Limited variable-force modals and the interactions of scal{ar,eless} implicatures[1]

Lydia NEWKIRK — *Rutgers University*

Abstract. In this paper I present novel data from Kinande (Bantu J, DRC) attesting a previously undiscussed type of variable-force modal: limited variable-force modals. The Kinande modal prefix *anga* can be interpreted as a possibility modal or as a weak necessity modal, but never as a strong necessity modal. I show the prior analyses, whether they focus on domain restriction or exhaustification, are alone insufficient to account for this kind of modal force pattern, and develop a unified account of modal force that explains the Kinande facts as well as the previously-attested modal systems, generating a typology of modal force.

Keywords: modals, modality, scalar implicatures, scaleless implicatures, exhaustification, Bantu, Kinande, typology

1. Introduction

Generally speaking, accounts of variable-force modals have considered a binary distinction between languages: either a modal is fully defined for its modal force, or it is variable across the whole scale of modal force (even if it is constrained in modal flavor). Kinande, a Bantu language spoken in the Democratic Republic of Congo, demonstrates that this binary does not consistently hold; Kinande has a modal prefix *anga* that is used for both possibility and weak necessity interpretations, but it is infelicitious under strong necessity interpretations. As a result, we will need a more refined theory of variable-force modality to allow for these limited variable-force modals.

The remainder of this paper is structured as follows: In Section 2 I give a brief overview of the prior data and analsyes for variable-force modals. In Section 3 I present the data from Kinande and discuss the issues it causes for those accounts. Section 4 diverts into a different literature, on exhaustification, and presents how it has and can be applied to variable-force modals. Section 5 then applies a combination of exhaustification and domain-restriction analyses to Kinande *anga*, and demonstrates how such an account derives the results we need. Finaly, Section 6 closes with some typological implications.

2. Prior accounts

The mainline semantic account for modals makes a fundamental distinction in modal force, giving English *can* and *must* meanings along the lines in (1):

[1]I would like to thank first and foremost Dr. Philip Mutaka for his language expertise and continuous and patient help, without which this project would be entirely impossible. I am also thankful to Simon Charlow, Augustina Owusu, Ang Li, and the attendees of the Rutgers Semantics reading group for their feedback at various stages of this project.

 In: Daniel Gutzmann and Sophie Repp (eds.): Proceedings of Sinn und Bedeutung 26, pp. 641–657. University of Cologne.

(1) a. $[\![\textit{can } p]\!]^w = \exists w'[w' \in \text{ACC}(w) \ \& \ p(w')]$
READ AS: There is some world w' that is accessible from world w such that p is true in w'.
b. $[\![\textit{must } p]\!]^w = \forall w'[w' \in \text{ACC}(w) \rightarrow p(w')]$
READ AS: For all worlds w' that are accessible from w, p is true in each of those worlds.

However, work on variable-force modals has established that some languages do not distinguish, descriptively, between possibility and necessity, as in the following example from St'at'imcets (Salish,BC), from Rullmann et al. (2008):

(2) St'at'imcets (Rullmann et al., 2008: p. 331):
Context 1: The cougar was on a rampage and was killing cats, dogs, and racoons, and it had a child cornered and was growling; it *would* have killed a child.
Context 2: You just know that sometimes cougars kill children when they venture into built-up areas; it *could* have killed a child.

zúqw-s-as **ka** ta sk'úk'wmi7t-a ti7 ku swúw'a lh-cw7áoz-as kw
die-CAUS-3ERG **IRR** DET child-DET DEIC DET cougar COMP-NEG-3CONJ DET
s-qus-cit-ítas
NOM-shoot-IND-3PL.ERG

'That cougar could/would have killed a child if they hadn't shot it.'

Depending on the context, the same modal morpheme (in this example the modal particle *ka*) can be interpreted as either a possibility modal or as a necessity one. This sort of modal, on its face, poses serious issues for the modal binary in (1), as well as issue for how to categorize these modals.

Deal (2011) analyzes the Nez Perce (Sahaptian, Columbia River plateau) modal *o'qa*, which is used in both possibility and necessity contexts as well:

(3) 'inéhne-**no'qa** 'ee kii lepít cíikan.
take-mmod you DEM two blanket

(Deal, 2011: ex. 1)

'You can/should take these two blankets'

Rather than propose that *o'qa* uses domain restriction to achieve variable force, Deal analyzes *o'qa* as not being variable at all: It's simply a possibility modal in a language that has no necessity modals. Since there's no pragmatic competitor in the modal domain, *o'qa* doesn't draw a scalar implicature of "not necessarily", and so remains compatible with necessity contexts. After all, if I am obligated to do something, then I am also allowed to do that thing. The infelicity of possibility modals in necessity contexts is due to a scalar implicature, which, however you derive it, depends on there being a stronger option (a necessity modal) to compare *o'qa* to.

Deal's analysis is supported by *o'qa*'s behavior in downward-entailing contexts. As soon as *o'qa* is embedded under negation, or in the antecedent of a conditional, or in the restrictor of a universal quantifier, it ceases to be ambiguous:

(4) **Context:** The referee is talking to an injured player.

tamáalwit-wecet wéet'u 'ee x̂eeleewi-**yo'qa** 'étke k'omáy'c 'ee wee-s-∅
rule-reason not you play-MOD because hurt you be-P-PRES
'áatim.
arm

'According to the rules, you can't play, because your arm is injured.'

(Nez Perce, Deal, 2011: ex. 46)

(5) **Context**: You are explaining to someone who thinks they have to leave that they are not in fact required to do so. It's not necessary for them to leave.

