On the natural language metaphysics of amounts[1]

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Abstract. In DPs like *the amount of nuts you ate* or *the number of cooks you hired*, a quantity noun (*amount*, *number*) combines with an entity noun (*nuts*, *cooks*). Such *quantity DPs* are surprisingly flexible: they can saturate not only quantity predicates, like *be 50 grams* or *be three*, but also predicates of ordinary entities, like *eat* or *hire*. To explain this flexibility, Scontras (2017) takes quantity DPs to denote not primitive quantities like 50g or 3, but set theoretic constructs that somehow *contain* the entities described by the entity noun (e.g., nuts or cooks). We explore a choice point in developing this approach: the relevant set theoretic structures—we call them *rich amounts*—can be construed either as sets of entities, given by the entity noun's extension, or as properties, based on the entity noun's intension. We show that this choice constitutes a dilemma. The dilemma arises with reference to quantity DPs without modifiers (e.g., *the amount of nuts*) and quantity DPs with relativization from an intensional context (e.g., *the amount of nuts you want to eat*). Construing rich amounts as sets of entities yields a credible analysis of the former data, but not of the latter. Construing them as properties can capture the latter data, but requires auxiliary assumptions without independent support to accommodate the former. As the two choices exhaust the relevant analytical possibilities, the dilemma questions the utility of rich amounts for semantic composition.

Keywords: amounts, quantities, predication, *de dicto* readings, maximization.

1. Introduction

A DP with *amount of*, like (1), seems to refer to an amount. But what is an amount? More precisely, how should amounts be construed for the purposes of semantic composition in natural language? What is, in the sense of Bach (1986), the natural language metaphysics of amounts?

(1) the amount of nuts you ate

One answer identifies amounts with the abstract quantities that the natural sciences posit in studying the physical world, quantities like 50g, 30ml, or the cardinality 3 (see, e.g., de Boer 1995). However, Scontras (2017) suggests that this answer is called into question by the distribution of phrases like (1), and the truth conditions that they can give rise to. These phrases can participate in both *quantity predication* and *entity predication*: they can saturate quantity predicates, like *is 50 grams* in (2a), but also predicates of ordinary entities, like *ate* in (2b).

(2) a. [The amount of nuts you ate] is 50 grams.
b. I ate [the amount of nuts you ate].

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For the quantity predication in (2a), the assumption that the subject DP denotes an abstract quantity is entirely plausible. But the entity predication in (2b) is puzzling. Abstract quantities cannot be eaten, and, yet, the sentence in (2b) is felicitous. This is so in virtue of (2b) describing not the eating of an abstract entity, but the eating of concrete nuts. But if the DP in (1) refers to an abstract quantity, how can such a meaning emerge compositionally?

Scontras (2017) proposes that, for the purposes of semantic composition, amounts are not primitives of the sort assumed in the natural sciences, but set theoretic constructs that are built from, hence contain, instantiating ordinary entities. On this view, the amount denoted by (1) is constructed from instantiating pluralities of nuts. We will refer to any constructs that broadly fit this description as *rich amounts*, and use the term *lean amount* to refer to the primitive quantities posited in the natural sciences.

In this paper, we scrutinize Scontras' proposal, call it the *rich amount strategy*, by exploring a choice point in the construction of rich amounts. How exactly does the entity nominal, here *nuts*, enter into the construction of a rich amount? Should rich amounts be construed as *sets*, built from the entity nominal's extension, or as *properties*, based on the entity nominal's intension? We will demonstrate that the choice between these two elaborations of the rich amount strategy, call them the *set solution* and the *property solution*, constitutes a dilemma.

The dilemma arises with reference to quantity DPs that lack a relative clause or other modifier, such as (3a), and quantity DPs that feature relativization from an intensional context, like (3b). We will see that the set solution affords a theoretically parsimonious analysis of DPs like (3a), but that DPs like (3b) are beyond its reach. Conversely, while the property solution captures the interpretation of DPs like (3b), to capture data like (3a), it would have to be supplemented with auxiliary assumptions that presently lack independent support.

(3) a. the amount of nuts
b. the amount of ambrosia that you want to eat

The choice between a set construal and a property construal of rich amounts exhausts the space of relevant analytical options, and, therefore, our argument represents a challenge to the rich amount strategy at large.

We will present the set and the property solutions in the next two sections. Sections 4 and 5 will then present the two horns of the dilemma that the data in (3) give rise to. In a preliminary exploration of possible solutions to the dilemma, Section 6 discusses possible amendments to the property solution, and Section 7 concludes.

Before proceeding, it will be useful to comment on the scope of the phenomenon to be discussed, and the type of data that we use as illustration. The noun *amount* is just one representative of a class of nouns, call them *quantity nouns*, that share the grammatical properties relevant for the present discussion. Other quantity nouns are, for example, *quantity*, *number*, *volume*, *range*, *share*, *allocation*, *quota*, *proportion*, and *ratio*. Such nouns have much the same syntactic distribution as *amount*. They can form DPs of a shape parallel to those in (1) and (3), and such DPs, which we will call *quantity DPs*, permit both quantity and entity predication. For the quantity noun *number*, this is illustrated in (4) and (5).

(4) the number of cooks you hired

(5) a. [The number of cooks you hired] is three.
 b. I hired [the number of cooks you hired].

For practical purposes, we will in the following focus on quantity DPs with *number*. This choice allows us to steer clear of a layer of complexity that plays no role in our argument, viz. the possible underspecification of the dimension of measurement. Like most quantity nouns, *amount* can invoke a range of different dimensions. In (2a), the unit noun *grams* in the predicate *be 50 grams* sets for (1) the dimension to weight. But *be 50 grams* could be replaced felicitously with predicates like *be half a cup* or *be ten*, illustrating that (1) permits other dimensions, including volume and cardinality. For quantity DPs with *number*, in contrast, the quantity noun itself fixes the relevant dimension, viz. to cardinality. While not essential, this limitation is convenient, as focusing on cases with *number* will simplify our discussion.

2. Rich amounts as sets

Under the rich amount strategy, quantity DPs denote rich amounts. What are those? In one answer, rich amounts are *sets of ordinary entities*. More specifically, *amount-uniform* sets of entities, that is, sets of entities that share the same measurement in a given dimension. Under this view, the quantity DP in (4) denotes a set of pluralities of cooks that have the same cardinality, in the sense of having the same number of atomic parts. To illustrate, consider a possible world w_1, where you hired exactly three cooks. In w_1, (4) will have the denotation in (6): the set of all pluralities of cooks in w_1 that have cardinality 3.[2]

(6) $[\![\text{the number of cooks you hired}]\!]^{w_1} = \{x\text{: } [\![\text{cooks}]\!]^{w_1}(x) \wedge |x| = 3\}$

The sort of denotation in (6) enables an elegant uniform account of both quantity predication and entity predication. Consider again the pair of sentences in (5), repeated in (7). As stated in (8), we will assume that in (7), the quantity DP composes with the predicate via functional application.

