On the semantics of unit fractions[1]

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Abstract. We present two novel observations concerning the linguistic behavior of unit fractions, e.g., *half, third* etc., which challenge their analysis as proportional quantifiers/modifiers, arguing instead that in certain environments fractions presuppose *contextually salient* partitions over individuals. We distinguish environments that require a salient partition from those that do not, and propose a syntax and semantics for fractions that derives the distinction.

Keywords: fractions, *half*, quantifiers, numerals, partitions.

1. Introduction

This paper is concerned with the semantics of unit fractions (hereafter, UFs); i.e., *half, quarter, eighth,* or generally fractions of the form $1/n$. In the generalized quantifier literature, UFs and numerals have been treated, explicitly or implicitly, as quantificational determiners (1) (cf. Keenan and Westerståhl 1997). This approach is supported by the ability of UFs and numerals to surface without an overt determiner (2), suggesting they themselves might be the determiner of the NP of which they are a part. However, since both numerals and UFs can surface under overt determiners as well (3)-(4), the determiner approach seems untenable.[2] Indeed, since Bartsch 1973, numerals are often treated as adjectival modifiers; an approach that has been extended to fractions in Ionin et al. 2006. In cases like (2) where no overt determiner appears, the modifier approach posits a silent existential quantifier above both numerals and UFs.

(1) a. $[\![half]\!] = \lambda f_{\langle e,t \rangle}.\lambda g_{\langle e,t \rangle}.\ |f \cap g| = (1/2 \times |f|)$
  b. $[\![five]\!] = \lambda f_{\langle e,t \rangle}.\lambda g_{\langle e,t \rangle}.\ |f \cap g| = 5$

(2) a. Half of the students passed the exam.
  b. Five (of the) students passed the exam.

(3) a. A half of the students passed the exam.
  b. The tall half of the students passed the exam.[3]

(4) The five students were found hiding behind a willow tree.

Under the modifier approach, *half of the students* denotes the set of all student pluralities whose cardinality is equal to half the cardinality of the maximal plurality of students (cf. Ionin et al.,

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[2]For a survey of arguments against the quantificational determiner approach see Bylinina and Nouwen 2020.

[3]Given the uniqueness presupposition introduced by definite determiners, embedding a UF under a definite determiner without an additional modifier – e.g., *the half of the students* – sounds odd out of context, as there is no contextually unique half to which the definite can refer. This, however, changes in a context that makes salient a unique $1/n^{th}$. For instance, if a quarter of the students and half of the post-docs in a department came to the field-trip, the following sentence seems felicitous: *The quarter of the students walked faster than the half of the post-docs.* To avoid specifying a context of utterance, we embed UFs under definite determiners only with an additional modifier, which seems to help accommodate a unique $1/n^{th}$ that satisfies the determiner's presupposition.

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2006: 6 ex. (18)). Similarly, *five (of the) students* denotes the set of all plural individuals whose cardinality is 5. A straightforward way of implementing this, which we term here *the standard approach*, involves assigning *half* and *five* the lexical entries in (5a)-(5b).[4]

Note that in (5) and for the remainder of the paper we assume: (i) that $|X|$ counts atomic parts of X if X is an individual, and counts the number of elements in X if X is a set; (ii) that the characteristic set of f (i.e., the $\langle e,t \rangle$ input to both numerals and UFs) is closed under sum-formation; (iii) that $\oplus f$ is the maximal element in f (the plural individual $x \in f$ such that every element in f is a part of x); and (iv) that $[\![students]\!] = [\![of\ the\ students]\!]$.[5]

(5) Unit fractions: *the standard approach*
  a. $[\![half]\!] = \lambda f_{\langle e,t \rangle}.\lambda x.\ f(x) \wedge |x| = (1/2 \times |\oplus f|)$
  b. $[\![five]\!] = \lambda f_{\langle e,t \rangle}.\lambda x.\ f(x) \wedge |x| = 5$

If the standard approach is correct, we expect certain logical equivalences between numerals and unit fractions to hold. In particular, given a domain which contains 10 students, the approach predicts *five (of the) students* and *half of the students* to both denote the set containing all student-pluralities of cardinality 5. This prediction can be generalized as in (6).

(6) **Prediction of the standard approach:** Given a set f, numeral n_i and UF $1/n_j$, such that the maximal element in f contains $n_i \times n_j$ atoms, $[\![n_i]\!](f) = [\![1/n_j]\!](f)$

This paper introduces several novel observations that constitute exceptions to the prediction in (6). We show that in certain environments using the phrase *half of the students*, requires that the context of discourse make salient a partition of the relevant students into two equal parts; a requirement that is not imposed on the corresponding numeral. We propose to account for the data by analyzing UFs as modifiers that take a variable ranging over partitions as a syntactic argument, and argue that in the particular environments where UFs differ from numerals, the value of their partition variable must be contextually determined. We then turn our attention to environments where UFs pattern with numerals and do *not* require a salient partition of their input set, and show that our partition-based semantics can account for UFs in those environments by allowing an existential quantifier over partitions to occupy the partition argument slot.

The idea that partitions play a role in the semantics of UFs is not itself new. In fact, Ionin et al. (2006) incorporate existentially closed partitions in the lexical entries of both UFs and numerals. However, we make two novel contributions to the debate over the semantics of UFs. First, we show that with respect to the prediction in (6), existentially quantifying over partitions is equivalent to the standard approach and is thus insufficient. Crucially, the partition in the semantics of UFs must be a syntactic variable, which in certain contexts is *not* existentially bound. Second, we present a novel generalization according to which whether UFs pattern like numerals or not (i.e., whether they verify (6) or not) is determined by their syntactic environment: only UFs under indefinite determiners pattern like numerals, while those under definite ones do not. Finally, we offer some remarks about how this generalization can be derived.

[4] In (5b) we are not committing to a non-intersective account of numerals, but merely having the numeral take the element that it modifies as an argument to maintain parallelism with (5a), where this is necessary.

[5] By assuming that $[\![of\ the\ students]\!] = [\![students]\!]$ we are not committing to the position that '*of*' and '*the*' in that PP are semantically vacuous. Rather, we adopt the following entry for partitive *of* from Ionin et al. 2006: $[\![of]\!] = \lambda x.\lambda y.y \leqslant x$. This entry, together with the assumption that a plural definite description like *the students* denotes the maximal student (i.e., $\oplus[\![students]\!]$), results in $[\![of\ the\ students]\!] = \lambda x.x \leqslant \oplus[\![students]\!] = [\![students]\!]$.

We proceed as follows: Section 2 presents constructions with UFs that challenge the standard numeral-style modifier approach. Section 3 then illustrates how the challenging data can be accounted for by introducing partition variables to the syntax of UFs, and how binding these variables existentially accounts for constructions that the standard approach captures correctly. In section 4 we provide a characterization that distinguishes the two kinds of constructions, and remark on how one might derive the distinction in section 5. Section 6 concludes.

