beta$ and $\neg(\alpha \wedge \neg\beta)$ have the same context potential. But the context change potentials of $\neg(\alpha \rightarrow \beta)$ and $\neg\neg(\alpha \wedge \neg\beta)$ come apart, no matter whether an unorthodox or an unorthodox Boethian model is chosen.

potentials in this way may help to marry our compositionality constraint with an unorthodox approach to the discourse dynamics of $\mathcal{L}_s$.

As a proof of concept, let us put one such approach on the table. The approach I have in mind can be developed in three steps.

Step 1. As noted above, there are many ways to interpret the kind of formal structure we called a model of conversation (Def. 2.1). In particular, if contexts are taken to be sets of possible worlds, they are the same kind of formal object we typically call a *proposition*. Hence, we may think of an update function, $\cdot[\cdot]$, as telling us about the proposition, $c[\phi]$, expressed by ϕ at c. Accordingly, an update function can be interpreted *non-dynamicaly*, as a theory of meaning according to which meanings are functions from contexts to propositions. A defender of a dynamic approach to meaning would, in addition, be committed to the claim that the proposition, $c[\phi]$, expressed by ϕ at a context c happens to be the context our context c will evolve to in light of an assertion of ϕ at c. On a non-dynamic interpretation of an update function, there is no such commitment. Let us refer to the update function from Def. 2.5 so interpreted as the *strict conditional view in its non-dynamic interpretation*.

Step 2. Contexts, as we have been using the term, are a special kind of state of information. Like other states of information, such as states of belief or knowledge, they settle some sentences about the world but not others. For instance, the context $\{w_d, w_g\}$ (consisting of the worlds in which the driver and the gardener are the sole culprits respectively) settles the sentence 'a member of the ground staff is the culprit' but it settles neither 'The gardener is the culprit' nor 'The gardener is not the culprit'. Any model of conversation for a fragment of language, whether interpreted dynamically or non-dynamically, induces a notion of of what it is for a sentence to be settled by a context, typically referred to as a notion of *support*.

The strict conditional view, for instance, gives rise to the following notion of support.

Definition 4.1 (Strict Support)**.** *Let $\cdot[\cdot]$ be the update function for the dynamic strict model of conversation $\langle C, \cdot[\cdot]\rangle$. For any context $c \in C$ and any $\phi \in \mathcal{L}^s$*

$$c \textit{ supports } \phi \textit{ iff } c[\phi] = c.$$

Such a notion of support is standard in the dynamic semantics literature and is typically used to define notions of semantic consequence. In this paper, however, we just take it as a claim about the conditions under which a state of information c settles a sentence ϕ.

Step 3. Let us take the strict conditional view in its non dynamic interpretation on board. Further, let us use its notion of strict support (Def. 4.1) to define the following notion of update:

Definition 4.2 (Informational Updating)**.** *Let a set of contexts $\mathcal{P}(W)$ be given and let our notion of support be defined as in Def. 4.1 . Then for any $c \in \mathcal{P}(W), \phi \in \mathcal{L}_s$, we define*

$$c[\phi]^{\ddagger} = \bigcup\{c' \subseteq c \mid c \textit{ supports } \phi\}. \qquad (\ddagger)$$

$\langle \mathcal{P}(W), \cdot[\cdot]^{\ddagger}\rangle$, so defined, is a model of conversation for our language $\mathcal{L}_s$. A notion of update similar to this one was recently put forward by Santorio (2022) (more on his account below) but

the idea of thinking about conversational update in terms of a notion of support, can be traced back to (Yalcin, 2007: p. 464).[12] The intuition behind the update function proposed in Def. 4.2 can be put as follows: From the perspective of the discourse participants, it is natural to assume that the speaker has information available to her that settles the sentence asserted and that she made her assertion with the aim of coordinating on a context that agrees with the information available to her. $\cdot[\cdot]^{\ddagger}$ models the idea that, in light of the just mentioned assumptions, discourse participants will rule out all those possibilities from the context that are incompatible with *any* way of coordinating on a context that settles the sentence asserted.[13]

While a thorough discussion of the predictions and consequences of the view is beyond the scope of this paper, let me highlight the following proposition.