(7) a. [The number of cooks you hired] is three.
 b. I hired [the number of cooks you hired].

(8) a. $[\![\text{[the number of cooks you hired] is three}]\!]^{w} =$
 $[\![\text{be three}]\!]^{w}([\![\text{the number of cooks you hired}]\!]^{w})$
 b. $[\![\text{I hired [the number of cooks you hired]}]\!]^{w} =$
 $[\![\text{hired}]\!]^{w}([\![\text{the number of cooks you hired}]\!]^{w})(\text{I})$

Now take again the particular world w_1 from above, where you hired exactly 3 cooks. The quantity predication in (7a) intuitively yields truth in such a world. Assuming (6) and (8a), this can be captured by positing for *be three* the denotation in (9), a function that applies to a set A of ordinary entities and demands that A have a member of cardinality 3. Since you *hired* three

[2]The rich amount strategy is similar to the reconstruction of degrees as equivalence classes of individuals proposed in, e.g., Klein 1980 and Bale 2006. Applied to numbers, it is also reminiscent of the view that cardinal numbers can be reconstructed in terms of sets of sets with the same cardinality, a view found in work by Frege and Russell (see e.g. Hatcher 1990; Dummett 1991). Note, however, that the rich amount strategy only makes a claim about how amounts, including numbers, are construed as semantic values for the purposes of semantic composition. Thanks to Kevin Klement for help navigating the philosophy literature.

cooks in w_1, there will exist in w_1 at least one plurality of cooks with cardinality 3, hence the set in (6) must be non-empty. Given (8a), this means that (7a) is true in w_1, as desired.

(9) $[\![\text{be three}]\!]^w(A) \Leftrightarrow \exists x[x \in A \wedge |x| = 3]$

Turning now to the entity predication in (7b), this sentence is intuitively true in w_1 just in case I hired (at least) three cooks in w_1. Given (6) and (8b), this intuition can again be captured with appeal to a suitable denotation for the predicate. Suppose that the domain of the function denoted by *hire* contains not only ordinary individuals, but also rich amounts, here sets of ordinary entities. If so, what does it mean for *hire* to relate a set A to an individual? According to (10), such a predication again yields existential truth conditions, here that the individual hired some ordinary entity that is a member of A. For (7b) in world w_1, given (8b), the resulting condition is that I hired some member of the set in (6), hence that I hired (at least) three cooks. This correctly captures intuitions about sentence (7b) in world w_1: under the assumption that you hired exactly three cooks, (7b) is intuitively true just in case I hired (at least) three cooks.[3]

(10) $[\![\text{hire}]\!]^w(A)(y) \Leftrightarrow \exists x[x \in A \wedge [\![\text{hire}]\!]^w(x)(y)]$

Extrapolating from w_1, we can now state in (11) the general denotation for the quantity DP in (4). In any possible world w, (4) denotes the set of all pluralities of cooks in w that have cardinality m, where m is the exact number of cooks that you hired in w.

(11) $[\![\text{the number of cooks you hired}]\!]^w =$
$\{x: [\![\text{cooks}]\!]^w(x) \wedge |x| = \max\{n: \exists y[[\![\text{cooks}]\!]^w(y) \wedge [\![\text{hire}]\!]^w(y)(\text{you}) \wedge |y| = n]\}\}$

The function max in (11) maps a set of cardinalities to its largest member. In (11), max applies to a set that contains a cardinality n just in case n is the cardinality of some plurality of cooks that you hired in w, and it outputs the greatest member of this set—the exact number of cooks that you hired in w. This derives (6) as a special case: having hired three cooks in w_1, you hired in w_1 cook pluralities of cardinalities 1, 2, and 3, and the greatest of these cardinalities is 3.

In concert with the entries for *be three* and *hire* in (9) and (10), the denotation in (11) moreover supports the intuitively correct general truth conditions for (7a) and (7b). (7a) is predicted true in w just in case max returns the cardinality 3 in (11), hence just in case you hired exactly three cooks in w. And (7b) comes out true in w just in case I hired a plurality of n cooks in w, where n is the exact number of cooks that you hired in w. This amounts to the condition that I hired (at least) as many cooks in w as you did.

Apart from supporting intuitively adequate truth conditions for the sentences in (7), the denotation in (11) lends itself to a syntactically parsimonious DP internal composition. To show this, it will be useful to restate the entry in (11) as in (12).[4]

[3] In Scontras (2017), equivalences like (10) are due to "derived kind predication", a composition principle adapted from Chierchia (1998), who in turn builds on Carlson (1977). We instead portray such equivalencies as being guaranteed by predicates' lexical meaning. We do so for expositional convenience only. The arguments made below are independent of how exactly equivalencies like (10) are derived.

[4] In (12), max applies to a set of number-uniform sets, rather than cardinalities, and references an ordering of such sets, where a set A counts as greater than a set B in virtue of the members of A having a greater cardinality than the members of B. Under this assumption, (12) is equivalent to (11). To illustrate, consider again w_1, where you hired exactly three cooks. When $w = w_1$, max applies to a set containing three sets, viz. for each cardinality $n \in \{1, 2, 3\}$, the set of cook pluralities of cardinality n. Among those three sets, the set of cook pluralities of cardinality 3

(12) $[\![\text{the number of cooks you hired}]\!]^w =$
$\max(\{\{x: [\![\text{cooks}]\!]^w(x) \wedge |x| = n\}: \exists y[[\![\text{cooks}]\!]^w(y) \wedge [\![\text{hired}]\!]^w(y)(\text{you}) \wedge |y| = n]\})$

Suppose now that (4) has the syntactic structure sketched in (13). We can then take the definite article *the* to denote the maximality operator max. Also, as stated in (14), the NP *number of cooks that you hired* can be taken to denote that set that max applies to in (12), so that *the* and the NP can compose via functional application.

(13) $[_{DP}$ the $[_{NP}$ $[_{NP}$ $[_{N}$ number] of cooks] $[_{CP}$ wh [you hired t]]]]

(14) $[\![\text{number of cooks you hired}]\!]^w =$
$\{\{x: [\![\text{cooks}]\!]^w(x) \wedge |x| = n\}: \exists y[[\![\text{cooks}]\!]^w(y) \wedge [\![\text{hired}]\!]^w(y)(\text{you}) \wedge |y| = n]\}$

With regard to the compositional derivation of the NP denotation, a natural possibility is that the NP *number of cooks* and the relative clause *you hired* also denote sets of sets, and that the two combine intersectively. Consider the denotations for the NP and the relative clause in (15).

