Questions in belief ascriptions[1]

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Abstract. Recent literature (Schmitt 2020; Haslinger and Schmitt 2021; Pasternak 2018b; Marty 2019) has discussed cases in which belief ascription to a plurality of attitude holders can be construed non-distributively. While some of these cases have been accounted for as instances of cumulativity between the attitude holders and a plurality of contents, the cases put forward in Pasternak (2018b) resist an analysis along these lines. This paper builds on Pasternak's (2018b) account, fixing its shortcomings and in so doing argues for a semantics of belief ascriptions in which belief is always interpreted relative to a question (cf. Yalcin 2016) and belief states are not just sets of possible worlds, but sets of classical propositions (as all propositions and questions are for Ciardelli et al. 2019).

Keywords: belief reports, question sensitivity.

1. Two kinds of non-distributive belief

This paper makes two claims about belief ascriptions. The first claim is that the belief state of an individual must be a more complex object than a set of possible worlds; the second, that "aboutness" must be built into the meaning of belief predicates. These claims are argued on the basis of a rather narrow fragment that has received some attention in the recent literature (Schmitt 2020; Haslinger and Schmitt 2021; Pasternak 2018b; Marty 2019), consisting of belief ascriptions to pluralities (sentences of the form NP_{PL} *think that p*). I will start by briefly reviewing what this literature has observed and by addressing the shortcomings of existing proposals we will get to the conclusions above. Without excluding the possibility of alternative implementations, this paper offers a rather conservative improvement over Pasternak's (2018b) approach that uses sets of sets of possible worlds to represent belief states and realizes the aboutness requirement as question sensitivity along the lines of Yalcin (2016).

Ordinarily, belief reports license a distributive inference on the attitude holder:

(1) Ann and Beth think Napoleon was Italian.
   a. ⇝ Ann thinks Napoleon was Italian.
   b. ⇝ Beth thinks Napoleon was Italian.

Recent literature (Schmitt 2020; Pasternak 2018b) however has noticed that sometimes a belief report is judged true even if its distributive inferences are false: in other words, that sometimes belief attribution can be construed non distributively.

1.1. Non-distributive belief as cumulativity

One kind of non-distributive (ND) reports is one in which a plural attitude holder seems to enter a cumulative relation with a plurality in the scope of *think*—(2) is such a case, adapted from Schmitt (2020).

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(2) SCENARIO: Independently of one another, Ann thinks she saw a griffin at the castle, Beth a zombie. Neither monster actually exists.
Ann and Beth think two monsters were in the castle. (true)

What makes (2) an ND report is precisely that it is judged as true in the given scenario, which at the same time does not support the truth of, for instance, *Ann thinks two monsters were in the castle*. There are two crucial features of (2). First of all, it is important that Ann and Beth make their observations independently and each excludes the possibility of other monsters roaming the castle—the notion of conceptual distinctness that is required to interpret (2) is characterized as a counterfactual one in Haslinger and Schmitt (2020, 2021): it must be the case that Ann would be able to distinguish griffins from zombies, if she believed zombies exist (and the same, in reverse, for Beth).

The second feature, more important for our purposes, is that the NP *two monster* is read de dicto, i.e., it is interpreted in the scope of the attitude predicate. As Schmitt (2020) argues, this makes an analysis of its truth conditions that relies on standard accounts of cumulativity problematic, because it would involve scoping both plural denoting expressions over the pluralized predicate *think* (see e.g. Beck and Sauerland 2000), thus failing to generate the de dicto reading which is the true one in the given scenario. Data like (2) are thus taken by Schmitt (2020) to motivate a particular theory of "plural projection" which, generalizing the notion of pluralities to all types of expressions, allows the denotation of *Ann and Beth* to enter a cumulative (belief) relation with a plurality of propositions.

1.2. Non-distributive belief as quantification over plural belief states

There are however ND belief reports of a different kind, first observed in Pasternak (2018a), that this approach cannot capture, because they do not involve any plural denoting expression in the complement of *think*.

(3) SCENARIO: Ann, Beth and Chloe never met Sarah's girlfriend (there is no girlfriend: Sarah is single but they don't know it). Ann and Beth think she is some French person, Chloe thinks she is a painter. This is all they think (they have no beliefs more specific than that).
   a. Sarah's cousins think she is dating a French painter. (% true)
   b. Sarah's cousins think she is dating a painter. (false)
   c. Sarah's cousins each think she is dating a French painter. (false)
   d. Ann thinks Sarah is dating a French painter. (false)

I give the judgment for (3a) with a "%" because not every speaker I have consulted is comfortable with saying that the truth of (3a) is supported by a scenario like (3). The analysis developed in section 5 will predict (3a) to be true "by default" in the given scenario, but it will permit derivations in which it is false, simply by allowing a predicate level distributivity operator scoping above the attitude predicate, thus deriving the same truth conditions that (3c) has. Speakers who systematically judge sentences like (3a) false can be thought of applying this distributivity operator in the interpretation of every belief report.

Certainly there are conversational contexts in which precision in reporting what each individual person thinks is required, and (3a) would then be, at best, a sloppy way of representing Ann,

Beth and Chloe's beliefs. It is important to note, however, that even restrictive speakers who do not judge (3a) as true in the given scenario find there is an obvious contrast between (3a) on one hand and (3b), (3c) and (3d) on the other. As Blumberg and Lederman remark, talking about what they call "revisionist reporting", even if an ascription like (3a) is to be considered false but marginally acceptable "for some kind of pragmatic reason" (Blumberg and Lederman 2020: 761), we need to make sense of why then the other ascriptions in (3) are sharply different in that they are just false.

For the rest of the paper, I will follow the existing literature and I will assume a judgment of "true" for sentences like (3a) in these kinds of scenarios: as the uncontroversial falsity of (3c) demonstrates, this true reading is one in which belief is construed non-distributively.