Proposition 4.1. *The model of conversation* $\langle \mathcal{P}(W), \cdot[\cdot]^{\ddagger} \rangle$ *for* $\mathcal{L}_s$ *as defined in Def. 4.2, is an unorthodox model of conversation.*[14]

So, in terms of a discourse dynamics for $\mathcal{L}_s$, $\cdot[\cdot]^{\ddagger}$ gives us what we want. But notice that ($\ddagger$) can be thought of as a bridge principle, linking a theory of meaning (the strict conditional view in its non-dynamic interpretation) to a theory about the discourse dynamics of the sentences in $\mathcal{L}_s$. While, by proposition 3.1, our new notion of update is not compositional, the theory of meaning in terms of which it is defined satisfies our compositionality constraint from Def. 3.4. Hence, on this view we have a non-semantic approach to discourse dynamics (as in Def. 2.3), an approach according to which meanings are compositional but context change potentials are not.

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[12] The discussion in Yalcin's paper is kept informal and no concrete update function is proposed.

[13] Let me highlight a feature of the view that, at a first glance, may seem odd. For sentences in our small conditional fragment, $\mathcal{L}_s$, asserting a sentence will always result in a context that supports the sentence asserted (in the sense of Def. 4.1). What may seem odd, however, is that this is not the case for conditional fragments larger than $\mathcal{L}_s$. For instance, (assuming we treat disjunction in terms of conjunction and negation in the usual way) assertions of compound conditionals of the form $(\alpha \to \beta) \vee (\alpha \to \gamma)$ may result in an updated contexts that do not support the sentence asserted. This entails that if the information available to the speaker settles the sentence asserted, the speaker must have more information about the world than is conveyed by her assertion. While a thorough discussion of this consequence of the view is beyond the scope of this paper, let me note that I do not think that it constitutes a problem. What it shows is that at some contexts, some sentences are not particularly effective means to communicate the information available to the speaker, a consequence that should not come as a surprise.

[14] Here is a proof. It is well known that, given Heim's entries. $c[\alpha] = c \cap W[\alpha]$, for any $\alpha \in \mathcal{B}$. It is not difficult to check that for any context c and any $\alpha \in \mathcal{B}$, $c \cap W[\alpha]$ will be the unique largest sub-context of c that supports α. Hence, for all $\alpha \in \mathcal{B}$, $\bigcup\{c' \subseteq c \mid c' \text{ supports } \alpha\} = c \cap W[\alpha] = c[\alpha]$. So, *Conservativity* follows.

Materiality follows since

$$\begin{aligned} c[\alpha \to \beta] = c \text{ iff } & c[\alpha][\beta] = c[\alpha], && \text{by (SC)} \\ \text{iff } & c = c - (c[\alpha] - c[\alpha][\beta]). && \text{by set theory \& Def. 2.5} \end{aligned}$$

That is, a conditional is supported on c exactly when $c = c - (c[\alpha] - c[\alpha][\beta])$. But, since for all contexts c, $c - (c[\alpha] - c[\alpha][\beta]) = c[\neg(\alpha \wedge \neg\beta)]$, a conditional is supported on c if and only if $\neg(\alpha \wedge \neg\beta)$ is supported. Since, $\neg(\alpha \wedge \neg\beta) \in \mathcal{B}$, the claim follows from *Conservativity*.

Strict Negation holds, trivially, whenever $c = \varnothing$. For non empty context c, c either does contain an α-world that is not a β-world or it does not. In the first case Def. 4.1 tells us that c itself is the (unique) largest sub-context of c that supports $\neg(\alpha \to \beta)$ while in the latter it tells us that it must be $\varnothing$. Hence, in the former case we have $\bigcup\{c' \subseteq c \mid c' \text{ supports } \neg(\alpha \to \beta)\} = c$ while in the latter we have $\bigcup\{c' \subseteq c \mid c' \text{ supports } \neg(\alpha \to \beta)\} = \varnothing$. So *Strict Negation* holds in each case.

Two remarks before we move on. First, it may be helpful to note that our input notion of meaning (the update function of the strict conditional view in its non-dynamic interpretation) need not be given in terms of a model of conversation. There are many truth-conditional approaches which give rise to the exact same support relation between contexts and sentence in $\mathcal{L}_s$ as the one given in Def. 4.1 (see Yalcin, 2007; Kolodny and MacFarlane, 2010: among others). Second, we may wonder whether there is a similar way of generating unorthodox Boethian models. Above we mentioned Santorio's paper (Santorio, 2022) which develops a new approach to the semantics of conditionals he refers to as a *path-semantics*. Introducing this view is beyond the scope of this paper. However, his semantics gives rise to a notion of update which does satisfy an analog of our Materiality constraint and an analog of Boethius' Theses. Hence, fans of unorthodox Boethian models may look to Satorio's notion of update as an option. Moreover, his path-semantics is, at its core, truth-conditional. It does satisfy a truth-conditional version of our compositionality constraint from Def. 3.3. Hence, his approach can be interpreted as another view according to which meanings are not context change potentials. On Satorio's approach too, a non-compositional notion of update is defined in terms of a compositional notion of meaning.

