A unified semantics for exceptive-additive *besides*[1]

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Abstract. Exceptive expressions like *except* and *but* are known to contribute an inference of exception when they occur with universal quantifiers and not to be compatible with non-universal quantifiers. Similarly to exceptives, exceptive-additive expressions like *besides* contribute an exceptive inference in universal statements. They, however, differ from exceptives in being able to co-occur with non-universal quantifiers. In such contexts they contribute an inference of addition. We propose a unified semantic treatment of exceptive-additives that derives their interaction with universal and non-universal quantifiers from independently motivated mechanisms. We extend the treatment of exceptives in terms of *Exh* (Gajewski 2013; Hirsch 2016; Crnič 2021) to exceptive-additives and propose that the difference between the two types of constructions lies in the way the alternatives are constructed. By reducing the set of alternatives, we can capture the additive inference of *besides* with *exactly n*-numerals. We extend this account to all other instances of modified numerals and existentials by adopting a decompositional approach to their semantics.

Keywords: *besides*, exceptives, exceptive-additive constructions, modified numerals, *Exh*, alternatives, quantification.

1. The puzzle

1.1. Parallels and differences between exceptive and exceptive-additive expressions

Combined with universal quantifiers such as in (1) and (2) the exceptive expressions *but*, *except*, and *besides* yield parallel inferences. For (1) this gives the following inferences: (i) a containment inference that Ann is a member of the restrictor set denoted by *girl*, (ii) a quantificational inference that the restrictor set minus Ann is a subset of the scope set denoted by *came*, and (iii) an exception inference that Ann is not a member of the scope set (Keenan and Stavi 1986; Hoeksema 1987; von Fintel 1993). Completely parallel inferences modulo the contribution of negation obtain for (2).

(1) Every girl *but/except/besides* Ann came.
$\rightsquigarrow$ Ann is a girl — containment
$\rightsquigarrow$ Every girl who is not Ann came — quantification
$\rightsquigarrow$ Ann didn't come — exception

(2) No girl *but/except/besides* Ann came.
$\rightsquigarrow$ Ann is a girl — containment
$\rightsquigarrow$ No girl who is not Ann came — quantification
$\rightsquigarrow$ Ann came — exception

Combined with non-universal quantifiers, as in examples (3) to (5), *but* and *except* result in

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ungrammaticality (Horn 1989; von Fintel 1993), whereas *besides* is grammatical. Interestingly, in all of these examples *besides* yields an additive interpretation. While the containment and quantification inferences remain as in the examples above, the exception inference changes to an additive inference to the effect that the predicate in the scope of the quantifier is true of Ann (von Fintel 1989; Sevi 2008; Vostrikova 2019a, b).[2]

(3) Some girl **but/*except/besides* Ann came.
⇝ Ann is a girl — containment
⇝ Some girl who is not Ann came — quantification
⇝ Ann came — addition

(4) At least/more than two girls **but/*except/besides* Ann came.
⇝ Ann is a girl — containment
⇝ At least/more than two girls who are not Ann came — quantification
⇝ Ann came — addition

(5) At most/fewer than two girls **but/*except/besides* Ann came.
⇝ Ann is a girl — containment
⇝ At most/fewer than two girls who are not Ann came — quantification
⇝ Ann came — addition

1.2. The standard approach to *but*-exceptives

The standard approach to the semantics of exceptives like *but* is due to von Fintel (1993, 1994). According to this approach, *but* does two things in (1) semantically: (i) it subtracts {Ann} from the restrictor set denoted by *girl*, and (ii) states that this is the minimal subtraction that is required to make the quantificational claim true. For (1) and (2) this can result in both truth and falsity, whereas for (3) to (5) it yields a contradiction.

More recent literature has tended to have (i) be contributed by *but* directly but lets (ii) be the contribution of an exhaustivity operator *Exh* strengthening the interpretation of its prejacent, i.e., the sentence without *Exh* (see Gajewski 2013; Hirsch 2016, also cf. Gajewski 2008). The LF for (1), for instance, would look as in (6).

(6) $[_{\mathrm{IP}_2}\ \mathrm{Exh}_{\mathrm{ALT}}\ [_{\mathrm{IP}_1}$ [every [girl [but $\mathrm{Ann}_{\mathrm{F}}$]]] came]]

The meaning for *but* is given in (7), following Hirsch (2016). *But* is looking to compose with a plural or atomic individual (denoted by the DP following *but*), then with a predicate of individuals (denoted by the restrictor of the quantifier). It introduces a presupposition that the predicate is true of this plural or atomic individual (thus, the containment inference is hard-wired into the lexical semantics of *but*). It outputs a new set of plural or atomic individuals such that the predicate is true of it and it does not overlap with the first plurality.

(7) a. $[\![but]\!]^{g,w} = \lambda x_e.\lambda f_{<et>}\text{: } f(x).\ \lambda y_e.\ f(y)\ \&\ \neg\mathrm{OVERLAP}(x)(y)$
b. $\mathrm{OVERLAP}(x)(y)$ iff $\exists z\ [z \leq x\ \&\ z \leq y]$

Given this, the denotation of the restrictor of the quantifier in (6) is as shown in (8): this predicate of individuals picks girls who are not Ann; Ann is subtracted from the domain of the

[2]Notice that (2) similarly to (3)-(5) carries the inference that Ann came. We will come back to the question of whether the status of the inference in (2) and in (3)-(5) is the same.

quantifier.

(8) $[\![\textit{girl but Ann}]\!]^{g,w} = \lambda y_e.$ y is a girl_w & ¬OVERLAP(Ann)(y)
$[\![\textit{girl but Ann}]\!]^{g,w}$ is defined only if Ann is a girl_w

Assuming there are four salient girls in the context (Ann, Bella, Carol, and Denise), the quantificational claim (the prejacent of *Exh*) is as follows.

(9) $[\![IP_1]\!]^{g,w}$ = T iff $\forall x[x \in \{\text{Bella, Carol, Denise}\} \rightarrow x\ \text{came}_w]$

We adopt the standard denotation for *Exh*, given in (10). *Exh* asserts its prejacent and negates all innocently excludable alternatives (IE) (the alternatives that are in every maximal set of alternatives that can be negated together with the assertion of the prejacent without introducing a contradiction (Fox 2007)).[3]

(10) a. $[\![Exh_{ALT}\ \phi]\!]^{g,w} = [\![\phi]\!]^{g,w}\ \&\ \forall q\ [q \in \text{IE}(\lambda w'.[\![\phi]\!]^{g,w'}, \text{ALT}) \rightarrow \neg q(w)]$
b. $\text{IE(p, ALT)} = \bigcap\{\text{ALT'} \subseteq \text{ALT: ALT' is a maximal subset of ALT s.t. } \text{ALT'}^{\neg} \cup \{p\} \text{ is consistent}\}$
c. $\text{ALT'}^{\neg} = \{\neg p' : p' \in \text{ALT'}\}$

This approach assumes the structural theory of focus alternatives (Katzir 2007; Fox and Katzir 2011). The alternatives used by *Exh* are determined by substituting the focused marked DP with other possible DPs of at most the same complexity.[4] The DP following *but* is marked with focus. The resuling set of alternatives is given in (11): in each of the alternative propositions a different individual is subtracted from the domain of *every*.