Again, in the examples in (3) there is no plurality that the plural attitude holder can enter in a cumulative relation with, so the mechanism of plural projection formulated in Schmitt (2020) falls short of accounting for this phenomenon. In fact, these examples lead Pasternak (2018a, b) on a very different analytical path: one in which belief predicates have a plural-sensitive meaning and belief states are defined of singular as well as of plural individual.

I think that indeed (2) and (3) are instances of two different phenomena. This paper will focus on belief ascription li (3), proposing an improvement of Pasternak's (2018b) analysis that accounts for (3) as well as other cases that neither account is equipped to deal with.

2. Pasternak's (2018b) recipe for plural belief states

Pasternak's (2018b) intuition about cases like (3) is that we can retain a mostly Hintikkian meaning for *think*, but with the important addition of belief states defined of plural individuals. The fundamental idea is that (3) is true in the given scenario because Ann, Beth and Chloe's individual belief states (each defined as the set of worlds compatible with someone's beliefs), as described in the scenario, "add up" to the content expressed by the proposition *Sarah's dating a French painter* (which is the complement of *think*).

The challenge Pasternak (2018b) faces, of course, consists in providing an adequate definition of this "adding up". An important observation is that compatible beliefs, such as those individually held in the scenario in (3) (there is no incompatibility, in ordinary context, between being French and being a painter), can be conjoined in an ND report, whereas incompatible beliefs end up being disjoined. This is shown by the following more involved example:

(4) SCENARIO: Ann, Beth and Chloe never met Sarah's girlfriend (there is no girlfriend: Sarah is single but they don't know it). Ann thinks she is some French person, Beth that she is a French painter, and Chloe thinks she is Dutch. This is all they think (they have no beliefs more specific than that).
Sarah's cousins think she is dating a French or a Dutch painter. (true)

Pasternak (2018b) takes this fact as evidence that the way individual beliefs are put together is through a mechanism of premise negotiation analogous to the one employed in standard analyses of modality (Lewis 1981; Kratzer 1981). Premise negotiation just means finding one proposition that characterizes all those worlds that best comply with a potentially inconsistent

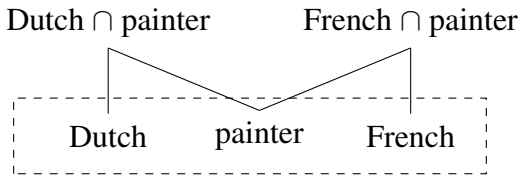
set of requirements (propositions), i.e., the premise set. If two requirements are compatible, then the possible worlds that comply best are those that comply with both (hence, the conjunction of the two premises, an intersection set-theoretically). As soon as two requirements are incompatible, then we have to be content with having worlds that respect one but violate the other (hence the disjunction, or union of the two sets of worlds characterized by the two premises). More generally, any set of propositions (i.e., of premises) induces a preorder on the domain of possible worlds, according to which world w' is better than w'' if and only if the set or premises satisfied in w' constitutes a proper superset of the one satisfied by w''''.

(5) Given a set of propositions B, we have an ordering $\prec_B$ s.t.
$\forall w, w'[w \prec_B w' \longleftrightarrow \{p : w \in p \wedge p \in B\} \supset \{p : w' \in p \wedge p \in B\}]$

(6) Given a set of propositions B and the set of all possible worlds W, Best(B) is the proposition $\{w : \neg\exists w' \in W[w' \prec_B w]\}$

If we take the individual beliefs as premises, the content that can be ascribed non distributively to the plurality must be entailed (Hintikka 1969) by the Best proposition in the sense of (6), which is the plural belief state. To take (4) as an example, we have (7) as the premise set.

(7) $\{p = \{w : \text{French}(w)\}, q = \{w : \text{Dutch}(w)\}, r = \{w : \text{painter}(w)\}\}$



No possible world satisfies all of the three premises: the French-and-painter and the Dutch-and-painter worlds are equally good and we cannot do better, so taking them together is the best we can do. Hence, the truth of (4).

This still needs a small fix: since any two people's belief states are very unlikely to have a non-empty intersection (they would have to agree on everything they happen to have an opinion about), we would almost never be able to obtain a plural belief state that is logically stronger than any of the individual ones. But in fact the judgments seen so far are preserved if Ann, Beth and Chloe happen to disagree about something irrelevant to the identity of the putative girlfriend of Sarah.

What Pasternak (2018b) assumes to avoid this problem is that every belief (state) is actually "about" a situation, in a way that, for example, allows Ann's belief state about Sarah's girlfriend to not discriminate between whether Toronto is in Canada or in the US (even though she has an opinion about that). Aboutness makes belief states coarse enough to make the premise negotiation mechanism viable, but I won't go into the details of how this aboutness relation may be implemented, because the analysis I will offer sidesteps this issue somehow. Next, we examine the shortcomings of Pasternak's (2018b) analysis.

3. Overgeneration problems

A key prediction of Pasternak's (2018b) can be schematized as:

(8) x thinks p
y thinks q
$p \cap q \neq \emptyset$

$\therefore$ x and y think $p \cap q$

Schmitt (2020) and Marty (2019) point out several cases in which the strong prediction represented by (8) is simply not borne out. The first kind of counterexample also tests the reliability of the aboutness requirement: (9), adapted from Schmitt (2020), shows that even when two compatible individual beliefs are about the same situation (dogs at the party), their intersection cannot be truthfully ascribed to the plurality.

(9) SCENARIO: Ann thinks that every dog at the party will play with Jane, Beth thinks that Fido will be the only dog at the party.
Ann and Beth think Fido will play with Jane at the party. (false)

It's quite clear that this is false: while the two beliefs are logically compatible and to some extent about the same thing, it feels like we cannot attribute to both a belief this specific.

