NPI *any* in the scope of *exactly n*[1]

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Abstract. Both the *even* theory of NPI *any* (Crnič 2011, 2014a, b, 2019a, b) and the exhaustification theory (Chierchia 2013) argue that *any* in non-monotonic contexts lexically requires an *even* operator to be felicitous. A counterexample to this argument is observed. In order to account for the counterexample, this paper proposes that *any* in the scope of a non-monotonic operator in the surface structure is actually located in the restriction of a definite plural description in the logical form. Assuming the generalized definition of Strawson-entailment (Guerzoni and Sharvit 2007; Gajewski and Hsieh 2014; Gajewski 2016), the proposed theory maintains Strawson-downward-entailingness as a necessary condition for the felicity of *any* even in the cases where *any* occurs in the scope of a non-monotonic quantificational determiner like *exactly n*.

Keywords: NPI, *any*, non-monotonicity, Strawson-entailment

1. Introduction

NPI *any* and minimizers share a lot of similarities in distribution. For example, they are both felicitous under a sentential negation, but are infelicitous in simple affirmative sentences, as shown in (1) and (2).

(1) a. John didn't read any book.
  b. John didn't read a single book.

(2) a. *John read any book.
  b. *John read a single book.

In the literature, there are three main views on *any* and minimizers. First, Heim (1984) argues that *any* and minimizers are different beasts. She observes that despite the similarities between the two, minimizers are more restricted in distribution. For example, while *any* is felicitous in the restrictor of a universal quantifier regardless of the main predicate, minimizers are not. As we can see, *any book* is felicitous in both (3)a and (3)b, but *even one book* is only felicitous in (4)a[2].

(3) a. Every student who read any book passed the exam.
  b. Every student who read any book wore blue jeans.

(4) a. Every student who read even one book passed the exam.
  b. ??Every student who read even one book wore blue jeans.

Crnič (2011, 2014a, b, 2019a, b) in a series of work have argued that *any* and minimizers are semantically the same. Both require *even* for their felicity. His theory is built upon Lahiri's

[1]This work was done during my stay at Harvard Linguistics department as a visiting fellow in 2022. I thank Sam Alxatib, Gennaro Chierchia and C.-T. James Huang for their criticisms and comments on this project. I also thank the participants of Sinn und Bedeutung 27 who asked very insightful questions that will help me dig deeper in this research project.

[2]The examples in (3) and (4) are from Crnič (2014a). I do not use the original examples from Heim (1984) because she did not keep the main predicate constant when comparing *any* and minimizers.

(1998) and Lee and Horn's (1995) theories, both of which reduce the semantics of *any* to that of the minimizers. The main motivation underlying the *even* theory of *any* comes from the robust cross-linguistic pattern of the morphological make-up of the NPI *any*. In (5) for example, the Hindi NPI *any* is followed by a morpheme *bhii* which is semantically akin to *even*, as the glosses show. This pattern of NPI formation has been widely observed cross-linguistically. We see the same morphological make-up of the NPI *any* in Japanese, Korean, Bangla, Malayalam and many more languages (Choi 2007; Jayaseelan 2011; Lee and Horn 1995; Ramchand 1997; Shimoyama 2006: a.o.).

(5) maiN-ne kisii-ko bhii nahiiN dekhaa
I any even not saw
I didn't see anyone.

(Lahiri (1998), glosses Crnič's (2019a))

Still a third theory on *any* and minimizers is the exhaustification theory proposed by Chierchia (2013). According to him, *any* is always exhaustified by O(nly) except that in non-monotonic environments, it is exhaustified by E(ven).

I summarize the three main theories on *any* and minimizers in the following table.

