It's not about *about* – comparatives, negation and intervals[1]

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Abstract. Solt (2014, 2018) discovered an intriguing pattern regarding the distribution of the approximator *about*. While *about n* is typically infelicitous under negation, this pattern is reversed with *more than about n*, which is fine under negation, but not in a simple, unembedded context. Solt proposed an ingenious account based on certain assumptions about the meaning of *about* and principles of language use, and, specifically, the fact that *about* is an approximator that manipulates a granularity parameter. I argue that the pattern uncovered by Solt is not specifically tied to approximators, as it can be reproduced with disjunctions of numerals and interval-denoting expressions (*between n and m*), and is therefore part of a broader generalization. I offer an account based on (a) the universal density of measurement scales (Fox and Hackl 2006), (b) a semantic analysis of degree constructions that involves in a crucial way the notion of *maximal informativity* (Buccola and Spector 2016, with roots in Rullmann 1995; Fox and Hackl 2006; Schlenker 2012; von Fintel et al. 2014) and (c) a pragmatic ban on redundant numerical expressions. I then discuss some limitations of the proposal, in comparison with Solt's.

Keywords: Numerals, Degrees, Intervals, Approximation, Comparatives, Negation, Maximal Informativity, Scope, Monotonicity.

1. Introduction

Solt (2014, 2018) noticed an intriguing pattern regarding the approximator *about*: while *about n* is typically degraded under negation (cf. (1)), *more than about n* is, on the contrary, degraded in a non-negative environment, and licensed in a negative environment (cf. (2)).

(1) a. Mary is about twenty years old
  b. *Mary is not about twenty years old

(2) a. *Mary is more than about twenty years old
  b. Mary is no more than about twenty years old
  c. (?) Mary's isn't more than about twenty years old.

Focusing mostly on the contrast in (2), I argue that Solt's account is not general enough: it crucially relies on the fact that *about* induces a kind of vagueness, but the very same pattern is observed for other degree expressions which denote sets of degrees, but are not semantically vague. I propose an alternative account based on the universal density of measurement scales and the notion of maximal informativity, in the spirit of Fox and Hackl (2006). I then discuss

[1]The title of this paper is inspired by that of another paper on approximators, by Paul Égré, entitled 'Around *around*' (Égré 2022), which briefly alludes to ideas I had at the time in connection with Solt's observations, but which I have since then become skeptical of. I am grateful to Paul Égré for the innumerable conversations we have had about *about*! I would like to also thank Danny Fox, Jeremy Kuhn and Stephanie Solt for extremely helpful exchanges, as well as the audience of *Sinn und Bedeutung 27* in Prague and four anonymous reviewers. I also acknowledge support from the Agence Nationale de la Recherche (FrontCog Project ANR-17-EURE-0017 and ProbaSem project ANR-19-CE28-0004-01).

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the pattern in (1).

Section 2 provides a summary of Solt's (2018) proposal in connection with (2).[2] Section 3 argues that the underlying generalization is not specifically related to approximators, which creates a problem for this account. In section 4, I advance my alternative proposal. Section 5 then discusses how the proposal could apply to (1) (and related data). Finally, section 6 discusses additional data that bear on the comparison of Solt's proposal and mine, in relation with monotonicity.

2. Solt's (2018) account

Solt (2018) accounts for the deviance of (2a) in terms of competition with the sentence that is obtained by deleting *about*, and specific pragmatic principles that regulate message choice in cases involving imprecision and approximators. Following Lasersohn (1999), Solt assumes that numerals are interpreted with respect to a certain contextually given granularity parameter i, and that an approximator such as *about* has the effect of turning a degree-refering expression into an expression that denotes a 'coarse-grained' degree, i.e. an interval centered on the relevant degree, with a length that itself depends on a granularity level i'. So the deviant sentence in (2a) and its countepart without *about* are interpreted as follows.

(3) a. *Mary is more than about twenty years old
$\rightsquigarrow$ Mary's age is above the interval $[20-i', 20+i']$, i.e. Mary's age $> 20+i'$
b. Mary is more than twenty years old
$\rightsquigarrow$ Mary's age is above 20, interpreted at some level of granularity i,
i.e Mary's age $> 20+i$

Now, nothing in the semantics imposes that i' should be greater than i. As a result of this, (3a) is not *definitevely more informative* than (3b) according to Solt's definition of *definitive informativeness*: a sentence S is definitively more informative than a sentence T if a) relative to any fixed granularity level for both sentences, S entails T, and b) there is a world where for every possible pair of granularity levels (i,i'), T is true relative to i and S is false relative to i'. Now, while (3a) does entail (3b) relative to any fixed granularity parameter (and is in fact equivalent to it), there is no world where, for every possible (i,i'), (3b) is true but (3a) is false, because it is always possible to set i' to 0, so (3a) is not definitively more informative than (3b). Furthermore, because (3a) is more complex than (3b) but is not definitively more informative, it is never a better message than (3b) if both are 'weakly assertable', namely if the speaker considers both to be true. So using (3a) should trigger the inference that the speaker does not take (3b) to be true. However, Solt also assumes a pragmatic principle whereby the use of an approximator signals that the bare numeral variant should be interpreted in a more precise way than the version with an approximator (i.e. $i' > i$). Given this, a speaker who takes (3a) to be true must take (3b) to be true, and so the inference that the speaker does not deem (3b) true is contradictory with her use of (3a). This explains the infelicity of (3a). In a nutshell, the gist of the account is the following:

1. Given the imprecise readings of round numbers, (3a) is not really more informative than

[2] I do not discuss here the more recent proposal made in Solt and Waldon (2019), which offers a completely different account for the contrast in (1), but does not address the one in (2).