2. Problems for the standard approach

According to the prediction of the standard approach in (6), given a domain with 10 students, (7a) and (7b) should be equivalent to each other. This is clearly borne out when UFs surface without an overt determiner, which on the standard approach, indicates that they are embedded under a covert existential. When there are ten students in a class, the sentence with the numeral *five* in (8a) is truth-conditionally equivalent to the sentence with the UF *half* in (8b).

(7) a. $[\![\textit{five of the students}]\!] = \lambda x.\ [\![\textit{students}]\!](x) \wedge |x| = 5$
b. $[\![\textit{half of the students}]\!] = \lambda x.\ [\![\textit{students}]\!](x) \wedge |x| = \frac{|\oplus[\![\textit{students}]\!]|}{2}$

(8) Context: There are 10 students in a class.
a. Five of the students passed the exam.
b. Half of the students passed the exam.

However, the equivalence breaks down in two environments. First, in *which*-questions. Consider (9), where it is stipulated that there are ten students in a class, five of whom passed the exam. In this context, the question with *five* (9b) is felicitous, given that the presupposition of the *which*-question that five students passed is met.[6] Yet the example with *half* (9a) is infelicitous, even though the presupposition that half of the students passed is also met.

(9) Context: A class has ten students. It is known that five of them passed the exam. I want a list of names of students who passed, so I ask:
a. #Which half of the students passed the exam?
b. Which five of the students passed the exam?

Interestingly, when the context partitions the students into two specific halves, the *which*-question in (9a) becomes felicitous. In (10), for instance, there is a contextually salient division of the students into equi-sized groups of computer science (henceforth, CS) majors and math majors, and the *which*-question with *half* can be used to inquire about which of the two groups passed the exam. As expected, the numeric counterpart is also felicitous. Thus, what sets UFs apart from numerals in *which*-questions, is that only the former require a contextually salient partition. This problem for the standard approach generalizes to any UF, like *third*, *fourth* etc.

(10) Context: A 10-student class consists of two groups: one group of 5 math majors, and another of 5 CS majors. It is known that one of these groups passed the exam.
a. Which half of the students passed the exam?
b. Which five of the students passed the exam?

[6] This presupposition follows from the requirement of questions to have a maximally informative true answer (Dayal, 1996)

The second challenge to the standard approach comes from the behavior of a specific UF, *half*, and its corresponding numeral in definite descriptions with a comparative/superlative modifier; i.e., in descriptions schematized in (11). To see why the standard approach fails here, some setup is required: it seems to be a general characteristic of comparatives that they are only licensed in definite descriptions when the predicate they modify denotes a set containing exactly two elements. Superlatives, on the other hand, seem to be licensed in definite descriptions only when their input set contains more than two elements. Consider (12): when there are only two students, only the comparative is licensed (12a), and when there are more than two only the superlative is (12b). We therefore take for granted the generalization in (13).

(11) $[_{DP}$ [the] $[[_{AP}$ A+{-er/-est}] $[_{NP}$ N]]]

(12) a. Of these two students, give the award to the {smarter / #smartest} student.
 b. Of these ten students, give the award to {#smarter / smartest} student.

(13) Given a predicate f of type $\langle e,t\rangle$ and a gradable predicate A of type $\langle d,et\rangle$:
 a. $[\![$*The A+er f*$]\!]$ is defined only if $|\{x : f(x) = 1\}| = 2$
 b. $[\![$*The A+est f*$]\!]$ is defined only if $|\{x : f(x) = 1\}| > 2$

Now, consider the behavior of *half* and a corresponding numeral in these constructions. In the context in (14), there are ten students. Thus the predicate $[\![$*five of the students*$]\!]$ is clearly true of more than one entity; it is true of any 5-sized plurality of students (so it is true of *10 choose 5*, or 252, entities). The principle in (13) thus correctly predicts that only the superlative modifier should be licensed in a definite description with the numeral *five*, as shown in (14a). Given that the standard approach predicts equivalence between $[\![$*five of the students*$]\!]$ and $[\![$*half of the students*$]\!]$ in this context, we expect that of the counterparts with *half* in (14b), only the superlative will be licensed as well. Yet the opposite pattern emerges with *half*.

(14) **Context**: A class consists of ten students. Five of them passed the exam.
 a. (i) ??The smarter five of the students passed the exam.
 (ii) The smartest five of the students passed the exam.
 b. (i) The smarter half of the students passed the exam.
 (ii) ??The smartest half of the students passed the exam.

Note, importantly, that if the generalization we take for granted in (13) is correct, what the felicity of the comparative modifier in (14b) teaches us is that the denotation of $[\![$*half of the students*$]\!]$ must be a set containing only two elements.

We argue next that these counterexamples to the standard approach demonstrate a unique property that distinguishes UFs from numerals; namely, that in *which*-questions and superlative/comparative DPs, UFs must be evaluated relative to a contextually salient partition.

3. Proposal: A partition-based semantics for UFs

To implement our account of the data above, we adopt the notion of *partition* of a plural individual. Informally, a set S partitions a plural individual x if every atomic part of x is a part of an element in S, and all the elements in S are disjoint; i.e., there is no individual that is a part of two distinct elements in S (cf. Higinbotham 1981; Schwarzschild 1996) – this is formally defined in (15). We then propose that a UF of the form ¹⁄n denotes the function in (16a) that takes as arguments a predicate f and a variable S over $\mathscr{D}_{\langle e,t\rangle}$, and presupposes that S is a specific kind of partition over the maximal individual in f – namely, a partitions that consists of *n equal* parts, as defined in (16b). If defined, the UF simply returns its partition argument S.

(15) S is a partition of an individual x iff: $\oplus S = x \wedge \forall s,s' \in S : \neg\exists y : y \leqslant s \wedge y \leqslant s'$

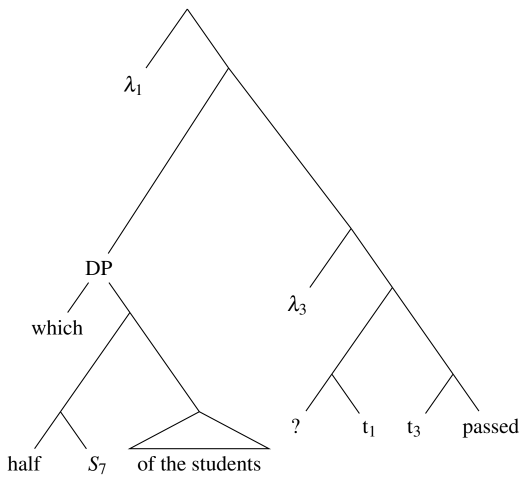
(16) a. $[\![1/n]\!] = [\![1/\Box]\!](n) = \lambda S_{\langle e,t\rangle}\lambda f_{\langle e,t\rangle}$: S is a partition$^{+}_{n}$ of $\oplus f$. S
b. S is a partition$^{+}_{n}$ of $\oplus f$ iff S is a partition of $\oplus f$, $|S| = n$, and $\forall s,s' \in S : |s| = |s'|$

In (16a) we decompose ¹⁄n into a fractionalizing function ¹⁄□ and the denominator n, but nothing below hinges on this decomposition. Given (16a), applying (e.g.) $[\![$ *half* $]\!]$ to $[\![$ *of the students* $]\!]$ derives a set containing two disjoint, plural individuals, each of cardinality half of $|[\![$ *the students* $]\!]|$.