(10) SCENARIO: Ann thinks that if Jane is a linguist, she is rich, Beth thinks that Jane is a linguist.
Ann and Beth think Jane is rich. (false)

(11) SCENARIO: Ann thinks that Jane is either Dutch or French, Beth thinks that Jane is French.
Ann and Beth think Jane is French. (false)

In the case of the last two examples it is really impossible to claim that the individual beliefs are not about the same situation, however this aboutness relation is defined—equally clear is that they are compatible and their intersection amounts to the proposition falsely ascribed to Ann and Beth in the two examples respectively.

What is the problem with (10) and (11)? In (10), we reject the ND report because Beth has no opinion about Jane being rich or not: in fact we are ignoring her opinion completely and ascribing to both the belief that she is rich. Similarly, in (11), we are completely erasing "half" of Ann's opinion: the possibility of Jane being Dutch, which she entertains, is completely missing in the belief ascription. Something true that could be said in the scenario in (11) is, perhaps, *Ann and Beth think that Jane is either Dutch or French*, in which we at least preserve both possibilities entertained by Ann. In a general sense, the problem is that the clauses embedded under *think* just do not represent the individual beliefs appropriately.

The conclusion that Schmitt (2020) draws from cases like (9) is that all of the Pasternak-type ND ascriptions (those that, as discussed in section 1, do not easily lend themselves to an explanation in terms of cumulativity, simply because there are no two pluralities that can enter such a relation) are "extremely rare" and "restricted to predicate modification configuration"

(Schmitt 2020: 19). In other words, we observe them when the contributions of the individual beliefs are properties that end up being intersected in the denotation of some nominal phrase.

I want to use this observation to start thinking about this problem in a different way: perhaps what we should take this restricted distribution to indicate is that this type of ND ascription involves "adding up" beliefs that are formally related to one another in a similar way as different answers to a same question are.

4. Belief and questions

We just saw that "being about the same thing" (or situation) is too weak a requirement on individual beliefs to avoid clear overgeneration problems when we attempt to ascribe belief to pluralities; and that perhaps we should think of this aboutness requirement as being a secondary effect of a more restrictive one, namely, that the individual beliefs be appropriate answers to the same question. Questions, intuitively and regardless of what approach one takes to their semantics, determine a class of possible answers and thus which propositions are relevant and which are not (a.o. Lewis 2008; Roberts 2012; Groenendijk and Stokhof 1984). We can tell that two propositions are "about the same thing" (they have the same subject matter) if they are different (partial or complete) answers to the same question.

Conveniently for us, Yalcin (2016) already makes the case (on different grounds than what we are concerned with here[2]) that belief is always defined relative to a question—arguing, in his words, for the question sensitivity of belief. The most straightforward formulation of this concept is assuming that a belief state of an individual x in world w (which we so far have taken to be the set of possible worlds compatible with x's beliefs in w, as per Hintikka (1969)) is always defined relative to a question. This means that any individual has, in any world, as many belief states as there are questions: B is a belief state relative to Q if B could be an answer to Q.

Now, several things should be noted before moving on to a precise formulation of this idea. Answering a question (be it an overt question or a so called Question Under Discussion), on an intuitive level, means providing some information that resolves an issue open in the conversational context. "Having an answer" to a question Q (not necessarily the true answer, but an answer) minimally means being opinionated about Q: if you are not opinionated, then you cannot exclude any answer to Q. But when it comes to thinking about belief states as answers to questions, then we want to represent unopinionatedness too (after all there are countless issues on which a regular person has no opinion). In that case we will say that the belief state is just not informative, in the sense that it does not exclude any possibility raised by the question.

We can think of an unopinionated state of x relative to Q as being the set of all possible worlds. In a sense, thus, it is strictly speaking improper to say that belief states in the system we are sketching are answers to a question: if I am asked a question that I have no answer to, I cannot signal my unopinionatedness by asserting a tautology (it would just not be an appropriate answer). Thus, the sense in which a belief state is an answer to a question is not the same as an

[2]Yalcin is primarily concerned with capturing the correct closure properties of belief. I will not review the motivations for his approach since the use I am making of it constitutes an independent motivation in itself, I think.

actual answer does. This is a caveat for the rest of the paper.

It is obvious at this point that retrieving the Hintikkian belief state, the one that represent beliefs in a maximally fine-grained way, is very straightforward. If $\mathrm{Dox}(x,w)$ is the set of worlds compatible with x beliefs in in world w, and $\mathrm{B}(Q,x,w)$ is x's belief states relative to Q in world w:

(12) $\bigcap\{\mathrm{B}(Q,x,w) : Q \in D_{((\mathtt{s}\to\mathtt{t})\to\mathtt{t})}\} = \mathrm{Dox}(x,w)$

One immediate payoff of using these question-sensitive belief states in our belief reports is of course that they represent a person's belief in a very coarse way: Ann's belief relative to *What is the capital of France?*, given that it is a proposition that answers this question, will entail nothing about any other issue Ann has an opinion about. Suppose that she believes that Nantes is the capital of France and Bonn of Germany: we want her belief state relative to *What is the capital of France?* to contain worlds where Bonn is the capital of Germany, but also any other city. It is straightforward to see how this sidesteps the issue of irrelevant disagreement which made Pasternak assume the existence of an "aboutness" relation that relates belief states and situations: in a way, we are directly building in this aboutness in the shape of question sensitivity.[3] So far, thus, we are just moving from a traditional Hintikkian entry as (13) to one like (14):

(13) $[\![\text{think}]\!]^w := \lambda p.\lambda x.\mathrm{Dox}(x,w) \subseteq p$