(3b); it is also more complex. Hence (3b) is preferable to (3a) if it is assertable.

2. If (3a) was used, (3b) was not assertable.
3. In a context where a speaker is in a position to say (3a), she is also in a position to say (3b), since the use of 'about' in (3a) indicates that the granularity level used when interpreting (3a) is greater than the one used when interpreting (3b). So if the speaker used (3a), the conclusion (from above) that (3b) is not assertable is necessarily false, which leads to a contradiction.

The fact that (2b) is felicitous results from the observation that the last step in the above reasoning is no longer valid when we turn to the negations of the two competitors:

(4) a. Mary is no more than about twenty years old.
b. Mary is no more than twenty years old.

Again, (4a) is neither definitively more informative nor simpler than (4b), so using (4a) triggers the inference that (4b) was not assertable. But this time, this conclusion is not contradictory, because entailment relations are reversed under negation. Assuming that the granularity level for (4a) is greater than the one for (4b), one can believe (4a) (i.e. that Mary's age no more than $20+i'$) without believing (4b) (i.e. without believing that Mary's age is no more than $20+i$, with $i<i'$), and so no contradiction ensues. Furthermore, this account predicts that (4a) triggers the implicature that its competitor (4b) was not assertable under its precise meaning, i.e. that Mary is possibly more than 20 years old – though very close to 20 years old.

3. Beyond Approximators

Disjunctions of numerals, especially if they are neighbors relative to a salient granularity scale (*twenty-two or twenty-three*, *thirty or forty*), appear to pattern just like *about n*:[3]

(5) Context: is Mary still in college?
a. ?? She's more than twenty-two or twenty-three years old, so probably she has graduated.
b. She's no more than twenty-two or twenty-three years old, so she might still be in college.

As Solt (2018) notes in her conclusion, her account does not extend to this kind of contrast involving disjunctions of numerals. Since (5b) is semantically equivalent (in her account) to the simpler *She's no more than 23 years old*, it should always be infelicitous.

Furthemore, a similar pattern is also observed with expressions that denote intervals but contain no approximator, such as *between n and m*, as evidenced by the following attested examples, all found by a Google Search, in which *more than between n and m* occurs in the scope of a negative element:

(6) a. You should load your dogs pack with **no more than between ten and twenty-five percent** of your dog's weight.

[3]The degraded sentence in (5a) becomes better in an echoic context, e.g., as a direct objection to something like (5b). We put such uses aside for the rest of this paper.

b. Experienced instructors felt students would **not** engage in any one media or text for **more than between ten and twenty** minutes.
c. During the five years of occupation, probably **not more than between ten and twenty** thousand people participated in any form of armed resistance.
d. The Flesher Trade **rarely** had **more than between ten and twenty Masters**
e. Most of your landmass is **no more than between ten and twenty** metres above sea level.

Introspectively, removing the negative element, in each of these cases, degrades the sentence. To further buttress this empirical claim, I report judgments on the following pair, collected from 18 self-reported native speakers of English, obtained through the Amazon Mechanical Turk platform.

(7) a. *Air Syldavia owns more than between 50 and 100 airplanes.
b. (?) Air Syldavia owns no more than between 50 and 100 airplanes.

Of the 18 native speakers tested, 3 gave the same rating to both sentences, 13 preferred (7b), and 2 preferred (7a), making the difference in favor of (7b) statistically significant by a sign-test.[4]

In the *no more more than* ... environment, disjunctions of numerals and *between*-phrases also pattern with *about n* regarding the kind of inferences they give rise to. That is, in the same way as (2b) (*Mary is no more than about twenty years old*) suggests that Mary is about 20 years old, (5b) (*Mary is no more than twenty-two or twenty-three years old*) and (7b) (*Air Syldavia owns no more than between fifty and one hundred airplanes*) suggest, respectively, that Mary is 22 or 23 years old, and that Air Syldavia owns between 50 and 100 airplanes. In all these cases, a sentence of the form ... *no more than* {*about n*}/{*n or m*}/{*between n and m*} ... is felt to imply the truth of ... {*about n*}/{*n or m*}/{*between n and m*} ....

In the next sections, I argue for an alternative proposal, based on Fox and Hackl (2006), and argue that it captures both the core distributional facts and the perceived readings of the relevant sentences.

4. The proposal: degrees, maximal informativity and universal density of measurement scales

My proposal relies on several ingredients: Fox and Hackl's (2006) *universal density of measurement scales* hypothesis, and a specific treatment of modified numerals that involve the notion of maximal informativity, based on Buccola and Spector (2016) (with roots in Rullmann 1995; Fox and Hackl 2006; Schlenker 2012; von Fintel et al. 2014). It also includes a pragmatic preference for simpler sentences, spelt out below.

[4] Each participant saw both sentences on the same screen, the order of the two sentences was randomized, and they had to rate each sentence from 1 to 7. I tested 24 participants, but excluded one who did not provide any answer and 5 who gave an incorrect answer to a forced choice question based on a clear acceptability constrast which was completely unrelated to the sentences of interest. Keeping all the 23 participants who gave answers, 3 gave the same rating to both sentences, 16 preferred (7b), 4 preferred (7a), making again the contrast in favor of (7b) significant by a sign test.

