Missing words and missing worlds[1]

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Abstract. In this paper, we discuss how a non-lexical account of neg-raising can be extended to explain a lexical gap in the domain of modals.

Keywords: modals, lexical gap, strong and weak modal.

1. Introduction: the problem

Traditionally, universal modal auxiliaries have been divided into two categories: *strong necessity* and *weak necessity modals*. They are called like that as strong necessity modals (such as *must* or *have to*) are semantically stronger than weak necessity modals (such as *should* or *ought to*), as the following examples show:

(1) a. You should/ought to leave but you don't have to leave.
     b. You should/ought to leave; in fact you have to leave.

Such a distinction cannot be made for existential modals. Despite morphological similarities, modals like *might/ could* do not stand in a similar strength relation with *may/can*:

(2) a. # You could/might leave but you can't/may not leave.
     b. # She might/could be in her office; in fact, she may/can be in her office.

In this paper, we argue that *might/could* pattern with *may/can* (outside of X-marked contexts). That is, they behave as existential duals of strong necessity modals and tend to yield a strong possibility reading. English appears to lack weak possibility modals. This is not a coincidence. Hardly any language seems to lexically distinguish between weak and strong possibility modals. Naturally, the question is why is that the case?

In addition, we argue that this question is related to another question, namely why weak necessity modals, when negated, can give rise to so-called Neg-raising (NR) readings, but strong necessity modals cannot do so.

Even though both *must* and *should* generally outscope negation (*Mary mustn't leave* means that it must be the case that Mary leaves, and *Mary shouldn't leave* means that it should be the case that Mary leaves, cf. Iatridou and Zeijlstra (2013); Homer (2015)), when embedded under a negated neg-raising predicate, only the latter but not the former is able to outscope the matrix negation:

(3) a. I don't think that John should marry Susan.   (✓ *should > not*)
     b. I don't think that John must marry Susan.   (# *must > not*)
                                                              (Homer 2015)

This shows that *should* is a neg-raiser but *must* is not. Similar observations can be made for

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other strong and weak necessity modals: weak necessity modals are neg-raisers, strong necessity modals are not.

In this paper, we aim to provide a strengthening account for neg-raising that explains why certain predicates may and others may not given rise to neg-raising readings. Moreover, we show how this approach can distinguish strong necessity and possibility modals from weak necessity modals and neg-raising predicates (NRPs). We then explain why weak possibility modals generally need to undergo strengthening, which makes them hard to detect.

2. Neg-raising

2.1. Existing approaches to NR and some challenges to them

Current standard approaches to NR, formulated in pragma-semantic terms, take NR readings to be the result of an excluded middle inference that is a special lexical property of NRPs (see Bartsch 1973; Horn 1989; Gajewski 2005a; Romoli 2013; Homer 2015, and Zeijlstra 2018, among others).[2] This approach has two versions:

(4)   a.   *The presuppositional approach* (Gajewski 2005b, 2007): NRPs come with an excluded middle *presupposition*.
      b.   *The implicature approach* (Romoli 2012, 2013): NRPs have excluded middle *alternatives*.

The presuppositional approach (Gajewski 2005b, 2007) takes NRPs to carry an excluded middle presupposition. That is, the speaker is presupposed to be opinionated about the truth or falsity of the embedded proposition. The NR reading is then a logical consequence of this presupposition and the literal meaning of the sentence, as shown in (5).

(5)   not [NRP [S]]
      Assertion: $\neg$ NRP (S)
      Presupposition: NRP (S) $\vee$ NRP $\neg$(S)
      $\therefore$ NRP $\neg$(S)   (Gajewski 2005a; p.14)

Under this account, the NR reading (6b) of (6a) follows straightforwardly.

(6)   a.   John doesn't think that Bill left.
      b.   John thinks that Bill didn't leave.

With the excluded middle presupposition that the speaker thinks that either Bill left or Bill didn't leave, (6a) entails (6b):

(7)   Assertion: It's not the case that John thinks Bill left.
      Presupposition: John thinks Bill left $\vee$ John thinks Bill didn't leave.
      $\therefore$ John thinks Bill didn't leave.

However, the universal projection of an excluded middle presupposition from the scope of negative indefinites turns out to be too strong in many contexts, as shown in the example below.

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[2]There are also a syntactic approach to NR on the market, most notably Collins and Postal (2014). However, this syntactic approach suffers from several problems that have been addressed in the literature (Romoli 2013; Zeijlstra 2018; Mirrazi and Zeijlstra 2021), which is why we do not discuss it here. See the aforementioned references for more discussion.

For the NR reading in (8) to be true, not only should everybody have an acquaintance relation with the addressee but they should also have an opinion about whether or not the addressee is stupid:

(8)   *It's the first day of school. Before entering the school your mom tells you:*
Remember, nobody here thinks you're stupid.

Apart from this, it is problematic for presuppositional approaches to NR that the NR reading does not always surface, as shown below (Homer 2015):

(9)   a.   Unlike many people nowadays, my great-grandparents didn't want to spend a lot of time on the internet.
         b.   ⇸My great-grandparents wanted not to spend all their spare time on the internet.

(10)   A*t a job interview:*
         a.   I don't want to make a lot of money, you know.
         b.   ⇸I want not to make a lot of money.

For Gajewski this has lead to arguing that the NR inferences must count as so-called *soft presuppositions* after Abusch (1993).

Instead of assuming that the excluded middle inference is a (soft) presupposition, Romoli (2012, 2013) proposes that NRPs take the excluded middle as a lexical alternative. A NRP like *think*, then, has ($\textbf{think}_x$p $\vee$ $\textbf{think}_x$¬p) as its lexical alternative, as shown in (11).

(11)   *Alt*($\textbf{think}$ p(x))={ $\textbf{think}_x$p, $\textbf{think}_x$p $\vee$ $\textbf{think}_x$¬p }

The set of alternatives of (12) is given in (13a). Exhaustification of these alternatives will result in the strengthened NR reading (13b).