#wéet'u 'ee kiy-**ó'qa**.
not you go-MOD

(Nez Perce, Deal, 2011: ex. 49)

Consultant: 'That's a different conversation, not this one. You're just saying *wéet'u 'ee kiyó'qa*, "you can't go".'

This behavior makes sense if *o'qa* is a possibility modal without any special domain restriction. Downward-entailing contexts reverse the relative strengths of existential and universal quantifiers; "not some (= no)" entails "not all", and parallel for other downward-entailing contexts. Since the negated possibility modal has such a strong interpretation, it's not compatible with the (relatively weak) negated necessity contexts, and so *o'qa* can't receive a necessity reading in these contexts.

3. Enter Kinande

Deal's account works well enough for Nez Perce, and might even extend to some of the languages previously analyzed with domain restriction accounts. But unfortunately it will not extend to Kinande. *Anga*, Kinande's variable-force modal, is not only ambiguous in simple positive sentences, but also in antecedents of conditionals:

(6) **Context:** You are at a bookstore looking for books that are interesting to you to read. The worker here is giving you recommendations. She picks up a book from a shelf that's in German, but she says that it's a very good book and you should get it if you can read German.

wamábyá	íwanganásóm'	ekitábw'	ekí,
u-a-ma-bi-a	i-u-anga-na-som-a	e-kitabu	eki,
SM.2sg-TM-TM-be-FV	C-SM.2sg-MOD-TM-read-FV	AUG-c7.kitabu	c7.this,
kúmbé	íwakígúlá	munábwíre	
kumbe	i-u-a-ki-gul-a	munabwire	
better.that	C-SM.2sg-TM-Om.c7-buy-FV	today	

"If you can read this book, you should buy it today."

(7) **Context:** You are shopping for books for school, and one of the workers at the book-store is helping you find the books you need. As you are walking past a shelf of books, the worker picks up a book and explains that it frequently sells out, so if it's one of the books you are supposed to read, you should buy it today while they still have stock. You could probably get by in the class without it, but you're more likely to do well if you have this book.

wamábyá	íwukándisyasom'	ekitábw'	ekí
u-a-ma-bi-a	i-u-ka-ndi-sya-som-a	e-kitabu	eki
SM.2sg-TM-TM-be-FV	C-SM.2sg-TM-TM-TM-read-FV	AUG-c7.book	c7.this
ky' ekalhási, kúmbé	íwakígúlá	munábwíre	
kyo ekalhasi, kumbe	i-u-a-ki-gul-a	munabwire	
PREp class, better.that	C-SM.2sg-TM-OM.c7-buy-FV	today	

'If you should read this book for school, you should buy it today."

It's also ambiguous in the restrictor of a universal quantifier:

(8) **Context**: Some celebrities published a list of "1,000 books that everyone should read before they die". Since then, Swera has been reading the books on that list, and only those books. She intends to read every single book on that list whenever she can get her hands on a copy, but she refuses to spend her time reading anything else.

Swera akásomá	óbuli	kitábu	ekyánganásóma
Swera a-ka-som-a	obuli	kitabu	e-kyo-a-anga-na-som-a
Swera SM.c1-TM-read-FV	every	c7.book	AUG-c7.REL-SM.c1-MOD-TM-read-FV

"Swera reads every book that she should read"

(9) **Context**: Masika is a very avid reader; she always has a book in her hands. She doesn't particularly follow recommendations or anything, she just reads every book she can get her hands on, as quickly as she can.

Masiká akásomá	óbuli	kitábu	ekyánganásóma
Masika a-ka-som-a	obuli	kitabu	e-kyo-a-anga-na-som-a
Masika SM.c1-TM-read-FV	every	c7.book	AUG-c7.REL-SM.c1-MOD-TM-read-FV

"Masika reads every book that she can read."

And even under negation, so long as that negation is not in the same clause as *anga*:

(10) omugalímu mwá**tétá**buga ati Másiká
omugalimu mo-a-**te-ta-**bug-a a-ti Masika
c1.teache TM-SM.c1-**TM.NEG-TM.NEG**-say-FV SM.c1-that Masik
anganátsíga
a-**anga**-na-tsig-a
SM.c1-**MOD**-TM-swim-FV
"The teacher didn't say that Masika can/should swim"

In fact, the only construction that disambiguates *anga* is when it occurs with clausemate negation:

(11) nga-oko reglema yi-ka-bug-a, si-u-anga-sat-a,
COMP-COMP c9.rules SM.c9-TM-say-FV, NEG-SM.2sg-MOD-dance-FV
kundi w-oyo u-kwa-ire oko ku-boko
because 2sg?-REPLRO SM.2sg-hurt-TM c17 c15-arm
"According to the rules, you can't play because your arm is injured."

(12) # sí-u-anga-sat-a
SM.2sg-MOD-dance-FV
Intended: 'you don't have to play"

The persistent ambiguity of *anga* indicates that we will need a different type of analysis for Kinande's modal system than for Nez Perce's.