4.1. Semantic and Pragmatic Assumptions

I adopt the following semantic assumptions (certain choices are made for convenience and are not crucial to the account):

1. Numerals are ambiguous: they can either denote degrees (type d) or Generalized Quantifiers over predicates of degrees (type $\langle d,t \rangle$):

   (8) a. $[\![\text{three}_d]\!]^w = 3$
   b. $[\![\text{three}_{\langle dt,t \rangle}]\!]^w = \lambda P_{dt}.P(3)$

2. When a numeral acts as a determiner (i.e. combines with a count noun), it needs first to combine with a silent existential closure operator ε, which creates a standard determiner (type $\langle et, \langle et,t \rangle \rangle$):

   (9) $[\![\varepsilon]\!]^w = \lambda n_d.\lambda P_{et}.\lambda Q_{et}.\exists X(\#X = n \wedge P(X) \wedge Q(X))$

3. *More than* combines with a degree-denoting expression to form a GQ over predicates of degrees (type $\langle dt,t \rangle$):

   (10) $[\![\text{more than}]\!]^w = \lambda n_d.\lambda P_{dt}.\exists m_d(m > d \wedge P(m))$

4. Type-mismatches are resolved by movement. That is, the analysis is couched within Heim and Kratzer's (1998) framework where type mismatches are resolved by covert LF movement rather than in terms of type-shifting, flexible types or function composition, but nothing essential hinges on this, and the analysis should be easy to translate into another framework.

5. A covert operator $\mathtt{max}_{\mathtt{inf}}$ that can freely be introduced takes the intension of a predicate of degree P and returns a predicate of degree P' defined as follows: P' is true a degree n if (a) P is true of n and (b) there is no true proposition of the same form that asymmetricallty entails the proposition that P is true of n. In other words, and very informally (and allowing for massive confusion between the metalanguage and our object language), '$\mathtt{max}_{\mathtt{inf}}(P)(n)$' states that the proposition expressed by '$P(n)$' is true and that there is no other true proposition expressible as '$P(m)$' which is both true and strictly more informative than the proposition expressed by '$P(n)$'. To put it in yet another way, '$\mathtt{max}_{\mathtt{inf}}(P)(n)$' states that n is the maximally informative numeral that can be truthfully used as an argument of P.[5]

   (11) $[\![\mathtt{max}_{\mathtt{inf}}]\!]^w = \lambda P_{\langle s,dt \rangle}.\lambda n_d.P(w)(d) \wedge \neg\exists m(\lambda v_s.P(v)(m) \subsetneq \lambda v_s.P(v)(n))$

6. Universal Density of Measurement Scales: for any two degrees m and n such that $m < n$, there exists a degree k such that $m < k < n$.

Furthermore, in the spirit of Buccola and Spector (2016), I adopt the following usage principle, which instantiates a general pragmatic preference for simpler expressions (in line with Solt's own account):

[5] $\subsetneq g$ represents asymmetric entailment: $f \subsetneq g$ if for every x such that $f(x) = 1$, $g(x) = 1$, and there is a y such that $f(y) = 0$ and $g(x) = 1$

Ban on redundant numerical expressions
A sentence S containing an occurrence of an expression of the form *m or n*, *about n*, or *between n and m* is infelicitous if for some numeral k, replacing the relevant occurrence with k yields a sentence S' that is equivalent to S.

4.2. Simple Comparative Numerals

Fox and Hackl (2006) noticed the following constrast:

(12) a. Gloria read more than three books.
$\rightsquigarrow$ does not implicate that John read exactly four books
b. Gloria read no more than three books.
$\rightsquigarrow$ implicates that John read exactly three books.

Our assumptions in the previous section allow us to explain this contrast, in exactly the same way as Fox and Hackl (2006), apart from implementation details (which is not surprising, since the above assumptions are directly inspired by Fox and Hackl 2006)

Starting with (12a), this sentence can receive many different parses, depending on whether and where $\mathtt{max}_{\mathtt{inf}}$ is introduced, as well as whether LF movement takes place. I will consider all and only structures that result from type-mandated LF movement and optional insertion of $\mathtt{max}_{\mathtt{inf}}$. In particular, not only must *more than three* raise for type-reasons (on top of 'standard' LF object movement for type reasons), *three* itself, when parsed as a GQ as in (8b) (type $\langle dt,t\rangle$), must raise out of *more than* ..., since *more than* combines with a type d expression. Then $\mathtt{max}_{\mathtt{inf}}$ can be freely inserted in any position where its type fits.[6] As a result, we have to consider six parses, as in (13) (even if we allowed additional parses, they would, as far as I can tell, always involve semantically vacuous movement, vacuous insertion of $\mathtt{max}_{\mathtt{inf}}$ or contradiction-inducing insertion of $\mathtt{max}_{\mathtt{inf}}$ and will essentially reduce to one of those parses):

(13) a. [more than three]$[\lambda n_d.[[\varepsilon\ n]$books$][\lambda x.$Gloria read $x]]$
b. three$_{\langle dt,t\rangle}[\lambda m_d[$[more than $m_d][\lambda n_d.[[\varepsilon\ n]$books$][\lambda x.$Gloria read $x]]]]$
c. [more than three]$[\mathtt{max}_{\mathtt{inf}}[\lambda n_d.[[\varepsilon\ n]$books$][\lambda x.$Gloria read $x]]]$
d. three$_{\langle dt,t\rangle}[\lambda m_d[$more than $m_d][\mathtt{max}_{\mathtt{inf}}[\lambda n_d.[[\varepsilon\ n]$books$][\lambda x.$Gloria read $x]]]]]$
e. three$_{\langle dt,t\rangle}[\mathtt{max}_{\mathtt{inf}}[\lambda m_d[$[more than $m_d][\lambda n_d.[[\varepsilon\ n]$books$][\lambda x.$Gloria read $x]]]]]$
f. three$_{\langle dt,t\rangle}[\mathtt{max}_{\mathtt{inf}}[\lambda m_d[$[more than $m_d][\mathtt{max}_{\mathtt{inf}}[\lambda n_d.[[\varepsilon\ n]$books$][\lambda x.$Gloria read $x]]]]]]$

Even though these parses are hard to make sense of, it should be clear that (13a) and (13b) are equivalent (they only differ with respect to the type of the numeral, which in the second case was forced to move for type reason, but with no intervening material). Likewise, (13c) and (13d) are equivalent. So we can focus on (13a), (13c), (13e) and (13f). I translate them into semi-formal representations in (15), in which only the most relevant elements are retained.