(11) $$\text{ALT} = \left\{\begin{array}{l} \lambda w.[\![\textit{[[every [girl [but Ann]]] came]}]\!]^{g,w} \\ \lambda w.[\![\textit{[[every [girl [but Bella]]] came]}]\!]^{g,w} \\ \lambda w.[\![\textit{[[every [girl [but Carol]]] came]}]\!]^{g,w} \\ \lambda w.[\![\textit{[[every [girl [but Denise]]] came]}]\!]^{g,w} \end{array}\right\}$$

The first alternative is the prejacent. All other alternatives are innocently excludable, meaning their negation is consistent with the assertion of the prejacent. *Exh* negates all of them. The overall predicted truth conditions for the entire sentence are given in (12). The negation of the alternatives results in the inference that in every set differing from the original only in the individual that gets subtracted, not every girl came. Given the truth of the prejacent, the latter can only be the case if it is Ann who did not come. This derives the exception inference.

(12) $$\begin{array}{l} [\![\textit{(6)}]\!]^{g,w} = \text{T iff } \forall x[x \in \{\text{Bella, Carol, Denise}\} \rightarrow x\ \text{came}_w]\ \& \\ \qquad \neg\forall x[x \in \{\text{Ann, Carol, Denise}\} \rightarrow x\ \text{came}_w]\ \& \\ \qquad \neg\forall x[x \in \{\text{Ann, Bella, Denise}\} \rightarrow x\ \text{came}_w]\ \& \\ \qquad \neg\forall x[x \in \{\text{Ann, Bella, Carol}\} \rightarrow x\ \text{came}_w] \end{array}$$

[3]This can also be implemented with *Exh* that negates all alternatives that are not entailed by the prejacent (see Chierchia 2013). We follow Hirsch (2016) in employing Fox's 2007 version of *Exh* based on innocent inclusion. The relevant difference here is that the application of the innocent inclusion based *Exh* for (3) to (5) is predicted to be vacuous and is ruled out by the constraint that blocks a vacuous application of *Exh*, whereas the *Exh* not based on the innocent exclusion would directly lead to contradictions in such cases.

[4]Formally, this can be implemented by introducing the requirement that the set of alternatives *Exh* employs has to be a subset of propositions denoted by the sentences in the set of structural alternatives:

(i) a. $[\![Exh_{ALT}\ \phi]\!]^{g,w}$ is defined only if $\text{ALT} \subseteq \{\lambda w'.[\![\phi']\!]^{g,w'} : \phi' \in \text{ALT}_{\text{str}}(\phi)\}$
b. $\text{ALT}_{\text{str}}(\phi) = \{\phi' : \phi' \text{ is derived from } \phi \text{ by replacing the focus marked x by y such that } y \lesssim x\}$

1.3. Deriving co-occurrence restrictions for *but*-exceptives

The standard approach derives the unacceptability of (3) to (5) with both *but* and *except*.

The LF predicted for the existential claim in (3) is as shown in (13).

(13) $[_{IP_2}$ Exh_{ALT} $[_{IP_1}$ [some [girl [but Ann_F]]] came]]

The meaning of the prejacent of *Exh* is as shown in (14) and the alternatives other than the prejacent are given in (15).

(14) $[\![IP_1]\!]^{g,w}$ = T iff $\exists x[x \in \{\text{Bella, Carol, Denise}\}$ & x $\text{came}_w]$

(15) a. $\lambda w.\exists x[x \in \{\text{Ann, Carol, Denise}\}$ & x $\text{came}_w]$
b. $\lambda w.\exists x[x \in \{\text{Ann, Bella, Denise}\}$ & x $\text{came}_w]$
c. $\lambda w.\exists x[x \in \{\text{Ann, Bella, Carol}\}$ & x $\text{came}_w]$

None of the alternatives in (15) is innocently excludable. Only one of them can be negated together with the assertion of the prejacent. For example, the negation of (15a) is compatible with the prejacent: this would mean that only Bella came. But the other two alternatives cannot be negated in this case, their negation would create a contradiction, as it is not compatible with Bella coming. The same goes for the second and the third alternative. Since none of the alternatives is innocently excludable, *Exh* has nothing to negate in this case. [5]

This LF is ruled out by the non-vacuity constraint (Spector 2013; Gajewski 2013; Fox and Spector 2018). Assuming that *but* obligatorily co-occurs with *Exh*, this is the the only possible LF for (3) with *but*. Considering that a sentence which is predicted to necessarily violate pragmatic constraints is perceived as ungrammatical, the ungrammaticality of (3) is explained.

(16) NON-VACUITY: *Exh[A]* is infelicitous if *Exh[A]* is equivalent to *A*.

Similar results obtain for the remaining examples in (4) and (5).[6] Let's consider *at most two* in (17) which is downward monotonic on its first argument.

(17) *At most two girls but Ann came.

The set of alternatives is given in (18). None of them is innocently excludable. Let's consider a situation where the negation of the second alternative is true, the situation where three girls came: Carol, Denise and Ann. This is compatible with the prejacent because there are two girls who came in the set {Bella, Carol, Denise} and not more. However, negating other alternatives cannot be done consistently with this, because in the sets {Ann, Bella, Denise} and {Ann, Bella, Carol} there will be exactly two girls who came. The same reasoning applies to other alternatives: the negation of at most one of them is compatible with the assertion of the prejacent.

[5] As shown in (Hirsch 2016), some additional assumptions have to be made about the case when there are exactly two salient individuals in the domain. In this case, the only alternative to the prejacent will be innocently excludable. If there are only two girls, say Ann and Bella, it is entirely possible that some girl in {Bella} came, but no girl in {Ann} came. This case is ruled out by the restriction on the use of an existential when it is known that there is exactly one individual satisfying the restrictor.
[6] Some of these cases also require some additional assumptions.

(18) $$\left\{\begin{array}{l} \lambda w.[\![[_{IP_1}\ [\ at\ most\ two\ [\ girls\ [but\ Ann]]]\ came\ ]]]\!]^{g,w} \\ \lambda w.[\![[_{IP_1}\ [\ at\ most\ two\ [\ girls\ [but\ Bella]]]\ came\ ]]]\!]^{g,w} \\ \lambda w.[\![[_{IP_1}\ [\ at\ most\ two\ [\ girls\ [but\ Carol]]]\ came\ ]]]\!]^{g,w} \\ \lambda w.[\![[_{IP_1}\ [\ at\ most\ two\ [\ girls\ [but\ Denise]]]\ came\ ]]]\!]^{g,w} \end{array}\right\}$$

Since there are no innocently excludable alternatives in this case, the application of *Exh* is predicted to be vacuous. Thus, this sentence is predicted to be ungrammatical in the same way as the example with the existential considered above.

This approach could be straightforwardly carried over to *except*, as done by Crnič (2021) for at least some such constructions. That being said, Moltmann (1995) and Vostrikova (2021) draw attention to contrasts like (19a) and (19b). While *except* allows for multiple remnants and prepositional phrases following it, *but* does not do so.

(19) a. Every boy danced with every girl except Bill with Ann.
b. *Every boy danced with every girl but Bill with Ann.