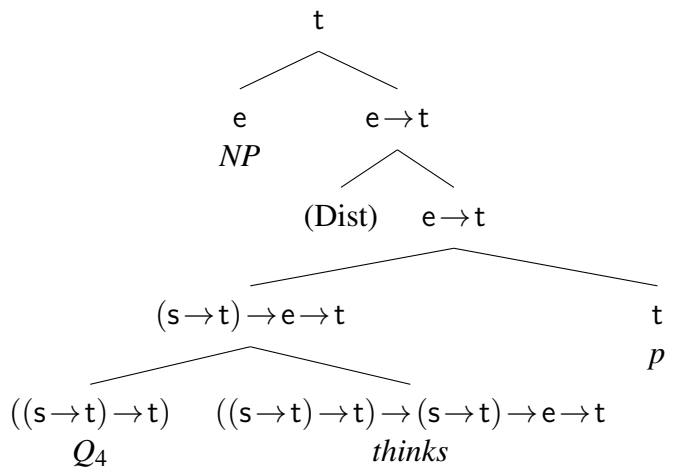
(14) Question sensitive denotation for *think* (to be revised):
$[\![\text{think}]\!]^w := \lambda Q.\lambda p.\lambda x.\mathrm{B}(Q,x,w) = p$ $\qquad ((\mathtt{s}\to\mathtt{t})\to\mathtt{t})\to(\mathtt{s}\to\mathtt{t})\to\mathtt{e}\to\mathtt{t}$

It is safe to assume that the value of the question, relative to which the belief report is to be interpreted, is contextually supplied, but of course it can be made explicit:

(15) As far as [who Jane is dating]$_{Q_j}$, Ann thinks Jane is dating Sarah.

Assuming for concreteness a system with the rules of composition of Functional Application and Intensional Functional Application (Heim and Kratzer 1998), the LFs of belief reports like *NP think(s) p* will have this form:

(16)



[3]This coarseness of representation is in fact the main motivation put forward in Yalcin (2016).

We can already make the point of why (9) is a false ascription. If belief is always relative to a question, it is fairly natural to assume that a plural belief belief state can be obtained only out of individual beliefs that are relative to the same question. Now, *that every dog at the party will play with Jane* and *that Fido will be the only dog at the party* are compatible, but are not answers to the same question: hence the truth of the belief report in (9) is not supported by the scenario given.

We have now taken care of one class of counterexamples to Pasternak's (2018b) proposal, just by translating the aboutness requirement into a formal (albeit still not explicitly stated) requirement that beliefs be about the same question: this gives us not only coarse belief states that can have non-empty intersections even if there is irrelevant disagreement, but also a condition strong enough to avoid counterexamples like (9). Now we must consider counterexamples like (10) and (11): the same-question requirement alone does not help us here, and this brings us to the second claim of this paper—that belief states cannot just be sets of possible worlds.

5. Belief states as higher order propositions

The goal now is to predict the falsity of (10) and (11) by maintaining the core of Pasternak's account, rooted in a plural sensitive meaning for *think* and belief summation as premise negotiation. Let's first consider (11).

5.1. Disjunctive belief

Intuitively, what makes (11) false is that we are completely disregarding the *Dutch* possibility that Ann entertains alongside the *French* possibility. Clearly the premise semantics mechanism rooted in the ordering of possible worlds allows this: *French*-worlds satisfy both premises (both Ann's and Beth's belief), and any *non-French*-world satisfies at most one of the two. I claim that what this case shows is that belief states must be objects that distinguish between determinate and indeterminate beliefs (in the scenario in (11), Beth's and Ann's beliefs respectively).

The belief that *Jane is either Dutch or French* is indeterminate because it involves entertaining multiple possibilities without any commitment to one. To be sure, Beth's belief that *Jane is French* also leaves many things open as to a plausible subject matter like *What kind of person is Jane?*, in that it entails nothing about, for instance, her profession, her age etc. But what the data suggest so far is that the presence of disjunction makes Ann's belief in (11) special as far as belief summation is concerned. Because her belief is expressed as a disjunction of two properties, we feel like summing her belief with Beth's in a way that results in one of the two possibilities being "erased" is not a licit move.

Once again, that the disjunction of properties is somehow the culprit here is suggested by contrasting this case with the one we started with, namely (3a). There too, the non distributive ascription can be said of erasing possibilities the individual attitude holders were entertaining. For example, Ann has no opinion about Sarah's girlfriend's profession (she just thinks she's French): so in a way, by ascribing to the plurality of which she is a part the *French painter* belief we are erasing some options that Ann was not excluding. But the difference in the

acceptability of the two reports is due to the fact that in (11) we are dealing with an individual belief that is expressed as a disjunction of properties.

In order to preserve Pasternak's (2018b) intuition that premise negotiation is what is involved in belief summation, we have to abandon the notion that belief states are sets of possible worlds, simply because these objects don't make the important distinction that underlies the contrast between (3a) and (11). In other words, we need a semantic object that reflects our intuition that the problem in (11) is the entertaining of different possibilities in a belief expressed disjunctively. The framework of Inquisitive Semantics (Roelofsen 2013; Ciardelli et al. 2019: a.o.) treats propositional content in a way that proves useful for our narrow purposes.

5.2. Content in Inquisitive Semantics

In Inquisitive Semantics (IS), both declaratives and questions characterize downward closed sets of states, where a state is a set of possible worlds. This means that both assertions and questions denote expressions of type $(\mathtt{s}\rightarrow\mathtt{t})\rightarrow\mathtt{t}$, which I shall refer to as the type of higher order propositions. The central notion that ties together assertions and questions is that both can be *settled* by an information state:

(17) An information state s settles a proposition A iff $s \in A$.

For example, the proposition denoted by *It's raining* is settled in w iff there is a state in the proposition that contains the world w. Downward closure is important, because we want it to be the case that if a state s settles A, any stronger state s' (i.e., such that $s' \subset s$) will settle A too:

(18) A set of states $A = \{s_1, \ldots, s_k\}$ is downward closed iff $\forall s \in A : s' \subset s \rightarrow s' \in A$.