(14) a. Gloria read more than three books
b. [more than three]$[\mathtt{max}_{\mathtt{inf}}[\lambda n_d.$Gloria read n books$]]$

[6]Since $\mathtt{max}_{\mathtt{inf}}$ must take as its argument something of type $\langle s,dt\rangle$, the denotation of its sister (of type dt) must be 'raised' to such an intensional type, i.e., within the Heim & Kratzer framework, we must resort to 'intensional functional application'.

c. $\text{three}_{\langle dt,t\rangle}[\mathtt{max_{inf}}[\lambda n_d.\text{Gloria read more than } n \text{ books}]]$
d. $\text{three}_{\langle dt,t\rangle}[\mathtt{max_{inf}}[\lambda m_d.[\text{more than } m_d][\mathtt{max_{inf}}[\lambda n_d.\text{Gloria read } n \text{ books}]]]]$

It turns out that (14a) and (14b) are equivalent, and just mean, unsurprisingly that the number of books read by Mary is greater than 3, while (14c) and (14d) are contradictory.

(13a) and (13b) are just equivalent to the plain, literal intuitive meaning of the sentence, represented in (14a), since they are essentially the most simple structures compatible with the types of the various elements and do not include $\mathtt{max_{inf}}$.

To interpret (14b) (which corresponds to (13c)), let us first focus on the subconstituent '$[\mathtt{max_{inf}}[\lambda n_d.\text{Gloria read } n \text{ books}]]$'. This denotes a predicate of degree which is true of a number n if (a) Gloria read n books and (b) there is no number m such that the proposition that Gloria read m books is true and asymmetrically entails the proposition that Gloria read n books. One can see that there is exactly one number that satisfies this condition, namely the maximal number n such that Mary read n books, i.e. the number of books that Mary read. This predicate then combines with *more than three*, giving rise to the proposition that there is a number greater than 3 which is the number of books that Mary read – so the resulting meaning is equivalent to that of (14a), making $\mathtt{max_{inf}}$ vacuous.

As to the third parse in (14c) (which corresponds to (13e)), let us see what the (semi-formal) predicate '$[\mathtt{max_{inf}}[\lambda m_d.\text{Gloria read more than } n \text{ books}]]$' is predicted to mean. This predicate is true of a number n if (a) Gloria read more than n books, and (b) there is no m such that the proposition that Gloria read more than m books is true and asymmetrically entails that Gloria read more than n books. Fox and Hackl's (2006) key insight is that, due to density, there cannot exist any such n. For suppose that, for some n, Gloria read more than than n books (which is a necessary condition for the predicate to hold of n, cf. clause (a)). Then there is a number x such that $x > n$ and Gloria read x books. But then take a number y such that $x > y > n$, which is guaranteed to exist given density. Since Gloria read x books, Gloria read more than y books. And since $y > n$, the proposition that Gloria read more than y books asymmetrically entails the proposition that Gloria read more than n books, and therefore n cannot meet the condition stated in clause (b). Since this reasoning holds for any n, the denotation of the predicate '$[\mathtt{max_{inf}}[\lambda m_d.\text{Gloria read more than } n \text{ books}]]$' is necessarily empty. Consequently, the whole parse in (14c) is necessarily false (i.e. contradictory), since it entails that the number 3 belongs to the denotation of a necessarily empty predicate.

Finally, we can see that (14d) is contradictory again. This is because the subconstituent '$[\lambda m_d.[\text{more than } m_d][\mathtt{max_{inf}}[\lambda n_d.\text{Gloria read } n \text{ books}]]$' is true of m just in case there is a number greater than m which is the number of books that Mary read. In other words, this predicate means *be a number greater that the number of books Gloria read*. As discussed in the previous paragraph, appending $\mathtt{max_{inf}}$ to this predicate results into a necessarily empty predicate; so the larger constituent '$[\mathtt{max_{inf}}[\lambda m_d.[\text{more than } m_d][\mathtt{max_{inf}}[\lambda n_d.\text{Gloria read } n \text{ books}]]]]$' is necessarily empty and the whole parse in (14d) is contradictory.

This explains why (12a) does not trigger the implicature that Gloria read exactly four books, on the assumption that $\mathtt{max_{inf}}$ is responsible for such implicatures.

Turning to (12b) (*Gloria read no more than three books*), I assume that *no more than n* is simply the (generalized) negation of *more than three*, i.e. $[\![\text{no more than } n]\!]^w = \lambda P_{dt}.\neg\exists n(n >$

$3 \wedge P(n))$. Consider then the following semi-formal parses for (12b), obtained from those in (14) by replacing *more than* with *no more than*

(15) a. Gloria read no more than three books
b. [no more than three][$\mathtt{max_{inf}}$[λn_d.Gloria read n books]]
c. three$_{\langle dt,t\rangle}$[$\mathtt{max_{inf}}$[λn_d.Gloria read no more than n books]]
d. three$_{\langle dt,t\rangle}$[$\mathtt{max_{inf}}$[λm_d.[no more than m_d][$\mathtt{max_{inf}}$[λn_d.Gloria read n books]]]]

The parses in (15a) in (15b) are simply equivalent to the negation of those in (14a) and (14b), and so give rise to the expected non-strengthened reading (true just in case the number of books read by Gloria, if any, is at most three). But let us now consider the parse in (15c).