(15) a. $[\![[_{NP} [_{N} \text{number}] \text{ of cooks}]]\!]^w = \{A: \exists n[A = \{x: [\![\text{cooks}]\!]^w(x) \wedge |x| = n\}]\}$
b. $[\![[_{CP} \text{wh [you hired t] }]]\!]^w = \{A: \exists y[y \in A \wedge [\![\text{hired}]\!]^w(y)(\text{you})]\}$

We can show that the intersection of the two sets in (15) yields the set in (14). First we attend to (15a). For any w, the family of sets in (15a) comprises all sets of number-uniform cook pluralities in w. In a rendition that more closely adheres to the notation in (14), this family of sets can also be described as in (16).

(16) $\{\{x: [\![\text{cooks}]\!]^w(x) \wedge |x| = n\}: n \text{ is a cardinality}\}$

Turning to the relative clause, (15b) equates its denotation in w with the family of sets which contain some element that you hired in w. Now, what does it take for a set S of pluralities to be in the intersection of the sets in (15a) and (15b)? By (15a), for some cardinality n, S must be the set of all cook pluralities in w of cardinality n; and moreover, by (15b), you must have hired in w one of S's members, that is, you must have hired in w a cook plurality of cardinality n. This transparently restates the membership condition for the NP denotation in (14). So we have succeeded in deriving this denotation from those in (15) through intersective composition.

To complete the compositional analysis of the quantity DP in (4), we observe that the equalities in (15) can be made to fall out from unexceptional assumptions about composition internal to the NP *number of cooks* and the relative clause *you hired*. (15a) is a straightforward consequence of the lexical entry for *number* in (17a), which assumes that its denotation applies to the set given by the entity nominal's extension, here the set of pluralities given by *cooks*. With regard to (15b), note that the entry for *hire* in (10) above in particular guarantees the equivalence in (17b). With this in mind, suppose that the variable that serves as the argument of *hire* in (15b) ranges over sets of pluralities. The abstraction triggered by the relative operator will then derive a set of sets of pluralities. In fact, given (17b), it will derive the intended target in (15b).

(17) a. $[\![\text{number}]\!]^w = \lambda B.\ \{A: \exists n[A = \{x: x \in B \wedge |x| = n\}]\}$
b. $\exists y[y \in A \wedge [\![\text{hired}]\!]^w(y)(\text{you})] \Leftrightarrow [\![\text{hired}]\!]^w(A)(\text{you})$

is greatest. So this is the set that max will output in (12) for $w = w_1$, deriving the special case in (6).

We conclude that for the data examined so far, the rich amount strategy as developed in the set solution has a certain utility. Construing rich amounts as amount-uniform sets of entities, it supports a natural account of the observation that quantity DPs can saturate both quantity predicates like *be three* and entity predicates like *hire*. Also, there is also a natural compositional path to the intended amount-uniform sets as the denotations of quantity DPs.

In this section, we largely traced the steps of Scontras (2017). While he does not spell out a denotation for quantity predicates, almost all the central assumptions we made are essentially his, setting aside inconsequential matters of notation and technical execution. We deviate from Scontras (2017) in one notable respect, though: Scontras does not construe rich amounts as sets of entities, but as *nominalized properties of entities*. While nominalization is irrelevant for the arguments to be presented below, the choice between sets of entities and properties of entities, the latter understood as functions from possible worlds to sets of entities, is important, and will be shown to lead to a dilemma. In the next section, we continue laying the groundwork for making this point, by reworking the analysis stated above, treating rich amounts as properties of entities.

3. Rich amounts as properties

The property solution construes rich amounts as properties of ordinary entities, understood as functions from possible worlds to sets of entities. With properties of entities taking the place of sets of entities, other assumptions made in the last section must be adjusted accordingly. But despite these adjustments, for the cases analyzed so far, the compositional derivation of meanings remains parallel to what we saw in the previous section.

We begin again by illustrating the proposal with reference to the particular world w_1 where you hired exactly three cooks. In w_1, the quantity DP in (4), repeated in (18), denotes the property of being three cooks, explicated as a function from worlds to sets of ordinary entities that maps any world w to the set of all cook pluralities in w that have cardinality 3.

(18) the number of cooks you hired

(19) $[\![\text{the number of cooks you hired}]\!]^{w_1} = \lambda v.\ \{x\colon [\![\text{cooks}]\!]^{v}(x) \wedge |x| = 3\}$

Generalizing from (19), the denotation of the quantity DP in (18) can be equated with the property of being n cooks, where n is the exact number of cooks that you hired in w. This can be stated as in (20), which minimally revises (11) in the last section. Note that for $w = w_1$, max in (20) will again output the cardinality 3, which derives the special case in (19), as intended.

(20) $[\![\text{the number of cooks you hired}]\!]^{w} =$
$\lambda v.\ \{x\colon [\![\text{cooks}]\!]^{v}(x) \wedge |x| = \max\{n\colon \exists y[[\![\text{cooks}]\!]^{w}(y) \wedge [\![\text{hire}]\!]^{w}(y)(\text{you}) \wedge |y| = n]\}\}$

Consider now again the quantity predication sentence in (7a) and the entity predication sentence in (7b), both repeated in (21). For these sentences, the denotation for the quantity DP in (20) can enter into a composition that yields the same truth conditions as those derived under the set solution in the last section. To achieve this result, it is sufficient to suitably adjust the entries for *be three* and *hire* stated above, in (9) and (10), respectively, as in (22).

(21) a. [The number of cooks you hired] is three.
b. I hired [the number of cooks you hired].

(22) a. $[\![\text{be three}]\!]^{w}(\alpha) \Leftrightarrow \exists x[x \in \alpha(w) \wedge |x| = 3]$
b. $[\![\text{hire}]\!]^{w}(\alpha)(y) \Leftrightarrow \exists x[x \in \alpha(w) \wedge [\![\text{hire}]\!]^{w}(x)(y)]$

According to these entries, the denotation of both predicates in a world w apply to a property α to yield truth conditions that quantify existentially over the set of pluralities that α outputs for w. In the composition for the sentences in (21), this set coincides with the set A referenced in the entries in (9) and (10) above.[5] Since the entries in (22) do not otherwise differ from those in (9) and (10) above, we can be sure that (20) and the entries in (22) correctly reproduce the truth conditions for the sentences in (21) that were derived under the set solution.

