Cumulative readings in focus contexts[1]

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Abstract. Cumulative readings of sentences containing plurals involve universal inferences. Under a classical approach, those inferences are introduced by the predicate which takes the plurals as arguments (e.g. Sternefeld 1998, Beck & Sauerland 2000). Recent work, however, has proposed that plurals are associated with a core existential semantics, and that universal inferences are introduced external to the predication as a grammatical implicature (e.g. Magri 2014, Bar-Lev 2018, 2020, Chatain 2022). We provide new support for the implicature view by establishing that cumulativity patterns like an implicature in the way it interacts with the focus operator *only*. When *only* occurs with a scalar term, implicatures are in general observed just in the *presupposition*, and we show that cumulativity is likewise observed just as presuppositional in *only* data. While the data follow for free from the implicature analysis, the predicate analysis does not directly predict the observed readings.

Keywords: cumulativity, focus, *only*, exhaustification, presuppositional implicature

1. Introduction

Statements containing multiple plurals can exhibit *cumulative* readings, involving two universal inferences (Kroch 1974, Scha 1981). The sentence in (1), for instance, has a reading whose truth conditions are given by the conjunction of the two inferences in (2), that each guide saw Amy or Bani, (2a), and that each of Amy and Bani was seen by a guide, (2b).

(1) The guides saw Amy and Bani.

(2) a. $\Rightarrow \forall y \leq_{AT} G [\ \exists x \leq_{AT} A+B [\ saw(y,x)\ ]\ ]$
b. $\Rightarrow \forall x \leq_{AT} A+B [\ \exists y \leq_{AT} G [\ saw(y,x)\ ]\ ]$

The traditional approach derives cumulativity in some way from the *predicate*. Building on Krifka (1986), Sternefeld (1998) and Beck & Sauerland (2000) propose that a covert operator ** applies to the predicate and introduces universal inferences. (1) has the LF in (3), where ** merges with *saw*. The lexical entry for ** is given in (4). Another option would be to encode universal quantification directly in the lexical entry for *saw* itself (e.g. Scha 1981). Throughout the paper, we will illustrate the predicate approach with the ** operator.[2]

(3) **Predicate analysis**
[[the guides] [**saw [Amy and Bani]]]

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[2]Instead of encoding cumulativity in the predicate, another possibility is to formulate the composition rules by which the predicate composes with its arguments to output cumulative truth-conditions for the predication. See Schmitt (2019, 2020), Haslinger & Schmitt (2020), and related work.

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(4) **Defining ****
$[\![**]\!] = \lambda f_{e,et} . \lambda X . \lambda Y . \forall y \leq_{AT} Y [\exists x \leq_{AT} X [f(y,x)]] \wedge \forall x \leq_{AT} X [\exists y \leq_{AT} Y [f(y,x)]]$

Bar-Lev (2018, 2020), however, has put forward a different analysis. Following Magri (2014), Bar-Lev takes plurals to be associated with a core existential semantics within the predication, so the predication in (1) says just that *a* guide saw Amy *or* Bani. Cumulative inferences are then introduced as a grammatical *implicature* by an exhaustivity operator external to the predication (see also Chatain 2022). The LF for (1) can be revised as in (5). Each plural merges with an existential distributivity operator, and Exh takes higher scope.

(5) **Implicature analysis**
[Exh [[$Op_{\exists}$ the guides] [saw [$Op_{\exists}$ Amy and Bani]]]]

Our aim is to provide novel evidence for the implicature analysis based on how cumulativity interacts with a separate operator in the clause: the exclusive focus particle, *only*. Empirically, we will establish a contrast between *presupposition* and *assertion* in *only* data. Cumulativity will be observed in the presupposition, but in the assertion, *only* will behave as if it quantifies over existential alternatives. As we will see, the implicature analysis has the necessary features to reconcile the conflicting meaning components.

To derive the target assertion, *only* will require compositional access to an existential meaning, which is possible in the LF in (5), but not (3). *Only* is a propositional operator, so must take scope over the predication, and that expresses an existential meaning only in (5). Concretely, we pursue an analysis based on the configuration in (6), where *only* occurs at an *intermediate* scope position: above the predication, but beneath the source of cumulativity, Exh. Moreover, it will follow from independent results in the literature on implicatures that Exh will introduce cumulativity just into the presupposition in construction with *only*. In this way, cumulativity will pattern exactly like an implicature in the *only* data.