(12)   John doesn't think that it is raining = ¬$\textbf{think}_j$ p

(13)   a.   *Alt*(¬$\textbf{think}_j$ p)={ ¬$\textbf{think}_j$ p, ¬($\textbf{think}_j$ p $\vee$ $\textbf{think}_j$¬p) }
         b.   ⟦EXH⟧(¬$\textbf{think}_j$ p)= ¬$\textbf{think}_j$ p $\wedge$ ¬¬($\textbf{think}_j$ p $\vee$ $\textbf{think}_j$¬p) =
             ¬$\textbf{think}_j$ p $\wedge$ ($\textbf{think}_j$ p $\vee$ $\textbf{think}_j$¬p) = $\textbf{think}_j$¬p

The scalar implicature account of NR has the advantage of not running into the projection problems of the presuppositional account. Moreover, as the generation of scalar implicatures depends on the contextual relevance of particular alternatives, the second problem addressed concerning the alleged presupposition failures does not arise either.

At the same time, Romoli's implementation of the implicature calculation is based on the assumption that NRPs have *lexical* alternatives, which are hardly pronounceable and are not attested elsewhere (Križ 2015).

A perhaps more pressing problem for both types of lexical approaches is that in certain contexts non-NRPs nevertheless get a NR reading, as illustrated below (where the lawyer must know what is constitutionally possible).

(14)   Trump: I can overturn the result of the election.
         Constitutional lawyer: I'm not sure that's constitutionally possible, sir.

(15)   Anthony: you know why?
        Uncle Junior: I don't know that I give a f***.   (Sopranos, S1.E6)

Summing up, the discussion above shows that these semantic-pragmatic approaches to NR face particular non-trivial problems. One of the reasons is that the property that some predicate is a NRP must be lexically encoded (either as a (soft) presupposition or as part of its alternatives).

2.2. Proposal

For these reasons, we present a semantic-pragmatic account of NR that is non-lexical in nature. As it turns out, not only do the problems mentioned before for the lexical approaches disappear under our approach, but it also predicts a number of novel facts that can be observed in this domain.

Inspired by the recent implicature approaches to Free Choice inferences (Bar-Lev and Fox 2017), and Homogeneity (Bassi and Bar-Lev 2018; Magri 2014; Bar-Lev 2020), there has been new attempts to derive the neg-raising reading using the machinery of exhaustification (EXH) (Mirrazi and Zeijlstra 2021; Staniszweski 2021; Jeretič 2021). Below, we spell out the details of our own proposal presented earlier in (Mirrazi and Zeijlstra 2021), indicating where it differs from other exhaustification-based accounts as well.

The first ingredient of our analysis is that operations that apply to the LF of a particular utterance may also apply LFs that are strictly equivalent to the original LF, where strict equivalence is defined as follows:

(16)   a.   p is strict equivalent to q ($p \Leftrightarrow_{strict} q$) iff p strictly entails q ($p \Rightarrow_{strict} q$) and q strictly entails p ($q \Rightarrow_{strict} p$)
        b.   p strictly entails q ($p \Rightarrow_{strict} q$) iff in every world where p is true, q is true as well[3]

The reason for this is that LF operations apply to the meaning of an utterance p, i.e. the set of worlds where p holds, and should be blind to the way this meaning was originally structured. Pragmatic reasoning or any other operation that applies to LFs cannot distinguish between strictly equivalent LFs.

Strict LF-equivalence also requires presupposition conservation. We cash this out in a trivalent system, where the possible truth-values are 1,0 and #, where presupposition failure is marked by the third truth-value. Given the rules of strict duality, operations like EXH can apply to the dual of a negated universal modal, $\neg\forall w: p(w)$, which is $\exists w: \neg p(w)$, if and only if the two are strictly equivalent. This is indeed the case for non-factive epistemic modals, such as *think*. By (16), $\neg\forall w \in W : p(w) \Leftrightarrow_{strict} \exists w \in W : \neg p(w)$. The meaning of negated NR predicates is thus strictly equivalent to $\exists w \in W : \neg p(w)$.

In what follows, we show that this existential LF, unlike the strictly equivalent negated universal counterpart, yields a strengthened NR reading under exhaustification. One of the major reasons to apply EXH to the existential dual $\exists w \in W : \neg p(w)$ is that existential quantifiers, unlike universal quantifiers, can take singleton sets as their restrictor. This is important, as exhaustification over a set of domain alternatives that lacks singleton alternatives, as we will see later, will not be able to yield the strengthened NR reading. Thus, strict duality has a major advantage over

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[3]We are grateful to Amir Anvari for his insightful comments that led us to adopt this notion of equivalence.

proposals that directly exhaustify LFs containing a negated universal quantifier over possible worlds (such as Jeretič (2021)).

Another advantage of strict duality is that it rules out certain predicates for being neg-raisers.[4] If strict duality does not apply, no NR reading can be yielded. Consequently, when some negated universal predicate does not have a strict existential dual, such a predicate cannot be a neg-raiser. This way, modals that carry presuppositions that block duality are excluded from NR. Factive *know* is a good example. Assume that $\Diamond K_p$ is the existential dual of the knowledge operator $\Box K_p$. When $\Diamond K_p$ also carries the factivity presupposition that the embedded p is true, strict LF-equivalence does not hold. While the negated universal version of know presupposes that its prejacent is true; the existential dual that outscopes negation presupposes that the same prejacent is false:

(17) $\underline{p(w) = 1}.\neg\Box K \underline{p(w)} \Leftrightarrow_{strict} \underline{\neg p(w) = 1}.\Diamond K \underline{\neg p(w)}$

Note that even when $\Diamond K_p$ doesn't carry any presupposition, strict LF-equivalence is still not valid. In a world where the factivity presupposition is not satisfied, $\neg\Box K$ p(w) is #, but $\Diamond K$ $\neg p(w)$ is true:

(18) $\underline{p(w) = 1}.\neg\Box K \underline{p(w)} \Leftrightarrow_{\text{STRICT}} \Diamond K \neg p(w)$

As $\Diamond K$ $\neg p(w)$ is not strictly equivalent to $\neg\Box K$ p(w), EXH cannot apply to $\Diamond K$ $\neg p(w)$. Given that, as we will see later on, a strengthened NR reading can only be derived when the existential LF-equivalent is exhaustified, such a strengthened reading cannot be derived for factives like *know*. This means that it is not NRPs that are special in allowing NR inferences; it is rather strictly *non-NRPs* that are special in not allowing them. Strict non-NRPs, i.e. predicates that never yield NR readings, may carry a presupposition that is incompatible with their dual form. Then no *existential* reading can be derived that can be further strengthened.