The predicate '$\mathtt{max_{inf}}[\lambda n$(Gloria read no more than n books)]' is true of a number n just in case Gloria read at most n books, and there is no number m such that Gloria read at most m books and saying that Gloria read at most m books asymmetrically entails that she read at most n books. It turns out that the denotation of this predicate is not necessarily empty, and will in fact consist, if Gloria read some books,[7] of the singleton whose only member is the unique number n such that Glorial read exactly n books. To see this, assume that Gloria read exactly n books. Then of course she read no more than n books (in the purely logical, mathematical normative sense of *no more than n*). Furthermore, for any $m > n$, it is also true that she read no more than m books, but stating this is strictly less informative than stating that she read no more than n books. And for any $m < n$, it is simply false that Gloria read no more than m books. Therefore, n is the unique number such that (a) Gloria read no more than n books and (b) no true proposition of the same form is logically stronger. The predicate '$\mathtt{max_{inf}}[\lambda n$(Gloria read no more than n books)]' is therefore equivalent to *being the number of books that Gloria read*. It then combines with *three*, and the resulting proposition is simply that Gloria read exactly three books. I leave it to the reader to check that the very same reading is derived for (15d). This is the desired result: even if this reading is not the only reading for (12b), it is nevertheless a possible inference that we can draw from it. On the view defended here, this is simply due to the fact that (12b) has several parses, some but not all of which mean that Gloria read exactly three books.

4.3. Disjunctions of Numerals

Everything is now in place to account for the observed pattern with disjunctions of numerals, repeated below:

(16) a. ??Mary is more than twenty-two or twenty-three years old.
b. Mary is no more than twenty-two or twenty-three years old.

I assume that *or* is a polymorphic disjunction that can conjoin two expressions of the same type that ends in t, in the familiar way. In *twenty-two or twenty-three*, the two numerals must therefore be interpreted as GQs over predicates of degrees (type $\langle dt,t\rangle$) and not as degree-denoting (type d). As a result, the denotation of *twenty-two or twenty-three* is the following:

(17) $[\![\text{[twenty-two or twenty-three]}_{\langle dt,t\rangle}]\!]^w = \lambda P_{dt}.P(22) \vee P(23)$

[7] I ignore here the complications that can arise from considerations of situations where Gloria read no books. See Buccola and Spector (2016) for a detailed discussion.

Since *more than* must combine with a degree-denoting expression, a disjunction of numerals in its scope needs to move out of it to be interpretable (type mismatch). We can therefore essentially consider two parses, depending on whether $\texttt{max}_{\texttt{inf}}$ is introduced (additional parses involing movement of *more than n* after the disjunctive phrases has raised are in principle possible, but they will not create new meanings):

(18) a. [twenty-two or twenty-three][λn_d.[Mary is more than n years old]]
b. [twenty-two or twenty-three][$\texttt{max}_{\texttt{inf}}$[$\lambda n_d$.[Mary is more than n years old]]]

The first parse is true if either 22 or 23 is a number n such that Mary is more than n years old, which is equivalent to saying that Mary is more than 22 years old. Hence this parse is infelicitous due to the ban on redundant numerical expressions, since one could replace the disjunction of numerals with simply *twenty-two* and get an equivalent sentence. The second parse is contradictory, for the very same reason why (14c) was contradictory. Namely, given the universal density of measurement scales, the predicate '[$\texttt{max}_{\texttt{inf}}$[$\lambda n_d$.[Mary is more than n years old]]]' necessarily has an empty denotation (as discussed above in connection to (14c)): if Mary is more than n years old, then she is also more than $n+\varepsilon$ years old, for some ε, and so the proposition that Mary is n years old, if true, is always asymmetrically entailed by some true proposition of the same form. So (16a) is necessarily infelicitous, since, depending on the parse, it either violates the ban against redundant numerical expressions, or is contradictory.

Let me now turn to why (16b), repeated below as (19), is felicitous.

(19) Mary is no more than twenty-two or twenty-three years old.

Let us consider the following two parses:

(20) a. [twenty-two or twenty-three][λn_d.[Mary is no more than n years old]]
b. [twenty-two or twenty-three][$\texttt{max}_{\texttt{inf}}$[$\lambda n_d$.[Mary is no more than n years old]]]

The discussion is essentially parallel to our discussion of the parses in (15) for *Gloria read no more than three books*. (20a) means that 22 or 23 is an n such that Mary is no more than n years old, which is just equivalent to saying that Mary is no more than 22 years old. Hence this parse is infelicitous given the ban on redundant numerical expressions. (20b), on the other hand, ends up meaning that Mary is 22 or 23 years old (an not older). Let us see why. The predicate '$\texttt{max}_{\texttt{inf}}$[$\lambda n_d$.[Mary is no more than n years old]]' is true of n is (a) Mary is no more than n years old, and (b) there is no m such that the proposition that Mary is no more than m years old is true and asymmetrically entails that Mary is no more than n years old. There is exactly one value that meets both condition, namely Mary's exact age. First, if a is Mary's age (measured in years), then Mary is no more than a years old. Second, the propositions of the form 'Mary is no more than m years old' that asymmetrically entail that Mary is no more than a years old are obtained by taking $m < a$; but these propositions are all false, hence condition (b) is satisfied for a. Therefore, the predicate '$\texttt{max}_{\texttt{inf}}$[$\lambda n_d$.[Mary is no more than n years old]]' is equivalent to 'being Mary's age', and (20b) means that Mary's age is either 22 or 23. Importantly, the ban on redundant numerical expressions is not violated, because if we replace *twenty-two or twenty-three* with *twenty-two* (resp. *twenty-three*), the resulting meaning is that Mary's age is 22 (resp. 23), which is not equivalent to saying that it is 22 or 23. Finally, this analysis explains why (19) is typically understood as conveying that Mary's age measured in years is 22 or 23.