3.1. Accounting for the challenges to the standard approach

We present above two enviornments in which the predicted equivalence between UFs and numerals seems to break down. Here is how our proposal accounts for that: First, in *which*-questions, according to the argument structure we assign UFs in (16a), and assuming a Karttunen (1977) syntax-semantics for questions (as implemented in von Fintel and Heim 2011), the *which*-question in (9a) (i.e., *which half of the students passed?*) has the structure in (17) – where the UF *half* takes a variable argument in addition to a set of individuals (which includes pluralities). This structure derives the truth-conditions in (18), which can be paraphrased as follows: The *which*-question is defined only if the variable argument of *half* (i.e., S_7 in (17)) is assigned to a partition of the maximal element in the set of students by the contextually determined assignment function g. If defined, the question denotes the set of propositions that are true only if there is an element in the contextually salient partition that passed the exam.

(17)



$[\![which]\!] = \lambda f_{\langle e,t\rangle}.\lambda g_{\langle e,t\rangle}.\ \exists x : f(x) \wedge g(x)$
$[\![?]\!] = \lambda p_{\langle s,t\rangle}.\lambda q_{\langle s,t\rangle}.\ p = q$
$[\![of\ the\ students]\!] = [\![students]\!]$ (see fn. 5)

(18) $[\![(17)]\!]^{g}$ is defined iff $7 \in \textsc{dom}(g)$ and $g(7)$ is a partition$^{+}_{2}$ of $\oplus[\![$ *students* $]\!]$; and
If defined, $[\![(17)]\!]^{g} = \lambda p_{\langle s,t\rangle}.\exists x \in g(7) : p = \lambda w.[\![passed]\!](w)(x)$

Given (18), the infelicity of the *which*-question in the context of (9), where context does not make salient a partition of the students into halves, is simply an instance of presupposition failure: Without a salient partition, the presupposition of the UF in the *which*-phrase is not met.

In the context of (10), however, the ten students are partitioned into two groups; one of five math majors and the other of five computer science majors. Suppose that $a \oplus b \oplus c \oplus d \oplus e$ are the math majors, and $f \oplus g \oplus h \oplus i \oplus j$ are the CS majors. If S_7 is assigned by the assignment function g to the partition $\{a \oplus b \oplus c \oplus d \oplus e, f \oplus g \oplus h \oplus i \oplus j\}$, $[\![(17)]\!]^g$ is defined, and denotes the set in (19). The question denotation in (19) can be paraphrased as *which of the two groups a,b,c,d,e and f,g,h,i,k passed*. This, intuitively, is exactly what we want to derive, as in (10), the inquirer wants to know which part of the salient partition passed the exam.

(19) $\{\lambda w.[\![passed]\!](w)(a \oplus b \oplus c \oplus d \oplus e), \lambda w.[\![passed]\!](w)(f \oplus g \oplus h \oplus i \oplus j)\}$

Our second challenge to the standard approach had to do with the behavior of *half* in particular, in definite descriptions with comparative/superlative modifiers. Recall that we show that even given a domain of ten students, $[\![$*half of the students*$]\!]$ is not equivalent to $[\![$*five of the students*$]\!]$ in that particular environment: While the latter behaves as if it is a set whose cardinality is *10 choose 5* (i.e., 252), the former seems to behave as if has cardinality 2. This is an immediate result of our analysis in (16a), according to which the number of elements in the set that results from applying the UF function to its input set is equal to the denominator of the UF. Thus, a phrase headed by *half* will always denote a set whose cardinality is 2. Note that we correctly predict that this phenomenon is restricted to *half*, and does not generalize to other UFs. Other UFs do not denote two-sized sets, and therefore, should only be compatible with a superlative modifier as per the generalization in (13). That this prediction is borne out is illustrated in (20).

(20)
a. The smartest quarter of the students passed.
b. ??The smarter quarter of the students passed.
c. $|[\![$*quarter of the students*$]\!]| = 4$, as a partition$^+_4$ of $\oplus[\![$*students*$]\!]$ has cardinality 4.

3.2. Accounting for cases that the standard approach captures correctly

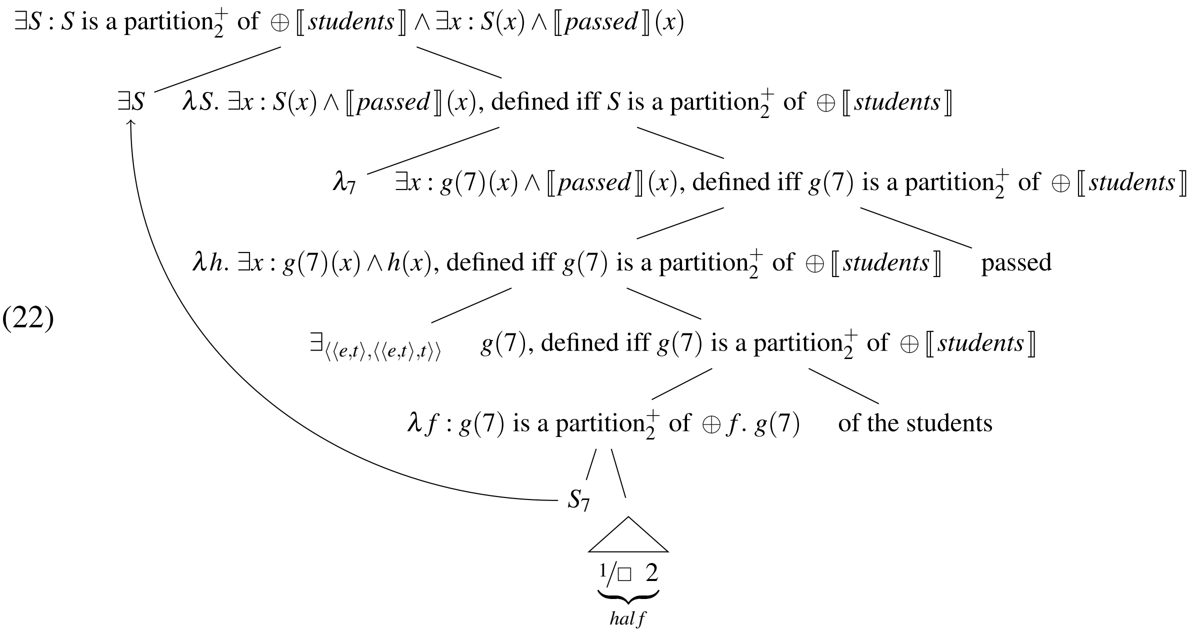
Recall that the standard approach was a good account of a subpart of the data; namely, of UFs that surface without an overt determiner above them (2). In those cases, it was assumed that the bare UFs are preceded by a covert existential quantifier, and the standard approach correctly predicts bare UFs to be equivalent to their corresponded numerals. In (21), for instance, using the UF *half* or a numeral to refer to sixteen students out of thirty-two does not seem to make a difference. Furthermore, contrary to what our entry for UFs in (16a) predicts, no salient partition of the students in the class is required for the use of *half* in (21a) to be felicitous.