Accordingly, Vostrikova (2019b, 2021), as well as Potsdam and Polinsky (2019), draw the conclusion that the *except* in (19a) syntactically coordinates two clauses. Vostrikova suggests a meaning for *except* that can be seen as a modification of von Fintel's 1993 one for *but* so as to be applicable at the clausal level. Thereby similar predictions regarding co-occurrence data are derived by the lexical meaning of *except*. Crnič (2021) explores the possibility that even in the clausal case, there is a place for strengthening through *Exh*.

1.4. The puzzle of *besides*

On the one hand, the standard recipe for *but* together with strengthening by *Exh* could be employed for *besides* as well. This would derive the facts in (1) and (2). Notice in this respect that, indeed, as far as syntax is concerned, *besides* appears to be phrasal just as much as *but*:

(20) *Every boy danced with every girl besides Bill with Ann.

On the other hand, extending this standard recipe to *besides*, all else being equal, would have undesirable consequences for (3) to (5). This is so because parallel to *but*, *besides* would be predicted to incur unacceptability through strengthening via *Exh* in these cases.

Notice that one cannot assume that *but* and *besides* are parallel in terms of subtraction but that only the former involves strengthening. While this would make *besides* acceptable with both universal and non-universal quantifiers, it would have the consequence that neither exceptive inferences would be drawn for the former nor additive inferences for the latter. Indeed, the exceptive or the additive inference seem to be non-removable and uncancellable parts of the meaning of sentences with *besides*. This is tested in (21a) and (21b) for (1) and (3) respectively. If the exceptive or additive inference were absent, it should be fine to precede the sentence with a sentence negating the supposed inference, but this is not the case. The sequences are degraded (Vostrikova 2019c).

(21) a. #Ann came. Every girl besides Ann came too.
b. #Ann didn't come. But some girl besides Ann came.

At this point, we can characterize the main semantic puzzle raised by exceptive-additive *besides* as follows:

(22) **Exceptive-additive puzzle (EAP):** *Besides* yields exceptive inferences precisely in those environments in which exceptive *but* is grammatical and yields additive inferences in precisely those environments in which exceptive *but* is not grammatical.

We need to ask why this should be the case. The EAP suggests that the behavior of *besides* is driven by factors of logicality. In this paper, we therefore adopt the following hypothesis to deal with the EAP:

(23) **Exceptive-additive hypothesis (EAH):** *Besides* yields additive inferences in those environments in which exceptive *but* is not grammatical in order to avoid ungrammaticality.

1.5. The previous treatments of *besides* and the plot

Previous approaches to the semantics of exceptive-additive *besides* provided an account for a subset of cases in (1)-(5).

Von Fintel (1989) proposed a unified treatment of *besides* with universal and negative quantifiers, as well as the non-monotonic quantifier *exactly n*. According to this analysis, *besides*, like any exceptive, contributes domain subtraction. Additionally, *besides* introduces the minimality condition, meaning that the subtracted set is the one with the minimal cardinality necessary for the quantificational claim to be true. The lexical entry proposed in (von Fintel 1989) is shown in (24): *besides* composes with a set denoted by the DP it introduces,[7] a set denoted by the restrictor, the determiner and the set denoted by the scopal argument. It subtracts the first set from the restrictor of the quantifier and adds the minimality condition (underlined in (24)).

(24) $$[\![\textit{besides}]\!]^g = \lambda C_{<et>}.\lambda A_{<et>}.\lambda D_{<<et><et,t>>}.\lambda P_{<et>}.\ D(A\text{-}C)(P)\ \&\ \underline{\forall Y\ [|C|>|Y| \rightarrow \neg\ D(A\text{-}Y)(P)]}$$

The work of the minimality condition can be appreciated when we look at the specific example in (25a). Its predicted interpretation is in (25b). The first conjunct states that if we remove Ann from the domain it holds that exactly two girls came. The second conjunct says that if a set with the cardinality less than the cardinality of {Ann} is subtracted, the *exactly*-claim will not be true. There is only one set with a smaller cardinality than the singleton set {Ann} and that is the empty set. Subtracting an empty set equals to no subtraction at all. Accordingly, the minimality condition states that it is false that exactly two girls came overall. If it is true that exactly two girls came who are not Ann, but not exactly two girls came overall, Ann must be the girl who came, as adding Ann to the domain cannot reduce the number of girls who came. Thus, the additive inference is captured.

(25) a. Exactly two girls besides Ann came.

b. $$[\![\textit{(25a)}]\!]^g = \text{T iff } |(\{z\text{: z is a girl}\} - \{\text{Ann}\}) \cap \{x\text{: x came}\}| = 2\ \&\ \underline{\forall Y[|\{\text{Ann}\}|>|Y| \rightarrow |(\{z\text{: z is a girl}\} - Y) \cap \{x\text{: x came}\}| \neq 2\ ]}$$

This treatment extends to cases with universal and negative quantifiers and accounts for the fact that in the first case the inference is negative, and in the second, it is positive. If every girl who is not Ann came, but not every girl came, Ann must be the girl who did not come. If no girl who is not Ann came, but some girl came, Ann must be the girl who came.

[7]Note that this approach assumes that the DP following an exceptive is a set denoting expression.

The problem with this approach observed by von Fintel (1989) is that it is not straightforwardly extendable to upward monotonic numerals like *at least two* in (4). It cannot be the case that at least two girls who are not Ann came, but not at least two girls came overall. For the same reason, it also does not extend to existentials in (3).

Vostrikova (2019a) proposes a unified compositional analysis of the additive uses of *besides* with existentials, *wh*-questions (like in (26a)), and in focus constructions (illustrated in (26b)). However, the proposed treatment considers *besides* as a pure additive element, and it does not account for the exceptive reading with universal quantifiers.

(26) a. Who besides Ann came?
b. Besides Ann, Bill danced with Mary$_F$

Vostrikova (2019c) proposes a unified treatment for the exceptive uses of *besides* with universal and negative quantifiers and the additive uses of *besides* with existentials, *wh*-questions and focus associates. Exceptive-additive flip is treated as a case of a structural ambiguity. This analysis, however, is not straightforwardly extendable to modified numerals and such cases are not discussed. In this paper we will not say anything about cases like (26) and will focus on quantificational cases and modified numerals.

The rest of the paper spells out a unified approach to the uses of *besides* in (1)-(5) relying on the standard mechanism introduced above. The work is distributed between *besides* contributing domain subtraction and *Exh* being responsible for the exceptive or the additive inference. We argue that the only difference from the exceptive *but* lies in the kind of alternatives used by *Exh*. This minimal change straightforwardly derives results von Fintel (1989) obtained for *exactly n* numerals. It is shown that this can be extended to other cases of non-universal quantifiers if they are allowed to be decomposed along the lines of what has been suggested in the literature for modified numerals (Hackl 2000; Heim 2000; Mayr and Meyer 2014; Buccola and Spector 2016), so that one part can scope below *Exh* and one above it.