Moreover, the property denotation in (20) can be derived compositionally in a way that minimally revises the composition detailed for the corresponding set denotation in (12). To see that, we begin with the observation that, paralleling the reformulation of (11) as (12) in the last section, the denotation for (18) in (20) can be rewritten as in (23).[6]

(23) $[\![\text{the number of cooks you hired}]\!]^{w} =$
$\max(\{\lambda v.\ \{x\text{: } [\![\text{cooks}]\!]^{v}(x) \wedge |x| = n\}\text{: } \exists y[[\![\text{cooks}]\!]^{w}(y) \wedge [\![\text{hired}]\!]^{w}(y)(\text{you}) \wedge |y| = n]\})$

Suppose now again that (18) has the structure in (13), repeated in (24). We can continue to assume that the definite article denotes the maximality operator max. Correspondingly, as stated in (25), we can assume that in (23), it is the denotation in w of the NP *number of cooks that you hired* that provides the argument of max.

(24) $[_{DP}$ the $[_{NP}$ $[_{NP}$ $[_{N}$ number] of cooks] $[_{CP}$ wh [you hired t]]]]

(25) $[\![\text{number of cooks you hired}]\!]^{w} =$
$\{\lambda v.\ \{x\text{: } [\![\text{cooks}]\!]^{v}(x) \wedge |x| = n\}\text{: } \exists y[[\![\text{cooks}]\!]^{w}(y) \wedge [\![\text{hired}]\!]^{w}(y)(\text{you}) \wedge |y| = n]\}$

Furthermore, the denotation in (25) can be derived compositionally under natural assumptions, recapitulating steps taken in the last section with suitable adjustments. For the NP *number of cooks* and the relative clause *you hired*, we can now assume the denotations in (26a) and (26b):

(26) a. $[\![[_{NP}\ [_{N}\ \text{number}]\ \text{of cooks}]]\!]^{w} = \{\alpha\text{: } \exists n[\alpha = \lambda v.\ \{x\text{: } [\![\text{cooks}]\!]^{v}(x) \wedge |x| = n\}]\}$
b. $[\![[_{CP}\ \text{wh [you hired t] }]]\!]^{w} = \{\alpha\text{: } \exists y[y \in \alpha(w) \wedge [\![\text{hired}]\!]^{w}(y)(\text{you})]\}$

According to (26a), we now take *number of cooks* to denote a family of number-uniform properties, viz. a family that contains for any cardinality n, the property of being n cooks. More in line with the notation in (25), this set could also be described as in (27).

[5]To illustrate, consider again the world w_1 where you hired exactly three cooks. In composing a truth value in w_1 for (21a) and (21b), A in (9) and (10) was equated with the set of all cook pluralities in w_1 of cardinality 3. Given (19), $\alpha(w_1)$ amounts to the very same set.

[6]This restatement again requires an adjusted understanding of max. This operator is now based on an ordering of number-uniform *properties*, that is, properties that for some given cardinality n, map any input world to a set of pluralities of cardinality n. We assume that for cardinalities m and n, the property of being m cooks counts as greater than the property of being n cooks in virtue of m being greater than n. The operator max in (23) is then to be understood as mapping a set of properties to a member that is greatest relative to this ordering.

(27) $\{\lambda v.\ \{x\colon [\![\text{cooks}]\!]^{v}(x) \wedge |x| = n\}\colon \text{n is a cardinality}\}$

As for the relative clause *you hired*, (26b) equates its denotation in a world w with the family of all properties α such that in w you hired a member of the set α(w). What does it take for a property to be in the intersection of the two sets in (26)? By (26a), for some cardinality n, the property must be the property of being n cooks; and by (26b), you must have hired one of the members that the property determines for w, that is, you must have hired a cook plurality in w of cardinality n. Since this amounts to the membership condition for the set of properties in (25), we have succeeded in deriving (25) through intersective composition.

Finally, the denotations in (26) are themselves subject to unexceptional compositional derivations. The denotation in (26a) can be derived by assigning *number* the entry in (28a). According to this entry for *number*, its denotation applies to the property given by the entity nominal's intension, here the intension of *cooks*. As for (26b), the entry for *hire* in (22b) guarantees the particular equivalence in (28b). Given this equivalence, (26b) falls out compositionally if relativization is assumed to leave a trace ranging over properties, and therefore to also result in abstraction over properties.

(28) a. $[\![\text{number}]\!]^{w} = \lambda\beta.\ \{\alpha\colon \exists n[\alpha = \lambda v.\ \{x\colon x \in \beta(v) \wedge |x| = n\}]\}$
b. $\exists y[y \in \alpha(w) \wedge [\![\text{hired}]\!]^{w}(y)(\text{you})] \Leftrightarrow [\![\text{hired}]\!]^{w}(\alpha)(\text{you})$

This completes our introduction of two possible elaborations of the rich amount strategy—the set solution and the property solution. We have seen that both solutions can capture the limited inventory of examples that we have focused on so far. But we should now ask how each of these solution fares when evaluated against a broader set of data. As announced before, we will see that the choice between the two solutions leads to a dilemma. Section 4 will discuss the predicted interpretation of quantity DPs where the quantity noun appears without a relative clause or other modifier. We will see that such cases behave as expected under the set solution, whereas the property solution faces a threat of undergeneration. While this result may favor the set solution, the findings in Section 5 seem to remove the set solution from contention, by identifying an undergeneration problem for this solution when applied to quantity DPs that contain a relative clause with an intensional verb.

4. Bare quantity DPs: the property solution and undergeneration

Our discussion has so far focused on quantity DPs that contain an NP with a relative clause modifier. However, as the cases in (29) illustrate, quantity DPs need not feature such a relative clause or other NP modifier.

(29) a. the amount of nuts
b. the number of cooks

Like their modified cousins, *bare* quantity DPs can participate in quantity predication, as illustrated in (30).[7]

[7] In philosophy, interest in bare quantity DPs that describe numbers can be traced back to Frege's writings (e.g., Frege 1884: §57). Frege's work triggered discussion of bare quantity DPs with *number* in English, with a focus on quantity predication in cases like *The number of planets is eight* (see, e.g., Moltmann 2013).

(30) a. [The amount of nuts] is 50 grams.
b. [The number of cooks] is three.

To be fully felicitous, sentences such as those in (30) often require contexts of utterance that raise to salience a proper subset of the entity noun's extension: the sentences in (30) are most naturally understood as reporting on the weight of a contextually salient portion of nuts or the cardinality of a contextually salient plurality of cooks. This is expectedly required to allow for such sentences to be judged true in realistic circumstances, since no plausible scenario will allow for the totality of all nuts in the world to weigh 50g or the totality of cooks in the world to have the cardinality 3. As far as we can see, this tacit domain restriction does not play a role in our arguments below.

Apart from the expected pragmatic condition on the use of bare quantity DPs, bare quantity DPs are subject to a further restriction: when saturating entity predicates, as in (31), they invariably result in deviant sentences. This deviance does not seem to merely reflect a need for domain restriction or other contextual support. The problem, it seems, is not just that felicitous uses of the sentences in (31) require contexts that are hard to imagine out of the blue. Rather, the oddness of those sentences appears to be robustly intuited regardless of context.[8]

(31) a. # I ate [the amount of nuts].
b. # I hired [the number of cooks].