If we indicate with $[\![\cdot]\!]_I$ the interpretation function that returns higher order propositions for assertions and questions, we can define the basic connectives, beside some other key notions, as in (19). Note that the informative content of a higher order proposition expressed by ϕ is, in this system, equivalent to the classical proposition expressed by ϕ: by taking the grand union of all the states in $[\![\phi]\!]_I$ we obtain the set of worlds where ϕ is true. For any proposition *phi* we also identify a special subset of its states: the set of alternatives $\mathrm{alt}(\phi)$, which is the set of the states that are not contained in any other state in $[\![\phi]\!]_I$.

(19)
a. $[\![\phi]\!]$ is the set of possible worlds where ϕ is true
b. $[\![\phi]\!]_I \coloneqq \{s : s \subseteq [\![\phi]\!]\}$ (abbreviated as $\{\phi\}^{\downarrow}$)
c. $\mathrm{info}(\phi) \coloneqq \bigcup \phi$ (the informative content of ϕ)
d. ϕ is true at w iff $w \in \mathrm{info}(\phi)$
e. $[\![\phi]\!]_I$ entails $[\![\psi]\!]_I$ iff $[\![\phi]\!]_I \subseteq [\![\psi]\!]_I$
f. $[\![\neg\phi]\!]_I \coloneqq \{s : s \cap \phi = \emptyset\}$
g. $[\![\phi \wedge \psi]\!]_I \coloneqq [\![\phi]\!]_I \cap [\![\psi]\!]_I$
h. $[\![\phi \vee \psi]\!]_I \coloneqq [\![\phi]\!]_I \cup [\![\psi]\!]_I$
i. $[\![\phi \rightarrow \psi]\!]_I \coloneqq [\![\neg(\phi \wedge \neg\psi)]\!]_I$
j. $\mathrm{alt}(\phi) \coloneqq \{s : s \in [\![\phi]\!]_I \wedge \neg\exists s'[s' \in [\![\phi]\!]_I \wedge s \subset s']\}$

(20)
a. ϕ is informative iff $\mathrm{info}(\phi) \neq \mathscr{W}$
b. ϕ is inquisitive iff $\mathrm{info}(\phi) \notin \phi$
c. ϕ is a question iff it is non-informative

d. ϕ is an assertion iff it is non-inquisitive
e. ϕ is hybrid iff it is both inquisitive and informative
f. ϕ is a tautology iff it is neither inquisitive nor informative

We can see from (20b) that as soon as a proposition has more than one alternative (more than one maximal state in its denotation), it is inquisitive. And from the other definitions we also see that the presence of disjunction guarantees inquisitiveness, and that a negated formula always denotes a proposition that is not inquisitive (by the definition of negation in (19f), we are guaranteed to have only one maximal state). The proposition expressed by *Ann lives in Paris or Nantes* is thus a hybrid, according to the definition above:

(21) $[\![\text{Ann lives in Paris or Nantes}]\!]_I = \{s : s \subseteq [\![\text{A. l. in Paris}]\!] \vee [\![\text{A. l. in Nantes}]\!]\}$

It is inquisitive, because it contains more than two alternatives and its informative content is not itself in (21), and informative because its informative content does not cover the logical space (for instance, there is no state in (21) of all worlds where Ann lives in London). Something non informative would be a question:

(22) $[\![\text{Where does Ann live?}]\!]_I = \{s : \exists x[x \text{ is a place} \wedge s \subseteq \{w : \text{live}_w(\text{Ann}, x)\}]\}$

If we union the states in (22), we get back the logical space: there are as many alternatives in (22) as there are places that Ann could live in. This is what makes (22) non-informative and thus a question. But this does not mean, of course, that all possible states are in (22): only those that entail Ann living somewhere. So to settle the issue raised by the question *Where does Ann live?* I must provide a piece of information that entails her living somewhere, which is guaranteed by (17). A proposition qualifies as an answer to (22) (for our purposes) if it is a subset of (22).

5.3. Back to belief

What is now a belief state relative to a question? We want it to be an answer to the question, because that encodes aboutness we were after, and to be in some relation with the individual's doxastic set. Here is a possible definition:

(23) Definition of a question-sensitive belief state:
$\text{B}(Q, x, w)$ is the logically strongest proposition p s.t.
$\text{Dox}(x, w) \subseteq \text{info}(p)$ and
$\forall s[s \in \text{alt}(p) \rightarrow \exists s', \ldots, s'' \in \text{alt}(Q)[s = \bigcap\{s', \ldots, s''\}]]$

To see what (23) gives us, suppose the question we are interested in is something along the lines of *What kind of person is Jane?*: this is a question that asks for a property holding of Jane.

(24) $[\![\text{What kind of person is Jane?}]\!]_I = \{s : \exists f_{\mathsf{s}\rightarrow\mathsf{e}\rightarrow\mathsf{t}}[s \subseteq \{w : f_w(\text{Jane}) = 1\}]\}$

Now suppose Ann has no clue as to who Jane is: the strongest proposition whose informative content is entailed by Ann's doxastic set and such that each of its alternatives is the result of the intersection of some alternatives in (24) is (24) itself. She cannot rule out any property when it comes to Jane, precisely because she is unopinionated on the matter.

If instead Ann thinks that Jane is a French painter, her belief state relative to (24) is $\{\text{French}\}^{\downarrow}\cap$

$\{\text{painter}\}^{\downarrow}$, the strongest proposition whose informative content is entailed by her doxastic set and whose only alternative is the result of the intersection of two alternatives in (24), the ones corresponding to the properties of being French and of being a painter. Here it's clear why we need the last clause of (23): since these are all downward closed sets, we cannot just demand the belief state be the strongest proposition, because that would pick out a proposition that is way too informative and we would lose on the coarse representation of content, for which we have moved towards question-sensitive belief states. For instance, what Ann thinks about what the capital of France is is irrelevant as far as (24) is concerned, but without the last clause in (23) we would get a belief state that represents Ann beliefs about that too.