Table 1: Three main theories of *any* and minimizers

	Need *even* for felicity?		
	Heim	Crnič	Chierchia
Any	No	Yes	Yes (only in non-monotonic cases)
Minimizer	Yes	Yes	Yes

In this paper, I will argue against both the uniform *even* theory of *any* as well as the mixed view of *any* in the exhaustification theory. Specifically, I will present a novel counterexample with *any* in a non-monotonic context on which both these theories make incorrect predictions. In order to account for the counterexample, I propose that *any* in the scope of a non-monotonic operator *exactly n* in the surface structure is actually located in the restriction of a definite plural description in the logical form. In §2, I will give some arguments against the uniform *even* theory of *any*. Specifically, I highlight an example where *any* is in the scope of *exactly n* as a challenge to the *even* theory. In §3, I will show that the same counterexample also poses a challenge to the mixed view of *any* in the exhaustification theory. §4 will propose a new theory that accounts for a counterexample. §5 discusses some remaining issues of the proposed theory and §6 concludes.

2. Against the uniform *even* theory of *any*

In a series of work, Crnič (2011, 2014a, b, 2019a, b) advocates a uniform *even* theory of *any*. Over the years, he has proposed two variations of the *even* theory. In the first version, *any* is decomposed to a propositional *even* operator and the quantificational determiner *one*, with *one* being the focus associate of *even*. The focus alternatives activated by *any*, therefore, are the numerals bigger than one. In the second version, *any* is decomposed to a propositional *even* operator and the existential quantificational determiner *a* which carries a contextually determined domain variable D. With the domain variable as the focus associate of the *even* operator, the subdomains $D' \subseteq D$ are activated as the focus alternatives. An assumption adopted in both two versions is the Entailment-Scalarity Principle, which states that for any propositions p and q, if p entails q, then p is at most as likely as q.

The uniform *even* theory of *any* accounts for the classical NPI examples like (1)a without any issue. When *any* is in a downward-entailing environment such as in the scope of negation, the prejacent of *even* is logically stronger than all the other alternatives. According to the assumed Entailment-Scalarity Principle, the least likelihood presupposition of *even* is automatically satisfied. The readers can verify this for both of the two versions of the *even* theory presented in (7) and (8) respectively.

(1a) John didn't read any book.

(6) $[\![even]\!] = \lambda p : \forall q \in ALT(p)[q \neq p \rightarrow p \leq_{likely} q].p$

(7) Crnič (2014a, b): any = even+one$_F$
  a. LF: Even [$\neg$ one$_F$ book λx John read x]
  b. ALT = $\{$John didn't read one book, John didn't read two books,...,John didn't read n books$\}$

(8) Crnič (2011, 2019a, b): any = even+a$_{D_F}$
  a. LF: Even [$\neg$ a$_{D_F}$ book λx John read x]
  b. ALT=$\{\neg$John read a book in $D' | D' \subseteq D\}$

Simply reducing the NPI *any* to the combination of a propositional *even* operator and the numeral one or the existential determiner *a*, however, makes incorrect empirical predictions. First of all, *any* and *even...one* have different distributions. For example, in (9), while *even...one* is felicitous, *any* is not. In (10)[3], on the other hand, *any* is felicitous but *even...one* is not (Chen 2019).

(9) Context: Mary asked John to count how many students were present at the seminar. John forgot about it. Mary complained,
  a. John didn't count even one student.
  b. *John didn't count any student.

(10) Context: Mary and Sue are talking about John.
  a. John wasn't born in any big city. (He was born in a small town.)
  b. *John wasn't born in even one big city. (He was born in a small town.)

Although analyzing *any* as *even...a$_D$* has no problem accounting for (9) and (10), it makes incorrect predictions elsewhere. According to Crnič (2019a; cf. Linebarger 1987), *any* in the scope of a non-monotonic operator like *exactly n* gives rise to a so-called "size effect". For example, in (11), when the number ensuing "exactly" is small relative to the context, *any* is felicitous. However, when the number ensuing "exactly" is big relative to the context, *any* is infelicitous.

(11) *Context: There are 12 graduate students in the department.*
  a. Exactly 2 students read any book.
  b. ??Exactly 10 students read any book.

[3]Mats Rooth raised an issue on the example sentence (10a). He said this sentence does not sound like a plain every day use of English. I will have to leave the investigation of this intuition to another occasion.