Our second ingredient finds a parallel in the implicature account of Free Choice (Fox 2007; Bar-Lev and Fox 2017), and Homogeneity (Bassi and Bar-Lev 2018; Magri 2014; Bar-Lev 2020). In line with this approach, we take strengthened readings to be the result of the application of an exhaustivity operator at LF. Modals trigger subdomain alternatives (Zeijlstra 2011; Bassi and Bar-Lev 2018; Staniszewski 2021). Thus, in the exhaustification of modals, we will only make use of domain alternatives and not scalar alternatives. We adopt the definition of the exhaustivity operator (EXH) by Bar-Lev and Fox (2017).[5]

(19)   Innocent Exclusion + Innocent Inclusionbased exhaustivity operator:
       $[\![\text{EXH}]\!]^{IE+II}(C)(p)(w) \Leftrightarrow \forall q \in \text{IE}(p,C)[\neg q(w) \wedge \forall \in \text{II}(p,C)[r(w)]$

(20)   Given a sentence p and a set of alternatives C:

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[4]Note that without adding more, the assumption that exhaustification applies to the LF after applying DeMorgan laws (per Jeretič's suggestion) overgenerates neg-raising for all negated universal modals. To get the right result, one would need to further assume that DeMorgan laws do not work for certain modals. To motivate this assumption, a notion similar to strict equivalence would have to be employed again.

[5]We would like to clarify that we use the exhaustification mechanism without necessarily committing to its grammatical status. For our purpose, strengthening is a pragmatic phenomenon that can be triggered via the general principle of the Strongest Meaning Hypothesis (Dalrymple et al. 1998; Winter 2001; Yoon 1996). However, we think that the Innocent Exclusion + Innocent Inclusion–based exhaustification mechanism proposed by Bar-Lev and Fox (2020) provides a useful tool to formally talk about how the strengthening proceeds.

a. $\text{IE}(p,C) = \bigcap \{C' \subseteq C : C'$ is a maximal subset of C, s.t.
$$\{\neg q : q \in C'\} \cup \{p\} \text{ is consistent }$$

b. $\text{II}(p,C) = \bigcap \{C'' \subseteq C : C''$ is a maximal subset of C, s.t.
$$\{r : r \in C''\} \cup \{p\} \cup \{\neg q : q \in \text{IE}(p,C)\} \text{ is consistent }$$

According to the definition above, EXH takes a proposition (p), and a set of alternatives (C) as arguments, and returns the conjunction of all of the negated innocently excludable (IE) alternatives, and all of the asserted (assigned *true*) innocently includable (II) alternatives. The IE alternatives are all those that can be assigned *false* consistently with the prejacent. The II alternatives are those that can be assigned *true* consistently with the prejacent and the falsity of all IE alternatives. The NR reading is then derived via application of EXH, starting with the LF corresponding to the basic existential reading ($\exists w \in W: \neg p(w)$).

Let's assume the speaker's belief worlds consists of three worlds w_1, w_2 and w_3.

The alternatives generated from replacing the domain variable with its subsets in the existential reading are given in (21).

(21) $\exists w \in \{w_1, w_2, w_3\}: \neg p(w)$, $\exists w \in \{w_1, w_2\}: \neg p(w)$, $\exists w \in \{w_1, w_3\}: \neg p(w)$,
$\exists w \in \{w_2, w_3\}: \neg p(w)$, $\exists w \in \{w_1\}: \neg p(w)$, $\exists w \in \{w_2\}: \neg p(w)$, $\exists w \in \{w_3\}: \neg p(w)$

No alternatives are IE. All alternatives are II.

Upon exhaustification, we will have (22), which is equivalent to the NR reading.

(22) $\text{EXH}^{IE+II}(\text{Alt}(\exists w \in \{w_1, w_2, w_3\}: \neg p(w))) (\exists w \in \{w_1, w_2, w_3\}: \neg p(w)) =$
$\exists w \in \{w_1, w_2, w_3\}: \neg p(w) \wedge \exists w \in \{w_1, w_2\}: \neg p(w) \wedge \exists w \in \{w_1, w_3\}: \neg p(w) \wedge$
$\exists w \in \{w_2, w_3\}: \neg p(w) \wedge \exists w \in \{w_1\}: \neg p(w) \wedge \exists w \in \{w_2\}: \neg p(w) \wedge \exists w \in \{w_3\}: \neg p(w)$
$= \forall w \in \{\boldsymbol{w_1, w_2, w_3}\}: \neg \mathbf{p}(w)$

Note though, that not every non-NRP have particular presuppositions that render existential LF-equivalents impossible. For instance, strong modals like *must* or *need* do not do so. Hence, at this stage our approach may still overgeneralize. However, as we will see in the next section, the same reason why exhaustifying negated universals does not lead to strengthened NR readings, also will turn out to apply to the existential LF-equivalents of such modals.

The reader may wonder why EXH applies do the existential dual and not the the original negated LF containing the negated universal (as proposed in Jeretič (2021)). After all, $\exists \neg p$ is strictly equivalent to $\neg \forall p$ in the NR examples provided. The reason is that a universal quantifiers, unlike existentials, cannot take a domain of quantification that is a singleton set as their first argument[6] (witness the oddity of sentences like *Every current pope lives in Rome*). Hence, the set of alternatives of a negated universal does not contain any singleton alternatives, as shown in (23). Consequently, exhaustifiyng $\neg \forall p(w)$ will not lead to the NR reading (24).