(21) Context: Of the 32 students in a class, 16 passed the exam. The TA tells the professor:
a. Half of the students passed the exam.
b. Sixteen students passed the exam.

To capture the truth-conditions of (21a) without giving up on the entry in (16a), we propose that this sentence has the LF in (22), where the partition argument of the UF is occupied by a generalized existential quantifier over partitions, as defined in (23). Since UFs, as we define them, are of type $\langle et, \langle et, et \rangle\rangle$ and the quantifier in (23) is of type $\langle\langle et, t\rangle, t\rangle$, the quantifier has to QR and leave a $\langle e, t\rangle$-type trace.

This results in the truth conditions in (24), as long as we assume that the presupposition of $[\![half]\!]$ is locally accommodated below the generalized quantifier over partitions. Informally, (21a) is predicted by (24) to be true as long as there is some partition of the students into two halves such that all the members of one of the halves passed the exam. Thus, if we allow the partition variable of a UF to be existentially closed in the case of (21a), we correctly predict the truth-conditions for (21a) and (21b) to be equivalent – as the standard approach does.

(22)



(23) $[\![\exists S]\!] = \lambda P_{\langle et,t \rangle} . \exists S : P(C) = 1$

(24) $[\![(22)]\!]^g = 1$ iff $\exists S : S$ is a partition$_2^+$ of $\oplus [\![students]\!] \wedge \exists x : S(x) \wedge [\![passed]\!](x)$

3.3. Restricting the distribution of existentially closed partitions

To account for the behavior of UFs in *which*-questions and certain definite DPs, we introduce a partition argument into their lexical entry and saturate it with a free variable. To account for the behavior of bare UFs (i.e., UFs under a covert existential determiner), we suggest that in these environments a generalized quantifier over partitions can also occupy the partition argument slot. However, once we introduce existentially closed partitions, an immediate worry arises: if existentially closed partitions and contextually-valued variables over partitions are in a free distribution, do we not lose our account of *which*-questions and definite DPs? In the remainder of this section we use the case of *which*-questions to illustrate that, indeed, we do lose our account of UFs in that environment if we allow free existential closure of the partition variable.

To show conclusively that no existential closure of the partition argument is possible in *which*-questions, we must determine what the predicted truth-conditions would have been had closure been allowed. If in the question *which half of the students passed the exam?* the partition argument is occupied by a generalized existential quantifier, that quantifier must QR for type reasons — but where to? There seems to be only one possibility, if we want to maintain Hamblin's (1976) insight that questions denote sets of propositions: $\exists S$ must QR to a position between the *which*-phrase and the propositional binder, as in (25). (25) is identical to the LF in (17) but with existential closure of the partition argument.

(25) $[\lambda_1\ \exists S\ \lambda_7[_{\text{DP}}\text{which}[[\text{half } S_7] \text{ of the students}]][\lambda_3[[?\ t_1][t_3 \text{ passed}]]]]$

The truth-conditions derived from (25) are in (26). Informally, the question denotes a set of propositions true if any plurality of students whose cardinality is half of $\oplus[\![\mathit{students}]\!]$ passed the exam. Crucially, (26) is equivalent to what the standard approach predicts here. Thus, if (25) were a possible LF for *which half of the students passed the exam?*, we would incorrectly predict that no salient partition is required for this question to be defined, contrary to fact. We conclude that the partition argument cannot be existentially bound for UFs in *which*-phrases.

(26) $[\![(25)]\!]^g = \lambda p.\ \exists S: \exists x \in S: S \text{ is a partition}_2^+ \text{ of } \oplus[\![\mathit{students}]\!] \wedge p = \lambda w.[\![\mathit{passed}]\!](w)(x)$

If we claim that an UF's partition argument is a bound variable in some environments but is *mandatorily* free in others, then the onus is on us to distinguish environments that require existential closure from those that require free variables. Next, we attempt to do just that.

4. Definiteness *vs* Indefiniteness of the partition

In order to account for the different behavior of UFs in indefinites, *which*-questions and the particular kind of definite DPs discussed above, we have proposed that the partition argument of the UF can either be existentially closed or supplied by the context. However, we also had to comit to the position that these two options are not freely available in all environments. For example, in *which*-questions, we argued that the value of the partition must be supplied by context. In this section, we show that whether or not the partition argument of a UF is existentially bound depends on the in/definiteness properties of the determiner that the UF is embedded under. In short, we argue that the generalization in (27) correctly captures the distribution of existentially bound *vs* contextually salient partitions.

(27) **(In)definiteness generalization:** UFs in definite DPs can only combine with *free* partition variables. UFs in indefinite DPs can only combine with $\exists S$ (defined in (23)).

4.1. Indefinite DPs

We show above that bare UFs (assumed to be indefinite) do not require a salient partition, but we did not determine whether the option of a contextually supplied partition is *also* available for UFs in indefinite DPs. We show next that existential closure in this environment is obligatory.

To see this, we have to examine UFs in environments that are not upward-entailing, as in upward-entailing contexts the interpretation derived when the partition argument of a UF is contextually salient entails the interpretation derived when the partition argument is existentially closed. If there is a contextually salient partition of a class into two halves, say, of math and CS majors, and one of those halves passed the exam, then – trivially – there is *some* partition of the students into two halves such that one of these halves passed the exam.

Consider, then, the example in (28a) where a UF is embedded under negation.

(28) **Context**: There are three books in box #1 and three books in box #2. Students in a reading competition are assigned points when they read all of the books in a box. Mary read two books from box #1 and one book from box #2. A asks: Does Mary get a prize? B responds:
a. #It is not the case that Mary read (a) half of the books.
b. #It is not the case that Mary read three books.

In this context, the books are partitioned into two halves, namely the half in box #1 and the one in box #2, and it is in fact true that Mary did not read all of the books in either of these two halves. Thus, if the partition argument of the UF in (28a) were allowed to be a free variable, we would have assigned the truth-conditions in (29b) to (28a), incorrectly predicting that the sentence is true in the given scenario.[7] To predict the infelicity of (28a) here, we therefore have to assume that existential binding of the partition argument is obligatory in this environment.