Crnič (2019a) takes the contrast between (11)a and (11)b as strong evidence supporting the uniform *even* theory of *any*. According to him, *any* is analyzed as *even...*a_D in this case. In a sentence like (11)a, in order for the least likelihood presupposition of *even* to be satisfied, we will need a context where the speaker expects a lot of students, say, 10 out of 12, to have read at least one book. Moreover, the larger the domain of books the speaker takes into consideration, the bigger the number of students who read at least one book from that domain. In such as context, relative to the subdomain alternatives, the proposition that exactly 2 students read a book in D will be less likely since the speaker will expect there to be more students who have read at least one book in D. Crnič argues that since this context that satisfies the least likelihood presupposition of *even* is a natural one and is easy to accommodate, *any* is felicitous.

(11a) Exactly 2 students read any book.

(12) ALT=$\{$Exactly 2 students read a book in $D'|D' \subseteq D\}$

However, when we replace the number 2 with 10, in order to satisfy the least likelihood presupposition of *even*, we will need a very different context. Basically, the speaker expects very few students, say 2 out of 12, to have read at least one book. Moreover, the larger the domain of books the speaker takes into consideration, the smaller the number of students who read at least one book from that domain. This kind of context, however, contradicts our common assumption that the more books we consider, the more readers there should be. Crnič argues that since this context that satisfies the presupposition of *even* is unnatural and extremely hard to accommodate, *any* is infelicitous.

(11b) Exactly 10 students read any book.

(13) ALT=$\{$Exactly 10 students read a book in $D'|D' \subseteq D\}$

If *any* always requires *even* for its felicity, then the so-called "size effect" observed in (11) should always exist. This prediction, however, is not empirically attested. Take a look at (14). In the given context, both the *exactly n* sentences in (14) are felicitous, no matter *n* is big or small relative to the context.

(14) *Context: John is watching a car racing game. There are 12 cars competing. From 100 miles on, there is a gas station every few miles. Bill asks John what the game is like right now.*
  a. Exactly 2 cars are close to any gas station.
  b. Exactly 10 cars are close to any gas station.

Based on the *even* theory of *any*, I give the logical form and the alternative set for (14)a in (15) and those for (14)b in (16). The readers can verify for themselves that in order for the prejacent of *even* in (15) to be less likely than the other alternatives, we need a context where the speaker expects there to be many cars, say, 10 out of 12 cars, to be close to a gas station. Moreover, the larger the domain of gas stations, the more cars that are close to a gas station. On the contrary, in order for the prejacent of *even* in (16) to be less likely than the other alternatives, we need a context where the speaker expects there to be few cars, say, 2 out of 12 cars, to be close to a gas station. Moreover, the larger the domain of gas stations, the less cars that are close to a gas station.

(15) a. LF: Even [exactly 2 students λx a_{D_F} book λy x read y]

b. ALT = $\{$Exactly 2 students read any book in $D'|D' \subseteq D\}$

(16) a. LF: Even [exactly 10 students λx a$_{D_F}$ book λy x read y]
b. ALT = $\{$Exactly 10 students read any book in $D'|D' \subseteq D\}$

The two contexts that can make the presupposition of *even* satisfied in (14)a and (14)b respectively contradict each other. Therefore, the *even* theory predicts that (14)a and (14)b can never be felicitous in one and same context. However, we have seen that they are actually felicitous at the same time in the given context in (14). Analyzing *even...a*$_{D_F}$ makes an incorrect prediction in this case.

Let's take stock. Crnič argues that an indispensable ingredient in the semantics of *any* is an *even* operator. He has analyzed *any* as *even...one*$_F$ or *even...a*$_{D_F}$. However, the *even* theory makes incorrect empirical predictions. First of all, we have seen that *any* and *even...one*$_F$ have different distributions. We find cases where *any* is felicitous but *even...one*$_F$ is not. We also find cases where *even...one*$_F$ is felicitous but *any* is not. Furthermore, analyzing *any* as *even...a*$_{D_F}$ predicts that in sentences where *any* locates in the scope of *exactly n*, there will be always a "size effect". However, in (14), we don't observe such a "size effect". Both the sentences are felicitous in the given context. Thus I conclude that *any* does not require *even* for its felicity.