2. The proposal

2.1. *Besides* and *every*

We'll assume *besides* is like *but*: it does domain subtraction and introduces the containment presupposition, as shown in (27). Like *but*, *besides* composes with a plural or atomic individual (denoted by the DP following *besides*), then takes a predicate of individuals (the restrictor of a quantifier) and outputs a new predicate of individuals. It introduces a presupposition that the restrictor predicate is true of the individual introduced by *besides*. The new predicate it outputs is true of atomic or plural individuals if they satisfy the restrictor predicate and do not overlap with the individual introduced by *besides*.

(27) a. $[\![besides]\!]^{g,w} = \lambda x_e.\lambda f_{<et>}$: f(x). λy_e.f(y) & ¬OVERLAP(x)(y)
b. OVERLAP(x)(y) iff $\exists$z [z $\leq$ x & z $\leq$ y]
c. $[\![girl\ besides\ Ann]\!]^{g,w} = \lambda y_e$. y is a girl$_w$ & ¬OVERLAP(Ann)(y)
is defined only if Ann is a girl$_w$

We build on Gajewski (2008, 2013); Hirsch (2016); Crnič (2021), and propose that the exceptive-additive inference is contributed by *Exh*. Thus, the structure of a sentence with *besides* with a universal quantifier in (28a) is as shown in (28b).

(28) a. Every girl besides Ann came.

b. $[_{IP_2}$ Exh$_{ALT}$ $[_{IP_1}$ [every [girl [besides Ann]$_F$]] came]]

However, we suggest that the alternatives are not computed by substituting the element following *besides* with its alternatives, as proposed for *but* and discussed above. Instead, *besides* makes use of the structurally simpler alternatives. Thus, the focus is not on the element following *besides*, but on the entire *besides DP* phrase. In this case the set of alternatives is like shown in (29).

(29) $$\text{ALT} = \left\{ \begin{array}{l} \lambda w.[\![\textit{every girl besides Ann came}]\!]^{g,w} \\ \lambda w.[\![\textit{every girl came}]\!]^{g,w} \end{array} \right\}$$

Exh asserts the prejacent and negates the only alternative distinct from the prejacent. The result of this is shown in (30a), or equivalently (30b). The presupposition of containment contributed by *besides* is in (30c).

(30) a. $[\![\textit{(28b)}]\!]^{g,w}$ = T iff $\forall$x [(x is a girl $_w$ & $\neg$OVERLAP(Ann)(x)) $\rightarrow$ x came$_w$] & $\neg$ $\forall$y[y is a girl$_w$ $\rightarrow$ y came$_w$]

b. $[\![\textit{(28b)}]\!]^{g,w}$ = T iff $\forall$x [(x is a girl & $\neg$OVERLAP(Ann)(x) $\rightarrow$ x came$_w$] & $\exists$y [y is a girl$_w$ & $\neg$ y came$_w$]

c. $[\![\textit{(28b)}]\!]^{g,w}$ is defined only if Ann is a girl$_w$

The resulting interpretation correctly captures the negative inference (28a) comes with. If every girl who is not Ann came, but some girl did not come, then Ann is the girl who did not come.

So far we have considered the case where the DP following *besides* denotes an atomic individual. When *besides* introduces a plural DP, the negative inference has to apply to every individual in this plurality. This is captured by including the alternatives where the plurality is substituted by individuals denoting the plurality subparts as shown in (31).

(31) a. Every girl [besides Ann and Bella]$_F$ came.

b. $$\left\{ \begin{array}{l} \lambda w.[\![\textit{every girl besides Ann and Bella came}]\!]^{g,w} \\ \lambda w.[\![\textit{every girl besides Ann came}]\!]^{g,w} \\ \lambda w.[\![\textit{every girl besides Bella came}]\!]^{g,w} \\ \lambda w.[\![\textit{every girl came}]\!]^{g,w} \end{array} \right\}$$

Exh asserts the prejacent and negates all the other alternatives as they are all innocently excludable. The overall interpretation predicted for (31a) is in (32). Given the truth-conditional content in (32a), the fact that the negative inference applies to both Ann and Bella is captured: all girls who are not Ann and Bella came, but there is a girl who is not Ann who did not come (this must be Bella) and there is a girl who is not Bella who did not come (this must be Ann). The presupposition introduced by *besides* captures the containment inference for both Ann and Bella.

(32) a. $[\![\textit{(31a)}]\!]^{g,w}$ = T iff
$\forall$x[(x is a girl$_w$ & $\neg$OVERLAP(Ann+Bella)(x)) $\rightarrow$ x came$_w$] &
$\exists$y[(y is a girl$_w$ & $\neg$OVERLAP(Ann)(y)) & $\neg$y came$_w$] &
$\exists$z[(z is a girl$_w$ & $\neg$OVERLAP(Bella)(z)) & $\neg$z came$_w$] &
$\exists$x[x is a girl$_w$ & $\neg$x came$_w$]

b. $[\![\textit{(31a)}]\!]^{g,w}$ is defined only if Ann and Bella are girls$_w$

There is another way of constructing alternatives that would result in the same overall denotation for the sentence. Specifically, one could say that *besides* itself is focus-marked (as shown in (33a)) and the alternatives are formed by substitution of *besides* with its alternative *including*. Then, assuming that the subdomain alternatives for the plural are available, the list of alternatives is like it is shown in (33b).

(33) a. Every girl besides$_F$ Ann and Bella came.

b. $\left\{ \begin{array}{l} \lambda \text{w}.[\![\textit{every girl besides Ann and Bella came}]\!]^{g,w} \\ \lambda \text{w}.[\![\textit{every girl besides Ann came}]\!]^{g,w} \\ \lambda \text{w}.[\![\textit{every girl besides Bella came}]\!]^{g,w} \\ \lambda \text{w}.[\![\textit{every girl including Ann and Bella came}]\!]^{g,w} \end{array} \right\}$

As far as we can see, both options are open possibilities, and since they lead to the same resulting interpretation we remain agnostic about which one is the right way to go.

For simplicity of exposition, we will only consider cases where the individual introduced by *besides* is atomic from now on, but the treatment can always be extended to plural cases along the lines shown above.

2.2. *Besides* and *no*

Just like the standard approach to the semantics of exceptives, this approach applies to a universal and a negative quantifier in a unified way. The set of the alternatives for the prejacent of *Exh* in (34b) is as shown in (34c).

(34) a. No girl besides Ann came.

b. $[_{\text{IP}_2}\ \text{Exh}_{\text{ALT}}\ [_{\text{IP}_1}\ [\text{no [girl [besides Ann]}_\text{F}\ ]]\ \text{came}\ ]]$

c. $\left\{ \begin{array}{l} \lambda \text{w}.[\![\textit{no girl besides Ann came}]\!]^{g,w} \\ \lambda \text{w}.[\![\textit{no girl came}]\!]^{g,w} \end{array} \right\}$

The predicted interpretation for the sentence is given in (35). The two conjuncts taken together entail that Ann is the girl who came. If no girl who is not Ann came, but some girl came (overall), this girl can only be Ann. This correctly captures the fact that (34a) comes with the positive inference that Ann came.