(29) Assume that the books in box #1 are a, b, and c and in box #2 are d, e, and f.
a. $S_7 = \{a \oplus b \oplus c, d \oplus e \oplus f\}$
b. $[\![\textit{It is not the case that Mary read half } S_1 \textit{ of the books}]\!]^g =$
1 iff $\neg\exists x \in g(1) : \forall y \leqslant_{AT} x : [\![\textit{read}]\!](y)(\textit{Mary})$
(where for any x, y, $y \leqslant_{\text{AT}} x$ iff $y \leqslant x \wedge \neg\exists y' : (y' \neq y \wedge y' \leqslant y))$

4.2. Definite DPs

Turning to definite DPs, we show next that in this environment, UFs can only take a contextually salient partition as an argument. For reasons to be discussed below, we use the example in (30a), in which there is universal quantification (‘*all*’) on the modifier, to illustrate this (rather than the more simple case in (30b)). When the UF’s partition argument is a free variable, we predict (31) to be the denotation of the definite DP in (30a). Thus, the DP denotes the unique plurality in the contextually-supplied partition S_1 whose atomic parts are all tall. The entire sentence is then predicted to be true iff that unique plurality read the books, as shown in (32).

(30) a. The half of the students who are all tall read the books.
b. The tall half of the students read the books.

(31) $[\![\textit{the half } S_1 \textit{ of the students who are all tall}]\!]^g = \iota x : \forall y \leqslant_{AT} x : [\![\textit{tall}]\!](y) \wedge x \in g(1)$,
defined iff $g(1)$ is a partition$^+_2$ of $\oplus[\![\textit{students}]\!]$ and $\exists! x : \forall y \leqslant_{AT} x : [\![\textit{tall}]\!](y) \wedge x \in g(1)$

(32) $[\![\textit{The half } S_1 \textit{ of the students who are all tall read the books}]\!]^g = 1$ iff
$[\![\textit{read}]\!]([\![\textit{the book}]\!])(\iota x : \forall y \leqslant_{AT} x : [\![\textit{smart}]\!](y) \wedge x \in g(1))$,
defined iff $g(1)$ is a partition$^+_2$ of $\oplus[\![\textit{students}]\!]$ and $\exists! x : \forall y \leqslant_{AT} x : [\![\textit{tall}]\!](y) \wedge x \in g(1)$

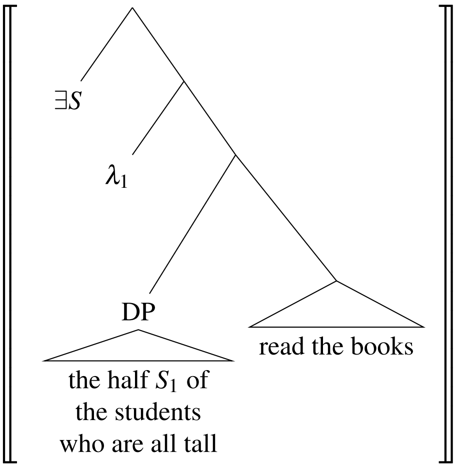
For the partition argument to be existentially closed, we stipulated that a generalized existential

[7]We assume no homogeneity effects arise when $[\![\textit{half}]\!]$ is in an indefinite. Otherwise, we would make the false prediction that (28a) is only true if Mary read none of the books, regardless of whether the partition argument is existentially closed or not. This assumption is shared with numerals in indefinites DPs, which also do not show homogeneity, and is presumably part of a larger puzzle of why certain constructions remove homogeneity (Križ, 2015 a.o.). It is of course logically possible within our analysis that we do get homogeneity in indefinite DP but only when the partition argument is not existentially closed, which would undermine our argument regarding (28), but we set this possibility aside for the purposes of this paper.

quantifier over partitions must reside in the UF's partition argument slot. For type reasons, this quantifier must then QR to a position outscoping the rest of the sentence. An LF for the sentence in (30a) in which this is the case is provided in (33), alongside the truth-conditions it derives. The LF is predicted to be true as long as there is some way to partition the students into two halves, one of which is all tall, such that the half which is all tall read the books.

The truth-conditions in (32) and in (33) are not equivalent. To see this, let us examine them with respect to the scenarios in (34a)-(34b). In (34a) there is no salient partition of the 10 students, so the LF with the free partition variable in (32) is undefined in this context. On the other hand, the truth-conditions in (33) hold in this scenario, since there is a way of partitioning the students into two halves such that the unique half which is all tall read the books. On the other hand, both sets of truth-conditions are true given the scenario in (34b).

(33)



$= 1 \text{ iff } \exists S : S \text{ is a partition}_2^+ \text{ of } \oplus[\![\textit{students}]\!] \wedge \exists! x[\forall y \leqslant_{AT} x : [\![\textit{tall}]\!](y) \wedge x \in S] \wedge [\![\textit{read}]\!]([\![\textit{the books}]\!])(\iota x : \forall y \leqslant_{AT} x : [\![\textit{tall}]\!](y) \wedge x \in S)$

(34) a. Context: There are ten students, eight of them are tall. Five of the eight tall students read the books.
b. Context: There are ten students, of whom five are CS majors and five are math majors. The CS majors are all tall but only three of the five math majors are tall. The CS majors read the books, but the math majors did not.

Now let us consider our judgements regarding (30a) – *the half of the students who are all tall read the book*: uttering this sentence seems infelicitous in the scenario in (34a), and felicitous (and true!) in the scenario in (34b). We thus conclude that the the sentence *cannot* be assigned the LF in (33), where the partition argument is existentially bound. This result falls under our generalization in (27), given that the UF in (30a) is embedded under a definite determiner.

The reason we use the sentence with *all* in (30a), rather than the simpler sentence in (30b) is that the latter obscures the truth-conditional difference between salient partitions and existentially closed ones. In particular, it seems that unlike (30a), (30b) presupposes that exactly half of the students are tall: it seems infelicitous in both the contexts in (34a) and (34b), where more than half of the students are tall,[8] and is felicitous only in a context like (35) where that is the case.

(35) Context: There are five tall students and five short students in the class.

Given (35), the truth-conditional differences between the salient partition reading (32) and its

[8]This is arguably due to homogeneity effects, which are removed by quantifiers like *all* (Löbner, 2000).

existentially closed counterpart (33) seem to disappear: both are true as long as the five tall students passed. This is because there is only one way to partition the students in (35) such that the presupposition of the definite that one of the halves in the partition is all tall is met; i.e., the partition into the five tall students and the five short ones. Therefore, the existential quantifier in (33) is trivial in this context and (33) becomes truth-conditionally equivalent to (32).

Indefinite DPs with UFs, headed either by a covert existential quantifier or by the determiner '*a*', thus only allow for existential closure of the UF's partition, while definite DPs headed by '*the*' only allow for a contextually salient partition, supporting the generalization in (27). Next, we provide evidence for our that generalization beyond '*a*' and '*the*', from *wh*-questions.