We will now evaluate how the set solution and the property solution apply to bare quantity DPs, focusing as before on DPs with *number* for exposition. We begin with the property solution. Without further additions, the property solution predicts bare quantity DPs to be deviant with both quantity and entity predicates. To see that this is the case, let us consider the expected syntactic structure for the quantity DP in (29b), sketched in (32). As it is developed in Section 3, the property solution assigns to the NP *number of cooks* the denotation in (33).

(32) $[_{DP}$ the $[_{NP}$ $[_{N}$ number] of cooks]]
(33) $[\![\text{number of cooks}]\!]^{w} = \{\lambda v.\ \{x\text{: } [\![\text{cooks}]\!]^{v}(x) \wedge |x| = n\ \} : n \text{ is a cardinality }\}$

For any world w, the NP denotation is the same set of number-uniform properties: the set of properties that contains for any cardinality n, the property of being n cooks. Evidently, for any two distinct cardinalities m and n, the property of being m cooks and the property of being n cooks are distinct. Therefore, since the set of cardinalities is infinite, so is the set of properties in (33). Given the assumed cardinality-based ordering of the properties in this set (see footnote 4), this means that none of its members counts as maximal. As a consequence, the denotation in (33) cannot be in the domain of the maximality operator max, as this operator cannot map (33) to any output. Since the property solution takes max to serve as the denotation of the definite article, this means that the structure in (32) turns out to be uninterpretable.

[8] Aligned with our data in (31), Scontras (2017) notes the oddness of (ia). He suggests that this oddness can be attributed to the maximizing semantics of the definite article and the concomitant need for domain restriction. However, this suggestion misses the fact that bare quantity DPs are routinely felicitous as arguments of quantity predicates. Paralleling the contrast between (31) and (30), (ia) clearly contrasts with, say, (ib). So accommodation of the requisite domain restriction is not in general hard enough to yield judgments of oddness. Given this, appeal to the need for domain restriction is insufficient to derive the oddness of cases like those in (ia) or (31).

(i) a. #John bought the amount of apples.
b. The amount of apples is 10 kilograms.

What emerges is that according to the property solution, bare quantity DPs, in virtue of being uninterpretable, should not be usable felicitously in any syntactic frame, regardless of whether they saturate a quantity predicate or entity predicate. This correctly predicts the oddness of the entity predication cases, but at the cost of wrongly excluding the quantity predication cases. Therefore, leaving the acceptability of quantity predication with bare quantity DPs unexplained, then, the property solution faces a threat of undergeneration.

Are there ways of meeting this challenge within the limits of a property solution, revising certain negotiable assumptions adopted in Section 3? We will postpone addressing this issue until Section 6. Instead, let us now turn to the set solution detailed on Section 2. We will see that the set solution can capture bare quantity DPs in quantity predication. To illustrate, (34) shows the denotation that the set solution assigns to the NP in (32).

(34) $[\![\text{number of cooks}]\!]^{w} = \{\{x\text{: } [\![\text{cooks}]\!]^{w}(x) \wedge |x| = n\}\text{: n is a cardinality}\}$

Unlike the NP denotation under the property solution in (33), the denotation in (34) is world dependent. For any realistic possible world w with only a finite number of cooks, the set of number-uniform sets in (34) is finite as well. Specifically, for a world where the set of all cooks has cardinality n, the set in (34) will have n+1 members. To illustrate, consider again world w_1 with exactly three cooks, say c_1, c_2 and c_3. For $w = w_1$, the set in (34) will contain the singleton set containing the plurality consisting of all those three cooks, in (35a), the set containing all smaller pluralities consisting of any two atomic parts of that plurality, in (35b), and the set containing all three atomic cooks, in (35c). In addition, (34) will of course contain the set of all cook pluralities with cardinality 4, and likewise for any cardinality greater than 4. Since there are no cook pluralities of cardinality greater than 3 in w_1, all of these sets coincide with the empty set, in (35d).

(35) a. $\{ c_1 + c_2 + c_3 \}$ c. $\{ c_1, c_2, c_3 \}$
b. $\{ c_1 + c_2, c_2 + c_3, c_1 + c_3 \}$ d. $\varnothing$

Turning to the quantity DP in (32) as a whole, the set solution assigns to it the denotation in (36), the number-uniform set of pluralities that the maximality operator max outputs for (34).

(36) $[\![\text{the number of cooks}]\!]^{w} = \max(\{\{x\text{: } [\![\text{cooks}]\!]^{w}(x) \wedge |x| = n\}\text{: n is a cardinality}\})$

To illustrate, let us again consider the case where $w = w_1$, for which (34) is the four-membered set whose elements are listed in (35). Relative to the cardinality-based ordering of number-uniform sets appealed to in Section 2, (35a) clearly counts as greater than both (35b) and (35c). But how does (35a) relate to the empty set in (35d)? Note that the empty set did not appear in the argument of max in the cases studied in Section 2, so this question did not arise there. However, elaborating on what we stated in Section 2, it seems natural enough to assume that every non-empty amount-uniform set of pluralities counts as greater than the empty set. If so, then for $w = w_1$, the singleton set (35a) is the greatest member of (34), and hence it is this singleton set that max will output in (36). Generalizing from this, we see that for any world w, the DP denotation in (36) is the singleton set that contains the plurality of *all cooks* in w.

Consider now again the entry for the quantity predicate *be three* assumed in Section 2, repeated in (37). According to (37), this predicate denotes the function that applies to a set of pluralities and returns truth just in case that set contains a plurality of cardinality 3.

(37) $[\![\text{be three}]\!]^{w}(A) \Leftrightarrow \exists x[x \in A \wedge |x| = 3]$

With reference to (37), we can now analyze the quantity predication sentence in (30b). Given (36), (37) ensures that this sentence is predicted to be true in w just in case the plurality of all cooks in w has cardinality 3. In other words, the set solution predicts (30b) to convey that there are exactly three cooks. This is intuitively correct, and in particular correctly renders (30b) true in w_1. We conclude, then, that the set solution succeeds in capturing quantity predication with bare quantity DPs.

What about the deviance of bare quantity DPs with entity predicates, as in (31b)? Does the set solution provide a reason for this deviance? Initially, it does not seem to, as the sentence is expected to be interpretable. Consider again the entry for *hire* assumed in Section 2, repeated in (38) from (10).