(23) $\neg \forall p w \in \{w_1, w_2, w_3\}: p(w)$, $\neg \forall w \in \{w_1, w_2\}: p(w)$, $\neg \forall w \in \{w_1, w_3\}: p(w)$, $\neg \forall w \in \{w_2, w_3\}: p(w)$
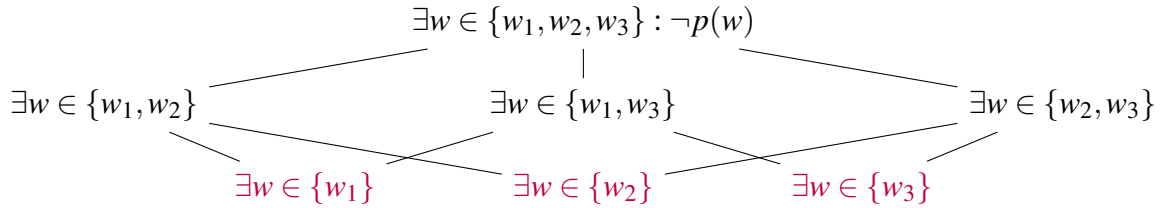
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[6] The exhaustification procedure that Jeretič (2021) proposes involves alternatives of the form $\neg \square_{\{w_1\}} p$, as shown below.

(i) $\text{Alt}(S') = \{\text{EXH}[\text{Alt}(S)][\neg\square_{\{w_1, w_2\}} p], \text{EXH}[\text{Alt}(S)][\neg\square_{\{w_1\}} p], \text{EXH}[\text{Alt}(S)][\neg\square_{\{w_2\}} p]\}$
$= \{\neg\square_{\{w_1, w_2\}} p, \neg\square_{\{w_1\}} p \wedge \square_{\{w_2\}} p, \neg\square_{\{w_2\}} p \wedge \square_{\{w_1\}} p\}$

(24) $\text{EXH}^{IE+II}(\text{Alt}(\neg\forall w \in \{w_1, w_2, w_3\}: (w)))(\neg\forall w \in \{w_1, w_2, w_3\}: p(w)) = \neg\forall w \in \{w_1, w_2,$
$w_3\}: p(w) \wedge \neg\forall w \in \{w_1, w_2\}: (w) \wedge \neg\forall w \in \{w_1, w_3\}: p(w) \wedge \neg\forall w \in \{w_2, w_3\}: p(w)) \neq$
$\forall w \in \{\boldsymbol{w_1, w_2, w_3}\}: \neg\boldsymbol{p(w)}$

Our approach solves one of the main issues with the existing pragmatic-semantic approach to NR, namely that non-NRPs may yield NR readings too. As long as strict duality is obeyed every predicate in the right context may give rise to a NR reading, including the examples, such as the examples in (14).

But, as addressed before, another challenge to these approaches is that the alleged excluded middle presuppositions can actually be violated. The NR reading does not emerge obligatory. This, for us, does not follow from having the exhaustifier apply optionally as that would render its inclusion arbitrary, but rather from the systematic *pruning* of alternatives. Concretely, we follow again Bar-Lev's (2018; 2020) account of non-maximal readings of definite plurals, where we take the non-NR reading to be the result of pruning all the subdomain alternatives which are singleton sets (i.e. $\{w_1\}$, $\{w_2\}$, $\{w_3\}$).

$$\exists w \in \{w_1, w_2, w_3\} : \neg p(w)$$

$$\exists w \in \{w_1, w_2\} \qquad \exists w \in \{w_1, w_3\} \qquad \exists w \in \{w_2, w_3\}$$

$$\textcolor{red}{\exists w \in \{w_1\}} \qquad \textcolor{red}{\exists w \in \{w_2\}} \qquad \textcolor{red}{\exists w \in \{w_3\}}$$

By applying EXH to the set of alternatives in (25), like in the case of exhastified negated universals discussed above, we get the weak non-NR reading, as shown in (26).

(25) $\exists w \in \{w_1, w_2, w_3\}: \neg p(w), \exists w \in \{w_1, w_2\}: \neg p(w), \exists w \in \{w_1, w_3\}: \neg p(w),$
$\exists w \in \{w_2, w_3\}: \neg p(w)$

(26) $\text{EXH}^{IE+II}(\text{Alt}(\exists w \in \{w_1, w_2, w_3\}: \neg p(w))) = \exists w \in \{w_1, w_2, w_3\}: \neg p(w) \wedge$
$\exists w \in \{w_1, w_2\}: \neg p(w) \wedge \exists w \in \{w_1, w_3\}: \neg p(w) \wedge \exists w \in \{w_2, w_3\}: \neg p(w)$

Pruning is a mechanism to reduce the set of alternatives to only those that are plausible and relevant in a given context, and it is governed by the following principles.

(27)   a.   **Maxim of Relevance:** Every utterance must be relevant to Q.
         b.   **Weakening:** Pruning can only weaken the meaning (Crnič et al. 2015).
         c.   **Minimal pruning:** Don't prune more than necessary to satisfy   (Bar-Lev 2020).

We saw that pruning the singleton worlds from the set of domain alternatives provides us with the right result, but under what conditions does pruning take place?