A problem this proposal encounters is the following: if someone utters (19) and we later learn that Mary is in fact only 15 years old, we typically do not judge the sentence false (though certainly misleading). I would like to advance two possible answers. One is that there is a parse that makes the sentence true in this situation, namely the parse in (20a). Even if this parse is pragmatically infelicitous, a charitable listener could agree that the sentence is true because it has a true parse. A second possible answer, which is even more speculative, is that the specific contribution of $\mathtt{max_{inf}}$ might not be 'at issue' and for this reason might be, under certain conditions, ignored when performing a truth-value judgment task. Given that $\mathtt{max_{inf}}$ can be viewed as an exhaustivity operator, this would be in line with some recent works claiming that the contribution of the exhaustivity operator is typically not at issue. (Bassi et al. 2021). Typically, a sentence such as (19) is used when there is a prior expectation that Mary is more than 22 or 23 years old, and seems to be used to convey that Mary is younger than one could have expected (rather than, say, older than one could have expected). The information that she is in fact 22 or 23 years old is from this point of view secondary.

4.4. *Between n and m* and *about n*

The account I have just proposed straightforwardly extends to interval-denoting expressions like *between n and m*, assuming that they are essentially equivalent to a (potentially infinite, given density) disjunction of degrees. That is:

(21) $[\![\text{between fifty and one hundred}]\!]^w = \lambda P_{dt}.\exists x_d(x \in [50,100] \wedge P(n))$ (if we ignore non-integers, this is just the same as the grand disjunction expressed by *fifty or fifty-one or ... or one hundred*)

Based on such a lexical entry, we can account for the judgments reported in Section 3 in exactly in the same way as we did in the previous section for disjoined numerals. Consider again the contrast in (7), repeated below in (22).

(22) a. *Air Syldavia owns more than between fifty and one hundred airplanes.
b. (?) Air Syldavia owns no more than between fifty and one hundred airplanes.

Essentially, parses that involve $\mathtt{max_{inf}}$ in the case of (22a) are contradictory or equivalent to a parse without $\mathtt{max_{inf}}$. And parses without $\mathtt{max_{inf}}$ express the proposition that there is a number n in $[50,100]$ such that Air Syldavia owns more than n air plances, which is equivalent to *Air Syldavia owns more than 50 airplanes*. Such parses are therefore ruled out by the ban on redundant numerical expressions. (22b), on the other hand, with a parse such as the one in (23), means that the maximally informative true proposition of the form *Air Syldavia owns n airplanes* is such that n is between 50 and 100. Since the maximally informative true proposition of this form is obtained with n being equated to the numbers of airplanes owned by Air Syldavia, the resulting meaning is that the numbers of airplanes owned by Air Syldavia is between 50 and 100 airplanes, which is the desired result.

(23) [between fifty and one hundred][$\mathtt{max_{inf}}$[λn_d.Air Syldavia owns no more than n airplanes]]

Regarding *about n*, I assume that it denotes an interval whose half-length i length is contextually given:

(24) $[\![\text{about } n]\!]^{w,i} = \lambda P_{dt}.\exists x_d(x \in [n-i, n+i] \wedge P(x))$

The account then applies in the same way. In particular, relative to a fixed i, a sentence such as *Mary is more than about 18 years old* is equivalent to *Mary is more than 18-i years old*, if $\texttt{max}_{\texttt{inf}}$ does not occur (and therefore violates the ban on redundant numerical expressions), or is contradictory if $\texttt{max}_{\texttt{inf}}$ occurs non-vacuously. But *Mary is no more than about 18 years old* is correctly predicted to be felicitous and to mean that Mary's age is in the interval $[18-i, 18+i]$, which is the desired result.

5. *About n*, *between n and m* and *n or m* without comparatives

Let me now discuss the contrast in (1), repeated in (25).

(25) a. Mary is about twenty years old.
b. ??Mary is not about twenty years old.

In the case of (25a), a parse that does not include $\texttt{max}_{\texttt{inf}}$ violates the ban on redundancy, since (24a) would then be equivalent to *Mary is 20-i years old* under an at-least readings, where i being the contextually given half-length of the interval denoted by *about twenty*, and thus would violate the ban on redundancy. But there is a parse that does not encounter this problem, provided in (26).

(26) [about twenty][$\texttt{max}_{\texttt{inf}}$[$\lambda n_d$.Mary is n years old]]

The predicate '$\texttt{max}_{\texttt{inf}}$[$\lambda n_d$.Mary is n years old]' means *being Mary's age*. As a result, (26) means that Mary's age is in the interval $[20-i, 20+i]$, which is the desired reading, and does not violate the ban on redundancy.

Given this observation, if nothing more is said, there is also a parse for (25b) that does not violate the ban of redundancy, and which is simply the negation of the parse in (26):

(27) not[[about twenty][$\texttt{max}_{\texttt{inf}}$[$\lambda n_d$.Mary is n years old]]]

To rule out this parse, we might posit that $\texttt{max}_{\texttt{inf}}$ does not like to occur under the scope of negation. As already noted, the operator $\texttt{max}_{\texttt{inf}}$ is in fact akin to the exhaustivity operator proposed in the scalar implicature and exhaustivity literature, which is typically viewed as not licensed under negation (see, e.g., Chierchia et al. 2012). One can check that all the parses that made felicitous the *no more than [m or n]/[about n]/[between m or n]* sentences in the previous sections are parses where $\texttt{max}_{\texttt{inf}}$ takes scope over the negative element *no more than*. Such a move is therefore consistent with my analysis so far.