4. Strengthening by exhaustification

The distribution of *anga*'s variability is in fact reminiscent of work by Jeretic (2020, 2021) (and similarly Staniszewski, 2020: for english *should*) recently proposed an exhaustification-based analysis for deriving variable-force modals in Ecuadorian Siona, and her analysis suggests a way forward for Kinande. Like Nez Perce, Siona has no necessity modal, only possibility modals. Unlike Nez Perce, but like Kinande, Siona *ba'iji* is ambiguous in downward-entailing contexts:

(13) Sai-ye ba-'i-to, sa-si'i
go-INF be-IPF-COND go-FUT-OTH
(Jeretic, 2021: ex. 7-8)
'If I can/must go, I will go'

But *ba'iji* is unambiguous in simplex clauses, where it has a strong necessity interpretation; for a possibility interpretation, *deoji* is used instead:

(14) Sai-ye ba-'i-ji
go-INF be-IPF-3S
(Jeretic, 2021: ex. 1)
'We must go.' (lit., 'there is to go')

(15) Sai-ye de'o-ji
go-INF good-3S
(Jeretic, 2021: ex. 2)
'We can go.' (lit., 'it is good to go')

And like Kinande, *ba'iji* is also unambiguous under clausemate negation:

(16) Sai-ye beo-ji
go-INF NEG.be-3S

(Jeretic, 2021: ex. 5)

'We mustn't go.'
'We don't have to go.'

Jeretic explains this peculiar pattern by analyzing Siona *ba'iji* as a possibility modal subject to exhaustification, drawing from Bowler (2014); Bar-Lev and Margulis (2014), among others to derive the strengthened meanings of *ba'iji* using the same mechanism that derives scalar implicatures when a stronger alternative is present in the grammar. Exhaustification is a semantic mechanism proposed (Fox, 2007; Bar-Lev and Fox, 2017; Chierchia et al., 2012) in order to account for scalar implicatures within the grammar, especially within embedded clauses. Essentially, the exhaustification operator (*exh*) is a covert instance of *only* that can occur on the left edge of a clause, where it contributes the meaning that while the proposition that it modifies is true, **only** that proposition is true, and none of its alternatives are true, so long as you can safely deny those alternatives without contradicting the original proposition.

(17) Exhaustification operator with Innocent Exclusion (Fox, 2007):
a. $[\![exh]\!](C)(p)(w) = p(w) \wedge \forall(q) \in IE(p,C)[\neg q(w)]$
b. $IE(p,C) = \bigcap\{C' \subseteq C : C'$ is a maximal subset of C, s.t. $\{\neg q : q \in C'\} \cup \{p\}$ is consistent.$\}$

More precisely, the *exh* operator takes a proposition, a world, and a contextually- and/or lexically-defined set of alternative propositions, the last of which can be roughly understood as the set of other things that the speaker could have said in the situation. Those alternatives can be derived, for example, by replacing specific lexical items with stronger versions of that item within the same category (e.g., replacing *some* with *every*). The *exh* operator then asserts p, passing along the original proposition, and then also asserts the negation of every proposition that can be **innocently excluded**. That is, for every proposition out of the alternative set that is not entailed by p, and also not entailed by p plus the negation of any combination of the other propositions in C. The innocently excludable alternatives are the alternatives that we can safely declare false without risking any contradiction with p.

Applying *exh* to English *can* thus derives a scalar implicature negating the *must* version of the proposition, giving us the meaning "you can, but you don't have to", as expected. But because Siona *ba'iji* has no stronger scalemate, there is nothing for *exh* to negate when applied to a proposition with *ba'iji*. So no scalar implicature is derived. That's not enough to strengthen *ba'iji* to a necessity modal, Jeretic needs the **subdomain alternatives** of *ba'iji*, not merely its scalar alternatives. Subdomain alternatives (first proposed in Chierchia, 2013: for NPIs) are obligatorily triggered for some lexical items, and optionally for others, and are alternatives formed not by altering lexical items in a proposition, but by creating propositions corresponding to each possible subset of the domain of a lexical item. For *ba'iji*, the subdomain alternatives are the powerset of the set of worlds accessible to *ba'iji*:

(18) $[\![ba'iji]\!] = \exists w'[w' \in \{w_1, w_2, w_3\} \wedge p(w')]$

(19) $[\![\mathrm{C}(ba'iji)]\!] =$
$\{\exists w'[w' \in \{w_1, w_2, w_3\} \wedge p(w')]$
$\exists w'[w' \in \{w_1, w_2\} \wedge p(w')]$
$\exists w'[w' \in \{w_2, w_3\} \wedge p(w')]$
$\exists w'[w' \in \{w_1, w_3\} \wedge p(w')]$
$\exists w'[w' \in \{w_1\} \wedge p(w')]$
$\exists w'[w' \in \{w_2\} \wedge p(w')]$
$\exists w'[w' \in \{w_3\} \wedge p(w')]\}$

If we add these to the alternative set for *ba'iji* and exhaustify, well, still nothing happens. None of the subdomain alternatives of *ba'iji* are innocently excludable (because while any single world could be excluded, you can't exclude them all at once while preserving the possibility claim, so none of them are in every possible consistent subset of alternatives). Jeretic follows Bowler (2014) and Fox (2007) in applying *exh* recursively to derive a strengthened meaning, but for the sake of clarity I will instead show the derivation using exhaustification with innocent **inclusion** from Bar-Lev and Fox (2017), which arrives at the same result in a more transparent way:[2]

(20) Exhaustification with Innocent Inclusion (Bar-Lev and Fox, 2017):

a. $[\![exh]\!](C)(p)(w) = \forall q \in IE(p,C)[\neg q(w)] \wedge \forall r \in II(p,C)[r(w)]$

b. $IE(p,C) = \bigcap\{C' \subseteq C : C'$ is a maximal subset of C, s.t. $\{\neg q : q \in C'\} \cup \{p\}$ is consistent.$\}$

c. $II(p)(C) = \bigcap\{C'' \subseteq C : C''$ is a maximal subset of C, s.t. $\{r : r \in C''\} \cup \{p\} \cup \{\neg q : q \in IE(p)(C)\}$ is consistent$\}$

Innocent Inclusion is an addition to Innocent Exclusion, and the IE step has priority, preserving any scalar implicatures that would be drawn. However, as an extra step, exhaustification with Innocent Inclusion also asserts all of the alternatives that can be added without risking contradiction with p or with any of the innocently excluded alternatives. Bar-Lev and Fox (2017) use this operator to derive free choice implicatures for disjunctions under possibility modals.