Following Kratzer (1989, 2012), we argue that singleton propositions predicated of the actual world are too specific to be cognitively viable. A plausible necessary condition for a cognitively viable proposition is that it should be possible for an actual human to believe it. Assuming that the person's beliefs are consistent, it follows that she has to be omniscient in a rather strong sense. Her beliefs have to be so specific that they are able to distinguish the actual world from all other possible worlds—including all of its perfect duplicates. We propose that the domain of alternatives can include singleton propositions if the actual world does not have to

be among the set of worlds in the quantification domain of a modal. That is, when believing in a singleton proposition does not require strong omniscience. Therefore, strengthened NR readings are predicted to be only possible for such modals. In such cases, consideration of a broader domain of alternatives leads to a stronger statement.[7]

Under this view, the (un)availability of strengthened NR readings for duality-allowing universal modals depends on which set of alternatives EXH applies over. When EXH applies over the whole set of subdomain alternatives, we get the strengthened reading. When EXH applies over the subset remained after pruning singleton sets, we get the weak reading. Reference to the actual world in the domain of quantification triggers the difference here. Singleton set alternatives are necessarily pruned when the quantification domain of modals includes the actual world. This is indeed the case with strong modals like *must* or *have to* and modals that expresses objectivity or evidentiality. The actual world is unique and its inclusion in the quantification domain of modal renders a non-homogeneous set. Distinguishing the actual world from all other possible worlds is not cognitively viable (Kratzer 2012). The question is to what extent this constraint on singleton alternatives can be extended to strong modals in general. In the rest of this paper, we will pursue this idea.

3. Modality and Anchor Semantics

In the previous section, we introduced a pruning system that is sensitive to the kind of worlds in the quantification domain of modals. A corollary of this proposal is that modals differ in whether or not their domain includes the actual world. In this section, we argue that this is what is behind the intuition about the weakness of *should* and *ought* (see also Silk (2016, 2018, 2022)). We follow Mirrazi (2022) who implements this insight about the semantics of weak necessity modals in the framework of Anchor Semantics (Arregui 2009; Kratzer 2013; Kratzer et al. 2014; Kratzer 2020). Before explaining the details of our proposal, we first need to lay out our assumption about the semantics of modals.

3.1. An Anchor Semantics for modals

In the standard Kratzerian framework, modals differ in two dimension: (i) the type of quantification over possible worlds (modal force), and (ii) the worlds included in their domain of quantification (modal base). The former is typically lexically encoded in the semantics of modals, but the latter is determined by an interplay of contextual factors, at least in most European languages.

We adopt the *Anchor semantics* for modals proposed by (Kratzer 2013), according to which the modal base is constructed out of two ingredients: a modal anchor which projects the initial domain, and a modal restriction that determines the final domain. The quantificational domain of modals is initially determined by taking a situation from the actual world (the *anchor situation*), and considering the set of possible worlds that have an exact match of that situation (Kratzer 2020). This conjecture, dubbed *factual domain projection*, is defined below.

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[7]Note that we are not suggesting that the reasoner (i.e. the hearer) has to decide on the truth or falsity of every singleton proposition separately. They can assign true to all singleton propositions when a set of worlds are all of a particular kind. The reasoning process breaks when the worlds are taken (by the reasoner/hearer) to be candidates for the actual world. A quantification domain that includes the actual world is not a homogeneous set as one (and only one) world necessarily differs from other worlds in being the actual world.

(28)   **Factual Domain Projection (Kratzer 2020)**
        For any part of a (maximal) situation s, $f_{act}(s)$ is the set of possible (maximal) situations that have an *exact match* of *s*.

Philips and Kratzer (2022) take this basic capacity of considering possible extensions of an actual anchor situation as a common component of many, apparently different, types of modal cognition. Moreover, the factual domain projection captures the intuitive idea that even in our modal claims, we are concerned with worlds that we take to be candidates for the actual world. It is clear that the initial domain of unembedded modals includes the actual world as it has the exact match of a piece of itself (unless the modal domain is projected from a particular individual's mental state that might be in conflict with the actual world (see Kratzer (2020) and Philips and Kratzer (2022) for examples of epistemicky anchors). Following Mirrazi (2022), we take the anchor situation to be the first argument of a modal, as shown below.

(29)
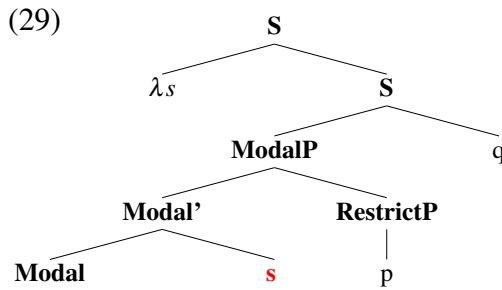
(Mirrazi 2022)

Like other variables, the value of the anchor situation depends on its place in the structure. In an unembedded sentence, the anchor situation takes its value from the evaluation situation.

(30)   **Modal Anchor Impact (Kratzer 2020)**
        The anchor situation of a modal is identical to the evaluation situation of the smallest constituent that contains the modal and its scope.

The choice of anchor situations is subject to the *Diversity Condition* that states that the anchor must be chosen such that the projected domain has both worlds where the modal's prejacent is true and worlds where it is false (see also Condoravdi 2002; Werner 2003; Giannakidou and Mari 2016). The initial modal domain projected from the anchor is further restricted by contextually supplied modal restrictions that allow for certain worlds in the projected domain to be ignored. Kratzer (2020) proposes that *contextual restrictions have to be provided from the prospective common ground*, which is the common ground as it stands after the claim in question has been made and negotiated (Stalnaker 2014; Mandelkern 2020).

(31)   **Prospective Contextual Modal Restrictions**
        Modal restrictions have to be provided from the *prospective common ground* (Kratzer 2020), which is the common ground as it stands after the claim in question has been made and negotiated (Stalnaker 2014; Mandelkern 2020).