3. Against the mixed view of *any* in the exhaustification theory

As reviewed in §1, the exhaustification theory (Chierchia 2013) argues for a mixed view of the NPI *any*. In most cases, *any* is argued to be exhaustified by an exhaustifier O with a semantics akin to *only* (cf. Krifka 1995). Basically, this exhaustifier O, when taking a proposition p as its argument, will assert the truth of p and negate any alternative q to p that is logically stronger than p. In the classical examples with an NPI *any* like (1)a, repeated here in (18), the exhaustification theory makes correct predictions without any issue. Since the prejacent of the exhaustification operator O entails all the other alternatives, O will just assert the truth of the prejacent. None of the alternatives will be negated. The resulting truth condition, therefore, is the same as the truth condition of the prejacent itself. The readers can verify this point using the semantics of O and the alternative set given below.

(17) $[\![O_{exh}]\!](p) = p \wedge \forall q \in ALT(p)[q \subset p \rightarrow \neg q]$

(18) John didn't read any book.

(19) LF: $O_{exh}\neg$ a$_{D_F}$ book λx John read x

(20) ALT = $\{O_{exh}\neg$ a$_{D'}$ book λx John read $x|D' \subseteq D\}$

In non-monotonic contexts, exhaustification via the O operator will result in incorrect truth conditions since the alternatives are logically independent of each other. Chierchia (2013) thus adopts Crnič's (2011) theory and proposes that *any* in non-monotonic contexts is exhaustified by an exhaustifier E with a semantics akin to *even*.

However, as §2 shows in detail, simply analyzing *any* as *even...a*$_{D_F}$ makes unattested empirical predictions. When *any* occurs in the scope of a non-monotonic quantifier like *exactly n*, the *even* theory predicts that there will always be a "size effect" of the number n. However, in the example (14), we find that both the sentence containing a small number and the sentence containing a big number are felicitous with *any*. Therefore, the same example also poses a challenge to the exhaustification theory.

4. Proposal

In the previous two sections, I have argued against the uniform *even* theory of *any* as well as the mixed view of *any* in the exhaustification theory. A conclusion we can draw from the discussion is — *any* does **not** require *even* for its felicity[4]. Specifically, the counterexample in (14) shows that *any* does not require *even* for its felicity in non-monotonic contexts. However, a question still remains unanswered. Why is *any* felicitous in non-monotonic contexts?

The good old theory of the NPI *any* is that *any* is licensed in downward-entailing environment (Ladusaw 1980). Later, Von Fintel (1999) updates the definition of downward-entailment to Strawson-downward-entailment. Basically, Strawson-DE assumes the truth of the presuppositions of the proposition under discussion as well as the alternatives when checking whether *any* is in a DE environment. Due to space constraint, I will not give a review of the development of this theory, but I would like to point out that the felicity of *any* in non-monotonic contexts is always a puzzle to the DE theory. Since down-entailingness is argued to be a necessary condition of the felicitous use of *any*, we will expect *any* to be unlicensed in the sentences in (11) and (14). Contrary to this prediction, however, we find that *any* is felicitous in (11)a and (14).

In the following, I will propose a theory that accounts for the felicity of *any* in (14). There are three main ingredients of this theory. First of all, I argue that *any* in the scope of a non-monotonic operator in the surface structure is actually located in the restriction of a definite plural description in the logical form. This means that a sentence like (14) is actually semantically interpreted as (21).

(21) The cars that are close to any gas station are exactly 2 in number.

Second, following Gajewski and Hsieh (2014) and Gajewski (2016), I argue that *any* in the restriction of a definite plural description is licensed very locally, via generalized Strawson-downward-entailment in the nominal domain. Last, *exactly n* comes in after *any* is already licensed.

The three ingredients of the proposed theory will work in tandem to make sure that *any* in the scope of a non-monotonic operator like *exactly n* is still in a Strawson-downward-entailing environment in the logical form. Therefore, the good old theory of the NPI *any* licensing can still be maintained.