(35) a. $[\![\textit{(34b)}]\!]^{g,w}$ = T iff $\neg\exists$y [(y is a girl$_w$ & $\neg$OVERLAP(Ann)(y)) & y came$_w$] & $\exists$x [x is a girl$_w$ & x came$_w$]

b. $[\![\textit{(34b)}]\!]^{g,w}$ is defined only if Ann is a girl$_w$

2.3. *Besides* and *exactly*

This approach is straightforwardly extendable to the non-monotonic quantifier *exactly n*. Like with the negative quantifier, we predict that *besides* gets the additive reading in such cases, which correctly captures the fact that (36a) comes with the positive inference that Ann came. (36b) provides the assumed LF for (36a). The alternatives used by *Exh* are given in (36c).

(36) a. Exactly one girl besides Ann came.

b. $[_{\text{IP}_2}\ \text{Exh}_{\text{ALT}}\ [_{\text{IP}_1}\ [\text{exactly one [girl [besides Ann]}_\text{F}]]\ \text{came}\ ]]$

c. $\left\{ \begin{array}{l} \lambda \text{w}.[\![\textit{exactly one girl besides Ann came}]\!]^{g,w} \\ \lambda \text{w}.[\![\textit{exactly one girl came}]\!]^{g,w} \end{array} \right\}$

For concreteness, we adopt the lexical entry for *exactly* given in (37). *Exactly* composes with a degree and two predicates of individuals and returns truth if and only if the degree is equal to the maximum cardinality for which a plurality satisfying both predicates exists. The definition for the metalanguage function *max*, which is employed in the denotation of *exactly*, is provided in (37b): this is a function that applies to a predicate of degrees and returns the unique largest degree for which the predicate holds, if such a degree exists; otherwise, it returns 0.[8]

(37) a. $[\![exactly]\!]^{g,w} = \lambda n_d.\lambda f_{<et>}.\lambda g_{<et>}.\text{max}(\lambda d_d.\exists x[|x|=d \;\&\; f(x) \;\&\; g(x)]) = n$
b. $\text{max}(P_{<dt>}) = \iota n[P(n) \;\&\; \forall m[P(m) \rightarrow m \leq n]]$ if $\exists d[P(d)]$, 0 otherwise

The prejacent of *Exh* gets the denotation in (38): it is true if and only if exactly one girl who is not Ann came.

(38) $[\![IP_1]\!]^{g,w}$=T iff $\text{max}(\lambda d.\exists x[|x|=d \;\&\; x \text{ is a girl}_w \;\&\; \neg\text{OVERLAP(Ann)}(x) \;\&\; x\text{ came}_w])=1$

Exh asserts the prejacent and negates the only alternative in (36c) distinct from the prejacent. The result of this is in (39): the sentence is predicted to be true if and only if exactly one girl who is not Ann came, but not exactly one girl came overall. Since addition Ann to the domain could not have possibly made the number of girls who came smaller than it was without taking Ann into consideration, (39) can hold only if Ann came. The containment inference is captured by the presupposition introduced by *besides*.

(39) $[\![(36b)]\!]^{g,w}$=T iff
$\text{max}(\lambda d.\exists x[|x|=d \;\&\; x \text{ is a girl}_w \;\&\; \neg\text{OVERLAP(Ann)}(x) \;\&\; x \text{ came}_w]) = 1 \;\&$
$\text{max}(\lambda m.\exists y[|y|=m \;\&\; y \text{ is a girl}_w \;\&\; y \text{ came}_w]) \neq 1$
$[\![(36b)]\!]^{g,w}$ is defined only if Ann is a girl_w

The theory we have developed here essentially implements von Fintel's (1989) proposal for *besides* in terms of domain subtraction and *Exh*. This theory is also considered by Gajewski (2013) for exceptives like *but* but is rejected because it incorrectly predicts their compatibility with *exactly*.

In the treatment we propose the positive inference of *besides* observed with negative quantifiers and numerals has the same nature. Thus, in the approach we develop here there are no separate paths for the exceptive reading with *no* and the additive reading with *exactly*. In these cases, along with *every*, the inference is the result of conjoining a quantificational claim with domain subtraction and the negation of the claim without subtraction. Whether the inference is positive or negative depends on the properties of a quantifier. We propose that there is no exceptive-additive ambiguity, just like there is no ambiguity of exceptives with negative and universal quantifiers.

In what follows, we will extend this treatment to all modified numerals, including the upward entailing ones, which were previously considered a major challenge for this approach.

2.4. *Besides* and upward entailing quantifiers

An attempt to give a parallel LF to the *at least n* case in (40a) does not lead to a well-formed meaning. This is because the quantifier is upward monotonic and both alternatives in (40b) are entailed by the prejacent. This means that *Exh* cannot negate anything in this case and

[8]The latter condition is needed to deal with *exactly 0* when there is no individual satisfying both of the predicates.

its application is predicted to be vacuous. Thus, this LF is predicted to be ruled out by the same non-vacuity constraint that rules out the use of exceptive *but* with upward monotonic quantifiers.

(40) a. $[_{IP_2}$ Exh$_{ALT}$ $[_{IP_1}$ [at least one [girl [besides Ann]$_F$]] came]]
b. $\left\{ \begin{array}{l} \lambda w.[\![\textit{at least one girl besides Ann came}]\!]^{g,w} \\ \lambda w.[\![\textit{at least one girl came}]\!]^{g,w} \end{array} \right\}$

Our proposal is built on the idea that modified numerals are quantifiers over degrees and as such they must undergo quantifier raising to be interpreted (see Hackl 2000; Mayr and Meyer 2014; Buccola and Spector 2016 on extending the idea of quantificational treatment of degree constructions along the lines proposed in (Heim 2000) to modified numerals).

At least one has the meaning given in (41b), which is obtained by putting together the meanings of *at least* in (41a) and *one*. Like any degree quantifier, it is looking to compose with a predicate of degrees. It returns truth if and only if there exists a degree greater than or equal to 1 of which the predicate of degrees is true.

(41) a. $[\![\textit{at least}]\!]^{g,w} = \lambda n_d.\lambda f_{<dt>}.\ \exists d[d \geq n\ \&\ f(d)]$
b. $[\![\textit{at least one}]\!]^{g,w} = \lambda f_{<dt>}.\ \exists d[d \geq 1\ \&\ f(d)]$

Given this semantics, *at least one* cannot be interpreted in its base position in (42a) because of the type mismatch. It undergoes quantifier raising and leaves a trace of type d. We propose that this trace is interpreted with a silent *exactly* below *at least one* left behind by QR, as shown in (42b). The numerical abstractor 1 is merged below the landing site of *at least one*.

(42) a. At least one girl came.
b. $[_{IP_2}$ at least one $[_{IP_1}$ 1 [exactly d_1 girl came]]]

Exactly has its standard denotation introduced in the previous section and repeated in (43) for convenience. It composes with the trace, then with the predicate denoted by *girl* and with the predicate denoted by *came*.

(43) $[\![\textit{exactly}]\!]^{g,w} = \lambda n_d.\lambda f_{<et>}.\lambda g_{<et>}.\max(\lambda d_d.\exists x[|x|{=}d\ \&\ f(x)\ \&\ g(x)]){=}n$

With these assumptions, the sister of *at least one* denotes the predicate of degrees shown in (44): this predicate is true of a degree if and only if exactly this many girls came.