Kratzer (2020) argues that since a Stalnakerian notion of common ground can have false presuppositions in it, the proposal, as it stands, predicts weak truth-conditions for strong modal claims with *must*. She proposes a constraint to allow strong modal claims to be false even when they would follow from speakers' false presuppositions. The constraint states that the world

of anchor must be in the context set ($w_s \in C$). Mirrazi (2022) points out that this constraint together with *Prospective Contextual Modal Restrictions* entails that the modal restrictions of strong necessity modals should be compatible with a factive common ground ($p \cap C \neq \emptyset$). Therefore, the actual world remains in the final quantification domain of strong modals. The denotation of a strong universal modal such as *must* is given below.[8]

(32) $[\![\Box_{strong}]\!] = \lambda s.\ \lambda p : p \cap C \neq \emptyset.\ \lambda q.\ (w_s \in C\ \wedge\ \forall w(w \in f_{act}(s) \cap p \to$
$\exists s' \exists s''(s' \leq w\ \wedge\ s'' \leq w\ \wedge\ Match(s',s)\ \wedge\ R(s'',s')\ \wedge\ q(s''))))$
                                                                                                         (Mirrazi 2022)

R represents a contextually supplied relation that maps the match of the anchor situation to a situation where the modal's prejacent is evaluated. The truth-conditions state that in all the worlds projected from the anchor situation ($\forall w(w \in f_{act}(s_0))$) and in which the modal restriction p holds ($\cap p$), the match of the anchor situation bears the contextual relation R to a situation in which the prejacent q is true. The constraint that the context (C) includes the world of anchor (w_s), together with the condition that modal restrictions should come from prospective Common Ground, keeps the truth-conditions of the modal claim strong.

3.2. Weak necessity modals

Traditionally, universal modals have been divided in two categories: strong necessity and weak necessity modals. They are called like that as statements containing strong necessity modals (such as *must* or *have to*) entail a corresponding claim with a weak necessity modal (such as *should* or *ought to*), but not vice versa. The following examples illustrate the entailment relation between strong and weak necessity modals.[9]

(33)   a.   You should/ought to leave but you don't have to leave / it's not that you must leave.
         b.   You have to/must leave, # but you shouldn't/ought not to leave.

(34)   a.   You should/ought to leave; in fact you have to/must leave.
         b.   You have to/must leave; # in fact you shouldn't/ought not to leave.

von Fintel and Iatridou (2008) and von Fintel and Iatridou (2020) observe that many languages morphologically use the same morphological strategy they use to mark the difference between X-marked and O-marked conditionals to distinguish between weak and strong universal modals. For instance, the strong necessity modal *must* in Hungarian can take the X-marker *-nA*, and express the meaning of weak necessity.

(35)   Péter-nek el   kell-**ene** mosogat-ni-a az edény-ek-et, de senki nem
         Peter-DAT PRT must-**nA** was-INF-3SG the dish-PL-ACC but noone not

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[8]Following von Fintel and Gillies (2010, 2021); Kratzer (2020); Silk (2016, 2018, 2022), among others, we take *must* to be a strong modal, but see Giannakidou and Mari (2016) and Lassiter (2016) for counter examples to this view, and Kratzer (2020) for an account of these apparently conflicting data.

[9]There are different approaches in the literature to explain the difference between the weak and strong necessity modals that we will not discuss here (see Rubinstein (2020) for an overview). Our aim here is to show that our proposal about the defining role of the actual world in the (un)availability of strengthening, which is in line with Silk's account of the distinction between weak and strong necessity modals, make correct predictions about the cross-linguistic picture of modals' strength.

<div style="margin-left:3em;">

köveleli   meg tőlle.
require-3SG.SUBJ-3.OBJ part 3.SG.ABL
'Peter ought to do the dishes, he is not obliged to.'

</div>

Without *-nA* on the strong necessity modal, the sentence will be a contradiction.

(36)   #Péter-nek el   kell mosogat-ni-a az edény-ek-et, de senki nem
         Peter-DAT PRT must was-INF-3SG the dish-PL-ACC but noone not
         köveleli   meg tőlle.
         require-3SG.SUBJ-3.OBJ part 3.SG.ABL
         'Peter has to do the dishes, he is not obliged to.

A somewhat similar pattern can be seen in English which uses its past tense morphology in X-marked contexts. Past form of modals *would* (for *will*) and *should* (for *shall*), which can appear in X-marked conditionals, imply weakness. Strong modals *must* and *have to* never appear in X-marked conditionals, as shown in (37). von Fintel and Iatridou (2020) argue that a common denominator of X-marking meaning in all of its occurrences is to indicate a departure from the default.

(37)   a.   If she had taken the train yesterday, she would/should have arrived by now.
       b.   *If she had taken the train yesterday, she must/had to have arrived by now.

Silk (2022) proposes that the difference between weak and strong modals is whether or not they predicate the necessity of their prejacent of "the actual world". While the truth of $\square_{strong}\phi$ depends on the value of ϕ at the evaluation world, $\square_{weak}\phi$ brackets away whether the necessity claim is verified in the actual world. Adopting the general insight that X-marking signals that the worlds being talked about needn't be candidates for actuality, Silk (2022) derives the apparent weakness of weak necessity modals from the meaning contribution of X-marking, i.e. canceling a presupposition that the set of worlds in a modal's domain of quantification is a subset of the context set. He frames this idea in a past-as-modal approach to X-marking. Here, we follow Mirrazi (2022) who implements this general insight in the framework of Anchor Semantics with a past-as-past approach to X-marking.[10]

Following Arregui (2009), Mirrazi (2022) takes the role of the past tense in X-marked modals to determine the temporal specification of the anchor situation. She proposes that the pastness of the anchor situation affects the semantics and pragmatics of X-marked modals. Unlike the anchor situations of strong modals, they do not invoke the condition that the world of the anchor has to be in the context set. This allows for the modal restrictions to be incompatible with the presuppositions of factive common ground[11], which in turn can lead to the exclusion of the actual world from the final quantification domain of these modals. According to this view, X-marked weak necessity modals contribute the same assertoric information as strong modals but yield weaker truth conditions because they lack the presupposition ($w_s \in C$) that strong modals carry. Compare the denotation of weak necessity modals given below with that of strong modals in (32).

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[10]We are using a past-as-past approach, which is compatible with the Anchor Semantics, but the general point we are trying to make here (defining the strength of modals in terms of the inclusion of the actual world) is independent of the role of past tense morphology in X-marking.

[11]Philips and Kratzer (2022) also take the weak necessity *should* to indicate the presence of restrictions that depart from the default.