Given this additional assumption, the only parse for (25b) that contains $\texttt{max}_{\texttt{inf}}$ is as follows:

(28) [about twenty][$\texttt{max}_{\texttt{inf}}$[$\lambda n_d$.not[Mary is n years old]]]]

We can show that (28) is contradictory (following again a core insight from Fox and Hackl 2006), because the predicate '$\texttt{max}_{\texttt{inf}}$[$\lambda n_d$.not[Mary is n years old]]' is necessarily empty, given an *at least*-meaning for the expression *n years old*.[8]. If a is Mary's age, then the set of degrees

[8]Following Heim (2000), I assume that lexical degree predicates are *monotonic*: if someone is d years old, then that person is also d' years old, for any $d' < d$, which makes *d years old* equivalent to *at least d years old*. This

that satisfy the predicate 'λn_d.not[Mary is n years old]]' is the open interval $(a, +\infty)$. Given density, this interval has no least element (since a does not belong to it). Now, for any two x and x' in this interval, the proposition that Mary is not at least $x - old$ asymmetrically entails that Mary is not at least $x' - old$ if and only if $x < x'$. But given density, there does not exist any x such that (a) Mary is not x-old and (b) there is no x' such that both $x' < x$ and Mary is x'-old. For any x such that Mary is not at least x-old, there is a smaller x' such that Mary is not at least x'-old (just pick x' between a and x). This means that the predicate '$\mathtt{max}_{\mathtt{inf}}[\lambda n_d$.not[Mary is n years old]]' is empty – there is no maximally informative true proposition of the form *Mary is not x-old*, because it is always possible, starting from such a true proposition, to find another true proposition that asymmetrically entails it. It follows that (28) is contradictory.

Now, this analysis makes completely parallel predictions when *about n* is replaced with *between m and n* and *m or n*. That is, the following patterns are *predicted*:

(29) a. Mary is twenty-two or twenty-three years old.
b. [Predicted: ??] Mary is not twenty-two or twenty-three years old.

(30) a. Mary is between twenty and twenty-five years old.
b. [Predicted: ??] Mary is not beween twenty and twenty-five years.

These judgments are quite unclear, possibly due to the possibility of interpreting the negative sentences as echoic (i.e. direct objections to their affirmative counterparts), in which case they are expected to be felicitous no matter what (it is as far as I know always felicitous to negate a sentence that has just been uttered – cf. also footnote 3). And (29b) and (30b) do in fact suggest, it seems to me, a situation where someone is directly replying to the corresponding affirmative sentence. This possibility however does not explain on its own why the judgments might be sharper, out of the blue, with (25b) than with these cases.

6. The role of monotonicity

Solt's (2018) proposal predicts that what matters to the felicity of *more than about n* is the global monotonicity of its environment: it needs to be in a monotone-decreasing environment. This is so because the key to her explanation for *Mary is no more than about 18 years old* is that this sentence can be true, relative to a certain granularity level *i*, while its counterpart without *about* is not true, under a more fine-grained granularity level. And indeed, we observe that *more than about n* is, for instance, felicitous in the scope of a universal quantifier:

(31) Everyone who is more than about twenty years old hated this movie.

However, a side effect of the fact that Solt's proposal is entirely pragmatic in nature is that it's only the *global* monotonicity of the environment that matters. That is, Solt predicts (32) to be infelicitous, contrary to fact:

(32) Everyone who is no more than about twenty years old hated this movie.

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does not entail that the English sentence *Mary is twenty-two years* old means that Mary is at least twenty-two years old, since it is possible that such a sentence always or preferably receives a parse that includes $\mathtt{max}_{\mathtt{inf}}$.

In (32), *more than about twenty* is in the scope of a negative element (*no*), which is itself in a decreasing environment, so the global environment for *more than about twenty* is itself increasing. For this reason, Solt's proposal, as far as I can tell, predicts (32) to be infelicitous.

In contrast with this, my proposal can predict both (31) and (32) to be felicitous, though in the case of (32), I need to make some quite counterintuitive assumptions, which are already present in Fox and Hackl (2006)).

Starting with (31), consider the following parse:

(33) [about twenty][$\mathtt{max_{inf}}$[λx_d.Everyone who is more than x years old hated this movie]]

Consider a model where there is someone who is x years old and who loved the movie, and where everybody who is more than x years old hated it. Then, quite obviously, the proposition that everyone who is more than x years old hated the movie is the strongest true proposition of this form. In such a situation, the predicate '$\mathtt{max_{inf}}$[λn_d.Everyone who is more than n years old hated this movie]' is true of x and only x. The important thing is that this predicate is not necessarily empty. When combining with *about twenty*, the resulting meaning (restricting ourselves to situations with finitely many individuals)[9] is that there is someone whose age is in the vicinity of 20 and who didn't hate the movie, and such that everybody older than this person hated the movie. What can be definitely concluded is that everybody whose age is significantly above 20 years old hated the movie, and that someone who is about 20 years old didn't hate it. This seems to be a plausible prediction. The reading predicted by Solt is that everyone whose age is above some interval $20+i$ for some i hated the movie, together with an implicature that it is possible that someone who is more than 20 years old didn't hate the movie (from the fact that the very same sentence without *about* is not assertable). These readings are very hard to tease apart in practice.