Finally, suppose that Ann is not sure whether Jane is Dutch or French, just like in the scenario in (11). This does not mean she is unopinionated: she clearly has some opinion but cannot commit to any one. In this case we get a belief state like (25). Once again this conforms with the definition in (23), because both the alternatives in (25) are the result of the intersection of some alternatives in (24) (the two alternatives being intersected with themselves).

(25) Ann's belief in (11):
$[\![\text{Jane is either Dutch or French}]\!]_I = \{\text{dutch}\}^{\downarrow} \cup \{\text{french}\}^{\downarrow}$

At this point we can give our final version of the meaning of *think*. So, *x think(s) p*, interpreted relative to a question Q, asserts that the informative content of x's belief relative to Q is p and that x or every member of the plurality x is opinionated as to Q.

(26) Question sensitive definition for *think*:
$[\![\text{think}]\!]^w := \lambda Q.\lambda p.\lambda x.\text{info}(\text{B}(Q,x,w)) = p \wedge \forall y \sqsubset x[\text{B}(Q,y,w) \subset Q]$

At this point we see that (26) and (23) together result in predictions that are seemingly somewhat counterintuitive. Consider the case in which Ann thinks Jane is a French painter, and let us indicate with Q_4 the question in (24).

(27) a. Ann thinks Q_4 that Jane is a French painter. (predicted true)
b. Ann thinks Q_4 that Jane is French. (predicted false)
c. Ann thinks Q_4 that Jane is a painter. (predicted false)

It is not clear that we want to predict (27b) and (27c) to be false, but it is certainly clear why they are, given our definitions: the embedded clauses there are not the strongest propositions among those that meet the requirements posed by (23). The issue here is that (27b) and (27c) are false interpreted relative to Q_4, which is the most general property-seeking question possible, but they are correctly predicted to be true if interpreted relative to *What country is Jane from?* and *What does Jane do for a living?* respectively.

5.4. Tackling the counterexamples

There is another concern with (26): why have we bothered introducing higher order propositions from the IS framework if what (26) does at the end is comparing the informative content with the embedded classical proposition? The reason is that now we have the tools to understand what goes wrong in the counterexamples to Pasternak (2018b) we have seen in section 3, and especially we are able to develop an empirically better mechanism for belief state summation. What makes Ann's belief in (11) crucially different from the other ones is that it is an

inquisitive proposition, where each of its two alternatives corresponds to one of the possibility that she entertains as to where Jane is from.

Intersecting (25) with Beth's belief (that is, $\{\text{french}\}^{\downarrow}$) simply gives us back $\{\text{french}\}^{\downarrow}$: once again the *Dutch*-possibility is erased. But this is not how belief summation is done—we do not intersect compatible belief, but rather, we make the best we can given the premises. And now we have have a way to do this that prevents the *Dutch*-possibility from being discarded.

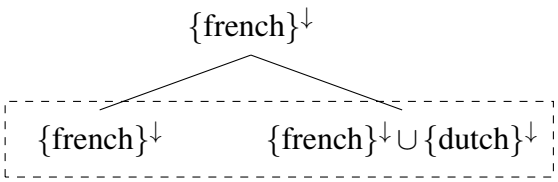
The ordering in (5), which Pasternak (2018b) takes to be the way in which plural belief states are derived, can be generalized to the domain of states, provided we have premises of a suitable type. Suppose that the premise set that is relevant for belief summation is indeed made of higher order propositions like the ones assumed across the board in the IS framework. Now we can order sets of states just like possible worlds are ordered in (5):

(28) Given a set of (higher order) propositions B, we have and order $\prec_B$ s.t.
$\forall s, s'[s \prec_B s' \longleftrightarrow \{p : s \in p \wedge p \in B\} \supset \{p : s' \in p \wedge p \in B\}]$

(29) Given a set of (higher order) propositions B and the set of all possible states $\mathscr{P}(W)$, Best(B) is the proposition $\{s : \neg\exists s' \in \mathscr{P}(W)[s' \prec_B s]\}$

This if not enough, however. If the two premises are the two individual belief states we still derive that the best proposition is the one expressed by *Jane is French*:

(30)



In the case of (11) we want to derive a plural belief state expressed by *Jane is either French or Dutch*: the crucial step to do this in terms of "best set of states" given an ordering like the one defined as in (28) is to make sure that in the premise set the *French*- and the *Dutch*-possibilities are each a premise in its own right. This means, on an intuitive level, that whenever one individual entertains different possibilities, each of them has to "count" whenever we derive a plural belief state.

(31) Definition of belief states for plural individuals:
$\mathrm{B}(Q,x,w) := \mathrm{Best}(\{\{s' : s' \subseteq s\} : \exists y_{\mathrm{at}} \sqsubset x[s \in \mathrm{alt}(\mathrm{B}(Q,y,w))]\})$

The key to accommodating counterexamples like (11) in a system based on premise negotiation is thus to have semantic objects representing belief states that allow us to treat the disjuncts can as premises in their own right, and the downward closed higher order propositions defined in IS are one possible such object.

This consequence of the system we have set up does not surface, however, only when an individual belief is expressed as a disjunction. Consider the following false ascription, which is predicted to be true if we take Pasternak's (2018b) system:

(32) SCENARIO: Ann thinks Jane is from Europe, but has no clue from what country. Beth thinks Jane is French.
Ann and Beth think Jane is French. (false)

(33) a. What continent is Jane from?
b. What country is Jane from?