(38) $[\![\text{hire}]\!]^{w}(A)(y) \Leftrightarrow \exists x[x \in A \wedge [\![\text{hire}]\!]^{w}(x)(y)]$

According to this entry, the denotation of *hire* in w maps a set to truth just in case you hired some member of this set in w. Given this, (31b) is predicted to be true in w_1 just in case you hired the plurality in (35a). More generally, the sentence is predicted to be true in a world w just in case you hired in w the plurality of all cooks in w.[9]

To be sure, these truth conditions do not by themselves shed light on the deviance of (31b). In fact, it is not hard to find sentences that intuitively have those truth conditions, and yet are not judged deviant. Most obviously, the relevant meaning is judged to be expressed by sentence (39), which of course can be fully felicitous.

(39) I hired [the cooks].

However, we could speculate that it is actually the very existence of sentences like (39) that contributes to the oddness of (31b). Sentence (31b) might be deviant in virtue of expressing the same truth conditions that (39) expresses while being more complex than (39) in terms of syntactic structure. In other words, sentence (31b) might be odd in virtue of being *blocked*. More generally, we could speculate that entity predication with bare quantity DPs is blocked by equivalent entity prediction with syntactically simpler DPs, DPs that are simpler in virtue of lacking a quantity nominal like *number* or *amount*. If this speculation turned out correct, the set solution would not only capture the truth conditions of quantity predication sentences with bare quantity DPs like those in (30), but would at the same time derive the oddness of entity predication sentences with bare quantity DPs, like those in (31).[10]

[9] We reported in footnote 8 that Scontras (2017) attributes the oddness of the entity predication sentence in (i) to a failure of domain restriction. We now add that, more specifically, he suggests that "in the absence of context, which could establish a salient restriction on the domain, [(i)] asserts that John bought some apples that measure the maximal degree, that is, he bought the totality of apples." Even though Scontras portrays this as a prediction of his version of the property solution (which construes rich amounts as nominalized amount-uniform properties) it is actually a prediction of the set solution. Like the version discussed in Section 3, Scontras' version of the property solution predicts that *the amount of apples* fails to denote.

(i) #John bought the amount of apples.

[10] To be sure, to defend this proposal in earnest, one would have to explain why the quantity predication in (30b), repeated in (i), is not likewise blocked by the sentence in (ii). Such an explanation could perhaps capitalize on the observation reported in, e.g., Buccola and Spector (2016: fn. 9), that sentences of the form in (i) are not in general fully acceptable, and depending on their content, can come close to being outright unacceptable. Buccola and Spector report that for them (ii) is "only marginally natural", and that *The books on that table are three* (which

Hence, the behaviour of bare quantity DPs can potentially adjudicate between the set and property construal. Even though Scontras (2017) advanced a version of the property solution, only the set solution immediately captures the behaviour of bare quantity DPs in both quantity predication and entity predication. However, we will in the next section introduce the second horn of the dilemma announced at the outset. We will identify a problem that we think closes the door to the set solution as a proper elaboration of the rich amount strategy. In Section 6, this result will then lead us to take a second look at the property solution.

5. *De dicto* readings: the set solution and undergeneration

In this section, we will identify a challenge to the set solution presented in Section 2. To set the stage, we will first revisit the basic data that we focused on there. Consider again the quantity DP in (40) and the sentence in (41), once again repeated from (18) and (21a).

(40) the number of cooks you hired

(41) [The number of cooks you hired] is three.

We would now like to draw attention to the type of inference in (42), which features as its premise the quantity predication sentence in (41).

(42) [The number of cooks you hired] is three
———————————————
There exist (at least) three cooks

Intuitively, this inference is valid, as (41) intuitively entails the existence of three cooks. This is in fact unsurprising under the set solution. We showed that, under this analysis, (41) is true just in case you hired exactly three cooks. Given these truth conditions, the inference is straightforwardly predicted to be valid. However, we would like to point out that the inference is predicted to be valid even without referencing those truth conditions in full. Specifically, given that the conclusion in (42) does not reference the content of the relative clause in the quantity DP, we can show that validity is expected regardless of the relative clause's content. More precisely, this holds as long as the relative clause is interpreted as restrictive, so that the set denoted by the NP after modification is a subset of the NP without the modifier. To prepare our case against the set solution, it will be useful to now spell out this point in some more detail.

To begin, recall that Section 2 assigned to (40) the interpreted syntactic structure sketched in (43), repeated from (13). The denotation given to the NP *number of cooks* contained in this structure, a family of sets, is repeated in (44) from (34). Also, recall again that Section 2 assigned the predicate *be three* the denotation in (45), repeated from (9).

(43) $[_{DP}$ the $[_{NP}$ $[_{NP}$ $[_{N}$ number] of cooks] $[_{CP}$ wh [you hired t]]]]

(44) $[\![\text{number of cooks}]\!]^w = \{\{x\colon [\![\text{cooks}]\!]^w(x) \wedge |x| = n\}\colon n \text{ is a cardinality}\}$

they mark as ??) is worse.

(i) [The number of cooks] is three.

(ii) The cooks are three.

For the moment, we will not scrutinize the blocking idea any further. However, we will return to the issue in Section 6, where we will see data that shed doubt on its viability.

(45) $[\![\text{be three}]\!]^{w}(A) \Leftrightarrow \exists x[x \in A \wedge |x| = 3]$

Let us now attend to the denotation of (43) as whole. In Section 2, the relative clause modifier was assumed to compose intersectively, hence restrictively. So modification by the relative clause in (43) yields an NP that in w denotes a subset of the set in (44). Further, the definite DP as a whole was taken to denote a member of the set denoted by NP. These assumptions ensure that the denotation of (43) in a world w will be a member of the family of sets in (44), hence will be a set of number-uniform cook pluralities in w. Therefore, given the existential meaning for *be three* in (45), the truth of (41) in w requires that the denotation of (43), hence the set of cook pluralities in w, contain a member with cardinality 3. It follows that there are at least three cooks in w, capturing the validity of the inference (42).

As announced above, the deduction just presented, while assuming that the modifying relative clause composes restrictively, does not otherwise make reference to the content of the relative clause. Hence the validity of inferences of the form in (42) is predicted regardless of the relative clause's content. We will now see, however, that this prediction is incorrect.

The quantity DP in (46) is like our running example (40) in that it features a modifying relative clause. The new feature in (46) is that relativization is from an intensional context, viz. from a dependent clause that serves as the complement of *want* (cf. Moltmann 2013). In some ways, the quantity DP in (46) is unexceptional. As illustrated in (47), (46) is like its simpler cousin (40) in that it can saturate both quantity predicates like *be three* and entity predicates like *hire*.

(46) the number of cooks you want to hire

(47) a. [The number of cooks you want to hire] is three.
b. I hired [the number of cooks you want to hire].

However, the quantity predication sentence (47a) differs from its counterpart in (41) with regard to intuitions about the inference in (48), parallel to (42). While (42) is unequivocally valid, the same is not true for (48), where sentence (47a) serves as the premise.