4.2. Holding on to the Guiding Slogan

Now, from the perspective of dynamic semantics, the above mentioned approaches are radical. According to these views, meanings are not context change potentials. Hence, it is natural to ask whether there are ways to capture what is appealing about unorthodox models while holding on to the guiding slogan. Let me close the paper by saying something about the two most promising routes fans of the guiding slogan could explore.

The first route starts with the observation that the arguments we looked at above are not arguments about conditionals alone but arguments about the *interaction* of conditionals with negation. Accordingly, we may hope to arrive at an unorthodox view by treating negation more flexibly.

Indeed, there are views in the dynamic semantics literature which do just that. For instance, Malte Willer recently defended a so-called *bilateral* approach to updating (see Willer, 2022). Like the views discussed above, Willer treats contexts as sets of possible worlds. However, on his view, we have *two* notions of update to work with: we have one update function that governs *coming to accept* a sentence and one that governs a notion of *coming to reject* a sentence. The first notion is intended to tell us about how assertions affect the context of a conversation. The latter plays a special role in telling us about the update potentials of negated sentences. On his view, coming to accept a negated sentence, $\neg\phi$, is coming to *reject* the non-negated sentence ϕ. This allows for a much more flexible treatment of negated sentences. Unfortunately, Willer's own view gives rise to neither an unorthodox nor an unorthodox Boethian model of conversation. The reason is that his entry for plain conditionals does not satisfy the materiality constraint but mimics the entry of the dynamic strict conditional view. Hence, the view does not help us out of the box.[15] However, there may be a way of modifying the view so that we

[15]Moreover, the view is subject to the same objection we put forward against the dynamic strict conditional view above.

do arrive at an unorthodox (Boethian) model of conversation. While such a view would have the advantage of providing an update function that fits our needs *without* the help of a bridge principle, it would (in light of Proposition 3.1) violate the compositionality constraint form Def. 3.4. While there may be weaker compositionality principles such a view would satisfy, note that the compositionality constraint form Def. 3.4 straightforwardly explains the rationale behind arguments from embedding behaviour. Hence, rejecting the principle would force us to either reject such arguments as a guide to meaning or to come up with an alternative story about why such arguments work. Whether such a response is ultimately successful will have to be explored in future work.

The second route builds on the observation that, for the most part, we made use of a very simple notion of context. We only looked at contexts as modelled by sets of possible worlds. While simple possible worlds models are adequate for modeling information gain in light of assertions, we may wish to capture other discourse effects such as changes in the question under discussion (Roberts, 2012) or changes in what possibilities are taken to be salient at a conversational context (see Willer, 2013: for instance). Once, we switch to such a more involved, and maybe more adequate, notion of context, we may be in a position to capture what made unorthodox models appealing, while rejecting one of the core assumptions involved.

This is certainly an avenue we may want to explore. But note that changing our notion of context *may* but *need not* marry our compositionality constraint with what is appealing about unorthodox (Boethian) models. Moreover, at this point I do not see how modeling changes in the question under discussion or the introduction of a notion of salience will help to resolve the problem. Maybe a third, hitherto unexplored, parameter may help to marry the guiding slogan with the compositionality constraint from Def. 3.4. Whether there is such a parameter that resolves the above mentioned tension in a satisfying way has to be explored elsewhere.

5. Conclusion

I have argued that for a small fragment of language $\mathcal{L}_s$, we should choose an unorthodox model of conversation; a model that combines the benefits and rids us of the drawbacks of both Heim's as well as the dynamic strict conditional view. At a minimum such a view makes superior predictions to the dynamic strict conditional view, a view that is widespread in the dynamic semantics literature. But we have seen that if we assume a dynamic approach to meaning, any such view is in conflict with a popular and widespread notion of what it is for meanings to be compositional. The conflict disappears if we give up on the guiding slogan of dynamic semantics; the claim that meanings are context change potentials. We can, as we have seen, provide a non-compositional theory of the discourse dynamics of $\mathcal{L}_s$ in terms of a compositional notion of meaning. I pointed to two ways in which defenders of the guiding slogan could respond to the tension between our compositionality principle and unorthodox approaches to the discourse dynamics of $\mathcal{L}_s$. Both are interesting avenues to pursue. Deciding whether one of those avenues leads to an alternative answer to the puzzle presented in this paper, an answer which is more friendly to dynamic approaches to meaning, will have to be explored in future work.

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