4.3. Wh-questions

To be able to test the (in)definiteness generalization in *wh*-questions, we need to be able to test whether a given *wh*-determiner is definite or not. One environment where definite and indefinite determiners are known to behave differently is the post-copular position of existential *there*-constructions (henceforth TCs). Milsark (1974) observes that while DPs headed by the indefinite '*a*' are licensed in TCs (36a), their definite counterparts are not (36b). Interestingly, Heim (1987) noticed that different *wh*-phrases behave differently in TCs as well. Particularly, *which*-phrases pattern with definites (37b), while *how-many*-phrases pattern with indefinites (37a). The observation *vis-a-vis which*-phrases is consistent with independent arguments for analyzing these elements as definite DPs (cf. Rullmann and Beck 1998).

(36) a. There is a car in your garage.
b. # There is the car in your garage.

(37) a. How many cars are there in your garage?
b. # Which cars are there in your garage?

We thus conclude that *which*-phrases are definite, while *how many*-phrases are indefinite,[9] and our (in)definiteness generalization predicts two things: first, UFs embedded in *which*-phrases are predicted to only combine with a contextually salient partition, and second, the partition argument of UFs in *how many*-phrases must be existentially bound.

We have already shown in section 3.3 that the former prediction is indeed borne out. If we allow an UF's partition argument in *which*-phrases to be existentially closed, the *which*-question no longer requires a salient partition, contrary to the observations we use to challenge the standard approach. To illustrate that the latter prediction is also borne out we show first that UFs in *how-many*-questions do not require a salient partition, and second, that they in fact disallow it.

To see that *how-many*-questions do not require a salient partition, consider (38), where it is common ground that a multiple of an eighth of the cars was sold, but there is no salient partition

[9] It is not clear whether it is definiteness *per se* that determines whether a DP is licensed in existential constructions (cf. Abbott, 2006). In fact, Milsark (1974) himself posits the well-known distinction between weak and strong determiners, which he argues captures the distribution of DPs in TCs. In any case, it is clear that definite DPs are a subset of Milsark's *strong* DPs which are not licensed in TCs. The felicity of *how many*-questions in these constructions therefore argues that they are not definite. And since *which*-questions have been independently argued to pattern like definite DPs in other respects, we attribute their infelicity in (37b) to their alleged definiteness.

of the products into eighths. We take the felicity of the *how-many*-question in this context to show that it is possible for the UF's partition argument there to be existentially closed.

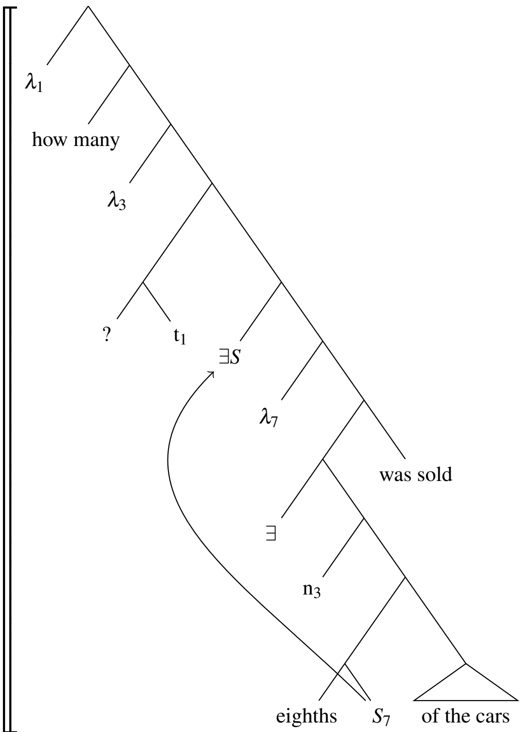
(38) Context: A car factory has hundreds of cars, but it is only worth it for them to sell cars in bulk. They therefore have a policy that they only sell multiples of an eighth of the cars they have at the beginning of the day. The boss asks one of his employees:
  a. How many eighths of the cars were sold today?

Note that deriving the correct reading for UFs in *how-many*-questions, even with an existential quantifier over partitions, requires some non-trivial assumptions about the semantics of *how-many*-questions. What is crucial for our present purpose, of course, is the correlation between the felicity of UFs in *how-many*-questions even when there is no contextually salient partition available, and the status of *how-many*-DPs as indefinites, given Milsark's test. However, it will serve us in the discussion below to have a working hypothesis in mind.

For illustrative purposes, then, we adopt the idea that '*how many*' is an existential quantifier over cardinalities, and that pied-piping in *how-many*-questions is undone at LF via reconstruction, which gives rise to the structure in (39) where the pied-piped phrase is interpreted in its base position while its *wh*-specifier is interpreted above Karttunen's question operator (an idea originally due to von Stechow (1996), see implementations in e.g., Beck and Rullmann 1999; Fox and Nissenbaum 2018; Gentile and Schwarz 2018). We can then assume that the generalized quantifier in the UF's partition argument slot QRs to a position below Karttunen's question operator for type reasons, deriving the set of propositions in (39).[10]

While the example in (38) shows that UF's in *how-many*-phrases do not *require* a contextually salient partition, determining whether a contextually salient partition is *allowed* in that environment raises its own complication. This is because the presupposition of the *how-many*-question with a salient partition entails the presupposition when the partition is existentially closed, and we therefore cannot simply examine a question's felicity conditions in order to determine whether there is a contextually salient partition option here.

[10]We have not commented on how the cardinal n_3 which is bound by the *wh*-phrase in (39) compositionally combines with the constituent headed by the UF. For the structure in (39) we must assume that cardinals are subsective modifiers that can count minimal elements in their input set, rather than atoms (cf. Ionin and Matushansky 2006).

(39) ⟦  ⟧ $= \{\lambda w.\ \exists S{:}S$ is a partition$^+_8$ of $\oplus[\![\textit{cars}]\!] \wedge$ n things in S were sold in $w \mid n \in \mathbf{N}^+\}$

To see this consider the truth-conditions for the question with a salient partition in (40).

(40) $[\![$*How many eighths* S_1 *of the cars were sold today?*$]\!]^g$ is defined iff $g(1)$ is a partition$^+_8$ of $\oplus[\![\textit{cars}]\!]$ and some element in $g(1)$ was sold today. If defined, is equal to: $\{\lambda w.\ n$-many elements in $g(1)$ were sold in $w \mid n \in \mathbf{N}^+\}$

Assuming again that questions are presupposed to have a true answer (Dayal, 1996), (40) presupposes that there is a salient partition of the cars into eighth, such that at least one eighth in this partition was sold today. This clearly entails the presupposition in (39), which is simply that at least an eighth of the cars was sold, without requiring a specific partition.