For an alternative set that includes the individual disjuncts as well as the conjunction, Innocent Exclusion will negate the conjunction, but the individual disjuncts are not innocently excludable. They are, however, innocently **includable**, because they can both be asserted (specifically with the possibility modal applied to them) while preserving the truth of $\Diamond(p \vee q)$ and the truth of $\neg\Box(p \wedge q)$, resulting in $\Diamond p \wedge \Diamond q$, where the disjunction has been strengthened to a conjunction.

In the absence of a scalemate, Innocent Inclusion will derive strengthening of an existential to the equivalent of a universal. As a schematic, Warlpiri is analyzed by Bowler (2014) as a language that has a disjunction operator *manu*, but no conjunction operator. As a result,

[2]As far as I have been able to determine, there are no predictive differences between applying exh^{IE} twice and applying exh^{IE+II} once, at least for Siona or Kinande. I use Innocent Inclusion here for the tidier notation, but all these derivations will work with recursive exhaustification without Innocent Inclusion, as well.

conjunction is missing from the set of *manu*'s scalar alternatives, and exhaustifying results in strengthening to conjunction:

(21) a. $[\![p \textit{ manu } q]\!] = p \vee q$
b. $C([\![p \textit{ manu } q]\!]) = \{p, q, p \vee q\}$
c. $[\![exh^{IE}(p \textit{ manu } q)]\!] = (p \vee q)$
d. $[\![exh^{IE+II}(p \textit{ manu } q)]\!] = (p \vee q) \wedge p \wedge q$
$\equiv p \wedge q$

Because there is no dedicated conjunction operator in Warlpiri, Innocent Exclusion doesn't negate the conjunction of p and q, like it does in English or other languages with dedicated words for *and*. As a result, Innocent Inclusion asserts the individual disjuncts, and can do so without running into contradiction with a negated conjunct. The result is simply asserting the conjunction that was missing from the language before.

In upward-entailing contexts, *manu* is interpreted as "and". But under negation, *manu* can only be interpreted as "or":

(22) Cecilia manu Gloria=pala yanu tawunu-kurra
Cecilia manu Gloria=3DU.SUBJ go.PST town-ALL
"Cecilia and Gloria went to town."

(23) Cecilia manu Gloria kula=pala yanu Lajamanu-kurra
Cecilia manu Gloria NEG=3DU.SUBJ go.PST LajamanuALL
"Neither Cecilia nor Gloria went to town."

The explanation for this is that a negated "or" is already exceedingly strong, and so *exh* can't draw a scalar implicature. It can draw a scaleless one, but only for implicatures that are already entailed by the original proposition anyway, and therefore no more informative than if *exh* had not applied at all:

(24) a. $[\![\neg(p \textit{ manu } q)]\!] = \neg(p \vee q)$
b. $C([\![\neg(p \textit{ manu } q)]\!]) = \{p, q, p \vee q\}$
c. $[\![exh^{IE}(p \textit{ manu } q)]\!] = \neg(p \vee q)$
d. $[\![exh^{IE+II}(p \textit{ manu } q)]\!] = \neg(p \vee q) \wedge \neg p \wedge \neg q$
$\equiv \neg(p \vee q)$

In general, this should follow in any downward-entailing context. However, *manu* is ambiguous in many such contexts, such as if-clauses and restrictors of universal quantifiers:

(25) Warlpiri (Bowler, 2014: ex. 9–10):

a. Kaji=npa kuyu manu mangarri ngarni ngula kapu=npa
IRR=2SG.SUBJ meat manu food eat.NPST that AUX.FUT=2SG.SUBJ
pirrjirdi-jarrimi.
strong-become.NPST
"If you eat meat and vegetables, you will become strong."

b. Kaji=npa jarntu pakarni manu window luwarni, ngula=ju
IRR=2SG.SUBJ dog hit.NPST manu window shoot.NPST that=TOP
Nungarrayi-rli kapi=ngki jirna-wangu-mani.
Nungarrayi-ERG AUX.FUT=2SG.NSUBJ scold-NPST
"If you hit the dog or break the window, then Nungarrayi will scold you."

However, this isn't surprising, given other evidence around scalar implicatures and exhaustification in the literature. While a purely pragmatic approach predicts that scalar implicatures should be unavailable in downward-entailing contexts, because they do not, in general, strengthen the overall utterance. But, as established in Chierchia et al. (2012); Chierchia (2017) and others, exhaustification can take place rather readily in downward-entailing contexts, resulting in ambiguous sentences, especially when the context is right:

(26) I don't expect that some students will do well, I expect that all students will. (Chierchia et al., 2012: p. 2305)

Such contexts, in fact, are among the strongest arguments for an exhaustification operator in the syntax, rather than deriving scalar implicatures purely in the pragmatics, without reference to the structure. In order to derive the expected meaning for (26), a scalar implicature of "some but not all students" must be drawn under the matrix negation, or else the second half of the sentence will contradict the first.