(38) $\quad [\![\Box_{weak}]\!] = \lambda s.\ \lambda p.\ \lambda q.\ \forall w(w \in f_{act}(s) \cap p \rightarrow \exists s' \exists s''(s' \leq w\ \wedge\ s'' \leq w\ \wedge$
$\quad\quad\quad Match(s',s)\ \wedge\ R(s'',s')\ \wedge\ q(s''))) \underline{\text{ where s is a past situation.}}$

$\hfill$(Mirrazi 2022)

Note that weak modals are semantically compatible with a strong reading. The actual world is not necessarily excluded from the domain. If there's a stronger alternative available, the use of weak modal generates an implicature that either the restrictions or prejacent of the modal are not compatible with the factive context set. Strong modals, on the other hand, are predicted to be infelicitous when the modal restrictions are not compatible with the factive context set.

Let us take stock here. In the previous section, we postulated that pruning mechanism is sensitive to whether or not the actual world is part of the quantification of modals. We have shown that the importance of the actual world in defining the properties of modals is not an ad hoc assumption, and has been independently proposed by Silk (2022); Mirrazi (2022) who aim to systematically derive the difference between weak and strong modals from the contribution of X-marking, as well as by Philips and Kratzer (2022) who are concerned with a more fundamental question about humans' cognitive capacity for modal thought. In this light, we propose that the nature of worlds in the domain of modals, in addition to the modal force, contribute to the strength of modals. We dub this notion of strength as *Actuality Strength*.[12]

(39) **Actuality Strength**
    a.   The final quantification domain of strong modals includes the actual world.
    b.   The final quantification domain of weak modals may or may not include the actual world.

4. Weak possibility modals

In principle, this distinction must be extendable to possibility modals. This way, four types of modals should expected to be attested across languages: strong and weak necessity modals, and strong and weak possibility modals. This seems to be a correct prediction when we look at X-marked contexts. Similar to what we observed about the contrast between weak and strong necessity modals in (37), only past form possibility modals *might* and *could* appear in X-marked, as shown below.

(40)   a.   If I had looked in my pocket, I might/could have found a penny.
    b.   *If I had looked in my pocket, I may/can have found a penny.

However, these X-marked possibility modals, unlike X-marked necessity modals, do not yield a weak reading outside of X-marked contexts. *Could* and *might* are not perceived as weaker alternatives to *can* and *may*. No entailment relationship holds between these two groups of possibility modals.[13]

---

[12]Not all languages morphologically distinguish between weak and strong necessity modals. In such languages, e.g. Farsi, the necessity modal is compatible with both interpretations. The use of adverbs such as *definitely* or *probably* can disambiguate between the two readings.
(i)   (šayad/hatman)   bayad ba-š   harf be-zan-i.
        probably/definitely NEC   to-her talk   IMPF.SUBJ-hit-2SG
        'You should talk to her./ You must to her.'
[13]Note that this is possible when they imply counterfactuality.
(i)   I'm always the one they really wish they could've been with but can't.

(41)   a.   You could/might leave # but you can't / may not leave.

            b.   She could/might be in her office; # in fact, she can/may be in her office.

Weak possibility modals appear to be rare cross-linguistically. Javanese provides a strong evidence for this cross-linguistic tendency. Vander Klok and Hohaus (2020) observe that Javanese has a dedicated functional morpheme, -NE, which combines with a strong necessity modal to derive a weak one. Crucially, Vander Klok and Hohaus (2020) show that -NE is not an X-marker.

(42)   a.   Bal-e   mesthi neng C.   b.   Bal-e   mesthi-ne   neng C.

                ball-def epis.nec in   C   ball-def epis.nec-NE in   C

                'The ball must be in C.'   'The ball should be in C.'

Interestingly, -NE cannot occur with possibility modals to derive weak possibility.

(43)   Aku iso-(*ne)   ngelangi.

         1sg circ.pos-NE av.swim

         'I can swim.'   (Vander Klok and Hohaus 2020)

The question naturally is how we can account for the rarity of weak possibility modals. We believe that the notion of Actuality Strength provides an answer to this question. Given Actuality Strength, a weak possibility modal is an existential quantifier over possible worlds that are not necessarily candidates for the actual world. However, not presupposing the inclusion of the actual world in the quantification domain of an existential modal leads to an extremely weak meaning: a proposition is merely true in *some* possible world, where the chosen world *doesn't have to be a candidate for the actual world*. If this proposition is not to imply a counterfactual reading (in a language like Javanese, for instance, which has a different X-marker), the meaning is trivially true for every proposition that is not an impossibility, and irrelevant to the question under discussion, as most of our utterances are concerned with what is true in the actual world. The reason is in general possibility modal claims are proposals to make the prejacent compatible with the common ground (Mandelkern 2020). A weak possibility modal claim (which does not imply X-marked interpretation) then would amount to saying that while p is not a live possibility in the common ground, it is compatible with the worlds outside of the context set. This would only be an informative assertion if it is stated as a rejection of a negated strong existential modal claim i.e. $(\neg\Diamond_{strong}p)$. Since natural language avoids triviality (Chierchia 2013 *et seq*), lexical items with a high probability of rendering triviality are expected to be rare.

As for X-marked possibility modals in English, one should note that there is nothing in the semantics of weak modals that forces the exclusion of the actual world. An integral part of the inquiry into the meaning of X-marked conditionals is the observation that they do not always imply falsity of their antecedent. For instance, the seminal observation by Anderson (1951) shows that they can be used to argue for the truth of the antecedent.

(44)   If Jones had taken arsenic, he would have shown just exactly the symptoms that he does in fact show.

Given that the resulting reading when the exclusion of the actual world is assumed is too weak, the statement containing an X-marked possibility modal is interpreted as having the strongest

meaning that is compatible with other assumptions in the context of utterance. There are two ways that can generate such a strong reading. The first option, which is exploited in English for the interpretation of X-marked possibility modals, is to take the actual world to be in the domain of quantification provided that the modal restrictions are compatible with the presuppositions of factive context set. This yield a strong possibility reading, as in (41). The second option, when the modal claim depends on restrictions beyond the default, is to strengthen the modal claim to a weak necessity reading along the line we proposed for the neg-raising reading. The latter strengthening mechanism is only expected outside of downward entailing contexts. Hungarian provides evidence for the second strategy, as shown below, though note that in Hungarian, a possibility modal in X-marked contexts also carry -nA.