Let us try to interpret this ascription relative to the question (33a): clearly, Ann is opinionated there, but Beth is not. The one alternative in Beth's belief cannot be obtained by intersecting some alternatives in (33a). Of course it is enough to assume that Beth knows France is in Europe to conclude that she is opinionated relative to (33a), but her belief in (32) is not relative to that question.

If we interpret (32) relative to (33b) the picture is different: now Beth is opinionated, but what about Ann? Her belief can be "translated" as *Jane is from* $x_1 \vee \ldots \vee x_k$ with $x_1, \ldots, x_k$ being all European countries. Now we are in the same configuration as in (11): we must sum a disjunctive belief and the belief in one of the disjuncts. Each disjunct (here: every European country) must be considered a premise in the calculation of the plural belief state. We then correctly predict that in the context sketched in (32) *Ann and Beth think Jane is from Europe* is true.

What about (10), where Ann thinks that *If Jane is a linguist, then she is rich*? If Beth thinks that *Jane is a linguist*, we cannot truthfully say that *Ann and Beth think Jane is rich*. Here, neither belief is inquisitive: the conditional, as defined in (19), does not introduce inquisitiveness, and Ann's belief in (10) is the proposition (34).

(34) $\{s : s \subseteq \{w : \neg(\text{linguist}_w(\text{Jane}) \wedge \neg\text{rich}_w(\text{Jane}))\}\}$

We are now back to a familiar problem: if we search for the best proposition in the sense of (29), we make the wrong prediction. The states that entail that Jane is a rich linguist satisfy both (34) and Beth's belief state $\{s : s \subseteq \{w : \text{linguist}_w(\text{Jane})\}\}$. Thus, from such a premise set we wrongly predict *Ann and Beth think Jane is a rich linguist* to be true.

Once again, however, it's question sensitivity, and specifically the assumption that beliefs have to be relative to the same question, that blocks this wrong prediction. Ann and Beth's beliefs, as presented in (10), are not relative to the same question.

Ann's conditional belief certainly is not an answer to *What kind of person is Jane?*: its only alternative is not in the denotation of that question. In fact, it can only be an answer to a conditional question like *If Jane is a linguist, is she rich?* or *If Jane is a linguist, what kind of person is she?* This is not just an artifact of how we have defined answerhood: Ann, in (10), is not committed to any property holding of Jane, quite differently from Beth. Beth is committed to a property: she thinks Jane is a linguist, and her belief cannot be relative to the conditional question.

Let's make another example that involves negation instead of conditional belief. Once again, the system in Pasternak (2018b) wrongly predicts (35) to be true.

(35) SCENARIO: Everyone thinks that Jane is either French, Belgian or Dutch. Ann thinks that Jane is not French, Beth thinks that Jane is not Belgian.
Ann and Beth think Jane is Dutch. (false)

It is tempting to say that in this case the individual beliefs are relative to the same question *Where is Jane from?*: unlike the conditional form in (10), both Ann and Beth are equally

committed to some answer. But what we have is this:

(36) a. $[\![\text{Where is Jane from?}]\!]_I = \{\text{French}\}^{\downarrow} \cup \{\text{Belgian}\}^{\downarrow} \cup \{\text{Dutch}\}^{\downarrow}$
b. Ann's belief in (35): $\{s : s \cap [\![\text{Jane is French}]\!] = \emptyset\}$
c. Beth's belief in (35): $\{s : s \cap [\![\text{Jane is Belgian}]\!] = \emptyset\}$

Neither Ann's nor Beth's beliefs are subsets of the question. One can see how these are not appropriate answers by imagining an actual question-answer pair:

(37) A: Where is Jane from?
B: ? She is not French.

An interaction like (37) is certainly not optimal, especially compared to an answer like *She is either Dutch or Belgian*. While both answers are equally informative (if we all accepts she can either be Dutch, Belgian or French), the acceptability of the answer in (37) improves much if B were to signal that they are actually answering a different question (namely, the polar question *Is Jane French?*) because they cannot completely answer A's question, as in:

(38) A: Where is Jane from?
B: Well, she is not French...

Going back to (35), we now see what the problem is: counterintuitive as it may seem, both Ann and Beth are unopinionated as to (36a). Both are opinionated instead about the respective polar question.

6. Forcing the non distributive construal

Just like the distributive construal of a belief report can be forced with an expression that marks the presence of a distributivity operator, as is the case in (3c), it is also possible to force the non distributive one (Martin Hackl, p.c.). For example, the use of certain adverbials can make the counterexamples seen in section 3 true. Consider, (39), which is judged true in the given scenario:

(39) SCENARIO: Ann thinks that every dog at the party will play with Jane, Beth thinks that Fido will be the only dog at the party.
Jointly/taken together, Ann and Beth think Fido will play with Jane at the party.

In discussing similar set ups concerning "distributed knowledge", von Fintel and Gillies (2011) use this real life example: *We knew more than is being owned up to. But nobody put the pieces together*. The second clause here clearly signals that that the knowledge is not to be intended distributively: the referent of *we* only has that knowledge if the ascription is intended non-distributively, like in (39). We want to have the flexibility to be able to make these belief reports where the logical conclusions of the individual beliefs are ascribed to the plurality.

Essentially, we want the system developed in the previous section to replicate Pasternak's (2018b) predictions, whenever needed (or rather, whenever this is forced). To do this, we may assume that, through these adverbials, the same-question requirement is circumvented. Cases like (39) can be taken to assert that there is a set of questions such that for each of Ann and Beth there is a question in the set relative to which they are opinionated, and taking both belief

states as premises results in the plural belief state whose informative content is the proposition embedded under *think*.