It's been noted, of course, that even embedded scalar implicatures seem to be dispreferred when they result in a sentence that is semantically weaker than if the implicature had not been calculated. But even then, it doesn't seem to be impossible, based on examples such as the following, from Chierchia (2017):

(27) a. If some students in your class are having difficulties, talk to them
b. . . . but if all of them do, do not talk to them; talk to me first.

On its own, perhaps (27a) tends toward an interpretation where you should always talk to any student having trouble, whether that's one out of the entire class or all of them. It seems that version where an embedded implicature of "some but not all students" is derived is dispreferred. But while that is certainly one possible interpretation of the sentence, and that interpretation requires the absence of an *exh* operator in the if-clause, the follow-up in (27b) shows that *exh* must be possible in (27a), in order for the follow-up sentence to be non-contradictory. Thus the restriction on the occurrence of *exh* is something like the following:

(28) **Parsing condition on** *exh* (Chierchia, 2017): If using *exh*, do so in a way that does not lead to weakening (unless weakening is necessary to avoid a contradiction).

This parsing constraint should be understood not as a structural constraint on when or where *exh* can be used; rather, it's a statement of relatively vague pragmatic preference, a weaker approach even than the Strongest Meaning Hypothesis from Chierchia et al. (2012: p. 2327). This sort of parsing constraint is much more of a suggestion than a rule, as befits the fuzziness of determining preferred interpretations out of context.

Given this discussion, I take the *exh* operator to be available at scope sites (clause boundaries), per Chierchia (2004); Chierchia et al. (2012), and freely so, as far as the syntax and semantics are concerned. Whether a given utterance appears to have an exhaustified meaning available or required, or neither, is a matter I take to be of pragmatics proper, and subject to considerations such as Gricean Implicature (Grice, 1975). But in principle, the semantics is capable of producing sentences either with or without an *exh* at any given CP layer.

These tools in our pockets, we can finally return to Jeretic's analysis for Siona. Defining *ba'iji* as a possibility modal in (30) and then exhaustifying over its subdomain alternatives, we directly derive a strengthened reading of *ba'iji*, because Innocent Inclusion lets us assert the existential claim over all of the singleton sets of worlds; taken together, this is logically equivalent to universal quantification:

(29) $[\![\, ba'iji \,]\!] = \exists w'[w' \in \{w_1, w_2, w_3\} \wedge p(w')]$

(30) $[\![\, \mathrm{C}(ba'iji) \,]\!] =$
$\exists w'[w' \in \{w_1, w_2, w_3\} \wedge p(w')]$
$\exists w'[w' \in \{w_1, w_2\} \wedge p(w')]$
$\exists w'[w' \in \{w_2, w_3\} \wedge p(w')]$
$\exists w'[w' \in \{w_1, w_3\} \wedge p(w')]$
$\exists w'[w' \in \{w_1\} \wedge p(w')]$
$\exists w'[w' \in \{w_2\} \wedge p(w')]$
$\exists w'[w' \in \{w_3\} \wedge p(w')]$

(31) $\exists w'[w' \in \{w_1\} \wedge p(w')] \wedge$
$\exists w'[w' \in \{w_2\} \wedge p(w')] \wedge$
$\exists w'[w' \in \{w_3\} \wedge p(w')]$
$\equiv \forall w'[w' \in \{w_1, w_2, w_3\} \rightarrow p(w')]$

The exhaustification analysis accounts for the ambiguities in embedded downward-entailing contexts nicely, since exhaustification is optional for downward-entailing contexts (Chierchia et al., 2012).

In a somewhat similar vein, Leffel (2012) describes a future marker *-ti* in Masalit (Nilo-Saharan, Maban, Sudan) which, in addition to indicating future, is also used for various possibility-modal readings:

(32) tísû tò-rón-tì
3sg goat.ACC 3sg-buy-ti
(Masalit, Leffel, 2012: 11b)
"He will buy a goat." or "He might buy a goat."

(33) ámá kómò á-kál-tì
1sg mountain.ACC 1sg-see-ti
(Masalit, ex. 15b)
"I will see the mountain" (e.g., tomorrow) or "I can see the mountain." (e.g., from where I'm standing)

Interestingly, *-ti* can also be used for epistemic necessity, but only if there is the particle *de* ('only') with it to get a necessity reading:

(34) tímàsàrà tú-**tì**
3sg Masalit 3sg-ti
(Masalit, Leffel, 2012: 17c)
"He might be Masalit"

(35) tímàsàrà **dè** tú-**tì**
3sg Masalit only 3sg-ti
(Masalit, Leffel, 2012: 17d)
"He must be Masalit" (lit. "He could only be Masalit.")

Leffel analyzes *-ti* as a possibility modal tied to a temporal base (allowing for futurate readings), and proposal that *de* operates just like *only* from Rooth (1985), asserting that all of the alternatives to the expression without *only* are false (=34). In sum: It ensures that (34) is true, and none of the other possibilities are true. This amounts to what is essentially universal force, because there is only one possibility left. Masalit might be analyzed as overtly expressing the exhaustification operator that is covert in Siona.

The pattern for Kinande *anga* differs from Siona and Masalit in a few important ways: Most obviously, *anga* shows a limited kind of variable force, varying only between possibility and weak necessity, and never getting an interpretation as a strong necessity modal. For another, and likely connected to the first difference, *anga* has at least one scalemate, *paswa*. We will see that the presence of a universal quantifier in the alternative set poses some challenges for straightforwardly applying an exhaustification analysis. Finally, *anga* is ambiguous in simple positive clauses as well as in downward entailing environments, distinct from Siona *ba'iji*. This likely comes from the lack of an unambiguously weak possibility modal in Kinande. In the next section I will address each of these problems while preserving the useful parts of the exhaustification approach.