4.1. The maximality component and the cardinality component in the semantics of *exactly n*

The three ingredients of the proposed theory are motivated independently. First of all, recent studies have argued that modified numerals are decomposed into three parts in semantics — an indefinite determiner $\mathbf{some}^u$ that introduces a discourse referent, a maximality operator $\mathbf{M}_u$ that selects the maximal plurality satisfying the conditions specified by the sentence containing the modified numeral and a cardinality component $\mathbf{2}_u$ that checks the cardinality of the maximal

[4]This conclusion does not exclude *any* from occurring felicitously in sentences with *even*. It simply argues that if *any* happens to be felicitous in a sentence with *even*, the *even* operator does not come from a lexical requirement of *any*.

plurality selected by $\mathbf{M}_u$ (Zhang 2020). The decompositional analysis of modified numerals is mainly motivated by their use in cumulative sentences (Brasoveanu 2013).

(22) Exactly three students watched exactly six movies.

In a cumulative sentence like (22), neither modified numeral is in the scope of the other. Instead, the interpretation of this sentence suggests that there are two contextually relevant maximal pluralities, one being *the* students who watched *a* movie and another being *the* movies that were watched by *a* student. The cardinality requirement imposed on these two pluralities by the two modified numerals comes after the two pluralities are already formed. This idea of delaying certain semantic evaluation is not unfamiliar. For example, in his analysis of the Haddock sentences like (23), Bumford (2017) argues that the two definite determiners **the** in this sentence should be decomposed into an indefinite determiner and a delayed maximality test.

(23) The rabbit in the hat

Zhang (2020) extends the idea of decomposition to her analysis of comparative sentences with non-monotonic modified numerals in the *than*-clause. In a sentence like (24), she argues that the contribution of the modified numeral *exacly 2* is three-fold. First, it introduces a discourse referent which satisfies the condition specified by the comparative. Second, it imposes a maximality test on the discourse referent. Last, it imposes a cardinality test on the the discourse referent. The latter two tests are delayed as a kind of post-supposed evaluation.

(24) Mary is taller than exactly 2 boys are.

Based on this split semantics of the non-monotonic quantifiers, the truth condition of sentence (24) can be paraphrased as: Mary is taller than *some* boys, and *the* boys are exactly 2 in number. This paraphrase presents the three-fold contribution of *exactly 2* in (24) in a straightforward way. The indefinite determiner *some* introduces a discourse referent. The definite determiner *the* imposes a maximality test and *exactly 2* imposes a cardinality test.

Inspired by Zhang's (2020) analysis of the non-monotonic modified numerals, I argue that *exactly n* in sentence (14) is decomposed in the same way. As we will see later, once we break *exactly n* into an indefinite determiner $\mathbf{some}_u$, a maximality operator $\mathbf{M}_u$ and a cardinality predicate $\mathbf{n}_u$, the NPI *any* in the scope of a non-monotonic quantifier in the surface structure turns out to be in the restriction of a definite plural description in the logical form. This is the first ingredient of the proposed theory.

4.2. Local licensing of *any* in the restriction of a definite plural

The observation that the NPI *any* in the restriction of a definite plural can be licensed very locally is not novel. A strong evidence supporting this hypothesis is that *any* is found to be felicitous in the restriction of a definite plural even if the main predicate is collective (Gajewski and Hsieh 2014; Gajewski 2016).

(25) a. The students with any knowledge of French formed a team.
b. The students with any knowledge of French in tense formed a team.

The felicity of *any* in (25)a is surprising if we assume Strawson-DEness as a necessary condition on the licensing of *any*. When we replace the restriction of the definite plural in (25)a with its subset, the resulting sentence (25)b is not logically entailed by (25)a. Instead, the two sentences are logically independent of each other. Therefore, the Strawson-DE theory of *any* predicts that *any* is infelicitous in (25)a, contrary to fact.

In order to reconcile between the Strawson-DE theory of *any* licensing and the felicity of sentence (25)a, Gajewski and Hsieh generalize the definition of Strawson-entailment to the nominal domain (Gajewski and Hsieh 2014; Gajewski 2016), an idea initially sketched in a footnote by Guerzoni and Sharvit (2007). According to the generalized definition of Strawson-entailment, as given in (26), for any two entities α and β of type $\langle e \rangle$, if β is a mereological part of α, then α Strawson-entails β.