7. The distribution of non distributive ascriptions

7.1. Other predicates

One obvious thing that we should ask ourselves is how what we have seen so far about *think* applies to other propositional attitudes. The first place to look is of course declarative embedding *know*. To the extent that the corresponding *think*-reports are possible (more on this in the next section), it seems to me that *know*-reports present the same possibility for ND ascription:

(40) SCENARIO: Sarah is dating a French painter. Ann only knows she is French, Beth only knows she is a painter.
Ann and Beth know that Sarah is dating a French painter. (true)

In connection with *know*, it should be noted that in ND *think*-ascriptions Maximize Presupposition (Heim 1991) is checked at the level of the individual beliefs:

(41) SCENARIO: Sarah is dating a Dutch painter. Ann mistakenly thinks she is French, Beth knows that she is a painter (and nothing else).
Ann and Beth think that Sarah is dating a French painter. (?)

The problem with (41) seems to be that MP is violated at the level of Beth's belief: in (41)'s scenario, *Beth thinks Sarah is dating a painter* is ruled out by MP (which favors *Beth knows that Sarah is dating a painter*), and this makes the ascription to *Ann and Beth* impossible.

What about other propositional attitudes like *want*, *hope*, *fear*, *wish* etc.? Focusing on *want* (which seems to me makes the point for all other), we can see they do not allow ND ascriptions of the kind that we have been concerned with here. If this were not the case, we would expect (42) to be true, while it clearly is not.

(42) SCENARIO: Ann wants to have a black pet, and she doesn't care about the species, as long as it is black. Beth wants to have a cat, and she doesn't care about the color.
Ann and Beth want to have a black cat. (false)

While I do not have anything compelling to say about this fact, I want to point out that the difficulty in building "plural desire" (and the corresponding mental states for the other attitudes) may lie in the fact that an attitude of desire (and fear, hope, etc.) is just more complex than the one of belief. Whether *want* is modeled through a comparative semantics (cf. Heim 1992) or not (cf. von Fintel 1999), an ascription of desire (and fear, hope etc.) necessarily involves projecting the agent's preferences over their doxastic domain.

7.2. The best questions for non distributive ascriptions

It is worth noting that the interesting aspect of ND ascriptions, namely that in certain cases the content ascribed to the plurality can be stronger than the contents believed by the individuals, only obtains if the belief states are non-exhaustive answers to the question. This is because two distinct and exhaustive answers to a question will never be compatible, and thus the mechanism

of premise negotiation will not deliver a stronger content to be the plural belief state.

(43) SCENARIO: There are five students: C, D, E, F and G. Ann thinks only C and D came. Beth thinks only E, F and G came.
Ann and Beth think every student came. (false)

(44) SCENARIO: There are five students: C, D, E, F and G. Ann thinks C and D came, and has no opinion as to the others. Beth thinks E, F and G came, and has no opinion as to the others.
Ann and Beth think every student came. (true?)

Whatever the status of the "true" judgment for (44), every consultant I asked agrees that there is a distinction between (43) and (44), and this is predicted. In (43), both belief states are clearly described as exhaustive answers to the question *Which students came?*, and premise negotiation will not derive a plural belief state that corresponds to *Every student came*. In the latter case, on the other hand, such belief state is indeed derived.

The relative difficulty one observes with (44), compared with other true ND ascriptions we have seen so far, can be explained once we recognize the difference between a question like *Which students came?* and the one we have been dealing with throughout the discussion, namely a *What-kind-of* question. While an answer to the former usually licenses an inference of exhaustivity (however this is modeled), an answer to a question that asks for a property holding of someone or something arguably can never be exhaustive:

(45) What kind of person is Jane dating?
a. A French person.
b. A French painter.
c. A rich French painter.
d. ...

One can add more and more properties to the answer, so that it becomes more and more specific, but every such answer feels equally appropriate. These are the questions relative to which ND ascriptions of the kind first observed in Pasternak (2018a) are most easily construed, precisely because the naturalness of non exhaustive answers to these questions allows the premise negotiation mechanism to surface, in that the belief ascribed to the plurality is stronger than the individual belief. At the same time, this also explains why this kind of ascription is "rare", as Schmitt (2020) points out: they are at least as rare as questions of this kind are.

8. Conclusion

This paper has argued that a plural-sensitive analysis for *think* in the spirit of Pasternak (2018a, b) is viable for non distributive reports that cannot be accounted for in terms of cumulativity, despite the problematic cases put forward by Schmitt (2020); Marty (2019). Building an adequate account has led us to make more general claims about belief ascription.

First, we have seen that we need more complex objects than just sets of possible worlds to adequately model one person's beliefs (about something). In particular, we need to be able to distinguish in the semantics, whether an opinionated agent can commit to one possibility and

when they cannot, and we need to access each individual possibility that an agent is entertaining in their beliefs, in order for plural belief states to be correctly derived.

The second claim is that aboutness must be encoded as a formal property that goes beyond the property of "being about something" and it must be part of the meaning of belief reports (Yalcin 2016; Mari and Portner 2021), because it has truth conditional import. Following much existing work, I have modeled the required aboutness property as a form of question-answer congruence. The theoretical relevance of the small fragment of ND ascriptions that was the focus of this paper is precisely that it is with these ascriptions to pluralities that this truth conditional import becomes visible.

I have implemented these two ideas in a rather conservative possible worlds semantics. In doing so, I have borrowed especially from Yalcin (2016), as far as subject matter sensitivity is concerned, and the large body of work in Inquisitive Semantics (Ciardelli et al. 2019) as far as the distinction between inquisitive and non inquisitive content is concerned, leveraging on one fundamental assumption of that framework (in particular, downward closure of propositions which allows a straightforward generalization of the premise negotiation mechanism to the ordering of classical propositions). Especially as far as the choice of the Inquisitive Semantics approach is concerned, alternative theoretical avenues are certainly viable. I contend, however, that any empirically adequate account of the fragment considered here must retain the results that the present implementation has delivered.

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