(44) $[\![IP_1]\!]^{g,w} = \lambda n.\max(\lambda d_d.\ \exists x[|x|{=}d\ \&\ x \text{ is a girl}_w\ \&\ x \text{ came}_w) = n$

Given this, the overall predicted meaning for (42b) is as shown in (45). The sentence is predicted to be true if and only if there is a degree equal to 1 or larger such that exactly this many girls came. This captures the meaning of the sentence: one or more girls came.

(45) $[\![\textit{(42b)}]\!]^{g,w}{=}$ T iff $\exists d[d \geq 1\ \&\ \max(\lambda n.\exists x[|x|{=}n\ \&\ x \text{ is a girl}_w\ \&\ x \text{ came}_w){=}d]$

Now, given that there is a constituent in the LF with the meaning of *exactly* and that we have an account of the interaction of *exactly* and *besides*, we can compute the additive inference of *besides* at the level of this constituent. The LF that we propose for (46a) is shown in (46b). As before, *besides* forms a constituent with the predicate denoted by the restrictor. *Exh* is merged below the abstraction over degrees and below the upward entailing quantifier *at least one*.

(46) a. At least one girl besides Ann came.

b. $[_{\text{IP}_3}$ at least one $[_{\text{IP}_2}$ 1 $[_{\text{IP}_1}\text{Exh}_{\text{ALT}}$ [exactly d_1 girl [besides Ann]$_{\text{F}}$ came]]]]

The predicted meaning of IP_1 is given in (47). It is true if and only if the degree denoted by g(1) is such that exactly this many girls came if Ann is not taken into account and not exactly this many girls came if Ann is included.

(47) $[\![IP_1]\!]^{g,w}$=T iff
$\max(\lambda d_d.\exists x[|x|=d \ \&\ x \text{ is a girl}_w \ \&\ \neg\text{OVERLAP(Ann)(x)} \ \&\ x \text{ came}_w])=g(1)$ &
$\max(\lambda n_d.\exists y[|y|=n \ \&\ y \text{ is a girl}_w \&\ y \text{ came}_w])\neq g(1)$

Accordingly, the sister of *at least one* is the predicate of degrees formed by abstraction over that degree, shown in (48).

(48) $[\![IP_2]\!]^{g,w}=\lambda n.\max(\lambda d_d.\exists x[|x|=d \ \&\ x \text{ is a girl}_w \ \&\ \neg\text{OVERLAP(Ann)(x)} \ \&\ x \text{ came}_w])=n$
$\&\ \max(\lambda m_d.\exists y[|y|=m \ \&\ y \text{ is a girl}_w \ \&\ y \text{ came}_w])\neq n$

At least one composes with this predicate of degrees and states that a degree satisfying this predicate exists and it is equal to 1 or larger. The overall meaning predicted for (46b) is shown in (49). The sentence is predicted to be true iff there is a number such that if we do not count Ann exactly this many girls came and if we count Ann not exactly this many girls came and this number is equal to 1 or larger. This correctly captures the overall meaning of this sentence including its additive inference. Adding Ann to the domain while keeping everything else the same cannot result in a smaller number of girls in the domain who came. Thus, the only way (49) can hold is if Ann came.

(49) $[\![(46b)]\!]^{g,w}$= T iff
$\exists n[n \geq 1 \ \&\ \max(\lambda d_d.\exists x[|x|=d \ \&\ x \text{ is a girl}_w \ \&\ \neg\text{OVERLAP(Ann)(x)} \ \&\ x \text{ came}_w])=n$
$\&\ \max(\lambda m_d.\exists y[|y|=m \ \&\ y \text{ is a girl}_w \ \&\ y \text{ came}_w])\neq n\ ]$
$[\![(46b)]\!]^{g,w}$ is defined only if Ann is a girl_w

The treatment we propose here crucially relies on the assumption that there is a constituent with the meaning *exactly* left behind by *at least one*. Now we will address the question of whether this assumption is well-grounded.

It is standardly assumed that degree quantifiers leave behind *many* and not *exactly* as shown in (50a) (Heim 2000; Hackl 2000). *Many* is an existential quantifier with the semantics given in (50b). As *many* is an upward monotonic quantifier, our approach predicts that *besides* is not able to operate on it.

(50) a. $[_{\text{IP}_2}$ at least one $[_{\text{IP}_1}$ 1 [many d_1 girl came]]]
b. $[\![many]\!]^{g,w} = \lambda n_d.\lambda f_{<et>}.\lambda g_{<et>}.\exists x[|x|=n \ \&\ f(x) \ \&\ g(x)]$

However, the approach we propose here is compatible with the idea that a degree quantifier leaves behind *many*. *Many* can be turned into *exactly* by an operator that contributes maximality. This is evident from the denotation of *exactly* in (43) we employ here, which includes both the existential quantifier and maximality as its meaning ingredients.

The idea that the existential *many* can be turned into *exactly* by inserting an operator that contributes maximality was independently proposed by Buccola and Spector (2016) for modified numerals. In a nutshell, they propose that a modified numeral can undergo QR twice, as shown in (51a). First, it moves above the existential *many*, creating a degree predicate within its scope.

Then, it undergoes a second movement, generating another predicate of degrees. The trace left behind by the second movement undergoes type-shifting through the shifter *MAX* shown in (51b), which uses the same *max* function we employed in the denotation of *exactly*.

(51) a. $[_{\text{IP}_2}$ at least one [2 [$d_{2\text{MAX}}$ $[_{\text{IP}_1}$ 1 [many d_1 girl came]]]]]
b. $[\![MAX]\!]^{g,w} = \lambda d_d.\lambda P_{<dt>}.\ \max(P)=d$

Alternatively, the same result can be achieved by inserting *Exh* as shown in (52) and forming the alternatives by substituting the trace by its focus alternatives, similar to how the meaning of *exactly* is derived for bare numerals like *one girl*.

(52) $[_{\text{IP}_2}$ at least one $[_{\text{IP}_1}$ 1 Exh [many $d_{1\text{F}}$ girl came]]]

We propose that whenever *besides* appears to operate on an upward monotonic quantifier, it is actually operating on the silent *exactly* within the scope of this quantifier.

Besides with another type of upward monotonic numerals - *more than n* - can also be treated in a straightforward manner along the lines suggested for *at least one*. The proposed LF for (53a) is given in (53b).

(53) a. More than one girl besides Ann came.
b. $[_{\text{IP}_3}$ more than one $[_{\text{IP}_2}$ 1 $[_{\text{IP}_1}$$\text{Exh}_{\text{ALT}}$ [exactly d_1 girl [besides Ann]$_\text{F}$ came]]]]

The resulting interpretation is given in (54). The sentence is predicted to be true if and only if there is a number such that if we do not include Ann then exactly this many girls came and if we include her not exactly this many girls came and this number is larger than 1.