5. Exhaustifying *anga*

Here is a summary table of Kinande's modal system, with *anga* highlighted:

	Epistemic	**Deontic**	**Circumstantial**	**Teleological**
Strong Necessity	*paswa*	*paswa*		*paswa*
Weak necessity	***anga***	***anga***	***anga***	
Possibility	***anga***	***anga***	***anga***	***anga***

Table 1: Kinande modal morphemes

Anga is felicitous across all modal flavors in both possibility and weak necessity contexts. It is very specifically infelicitous in strong necessity contexts, where generally speaking *paswa* ('must') is used instead. I propose that we account for *anga*'s variance, on the one hand, and its lack of compatibility with strong necessity contexts, on the other, with the same exhaustification

operator. *Anga* generates draws a scalar implicature by the familiar methods we have seen, with the alternative *paswa* expression negated by Innocent Exclusion. At the same time, *anga* optionally triggers subdomain alternatives, and since exhaustification also contains Innocent Inclusion, *exh* will assert as many of those subdomain alternatives as it can, resulting in a weak necessity reading.

This solution is nice and tidy; it lets us use the same mechanism to derive both where *anga* is felicitious and where it is not. It's also sensitive to the lexical inventory of Kinande, as suits our descriptive observations, that variable-force modality requires gaps in a modal paradigm, but can work in whatever size gap there may be. What distinguishes Kinande from Siona, St'at'imcets, or Nez Perce is that Kinande *anga* is limited in its variable force, and that Kinande has an unambiguous strong necessity modal. In this analysis here, those two facts are linked, and we expect them to be linked cross-linguistically. The result is an informative and constrained typology of variable and non-variable force.

Understanding that once *anga*'s meaning is strengthened it arrives at a weak necessity reading, we can solve our issue by simply incorporating this domain restriction into *anga* itself. Lexically, *anga* is a domain-restricted possibility modal:

(36) $[\![anga(p)]\!] = \exists w'[w' \in OS_2(OS_1(\bigcap(MB(w)))) \wedge p(w')]$

Domain restriction of a possibility modal has very little, if any, discernible effect. If something is still possible given some additional (perhaps unshared) assumptions or conditions, then well, it's still possible. But we are a long way from likely, especially when those assumptions are not shared by everyone in the discourse, per Rubinstein (2012).

But when this domain-restricted modal's subdomain alternatives are exhaustified, it no longer turns into a strong necessity modal; the *anga*-expression becomes a weak necessity expression, just like we want:

(37) a. $[\![exh^{IE}(anga(p))]\!] = \exists w'[w' \in \{w_1, w_2, w_3\} \& \ p(w')]$
$\wedge \neg \forall w'[w' \in \{w_1, w_2, w_3, w_4, w_5\} \rightarrow p(w')]$

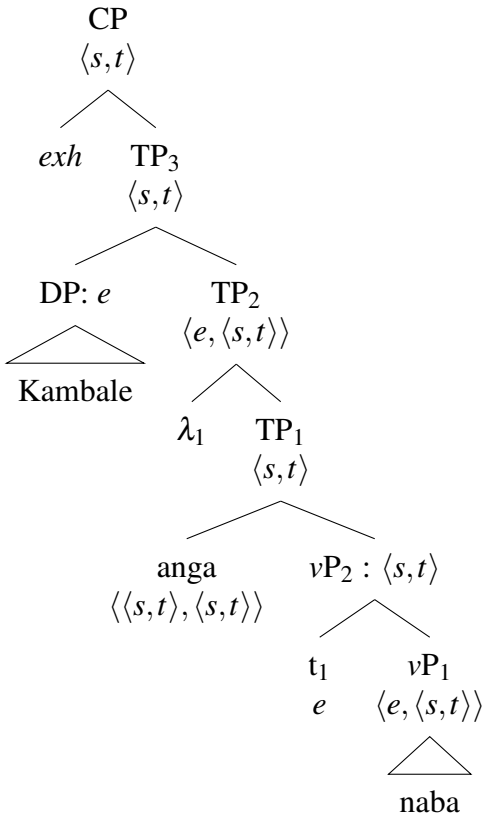
b. $[\![exh^{IE+II}(anga(p))]\!]$: $\exists w'[w' \in \{w_1, w_2, w_3\} \& \ p(w')]$
$\wedge \neg \forall w'[w' \in \{w_1, w_2, w_3, w_4, w_5\} \rightarrow p(w')]$
$\wedge \exists w'[w' \in \{w_1, w_2, w_3\} \& \ p(w')]$
$\wedge \exists w'[w' \in \{w_1, w_2\} \& \ p(w')]$
$\wedge \exists w'[w' \in \{w_1, w_3\} \& \ p(w')]$
$\wedge \exists w'[w' \in \{w_2, w_3\} \& \ p(w')]$
$\wedge \exists w'[w' \in \{w_1\} \& \ p(w')]$
$\wedge \exists w'[w' \in \{w_2\} \& \ p(w')]$
$\wedge \exists w'[w' \in \{w_3\} \& \ p(w')]$
$\equiv \forall w[w' \in \{w_1, w_2, w_3\} \rightarrow p(w')]$

5.1. *Anga* in simplex clauses

With these tools, it's simple to derive the ambiguity of *anga* in simple positive clauses. If we assume that it's optional to activate *anga*'s subdomain alternatives, we can derive the scalar

implicature for possibility-interpreted *anga*, as well as with the strengthened interpretation. In either case, I take the structure of an *anga*-expression to be something like the following:

(38) Kámbale a-angá-naba
Kambale SM.c1-MOD-wash
'Kambale can/should wash himself.'