In order to show whether a free variable over partitions is available for UFs in *how-many*-DPs, we will therefore consider cases where the *how-many*-question is in an embedded environment. Consider the example in (41) with the question-embedding predicate *ask*. Given the context in (42), (41) can be felicitously uttered if the boss wants to know the number of cars sold today, as in (42b). On the other hand, (41) is infelicitous in the scenario in (42a), where the boss only cares about how many lots were emptied. If the embedded question in (41) were able to have the denotation in (40) with the salient partition, (41) would be incorrectly predicted to be true in (42a). In particular, the context in (42) makes salient the partition in (43) into the cars in different lots. Given this partition, under the denotation in (40), (41) is true when the boss wants to know how many elements in this partition, namely how many lots, were sold.

(41) The boss asked how many eighths of the cars were sold today.

(42) **Context**: The cars produced by a car factory are stored in eight lots, each housing an eighth of the cars available at any given time. As in (38), the factory only sells cars in bulks whose size is an eighth of the product. However, the division of cars into lots is random. Therefore, a buyer might buy a bulk of cars consisting of cars stored in different lots. We know that at least one lot of cars was sold in full today.

a. **Scenario 1:** The boss wants to know how many of the lots in the factory were sold in full today.
b. **Scenario 2:** The boss wants to know how many cars were sold today.

(43) $[\![S_1]\!]^g = \{$the cars in lot #1, the cars in lot #2, ..., the cars in lot #8$\}$

In order to explain the infelicity of (41) in the scenario (42b), we therefore have to conclude that, as predicted by our (in)definiteness generalization, UFs in *how-many* questions cannot combine with a contextually salient partition. In conclusion, if (as suggested by Heim's implementation of Milsark's diagnostic with *wh*-words) *which*-phrases are definite while *how-many*-phrases are indefinite – then both of these elements conform to the (in)definiteness generalization we argue for in this section.

5. Deriving the (in)definiteness generalization

We argued that the distribution of contextually salient partitions *vs* existentially closed ones with UFs follows the (in)definiteness generalization, repeated in (44). In this section, we offer some tentative remarks regarding potential ways of accounting for this generalization.

(44) **(In)definiteness generalization:** UFs in definite DPs can only combine with *free* partition variables. UFs in indefinite DPs can only combine with $\exists S$ (defined in (23)).

5.1. Contextually salient partitions as a last resort

The first idea we consider is to tie the generalization in (44) to the inability of quantifiers to move out of definite islands. As we saw in section 3.2, when the sister of a UF is an existential quantifier over partitions, this quantifier has to move out of the host DP in order to be interpreted and avoid a type mismatch. Given that definite DPs are islands for movement (Chomsky, 1973), it is therefore possible that the existential quantifier option is ruled out in definites due to the inability of the quantifier to scope out of the definite DP.

If the above assumption about definiteness islands applying to our existential quantifier is correct, the principle in (45) predicts our (in)definiteness generalization. When a UF is in an indefinite DP, its sister is obligatorily an existential quantifier which takes scope outside the DP to avoid type mismatch. On the other hand, when the UF is in a definite DP, having its sister be a quantifier over partitions leads to ungrammaticality: the quantifier cannot be interpreted in its base position as this will lead to type mismatch, but at the same time it cannot move out of the definite DP due to definiteness island effects.

(45) **Last resort principle:** By default, the sister of a UF has to be an existential quantifier over partitions, *Some C*. If the LF with *Some C* leads to ungrammaticality, the sister of the UF is a free variable over partitions whose value is contextually supplied.

In what follows, we discuss the necessary assumption here that our existential quantifier over partitions cannot raise above its host DP. It has been observed since Chomsky (1973) that definite DPs are islands for movement. This is evidenced by the contrast in (46): in (46a), *who* can move out of the indefinite DP object (*a picture of* __), while the counterpart with a definite DP (*the picture of* __) is ungrammatical.

(46) a. Who did Mary see a picture of?
b. ??Who did Mary see the picture of?

Just like with overt wh-movement in (46), it seems that QR is also not possible out of a definite DP. In (47a), *every student* can take scope above the indefinite DP that is its host. This is evidenced by the fact that (47a) is true in the context in (48), where there is a different picture for every student such that Mary saw them all. On the other hand, in (47b), *every student* cannot take scope outside the definite DP: (47b) is not true in (48), but is rather only true if there is a single picture that every student is in, and Mary saw *that* picture (i.e., *every student* scopes inside the definite DP hosting it). We can therefore conclude that QR out of a definite is not possible (though see caveat below).

(47) a. Mary saw a picture of every student. $\exists > \forall, \forall > \exists$
b. Mary saw the picture of every student. $\mathit{the} > \forall, ??\forall > \mathit{the}$

(48) **Context:** The students each submitted a picture for the yearbook, and Mary, the yearbook editor, looked through all the pictures.

Under the assumption that our existential quantifier over partition patterns with the universal quantifier in (47) in being unable to scope outside a definite DP, the last resort principle in (45) predicts our (in)definiteness generalization.

This approach faces several problems. First, there are counterexamples to the claim that QR is subject to definie islands. In (49), *every* seems to scope above the definite DP, for instance. Furthermore, we know that indefinites in particular seem to be able to take scope outside of islands (Reinhart, 1977 a.o.). Since our existential quantifier over partitions is arguably an indefinite, one might be skeptical that the scope constraint illustrated in (47b) applies to it.

(49) I cleaned the top of every table.

Finally, there are conceptual problems with the last resort principle in (45). In particular, it is not clear why the existential quantifier over partitions should be the default option. Moreover, the constraint in (45) is transderivational, since the grammaticality of the free variable option here depends on whether an alternative LF with existential closure is grammatical. One has to therefore posit a mechanism in the grammar that licenses certain LFs only if certain alternative LFs behave in a particular way. While this is not unheard of, further work is required to determine whether this approach is justified in the context of the semantics of UFs.

5.2. Inherited (in)definiteness

It has been noted at least since Jackendoff (1977) that the (in)definiteness of a possessive DP in the Saxon genitive is determined by the (in)definiteness of its possessor. Jackendoff illustrates that DPs with an indefinite possessor are indeed indefinite by applying Milsark's diagnostic for indefiniteness; i.e., by showing that these DPs are licensed in TCs and thus cannot be definite.

(50) a. John's book → [+def]
b. A boy's book → [−def]

(51) a. #There is John's book on the table.
b. There is a boy's book on the table.

Similarly, it is argued that the Semitic construct state (hereafter, CS) also presents a case of inherited (in)definiteness (e.g., Borer 1984; Hazout 1991; Ritter 1991; Siloni 1997; Dobrovie-Sorin 2000). In the CS, a DP's head noun lacks an overt article and its definiteness feature is determined by the genitive; when the genitive is indefinite, the whole DP is indefinite, and when it is definite so is the whole DP (see overview in Alexiadou 2005). In (52) is an example from Hebrew, which lacks indefinite articles. Thus, when the genitive is bare (52a), the whole DP is indefinite and when the genitive has a definite article (52b), the whole DP is definite.