Let me now consider (32), which Solt (2018) does not predict to be felicitous to begin with, under the following parse:

(34) [about twenty][$\mathtt{max_{inf}}$[λx_d.Everyone who is no more than x years old hated this movie]]

The predicate 'λx_d.Everyone who is no more than x years old hated movie' is true of a degree x just in case everyone who is at most x years old hated this movie. Now, imagine a situation where someone whose age is x hated the movie and so did everyone younger, but everyone older loved it. Consider then the youngest person who is older than x (and therefore didn't hate it), call her age y. Then, *Everybody who is no more than x years old hated this movie* is true, and *Everybody who is no more than y years old hated this movie* is false (since someone who is y years old didn't hate the movie.) But then, for every x' in $[x,y]$, it is also true that everyone who is no more than x' years old hated this movie (since no one's age is between in $(x,x']$), and this last proposition asymmetrically entails the one based on x. Therefore, in a situation where there are finitely many individuals (so that we can talk about the youngest person who is older than the oldest person who hated the movie), given density, there can be no x such that *Everyone who is no more than x years old hated this movie* is the maximally true proposition of this form. So it looks like the predicate obtained by appending $\mathtt{max_{inf}}$ is necessarily empty, which would fail

[9]Taking into account density, the truth-conditions are in fact significantly more complicated if we allow for models with infinitely many individuals, in a way that is connected with our discussion of (34) below, but I do not discuss these complications here.

to explain the felicity of (34) (since a parse without $\texttt{max}_{\texttt{inf}}$ would violate the ban against redundancy, and other parses with $\texttt{max}_{\texttt{inf}}$ would violate the condition that $\texttt{max}_{\texttt{inf}}$ is excluded in decreasing environments). However, consider a model with infinitely many individuals, in which someone whose age is *x* hated the movie and everybody who is at most *x* years old hated the movie, but for every *y* above *x*, there is someone whose age is *y* and who loved the movie. In this case the predicate '$\texttt{max}_{\texttt{inf}}[\lambda x_d$.Everyone who is no more than *x* years old hated this movie]' is not empty, and is exactly true of *x*. And then the sentence is true if *x* is in the vicinity of 20 (a reading that does not violate the ban on redundancy). Following Fox and Hackl (2006), I need to assume that whether a sentence is ruled out as contradictory is evaluated not on the basis of underlying common knowledge (for instance we might know that there aren't infinitely many individuals), but on a purely logical basis, blindly to contextual information. When a sentence has passed this test, it can then be interpreted relative to a specific context, in which it is known that there are finitely many individuals, and scales are not necessarily treated as dense anymore. So the above parse would be felicitous and convey that there is an age *a* in the vicinity of 20 years old such that everybody who is at most *a*-years old hated the movie, but the youngest person whose age is more than *a* didn't hate the movie – which is, in fact, the intuitive reading of the sentence.

So it may seem that, on top of providing an account that extends to *between n and m* and *n or m*, the proposal advanced here has another advantage over Solt's (2018), namely it does not predict that global monotonicity is the only relevant factor that regulates the distribution of *more than about*.

That being said, my proposal seems to suffer from overgeneralization compared to Solt's. In particular, I predict the following to be felicitous:

(35) ??(In order to be allowed to drink alcohol in this country), one must be more than about eighteen years .

Because *more than eighteen* occurs in an increasing environment, Solt (2018) correctly predicts (35) to be infelicitous. I predict it to be fine, however, under the following parse:

(36) [about eighteen][$\texttt{max}_{\texttt{inf}}[\lambda x_d$.one must be more than x years old]]

The predicted reading is the following: there is an age *x* such that one must be above *x* in order to be allowed to drink alcohol, and for any age *y* greater than *x*, it is not the case that one must be older than *y* to be allowed to drink alcohol, and *x* is in the vicinity of 18. This describes a situation where the only age-related rule has the form *You must be more than x years old to be allowed to drink alcohool* and *x* is in the vicinity of 18 (typically the speaker would not know the exact value of *x*).

Now, I should note that the corresponding structure with a disjunction of numerals seems quite felicitous, unlike the one with a *between*-phrase

(37) a. (In order to be allowed to drink alcohol in this country), one must be more than twenty or twenty-one years old (I don't remember)

b. ??(In order to be allowed to drink alcohol in this country), one must be more than between 18 and 20 years old (I don't remember).

The predicted meaning of (37a), under a parse analogous to the one in (36), is simply that the underlying rule states *One must be more than x years old*, with $x = 20$ or $x = 21$ (and the speaker does not remember whether $x = 20$ or $x = 21$). The predicted reading for the infelicitous sentence in (37b) is completely similar, except that the condition on x is now that x should belong to the interval $[18, 20]$ (and the speaker does not know more).

The fact that my proposal incorrectly predicts (35) and (37b) to be potentially felicitous is a clear weakness. One unsatisfying move I could make is to argue that there are certain syntactic constraints on the scope of *about n* and *between n and m* that would rule out a parse analogous to the one in (36) in the case of these two expressions, but not in the case of disjunctions of numerals. But given the kind of parses that I need to assume in connection with the sentences in (31) and (32), this does not seem to be a very promising avenue.

7. Conclusion

In this paper, I showed that the universal density of measurement hypothesis (Fox and Hackl 2006), together with independently motivated assumptions, can account for the pattern uncovered and discussed in Solt (2014, 2018) regarding the distribution of *about* used as a degree approximator. I argued that this account has the advantage of generalizing to *between*-degree phrases as well as to disjunctions of numerals. Furthermore, where Solt (2014, 2018) predicts that the distributional facts depend on the global monotonicity of the environment in which *more than about* and *about* find themselves, I argued that this was not in general a good prediction, and that the proposal I made fared better in this respect. However, it also appears that my proposal overgenerates: some sentences that Solt correctly predicts to be infelicitous are predicted to be felicitous (in some contexts) by my proposal, suggesting that additional work is needed if we want to fully understand the patterns discussed by Solt and the related ones discussed in this paper.

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