(48) [The number of cooks you want to hire] is three
——————————————————
There exist (at least) three cooks

Intuititively, the validity of (48) depends on how its premise, sentence (47a), is understood. In one possible reading, call it *de re*, (47a) entails the existence of a particular plurality of three actual cooks that you want to hire. Under this reading of the premise, the inference in (48) is valid. However, setting this *de re* reading aside, (47a) also permits a reading, call it *de dicto*, which does not come with any implications about actual cooks, hence renders (48) invalid. To confirm the existence of such a reading, it may help to imagine replacing *cooks* in (47a) with, say, *vampires*. Aligned with our claim about (48), it is clear that the resulting variant of (47a) could well be considered true by someone who at the same time adheres to the sensible view that there are no actual vampires at all, let alone three.

Sentence (47a) seems to inherit the ambiguity just described from an ambiguity of the quantity DP in (46). Intuitively, this quantity DP itself can be read as *de re* or as *de dicto*. What we just saw is that the *de dicto* reading is not captured by the set solution detailed in Section 2.

Stepping back, though, we should ask whether it is possible to derive such readings by revising the assumptions in Section 2 while preserving the construal of rich amounts as sets of entities. Such a revised account would posit that the denotation of (46) in a world w is a set of pluralities of ordinary entities, but *not* a set of pluralities of *cooks* in w, and, hence, it is not drawn from the family of sets in (44). One attempt to pursue this option might posit that the denotation of (46) is instead a member of the family of sets in (49).

(49) $\{\{x: |x| = n\}: n \text{ is a cardinality}\}$

The non-empty sets in the family of sets in (49) partitions the set of *all pluralities* of ordinary entities, including both cooks and non-cooks. Hence the denotation of (46) would be, for some cardinality n, the set of all pluralities of that cardinality. The immediate benefit of this revision would be that, given the entry in (45), (47a) would no longer be predicated to entail that there exist at least three cooks, but merely that there exist three individuals. This would accommodate the intuition that (48) need not be valid.

However, the proposal that (46) denotes a member of (49) is not in fact a viable way of capturing intuitions about (48). One question is how such a denotation would arise compositionally. In particular, it is unclear on what grounds the denotation of the quantity DP in (46) might wind up not making any reference to the meaning of *cooks*. But we see a more decisive objection to the proposal that (46) denotes a member of (49). This proposal has unwanted consequences for the inference in (50).

(50) [The number of cooks you want to hire] is three
I hired [the number of cooks you want to hire]
―――――――――――――――――――――
I hired (at least) three cooks

The inference in (50) is intuited to be unambiguously valid. Given the denotations for *be three* in (45) and *hire* in (51), this should not be so if the quantity DP in (46) could denote a member of (49). If (46) had such a denotation, then the two premises in (50) would be *not* predicted to entail that there are three *cooks*. They should merely support the weaker conclusion that there are at least three *individuals* (that the speaker hired), who may or may not be actual cooks.

(51) $[\![\text{hire}]\!]^{w}(A)(y) \quad \Leftrightarrow \quad \exists x[x \in A \wedge [\![\text{hire}]\!]^{w}(x)(y)]$

We in fact do not see a principled development of the set solution that would reconcile intuitions about the inference in (48) with those about the inference in (50). We therefore conclude that quantity DPs with intensional relative clauses seem beyond the reach of the set solution.

So we now see in full the dilemma for the rich amount strategy that we announced at the outset. The option of construing rich amounts as sets is precluded by *de dicto* readings. At the same time, construing rich amounts as properties leads to an undergeneration challenge with bare quantity DPs. How is this dilemma to be resolved? While we will not attempt a comprehensive and final answer to this question in this paper, we will briefly discuss one avenue of resolution, viz. the possibility of amending the property solution so as to fit all the data we have seen. We will propose in the next section that such an amendment can be devised, but that it requires stipulations that would remain to be derived from more principled and independently motivated assumptions.

6. Tailoring the property solution?

Having seen that the set solution fails to deliver *de dicto* readings, we now add that such readings can be accommodated under a property solution. For reasons of space, we will demonstrate this here in a somewhat compressed format. Consider again the quantity DP with *want* in (46), repeated here as (52). Given assumptions stated in Section 3, (52) is expected to allow for a denotation that, for any world w, can be stated as in (53).

(52) the number of cooks you want to hire

(53) $[\![\text{the number of cooks you want to hire}]\!]^{w} =$
$\lambda v.\ \{x: [\![\text{cooks}]\!]^{v}(x) \wedge |x| = \max\{n: \forall u[u \in Acc_{w} \rightarrow \exists y[[\![\text{cooks}]\!]^{u}(y) \wedge [\![\text{hired}]\!]^{u}(y)(\text{you}) \wedge |y| = n]]\}$

Encoding content contributed by *want*, Acc in (53) maps any world w to the set of all your desire worlds in w, the worlds where what you want in w is realized. According to (53), the denotation of (52) in w is then the property of being m cooks, where m is the greatest cardinality n such that you hire (at least) n cooks in all of your desire worlds in w.

With this in mind, we now return to the quantity predication sentence in (47a), repeated here as (54). We noted that in the reading of this sentence that we are interested in, its *de dicto* reading, the sentence does not intuitively entail that there exist three cooks.

(54) [The number of cooks you want to hire] is three.

Now, we *would* derive this unwanted existence inference if we insisted on the entry for *be three* assumed in Section 3, repeated in (55a) from (22a). This is so because the truth conditions derived would still wind up quantifying existentially over a set of pluralities of cooks *in the world of evaluation w*. However, the unwanted existence inference can be eliminated with a suitable revision of the entry for *be three*. Specifically, the inference is removed if the condition "$x \in \alpha(w)$" in (55a) is replaced with the weaker condition "$\exists v[x \in \alpha(v)]$", as in (55b).

(55) a. $[\![\text{be three}]\!]^{w}(\alpha) \Leftrightarrow \exists x[x \in \alpha(w) \wedge |x| = 3]$
b. $[\![\text{be three}]\!]^{w}(\alpha) \Leftrightarrow \exists x[\exists v[x \in \alpha(v)] \wedge |x| = 3]$

Given this replacement, the condition for truth in (55b) does not impose any condition on w specifically, and so allows for its input property to map w to the empty set. As a consequence, as desired, the resulting meaning for (54) will no longer entail that there are cooks, let alone three. At the same time, for an input property that is amount-uniform, this condition requires that this property be one that is based on cardinality 3. For (54), assuming (53), this amounts to the condition that the quantity DP denote the property of being three cooks. Hence, (54) is now predicted to have a reading that is true in w just in case the largest cardinality n such that you hire (at least) n cooks in all of your desire worlds in w is 3. We take it that these truth conditions correctly capture the *de dicto* reading of sentence (54).