(39)


And as a reminder, we're working with these definitions for our modals:

(40) $[\![anga]\!] = \lambda p.\lambda w.\exists w'[w' \in OS_2(OS_1(\bigcap(MB(w)))) \wedge p(w')]$

(41) $[\![paswa]\!] = \lambda p.\lambda w.\forall w'[w' \in OS_1(\bigcap(MB(w))) \rightarrow p(w')]$

For possibility-interpreted *anga*, the subdomain alternatives for *anga* aren't used, and so the alternative set is just *paswa*, and the composition proceeds as follows:

(42) $[\![\mathrm{vP}_1]\!] = \lambda x.\lambda w.\mathsf{wash}(x)(w)$
$[\![\mathrm{vP}_2]\!] = \lambda w.\mathsf{wash}(t_1)(w)$
$[\![\mathrm{TP}_1]\!] = \lambda w.\exists w'[w' \in OS_2(OS_1(\bigcap(MB(w)))) \wedge \mathsf{wash}(t_1)(w')]$
$[\![\mathrm{TP}_2]\!] = \lambda x.\lambda w.\exists w'[w' \in OS_2(OS_1(\bigcap(MB(w)))) \wedge \mathsf{wash}(x)(w')]$
$[\![\mathrm{TP}_3]\!] = \lambda w.\exists w'[w' \in OS_2(OS_1(\bigcap(MB(w)))) \wedge \mathsf{wash}(\mathsf{Kambale})(w')]$
$[\![\mathrm{CP}]\!] = \lambda w.\exists w'[w' \in OS_2(OS_1(\bigcap(MB(w)))) \wedge \mathsf{wash}(\mathsf{Kambale})(w')]$
$\wedge\neg\forall w'[w' \in OS_1(\bigcap(MB(w))) \rightarrow \mathsf{wash}(\mathsf{Kambale})(w')]$

Anga remains domain-restricted compared to the corresponding *paswa* sentence, even in this un-exhaustified derivation, but that doesn't mean much when there is no "weak possibility" modal to compare with *anga*. However, if the interpretation proceeds with *anga*'s subdomain alternatives included in the alternative set, the composition goes as follows:

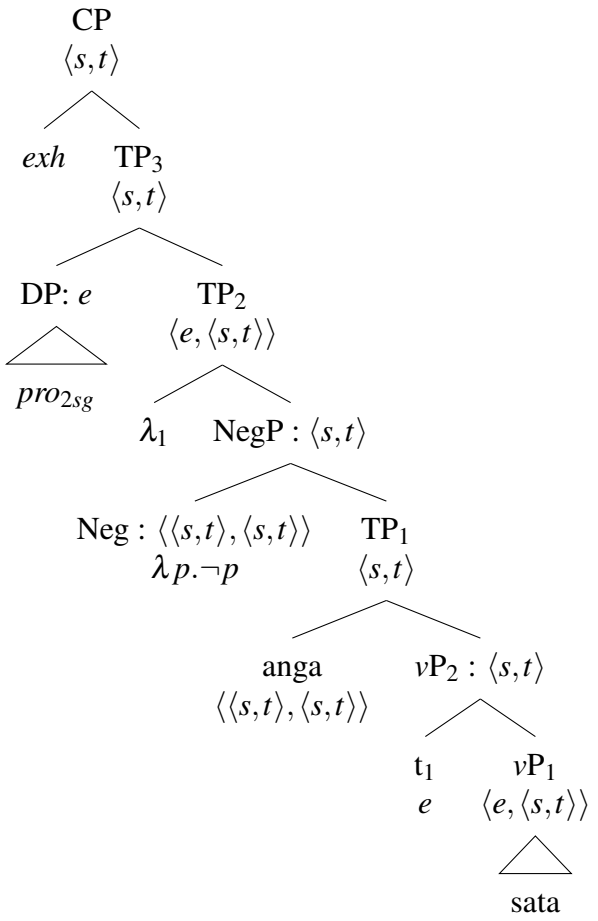
(43) $[\![vP_1]\!] = \lambda x.\lambda w.\mathsf{wash}(x)(w)$
$[\![vP_2]\!] = \lambda w.\mathsf{wash}(t_1)(w)$
$[\![TP_1]\!] = \lambda w.\exists w'[w' \in OS_2(OS_1(\bigcap(MB(w)))) \wedge \mathsf{wash}(t_1)(w')]$
$[\![TP_2]\!] = \lambda x.\lambda w.\exists w'[w' \in OS_2(OS_1(\bigcap(MB(w)))) \wedge \mathsf{wash}(x)(w')]$
$[\![TP_3]\!] = \lambda w.\exists w'[w' \in OS_2(OS_1(\bigcap(MB(w)))) \wedge \mathsf{wash}(\mathsf{Kambale})(w')]$
$[\![CP]\!] = \lambda w.\forall w'[w' \in OS_2(OS_1(\bigcap(MB(w)))) \rightarrow \mathsf{wash}(\mathsf{Kambale})(w')]$
$\wedge\neg\forall w'[w' \in OS_1(\bigcap(MB(w))) \rightarrow \mathsf{wash}(\mathsf{Kambale})(w')]$

5.2. *Anga* with clausemate negation

Kinande negation is fairly high in the clause, and specifically seems to outscope *anga*, as well as preceding it in the modal template:

(44) nga-oko reglema yi-ka-bug-a, si-u-anga-sat-a,
COMP-COMP c9.rules SM.c9-TM-say-FV, NEG-SM.2sg-MOD-dance-FV
kundi w-oyo u-kwa-ire oko ku-boko
because 2sg?-REPLRO SM.2sg-hurt-TM c17 c15-arm
"According to the rules, you can't play because your arm is injured."