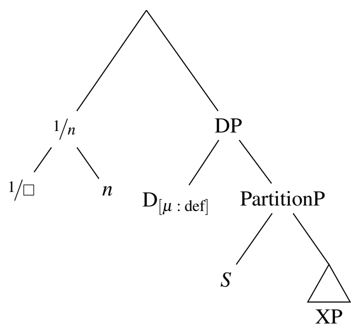
(52) a. beyt more
house teacher
'a teacher's house'

b. beyt ha- more
house DEF- teacher
'the teacher's house'

This phenomenon has been dubbed *(in)definiteness spread*. And, while the mere existence of this phenomenon is still up for debate (see, e.g., Danon 2001; Heller 2002 for arguments against (in)definiteness spread in the CS), if it does exist it seems suspiciously similar to our generalization *vis-a-vis* partition variables (44). A review of how inherited (in)definiteness has been accounted for in different languages is beyond the scope of this paper (cf. Alexiadou 2005). For the remainder of this section, we toy with one way of implementing (in)definiteness spread for UFs, using agreement of definite features of nested DPs.

First, let us modify our syntax of UFs as in (53), where instead of taking two arguments, a partition and a set of individuals, the UF selects for a DP headed by a determiner of partitions, whose restrictor is a partition phrase with a head *S* and a set of individuals in its complement. The head *S*, defined in (54), denotes a function from a set f, to the set of all possible partitions of $\oplus f$. Thus, $[\![$*S of the students*$]\!]$ denotes the set of all partitions of $\oplus[\![$*students*$]\!]$.

We assume that the determiner of partitions D carries a feature $[\pm def]$, and denotes the definite article when the feature's value is $[+def]$ (55a), and an existential quantifier when that value is $[-def]$ (55b) ((55a)-(55b) are higher-type counterparts of the definite and indefinite articles as formalized in, e.g. Heim and Kratzer 1998,[11] with context-dependency introduced as an index on the definite article). This allows the sister of the UF to denote a contextually salient partition when the feature on D is $[+def]$ and an existential quantifier over partitions when the feature is $[-def]$. Finally, we posit the modified lexical entry for UFs in (56), according to which a UF like *half* takes as its input a set of individuals (a partition), "checks" that this set consists of two equi-sized elements, and if it does, simply returns that set.

(53)


[11]We need higher types here given that the input to a determiner whose restrictor is a partition phrase is of type $\langle et,t\rangle$, unlike determiners of $\langle e,t\rangle$-denoting elements.

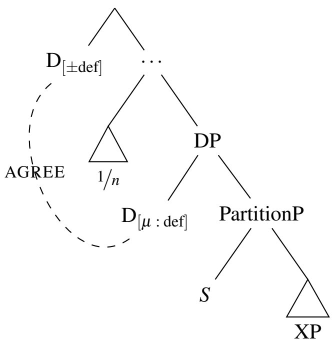
(54) $[\![S]\!] = \lambda f_{\langle e,t\rangle}\lambda g_{\langle e,t\rangle}.\ 1$ *iff* g is a partition of $\oplus f$ (see definition of partition in (15))

(55) a. $[\![D_{[+def]}]\!]^g = [\![the_i]\!]^g = \lambda f_{\langle et,t\rangle} : \exists! x \in g(i)[f(x)].\ \iota x : f(x)$
where $g(i)$ is a contextually salient subset of $\mathscr{D}_{\langle e,t\rangle}$

b. $[\![D_{[-def]}]\!]^g = [\![\exists]\!]^g = \lambda f_{\langle et,t\rangle}\lambda h_{\langle et,t\rangle}.\ \exists x \in \mathscr{D}_{\langle e,t\rangle} : f(x) \wedge h(x)$

(56) $[\![{}^1\!/\!_n]\!] = \lambda f_{\langle e,t\rangle} : |f| = n \wedge \forall x,y \in f : |x| = |y|.\ f$

The final assumption we make is the one in charge of deriving the generalization in (44). As illustrated in (53), UFs select for DPs whose head D is unvalued for [±def]. We then stipulate that the value of [±def] is determined via agreement with the host DP of the UF as in (57).

(57)


We thus achieve the desired result as follows: Under definite determiners, a UF's input DP inherits a $[+def]$ feature, and the uniqueness presupposition that it enforces – demanding that among the set of contextually salient partitions (i.e., the contextually salient subset of $\mathscr{D}_{\langle e,t\rangle}$), only one will partition of the maximal element in its restrictor. The input to the UF as defined in (56) will then be *that* partition. On the other hand, under indefinite determiners, a UF's input DP inherits a $[-def]$ feature, thus denoting an existential quantifier over partitions. To avoid type-mismatch this quantifier will need to raise above the UF for interpretation, and the result would be the truth conditions for indefinite sentences detailed in section 3.

6. Conclusion

We argue for a novel semantics for unit fractions, which has them take a partition as one of their arguments, and allows us to solve two novel puzzles involving UFs. We show that the partition argument must be existentially closed when the UF is in an indefinite DP, and a free variable whose value is contextually assigned when the UF is in a definite DP.

In our discussion, we focus only on a subclass of partitive constructions with UFs; namely, only on cases where the partitive phrase denotes a plural individual, as in (58a). In that case, the number of atoms in each element of the UF's partition argument is counted to ensure that the elements in the partition are equal in size. Our analysis can be extended, however, to cases where the partitive phrase denotes an atomic individual, like (58b). In such cases, rather than counting atoms, some other measure of the size of the relevant parts is needed. This has already been observed, and implemented by Ionin et al. (2006) and their implementation could be incorporated into our semantics for UFs. Note that the puzzles that motivated our partition-based analysis of UFs to begin with can both be replicated with the singular partitive *half of the orange*. First, the *which*-question in (59a) is only felicitous in a context like (59), where there is a salient partition of the orange into two halves. Our second puzzle, namely, the behavior of *half* in descriptions with superlative and comparative modification, also extends to UFs in partitives with a singular NP. This is illustrated by the contrast in (60), where only a comparative modifier seems to be licensed with *half* inside a definite description.

(58) a. Jane ate half of the oranges.
b. Jane ate half of the orange. (Adapted from Ionin et al., 2006)

(59) Context: Mary cut the orange into two halfs, one of them was a bit rotten but the other was good. Someone asks:
a. Which half of the orange did Jane eat?

(60) a. The tastier half of the orange.
b. ??The tastiest half of the orange.

An important implication of the data we present in this paper is that it posits a challenge to theories that aim to unify the semantics of numerals and fractions (e.g., Ionin et al. 2006). Only one of these, namely UFs, are shown to be subject to a felicity constraint, requiring that context make salient a particular partition, when embedded under a definite determiner. It is at least prima facie surprising that this is the case, given that UFs and numerals (in some pre-theoretical sense) seem to be used to "do the same thing," namely, to count elements in their input sets. Our observations thus raise some obvious conceptual questions like why numerals and UFs should differ in this way, and why do they differ only in particular environments.

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