Notably, replacing (55a) with (55b) does not have adverse effects for the analysis of quantity prediction in basic cases like (56), which repeats (41). The denotation that Section 3 derived for the quantity DP in (56) is repeated from (20) in (57). What does it take for (57) to map a given world v to a set that contains a plurality of cardinality 3? It must be the case that *in world* *w* you hired exactly three cooks. Hence (56) is still predicted to entail that there are (at least) three cooks in w, so that the intuitive validity if the inference (42) continues to be captured.

(56) [The number of cooks you hired] is three.

(57) $[\![\text{the number of cooks you hired}]\!]^{w} =$
$\lambda v.\ \{x\text{: } [\![\text{cooks}]\!]^{v}(x) \wedge |x| = \max\{n\text{: } \exists z[[\![\text{cooks}]\!]^{w}(z) \wedge [\![\text{hire}]\!]^{w}(z)(\text{you}) \wedge |z| = n]\}\}$

Having seen that the property solution can accommodate *de dicto* readings, let us now revisit the undergeneration challenge for this approach from bare quantity DPs that we identified in Section 4. Consider again the bare quantity DP in (58), which repeats (29b). The expected denotation for the NP *number of cooks* under the property solution in Section 4 is repeated from (33) in (59). We noted in Section 4 that the set of properties in (59) lacks a greatest element, so that (58) is predicted to be uninterpretable. And we noted that this is problematic, since a quantity predication sentence like (60), repeated from (30b), is actually judged to be felicitous.

(58) the number of cooks

(59) $[\![\text{number of cooks}]\!]^{w} = \{\lambda v.\ \{x\text{: } [\![\text{cooks}]\!]^{v}(x) \wedge |x| = n\}\text{: n is a cardinality}\}$

(60) [The number of cooks] is three.

We now acknowledge that, while this observation presents a challenge to the property solution as detailed in Section 4, it does not by itself exclude an implementation of the rich amounts strategy that construes rich amounts as properties. As one possibility, consider a conceivable denotation for (58) that can be stated as in (61).

(61) $[\![\text{the number of cooks}]\!]^{w} =$
$\max(\{\lambda v.\ \{x\text{: } [\![\text{cooks}]\!]^{v}(x) \wedge |x| = n\}\text{: } \exists x[[\![\text{cooks}]\!]^{w}(x) \wedge |x| = n]\})$

In (61), the argument of max is a set of properties which, for a given world w, includes the property of being n cooks only if there are (at least) n cooks in w, hence if the property maps w to a non-empty set. As long as the number of cooks in w is finite, this set *does* have a greatest element for max to output, and sentence (60) is correctly predicted to be true in w just in case there are exactly three cooks in w.

One question that remains is how the denotation in (61) might be derived compositionally. In an answer that sticks close to the assumptions in Section 3, the argument of max in (61) is the denotation of the NP sister of the definite article. In turn, that answer leads to the question of how the NP denotation could be derived compositionally. Again staying close to Section 3, one answer posits a covert predicate Δ, which, as stated in (62), denotes in a world w the set of all properties that map w to a non-empty set. Suppose that this covert predicate modifies the NP *number of cooks*. As stated in (63), given (59), intersective composition will then have the intended effect, deriving the argument of max in (61).

(62) $[\![\Delta]\!]^{w} = \{\alpha\text{: } \exists y[y \in \alpha(w)]\}$

(63) $[\![[_{\text{NP}}\ [_{\text{NP}}\ [_{\text{N}}\ \text{number}]\ \text{of cooks}]\ \Delta]]\!]^{w} =$
$\{\lambda v.\ \{x\text{: } [\![\text{cooks}]\!]^{v}(x) \wedge |x| = n\}\text{: } \exists x[[\![\text{cooks}]\!]^{w}(x) \wedge |x| = n]\}$

What we just outlined is a possible resolution of the undergeneration challenge identified for the property solution in Section 3. Of course, this resolution relies on a premise that would remain to be motivated, viz. the assumption that a silent predicate like Δ is indeed made available by grammar. The proposal furthermore raises questions about the distribution of Δ. If modification by Δ were obligatory, then the intended *de dicto* reading for (54) above could no longer be derived, as the resulting truth conditions would carry as an unwanted additional entailment the entailment there are three cooks. In fact, since (54) does not seem to even *permit* a *de dicto* reading with such an additional existence entailment, modification by Δ in (54) would need not be somehow excluded.

Further questions about the distribution of Δ arise from the central observation reported in Section 4 that while bare quantity DPs permit quantity prediction, they do not permit entity predication. Consider again the deviant sentence in (64), which repeats (31b). As noted in Section 4, this deviance could be derived with reference to the NP denotation in (59), which would render (64) uninterpretable. However, such a derivation of the deviance of (64) requires that in this sentence, the NP *number of cooks* resists modification by Δ.

(64) # I hired [the number of cooks].

As long as it is assumed that the quantity predication sentence (56) permits modification by Δ in the quantity DP, it seems implausible that the presence of Δ in the same quantity DP in (64) is excluded for syntactic reasons. However, there is another possibility, already entertained in Section 4 in our evaluation of the set solution. On a parse of (64) with Δ modifying NP, the sentence would convey that I hired all the cooks. Given this, it is conceivable that such a parse is blocked by the equivalent and less complex sentence in (65), which repeats (39).

(65) I hired [the cooks].

However, this proposal is once again called into question by further data. An overt relative clause like *that there are* is expected to permit a meaning much like the meaning of Δ in (62). Therefore, given the hypothesis that modification by Δ in (64) is subject to blocking, we should expect the same to hold for the overt relative clause *that there are*. However, this prediction does not seem to be borne out. In clear contrast to (64), (66) seems to be felicitous, no less so than the other quantity DPs with relative clauses that we have presented in this paper.

(66) I hired [the number of cooks [that there are]].

In sum, the success of an analysis of bare quantity DPs that relies on a covert modifier like Δ depends on the availability of a satisfactory account of this predicate's distribution. Stepping back, to show that the property solution is viable in some form or other, it remains to be shown how this approach can capture the distribution of bare quantity DPs, reconciling the deviance of such DPs in entity predication with their felicitous uses in quantity predication, while at the same time accommodating quantity DPs with *de dicto* readings.

7. Conclusion

We have explored the prospects of the rich amount strategy as an analysis of quantity DPs. We found that this approach faces a dilemma, which arises from quantity DPs with *de dicto* readings and bare quantity DPs. *De dicto* readings exclude a set construal of rich amounts, but bare quantity DPs introduce challenge for a property construal that remains to be resolved. We suggest that possible resolutions should be evaluated by comparison with an alternative approach to quantity DPs proposed in Alonso-Ovalle and Schwarz 2023, an approach where quantity predication and entity predication arise from different structures of DP, and the amount that enter semantic composition are the type of abstract *lean* quantities proposed in the natural sciences, rather than rich amounts.

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