(26) a. **If α and β are of type e, then $\alpha \rightarrow_S \beta$ iff $\beta \sqsubseteq \alpha$**
b. If α and β are of type t, then $\alpha \rightarrow_S \beta$ iff α=0 or β =1
c. If α and β are of type $\langle \sigma\tau \rangle$, then $\alpha \rightarrow_S \beta$ iff for all $x \in dom(\alpha) \cap dom(\beta), \alpha(x) \rightarrow \beta(x)$

Coming back to sentence (25)a, we see that this generalized definition of Strawson-entailment allows *any* to be licensed locally in the restriction of a definite plural description. Since the plurality consisting of the students with knowledge of French in tense is a mereological part of the plurality consisting of the students with knowledge of French, the definite plural subject in (25)a Strawson-entails the definite plural subject in (25)b. This means that we do not need to look beyond the subject in the sentence because *any* already finds itself in a Strawson-DE environment.

4.3. Putting the ingredients together

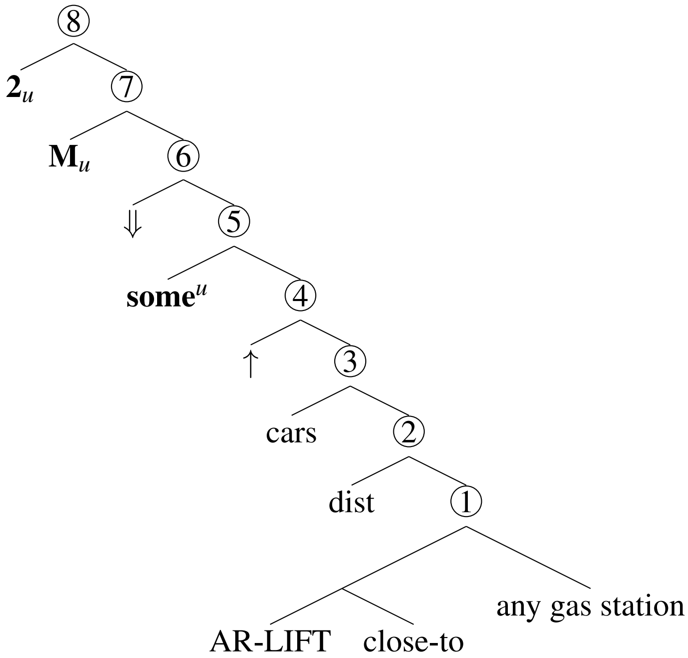
The previous two sections present all the ingredients of the proposed theory. Essentially, the non-monotonic quantificational determiner *exactly n* is decomposed into three parts: an indefinite determiner $\mathbf{some}^u$ that introduces a discourse referent; a maximality operator $\mathbf{M}_u$ that selects the maximal plurality satisfying the conditions specified by the sentence, and a cardinality predicate $\mathbf{n}_u$ that counts the number of the atoms in the plurality. Let's put everything together and see how the proposed theory can account for the sentence (14)a. The logical form and the step-by-step composition of the sentence are given on the next page.

From the semantic composition, we can see that a sentence is interpreted as an assignment function dependent set of ⟨Truth value, assignment function⟩ pair. The assignment function carries the information about the discourse referents introduced in the sentence and the predicative content in the sentence applies to the discourse referent as restricting conditions. In the example sentence (14)a, a discourse referent u is introduced by the indefinite determiner $\mathbf{some}^u$. The sentence specifies that u are cars and each atomic part of u is close to a gas station. The maximality operator $\mathbf{M}_u$ then selects the maximal plurality consisting of the cars that are close to a gas station. Last, $\mathbf{2}_u$ makes sure that the maximal plurality of cars selected by $\mathbf{M}_u$ has 2 atomic parts in it.

What is most important here is that at the node 7, the NPI *any* is in the restriction of a definite plural. As has been discussed in §4.2., the empirical fact that *any* is felicitous in the restriction of a definite plural regardless of the predicate type strongly suggests that *any* is licensed very locally in this case instead of at the propositional level. I take the arguments in Gajewski and Hsieh (2014) and Gajewski (2016) as an assumption and adopt their generalized definition of

Strawson-entailment given in (26). Once the generalized Strawson-entailment is adopted, it is a natural consequence of this assumption that *any* is felicitous in (14)a, since the restriction of a definite plural is a Strawson-DE environment.