(54) $[\![\textit{(53b)}]\!]^{g,w}$= T iff
$\exists n[n>1\ \&\ \max(\lambda d_d.\exists x[|x|=d\ \&\ x \text{ is a girl}_w\ \&\ \neg\text{OVERLAP(Ann)}(x)\ \&\ x\ \text{came}_w])=n$
$\&\ \max(\lambda m_d.\exists y[|y|=m\ \&\ y \text{ is a girl}_w\ \&\ y\ \text{came}_w])\neq n\ ]$
$[\![\textit{(53b)}]\!]^{g,w}$ is defined only if Ann is a girl_w

We extend this approach to cases when *besides* seems to operate on an indefinite like in (55a). Thus, they get the LFs shown in (55b). We propose that *some* can be interpreted as *at least one*, as shown in (55c), when the NP predicate is singular, or as *more than one*, as shown in (55d), when the NP predicate is plural. Thus, we suggest that indefinites are degree quantifiers, they do not quantify over individuals like it is standardly assumed (or at least they can be degree quantifiers). We apply this treatment to *several* in (55e) as well: it gets the same interpretation as *more than one*. In all these cases, the semantic contribution of *besides* is computed on the constituent with the meaning *exactly*, thus correctly predicting the additive inference.

(55) a. Some girl(s) besides Ann came.
b. $[_{\text{IP}_3}$Some $[_{\text{IP}_2}$ 1 $[_{\text{IP}_1}$$\text{Exh}_{\text{ALT}}$ [exactly d_1 girl(s) [besides Ann]$_\text{F}$ came]]]]
c. $[\![\textit{some}]\!]^{g,w} = \lambda f_{<dt>}.\ \exists d[d\geq 1\ \&\ f(d)]$
d. $[\![\textit{some'}]\!]^{g,w} = \lambda f_{<dt>}.\ \exists d[d>1\ \&\ f(d)]$
e. Several girls besides Ann came.

2.5. *Besides* and downward monotonic numerals

The account presented here extends straightforwardly to downward entailing modified numerals like *fewer than n* and *at most n*.

We follow (Buccola and Spector 2016) in treating them as existential quantifiers over degrees as shown in (56) and (57), similarly to *at least n* and *more than n*.

(56) a. $[\![\textit{fewer than}]\!]^{g,w} = \lambda n_d.\lambda f_{<dt>}.\ \exists m[m<n \ \&\ f(m)]$
b. $[\![\textit{fewer than two}]\!]^{g,w} = \lambda f_{<dt>}.\ \exists m[m<2 \ \&\ f(m)]$

(57) a. $[\![\textit{at most}]\!]^{g,w} = \lambda n_d.\lambda f_{<dt>}.\ \exists m[m\leq n \ \&\ f(m)]$
b. $[\![\textit{at most two}]\!]^{g,w} = \lambda f_{<dt>}.\ \exists m[m\leq 2 \ \&\ f(m)]$

Buccola and Spector (2016) show that if the sister of a degree quantifier has the maximality operator in it, the overall predicted interpretation of a sentence containing *fewer than n* or *at most n* with the semantics in (57) and (56) is equivalent to the interpretation resulting from applying the lexical entries in (58) (where they are treated as negative quantifiers) to a constituent without the maximality operator.

(58) a. $[\![\textit{fewer than two}]\!]^{g,w} = \lambda f_{<dt>}.\ \neg\exists m[m\geq 2 \ \&\ f(m)]$
b. $[\![\textit{at most two}]\!]^{g,w} = \lambda f_{<dt>}.\ \neg\exists m[m>2 \ \&\ f(m)]$

The truth conditions of (59a) can be expressed as (59b) or, equivalently, as (59c). Both entries make the correct prediction that the sentence is true if 0 or 1 girl came and it it false if 2 or more girls came. (59b) states that there is no degree equal to 2 or larger than 2 such that there is a plurality of girls who came with this cardinality. (59c) says that the cardinality of the maximal plurality of girls who came is 0 or 1. This amounts to the same thing.

(59) a. Fewer than two girls came.
b. $\neg\exists d[d\geq 2 \ \&\ \exists x[|x|=d \ \&\ x \text{ is a girl}_w \ \&\ x \text{ came}_w]]$
c. $\exists d[d<2 \ \&\ \max(\lambda n.\exists x[|x|=n \ \&\ x \text{ is a girl}_w \ \&\ x \text{ came}_w])=d]$

Given the discussion in the previous subsection, we propose that (60a) has the LF shown in (60b) (we are glossing over the internal composition *exactly* here).

(60) a. Fewer than two girls besides Ann came.
b. $[_{IP_3}$ fewer than two $[_{IP_2}$ 1 $[_{IP_1}$ Exh_{ALT} [exactly d_1 girl [besides Ann]$_F$ came]]]]

The sister of *fewer than two* has the denotation shown in (61). This is a predicate of degrees that is true of degrees for which it holds that exactly this many girls came without counting Ann and not this many girls came overall.

(61) $[\![IP_2]\!]^{g,w}=\lambda n.\max(\lambda d.\exists x[|x|=d \ \&\ x \text{ is a girl}_w \ \&\ \neg\text{OVERLAP}(\text{Ann})(x) \ \&\ x \text{ came}_w])=n$
$\&\ \max(\lambda m.\exists y[|y|=m \ \&\ y \text{ is a girl}_w \ \&\ y \text{ came}_w])\neq n$

Fewer than two with the denotation in (56b) composes with this predicate and states that such degree exists and it is 0 or 1. The predicted truth conditions for (60b) are given in (62). The sentence is predicted to be true if and only if either exactly zero girls who are not Ann came and not exactly zero came if we take Ann into account or exactly one girl who is not Ann came and not exactly one if we count Ann. Adding Ann to the domain while keeping everything else the same cannot make the overall number of the girls who came smaller, it can only make this number larger. Thus, we correctly capture the fact that the sentence in (60a) comes with the positive inference that Ann came.

(62) $[\![\textit{(60b)}\,]\!]^{g,w}$=T iff

$\exists$n[n<2 & max(λd.$\exists$x[|x|=d & x is a girl$_w$ & $\neg$OVERLAP(Ann)(x) & x came$_w$])=n & max(λm.$\exists$y[|y|=m & y is a girl$_w$ & y came$_w$])$\neq$n]

The same goes for the *at most two* case in (63a). It has a parallel LF shown in (63b). *At most two* composes with the same predicate of degrees shown in (61). *At most two* states that such a degree exists and it is equal to 0, 1 or 2. The predicted truth conditions are in (64). They capture the additive inference in exactly the same way.

(63) a. At most two girls besides Ann came.
b. [$_{IP_3}$ at most two [$_{IP_2}$ 1 [$_{IP_1}$ Exh$_{ALT}$ [exactly d$_1$ girl [besides Ann]$_F$ came]]]]

(64) $[\![$*(63b)* $]\!]^{g,w}$=T iff
$\exists$n[n$\leq$2 & max(λd.$\exists$x[|x|=d & x is a girl$_w$ & $\neg$OVERLAP(Ann)(x) & x came$_w$])=n & max(λm.$\exists$y[|y|=m & y is a girl$_w$ & y came$_w$])$\neq$n]

Given our assumptions about the contribution of *besides* and *Exh*, the decomposition account of modified numerals is the only way to produce a well-formed meaning for an upward entailing quantifier. However, the situation is different with downward entailing modified numerals. The application of *Exh* is not predicted to be vacuous if the sentence with *fewer than two* in (60a) gets the LF shown in (65), where *Exh* scopes over the entire modified numeral.