(45) Péter el-mosogat-hat-ná az edényeket.
      Péter prt-was-can-cond the dishes.acc
      'Peter should wash the dishes.'
      (X-marked interpretation: Peter could wash the dishes (but he isn't).)

(46) Ha belenéz-t-em   vol-na   a zsebembe, talál-hat-t-am   volna   egy pennyt.
      If   in-look-pst-1sg was-cond the into.pocket find-can-pst-1sg was-cond a   penny
      'If I had looked into my pocket, I could have found a penny.'[14]

In downward entailing contexts, however, only a possibility interpretation is available.

(47) Az iskolában Péter nem mosogat-hat-ná el az edényeket.
      In the.school Péter not   wash-can-cond prt the dishes
      'In the school, Peter isn't allowed to wash the dishes.'

We also expect to find languages with possibility modals that are lexically underspecified with respect to whether they makes reference to the actual world. We predict these modals to be ambiguous between a strong possibility reading (if the actual world is in the final domain) and a weak necessity reading outside of downward entailing environments (if the existence of a modal restriction beyond the default leads to the exclusion of the actual world from the final modal domain). In terms of our world-sensitive pruning, the inclusion of the actual world in the modal base should lead to the unavailability of necessity readings; only modals that do not presuppose the inclusion of the actual world in their domain of quantification can give rise to a universal reading via exhaustification. This is precisely the pattern Newkirk (2022a, b) reports on Kinande, for which she shows that it has a variable-force modal prefix *anga* whose meaning only varies between possibility and weak necessity, never to strong necessity.[15]

(48) Kabunga a-anga-na-sya   oko kalhasi ko   munabwire.
      Kabunga 3SG-MOD-T-come PREP class   PREP today
      'Kabunga might come to class today'
      'Kabunga should be coming to class today'
      #'Kabunga must be coming to class today'   (Newkirk 2022a)

---

[14]We are grateful to János Egressy for the Hungarian data.

[15]Similarly, Staniszweski (2021) proposes that weak necessity modals like *should* and *supposed to* are existential quantifiers over possible worlds. In upward entailing environments, this existential reading is strengthened to a weak necessity modal via EXH^{IE+II}. He assumes that the weakness is due to pruning of all alternatives that quantify over worlds in which one doesn't feel good (equivalent to the secondary ordering source in the system of von Fintel and Iatridou (2008)).

As predicted, *Anga* loses its ambiguity when negated.

(49) Kambere si-anga-bi-a   eká yó   lino.
      Kambere NEG-MOD-be-FV home PREP now
      #'*Kambere doesn't have to be at home now.*'
      'Kambere can't be at home now.'   (Newkirk 2022a)

5. Further Predictions

Introducing the notion of Actuality Strength, we predict that there should be cases where strong modals (possibility and necessity) pattern together to the exclusion of weak modals. This is precisely what we observe in (50).[16] While both strong possibility and necessity modals are incompatible with the denial of the prejacent, weak necessity modals are felicitous in such a context. In some sense, weak necessity modals are perceived as weaker than a possibility modal claim.

(50) a. # It must/may/might be raining, but it isn't.[17]
      b. It should/ought to be raining, but it isn't.

This follows from the notion of Actuality Strength. Strong modals have the actual world in their final domain of quantification, and thus cannot be followed with a contradictory claim that the prejacent is false in the actual world.

Another place where strong possibility and necessity modals pattern together is in neg-raising contexts. Note that weak necessity modals, unlike strong necessity and possibility modals align with NRPs.

(51) a. I don't think that John should marry Susan (OK NR with *should > not*)
      b. I don't think that John must marry Susan (# NR with *must > not*).
      c. I don't think that John may marry Susan (# NR with *may > not*).

                                                              (Homer 2015)

This indeed is fully in line with our proposal. As strong, but not weak necessity modals have the actual world in their domain of quantification, the singleton alternatives of negated strong necessity modals (but not of weak necessity modals) will be pruned under exhaustification. Hence, negated necessity modals can never give rise to NR readings.

6. Conclusions

In this paper, we have provided a non-lexical account of NR in terms of exhuastification (Bar-Lev 2018, 2020). We also introduced a world-sensitive pruning mechanism according to which the inclusion of the actual world in the final domain of NRPs leads to the pruning of all singleton alternatives. Applying EXH to the set of alternatives after pruning singleton alternatives does not yield a NR reading. We motivated this pruning mechanism with appealing to Kratzer's notion of cognitively viable propositions. Assertion of singleton propositions whose worlds in their domain are taken to be candidates for the actual world (strong modals) need omniscience in a rather strong sense, and are thus not cognitively viable. Holding that duals are the same in

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[16]A full-fledged account of this contrast is beyond the scope of this paper but we think the solutions offered by Kratzer (2020) for (50) within the Anchor Semantics, and by Silk (2022) for the difference between weak and strong necessity modals in (50) can be consistently adopted in our system.

[17]See Yalcin (2007) for an account of this anomaly in terms of *epistemic contradiction*.

the Actuality Strength (inclusion of the actual world), our account derives Horn's generalization that strong universal modals are never neg-raising predicates.

We then extended this idea about the importance of the actual world in determining NRPs to properties of modals in general. We showed that our account makes several predictions that are borne out:

- Only possibility modals whose domain doesn't include the actual world can and in principle must be strengthened (unless the superweak meaning is clearly intended) via exhaustification along the lines of Bar-Lev (2018, 2020).

- The strengthening process does not change the notion of strength in terms of the inclusion of the actual world. Thus, weak possibility modal are predicted to be strengthened to weak necessity modals.

- Strong modals share properties that weak modals lack, irrespective of the modal force.

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