(14a) Exactly two cars are close to any gas station.



(27) $[\![any\ gas\ station]\!] = \lambda P.\exists x[gs(x) \wedge P(x)]$

(28) $[\![close - to]\!] = \lambda y \lambda x.close(y)(x)$

(29) $[\![\text{AR}-\text{LIFT}\ close\ to]\!] = \lambda P \lambda x.P(\lambda y.close(y)(x))$

(30) $[\![①]\!] = \lambda x.\exists y[gs(y) \wedge close(y)(x)]$

(31) $[\![dist]\!] = \lambda P_{et}.\lambda X.\forall x[x \sqsubseteq_{atom} X \rightarrow P(x)]$

(32) $[\![②]\!] = \lambda X.\forall x[x \sqsubseteq_{atom} X \rightarrow \exists y[gs(y) \wedge close(y)(x)]]$

(33) $[\![③]\!] = \lambda X.cars(X) \wedge \forall x[x \sqsubseteq_{atom} X \rightarrow \exists y[gs(y) \wedge close(y)(x)]]$

(34) $[\![\uparrow]\!] = \lambda P \lambda x \lambda g.\{\langle P(x), g\rangle\}$ $\quad [\![\Downarrow]\!] = \lambda m.m(\eta)$ $\quad [\![\eta]\!] = \lambda x \lambda g.\{\langle x, g\rangle\}$

(35) $[\![④]\!] = \lambda X \lambda g.\{\langle cars(X) \wedge \forall x[x \sqsubseteq_{atom} X \rightarrow \exists y[gs(y) \wedge close(y)(x)]], g\rangle\}$

(36) $[\![some^u]\!] = \lambda c \lambda k \lambda g.\bigcup\{k(x)(g')|x \in D_e, \langle T, g'\rangle \in c(x)(g^{u \rightarrow x})\}$

(37) $[\![⑤]\!] = \lambda k \lambda g.\bigcup\{k(X)(g^{u \rightarrow x})|cars(X) \wedge \forall x[x \sqsubseteq_{atom} X \rightarrow \exists y[gs(y) \wedge close(y)(x)\}$

(38) $[\![⑥]\!] = \lambda g.\{\langle X, g^{u \rightarrow X}\rangle|cars(X) \wedge \forall x[x \sqsubseteq_{atom} X \rightarrow \exists y[gs(y) \wedge close(y)(x)\}$

(39) $[\![M_u]\!] = \lambda m \lambda g.\{\langle X, h\rangle|\langle X, h\rangle \in m(g) \wedge \neg\exists\langle Y, h'\rangle[\langle Y, h'\rangle \in m(g) \wedge h(u) \sqsubset h'(u)]\}$

(40) $[\![⑦]\!] = \lambda g.\{\langle X, g^{u \rightarrow X}\rangle|X = \oplus\lambda X.[cars(X) \wedge \forall x[x \sqsubseteq_{atom} X \rightarrow \exists y[gs(y) \wedge close(y)(x)]\}$

(41)

$$[\![2_u]\!] = \lambda m \lambda g. \begin{cases} \{\langle T, g' \rangle | \langle X, g' \rangle \in m(g)\} & \textbf{if } |atoms(\oplus G_u)| = 2, where\ G = m(g), \\ & G_u = \{g'(u) | \exists \beta . \langle \beta, g' \rangle \in G\} \\ \{\langle F, g \rangle\} & \textbf{otherwise} \end{cases}$$

(42)

$$[\![⑧]\!] = \lambda g. \begin{cases} \{\langle T, g^{u \to X} \rangle\}\ \textbf{if } |\text{atoms}(\oplus \{X | \text{cars}(X) \wedge \forall x [x \sqsubseteq_{\text{atom}} X \to \\ \qquad \exists y [gs(y) \wedge close(y)(x)\}) | = 2 \\ \{\langle F, g \rangle\} \qquad \textbf{otherwise} \end{cases}$$