(45)



Because the exhaustification operator is applied at the clause boundary, and not completely freely, there's no position between *anga* and negation to apply exhaustifcation. The result is a loss of ambiguity for *anga*. It doesn't matter whether we include subdomain alternatives or not, nor even whether we exhaustify or not; the semantic value of negated *anga* is already stronger than any of its alternatives, and so exhaustification does nothing at all:

(46)
$$[\![\text{vP}_1]\!] = \lambda x.\lambda w.\mathsf{play}(x)(w)$$
$$[\![\text{vP}_2]\!] = \lambda w.\mathsf{play}(t_1)(w)$$
$$[\![\text{TP}_1]\!] = \lambda w.\exists w'[w' \in OS_2(OS_1(\bigcap(MB(w)))) \wedge \mathsf{play}(t_1)(w')]$$
$$[\![\text{NegP}]\!] = \lambda w.\neg\exists w'[w' \in OS_2(OS_1(\bigcap(MB(w)))) \wedge \mathsf{play}(t_1)(w')]$$
$$[\![\text{TP}_2]\!] = \lambda x.\lambda w.\neg\exists w'[w' \in OS_2(OS_1(\bigcap(MB(w)))) \wedge \mathsf{play}(x)(w')]$$
$$[\![\text{TP}_3]\!] = \lambda w.\neg\exists w'[w' \in OS_2(OS_1(\bigcap(MB(w)))) \wedge \mathsf{play}(2\mathsf{sg})(w')]$$
$$[\![\text{CP}]\!] = \lambda w.\neg\exists w'[w' \in OS_1(\bigcap(MB(w))) \wedge \mathsf{play}(2\mathsf{sg})(w')]$$

As an illustration of how this works, here is a schematic exhaustificatio derivation for negated *anga* with subdomain alternatives, and toy domains $D(paswa) = \{w_1, w_2, w_3, w_4, w_5\}$ and $D(anga) = \{w_1, w_2, w_3\}$:

(47) a. $ALT[\![\neg anga(p)]\!] = \{\neg\forall w'[w' \in \{w_1, w_2, w_3, w_4, w_5\} \rightarrow p(w')]$,
$\neg\exists w'[w' \in \{w_1, w_2, w_3\} \& p(w')]$
$\neg\exists w'[w' \in \{w_1, w_2\} \& p(w')]$
$\neg\exists w'[w' \in \{w_1, w_3\} \& p(w')]$
$\neg\exists w'[w' \in \{w_2, w_3\} \& p(w')]$
$\neg\exists w'[w' \in \{w_1\} \& p(w')]$
$\neg\exists w'[w' \in \{w_2\} \& p(w')]$
$\neg\exists w'[w' \in \{w_3\} \& p(w')]\}$

b. $[\![exh^{IE}(anga(p))]\!] = \exists w'[w' \in \{w_1, w_2, w_3\} \& p(w')]$

c. $[\![exh^{IE+II}(anga(p))]\!]$: $\exists w'[w' \in \{w_1, w_2, w_3\} \& p(w')]$
$\wedge\neg\forall w'[w' \in \{w_1, w_2, w_3, w_4, w_5\} \rightarrow p(w')]$
$\wedge\neg\exists w'[w' \in \{w_1, w_2, w_3\} \& p(w')]$
$\wedge\neg\exists w'[w' \in \{w_1, w_2\} \& p(w')]$
$\wedge\neg\exists w'[w' \in \{w_1, w_3\} \& p(w')]$
$\wedge\neg\exists w'[w' \in \{w_2, w_3\} \& p(w')]$
$\wedge\neg\exists w'[w' \in \{w_1\} \& p(w')]$
$\wedge\neg\exists w'[w' \in \{w_2\} \& p(w')]$
$\wedge\exists w'[w' \in \{w_3\} \& p(w')]$
$\equiv \forall w[w' \in \{w_1, w_2, w_3\} \rightarrow p(w')]$

Even just looking at the alternative set in 47a shows that exhaustification won't be able to adjust negated *anga*-sentences at all: the semantics of negated possibility already entail all of the alternatives, so there are none that are innocently excludable, and in fact all of them are innocently includable, though they don't alter the meaning of the original expression at all.

6. Typological implications

The connection between gaps in the modal paradigm and variable-force modality, which seemed natural enough, now has a mechanical explanation, with exhaustification. Modals are variable-force by mechanisms of comparison on lexical scales. It's just that when there's nothing on that scale, you can get strengthening to fill that spot, and when there is something there, you strengthen the weaker terms to assert that competitor's negation. One is a scaleless implicature, the other a scalar implicature, but they come from the same operation, just on different scales.

As a result, we can expect to find finer gradations in variable-force modals than we previously thought. More specifically, they should be able to be as fine-grained as scalar implicatures elsewhere, modulo meanings specific to those categories and how they are distinct from modals in the first place. At the same time, however, clausemate negation should quite consistently prevent an existential modal from strengthening to a necessity modal of any type, again by simple virtue of the mechanisms themselves at play.

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