(65) [$_{IP_3}$ Exh$_{ALT}$ [$_{IP_2}$ fewer than two [$_{IP_1}$ 1 [exactly d$_1$ girl [besides Ann]$_F$ came]]]]

The meaning resulting from interpreting of this LF is shown in (66). The sentence is predicted to be true if 0 or 1 is the maximal number of girls who are not Ann who came, but neither 0 nor 1 is the maximal number of girls who came overall. This is possible if Ann came and exactly one other girl came. This reading is strictly stronger than the reading resulting from the LF in (60b) and it is hard to empirically establish if this is also a possible LF for this sentence. It is definitely not the only available reading: the sentence does not require that exactly one girl who is not Ann came, it is compatible with Ann being the only girl who came.

(66) $[\![$*(65)*$]\!]^{g,w}$=T iff
$\exists$n[n<2 & max(λd.$\exists$x[|x|=d & x is a girl$_w$ & $\neg$OVERLAP(Ann)(x) & x came$_w$])=n] & $\neg\exists$m[m<2 & max(λd'.$\exists$y[|y|=d' & y is a girl$_w$ & y came$_w$])=m]

Similarly, the application of *Exh* is not predicted to be vacuous if sentence with *at most* in (63a) gets the LF where *Exh* scopes over the entire modified numeral, as shown in (67).

(67) [$_{IP_3}$ Exh$_{ALT}$ [$_{IP_2}$ at most two girl [$_{IP_1}$ 1 [exactly d$_1$ girl [besides Ann]$_F$ came]]]]

The sentence is predicted to be true if and only if at most two girls came if we don't count Ann and not at most two girls came overall. This is possible if Ann came along with exactly two other girls. However, *at most n* numerals mandatorily come with the uncertainty inference (Geurts and Nouwen 2007; Nouwen 2010; Mayr and Meyer 2014) and the sentence in (68a) definitely cannot mean that exactly two girls who are not Ann came. This LF is ruled out due to its incompatibility with the uncertainty inference.

(68) $[\![$*(67)*$]\!]^{g,w}$=T iff
$\exists$n[n$\leq$2 & max(λd.$\exists$x[|x|=d & x is a girl$_w$ & $\neg$OVERLAP(Ann)(x) & x came$_w$])=n] & $\neg\exists$m[m$\leq$2 & max(λd'.$\exists$y[|y|=d' & y is a girl$_w$ & y came$_w$])=m]

2.6. The difference between exceptive and exceptive-additive constructions

The crucial difference between exceptives and exceptive-additives lies in the construction of alternatives. As discussed in Section 1, we adopt Hirsch's (2016) proposal for exceptives, where the alternatives are formed by substituting the DP following the exceptive by other DPs of at most equal complexity. Hirsch's approach correctly captures the fact that exceptives are incompatible with *exactly*, as illustrated in (69). The predicted LF for (69a) is shown in (69b).

(69) a. *Exactly two girls but/except Ann came.
b. $[_{IP_2}$ Exh$_{ALT}$ $[_{IP_1}$ [exactly two girls but Ann$_F$ came]]]

Assuming again that there are four salient girls (Ann, Bella, Carol, Denise), the alternatives for *Exh* in (69b) have the meanings shown in (70). The alternative in (70a) is the prejacent. None of the remaining alternatives is innocently excludable. We can negate maximally two of the alternatives together with the assertion of the prejacent. For example, the negations of (70b) and (70c) are compatible with the assertion of (70a) if only Bella and Carol came because in {Ann, Carol, Denise} and {Ann, Bella, Denise} there is only one girl who came and not two. But the alternative in (70d) cannot be negated as both girls who came (Bella and Carol) are in {Ann, Bella, Carol}. The same reasoning applies to other alternatives: none of them is in every maximal set of alternative propositions that can be negated together with the assertion of the prejacent. Consequently, the application of *Exh* is predicted to be vacuous, and this LF is ruled out by the non-vacuity constraint.

(70) a. λw.max(λd.$\exists$x[|x|=d & x$\in${Bella, Carol, Denise} & x came$_w$) = 2
b. λw.max(λd.$\exists$x[|x|=d & x$\in${Ann, Carol, Denise} & x came$_w$) = 2
c. λw.max(λd.$\exists$x[|x|=d & x$\in${Ann, Bella, Denise} & x came$_w$) = 2
d. λw.max(λd.$\exists$x[|x|=d & x$\in${Ann, Bella, Carol} & x came$_w$) = 2

3. For future research

In this paper we focused on *besides*-phrases occurring in the connected position (the position adjacent to the restrictor of a quantifier). This is not the only position *besides*-phrases can occupy in a sentence. There is an interesting pattern with *besides*-phrases occurring in the fronted position, as in (71). Sentence (71a) comes with the inferences discussed in this paper: Ann is a girl, Ann did not come and every other girl came. Something else is going on in (71b). Mark is not a girl but the sentence is acceptable and it comes with the additive inference: Mark came. The additive reading is not available for (71a) (assuming Ann is a girl) and the exceptive reading is not available in (71b) (Vostrikova 2019c). Vostrikova (2019c) proposes that in (71b) the *besides*-phrase does not operate on the domain of the quantifier *every girl*, but serves the same function as *besides* in (71c), where no quantificational determiner is present at all. We leave the question of the interaction of the fronted *besides*-phrase and *every* for future research, as well as the question of how the meaning of (71c) is derived. Our focus here is on *besides*-phrases in connected positions; the containment inference is mandatory in such cases (as shown in (71d)), as well as the negative inference and this is what our analysis captures.

(71) a. Besides Ann, every girl came.
b. Besides Mark, every girl came.
c. Besides Mark, John came.
d. #Every girl besides Mark came.

There is also an interesting pattern that fronted *besides*-phrases show with existentials: they come with an anti-containment inference as the contrast between (72a) and (72b) shows; the additive inference that Mark came is present in (72a). At this point we do not have a proposal about how the meaning of (72a) is derived and we do not have an explanation for the anti-containment inference in (72b). We leave the interaction of the fronted *besides* phrases with existentials for future research. We will only point out that the containment inference is mandatory with connected *besides*-phrases, as (72c) shows, which our analysis correctly captures.

(72) a. Besides Mark, some girls came.
b. #Besides Ann, some girls came.
c. #Some girls besides Mark came.

4. Conclusion

In this paper we proposed a unified compositional treatment to the exceptive and additive uses of *besides*. To our knowledge, this is the first approach to the semantics of *besides* that correctly captures the interaction of *besides* with the negative and universal quantifier and all cases of modified numerals. We proposed that the exceptive or the additive inference results from comparing the quantificational statement with domain subtraction done by *besides* and the statement without this subtraction. Our starting point is the unified treatment of *besides* with the universal and negative quantifier and the non-monotonic *exactly n*. The resulting positive or negative inference depends on the properties of the quantifier: with the universal quantifier the inference is predicted to be negative, with the negative and non-monotonic quantifier it is predicted to be positive. We extended this approach to all cases of modified numerals by adopting a decompositional account of them assuming that there is a constituent in the scope of the numeral with the meaning equivalent to *exactly* and by showing that the additive inference can be computed on that constituent. The advantage of the approach proposed here is that it captures the exceptive and the additive uses of *besides* using the theoretical mechanisms familiar from the discussion of other empirical phenomena in the literature: focus marking, structural alternatives, and an exhaustivity operator (*Exh*).

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