5. Remaining issues

There are some remaining issues on the proposed theory. First of all, the proposed theory can well explain why both (14)a and (14)b are felicitous in the given context, but it also predicts that *any* in the scope of *exactly n* in the surface structure will never have the so-called "size effect". Since *any* is licensed very locally, its felicity is already guaranteed before the cardinality predicate $\mathbf{n}_u$ comes in. No matter the number after *exactly* is big or small relative to a given context, *any* is predicted to be always felicitous. However, the "size effect" observed in sentences where *any* is in the scope of *exactly n* has long been accepted as an established empirical fact since Linebarger (1987). This means that either sentence (11)a is a different beast than (14)a and (14)b despite the fact that they look very similar on the surface or the "size effect" is actually illusionary.

Here, I will not decide between these two possibilities, but I would like to point out that a recent experimental study on the licensing of the NPI *any* challenges the *even* theory of *any* and as a result also challenges the accepted "size effect" (Alexandropoulou et al. 2020). The authors give the participants a prompt as given in (43). In the place of QUANT, they put different quantifiers and in the place of $(\text{ANY})_{pol}$, they either keep it unfilled or fill it with *any*. The authors say that the number they use in the quantifiers are all small numbers.

(43) I didn't expect this, but QUANT products had $(\text{ANY})_{pol}$ artificial sweeteners in them. What do you think the writer of the sentence expected? (click on your answer)
- that more products had artificial sweeteners in them
- that fewer products had artificial sweeteners in them

The authors find that when the prompt has *any* in it, about half of the participants choose "more products" and the rest half choose "fewer products". This is very surprising according to the *even* theory because a sentence containing *exactly n* with *n* being small and with *any* in the scope of *exactly n* is predicted to be felicitous only in a context where the speaker expects there to be more products had artificial sweeteners in them. The experimental results, however, suggest that only about half of the participants require such a context to be able to parse the prompt sentence. Moreover, compared with the results from the prompt without *any*, the difference is not significant. This means that the insertion of *any* actually does not bring with it an *even* operator, otherwise significantly more participants will choose "more products". If the "size effect" is a byproduct of the lexical requirement of *any* that the presupposition of *even* has to be satisfied, the experimental results reviewed above suggest that the "size effect" may not be real.

Another remaining issue is the contrast between (44) and (45) in terms of their felicity. It has been observed that *any* in the restriction of a definite singular description is infelicitous when the sentence is interpreted episodically, as shown in(44)). However, *any* is still licensed in the scope of *exactly one*, as shown in (45). This is surprising based on the proposed theory because (45) is analyzed as (44) at logical form. How is it possible that one is felicitous and the other is not?[5]

(44) *The student who read any book is exactly one in number.

(45) Exactly one student read any book.

Here I sketch an idea to resolve this issue. Basically, at the logical form, the nominal predicate of the definite description can be intensionalized. Therefore, we could still get a definite plural, not an extensional definite singular. This idea is actually already discussed in Gajewski and Hsieh (2014). The interested readers can refer to their paper for more details. Still another issue is whether the current proposal gives any insight in the interaction between *any* and other modified numerals[6]. I do not yet have an answer to this question. According to my preliminary survey, my informants who are native English speakers have diverging opinions on the felicity of *any* in the scope of other modified numerals. I have not been able to find a pattern.

6. Conclusion

In this paper, a novel counterexample is presented that challenges both the uniform *even* theory of *any* and the mixed view of *any* in the exhaustification theory. In order to account for the counterexample, I argue that *any* in the scope of *exactly n* in the surface structure is located in the restriction of a definite plural in the logical form. This theory is made possible by the recent decompositional analysis of non-monotonic quantifiers where *exactly n* is broken down into an indefinite determiner, a maximality component and a cardinality component. A consequent of the proposed theory is that the so-called "size effect" is predicted to be non-existent. I give a tentative discussion on this prediction, showing that this prediction may not be so non-orthodox given the recent experimental results in NPI licensing.

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[5]Thanks to Itai Bassi for raising this question to me.

[6]Thanks to Manfred Krifka for asking this question.

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