(6) **Skeletal structure: Exh $>$ *only* $>$ $\exists$**
[Exh [only [[$Op_{\exists}$ PL] [PRED [$Op_{\exists}$ PL]]]]]

The discussion will be organized as follows. In Section 2, we present our central puzzle, and show that the data are not directly predicted in the predicate analysis. In Sections 3 and 4, we present our solution based on the implicature analysis. In Section 5, we identify a potential alternate solution, based on the predicate analysis, but with revised trivalent meanings for ** and *only*. We suggest, however, that this avenue does not have the independent support that motivates the implicature approach to our data.

2. Puzzle

Our central data point is (7). *Only* is introduced into the earlier example in (2), and its associate is the object plurality, now under focus. The relevant interpretation is given by (8) and (9). *Only* yields a presupposition that its prejacent is true, and cumulativity can still be observed in the presupposition. That is, (7) exhibits a reading on which it presupposes that *each* guide saw Amy or Bani, and that *each* of Amy and Bani was seen by a guide, as in (8). At the same time, the intuited exclusive inference is that *no* guide saw anyone other than Amy or Bani. Assuming

that the only other salient individual is Carl, the assertion is (9).

(7) The guides only saw [Amy and Bani]$_F$.

(8) **Presupposition (cumulativity)**
$\forall y \leq_{AT} G$ [$\exists x \leq_{AT}$ A+B [saw(y,x)]] $\wedge$ $\forall x \leq_{AT}$ A+B [$\exists y \leq_{AT} G$ [saw(y,x)]]
(the guides saw Amy and Bani between them)

(9) **Assertion (exclusive inference)**
$\neg\exists y \leq_{AT} G$ [saw(y, Carl)]
(no guide saw Carl)

The challenge will be to capture both cumulativity in the presupposition, and the exclusive inference, together. To develop the puzzle in the remainder of this section, we will make explicit how the example in (7) would be analyzed in the predicate analysis, and demonstrate that the target exclusive inference is not readily predicted in that approach.

2.1. Challenge for the predicate analysis

If cumulativity is sourced to the ** operator, (7) would have the LF in (10), where ** merges with *saw*. *Only* takes scope over the predication, and therefore over **.

(10) **LF for (7): *only* > ****
[only [[the guides] [**saw [Amy and Bani]$_F$]]]

At this point, we assume the classical lexical entry for *only* in (11). *Only* presupposes that its prejacent is true, and negates any alternative that is not entailed by the prejacent. The target presupposition follows straightforwardly from *only*'s prejacent presupposition. Because *only* takes scope over **, its complement expresses the cumulative meaning in (8), and that constitutes the presupposed prejacent of *only*.

(11) **Defining *only* (first version)**
$[\![\text{only}]\!](C) = \lambda p\,.\,\lambda w : p(w)\,.\,\forall q \in C$ [$p \not\subseteq q \rightarrow \neg q(w)$]

But, what is (10) predicted to assert? To assess the meaning, it is necessary to determine the alternative set, C, over which *only* quantifies. We assume with Katzir (2007) and Fox & Katzir (2011) that alternatives are computed as syntactic objects by replacing the focused constituent with elements of equal or lesser structural complexity. We will confine our attention to those alternatives where the conjunction, *Amy and Bani*, is replaced with a simplex DP. In (12), the conjunction is replaced with one of its component conjuncts. In (13), the conjunction is replaced by a separate element, the DP *Carl*. We refer to (12a) and (12b) as the *internal* alternatives, and (13) as the *external* alternative.

(12) **Internal alternatives**
a. [[the guides] [**saw Amy]]
b. [[the guides] [**saw Bani]]

(13) **External alternative**
[[the guides] [**saw Carl]]

Because ** is present in the complement of *only* and is not focused, it is also present within each of the alternatives. Each alternative thus carries a *universal* entailment. (12a) conveys that *each* guide saw Amy, and likewise for the other alternatives, as in (14) and (15).

(14) **Internal alternatives**
a. $\forall y \leq_{AT} G$ [saw(y, Amy)]
b. $\forall y \leq_{AT} G$ [saw(y, Bani)]

(15) **External alternative**
$\forall y \leq_{AT} G$ [saw(y, Carl)]

Herein lies the challenge: if *only* negates universal alternatives, an off target assertion results. Two problems arise, stemming from the external and internal alternatives.

First, the external alternative gives rise to a *too weak* inference. Negating (15) introduces just the entailment that *not every* guide saw Carl, as in (16). Suppose there are two guides. The derived entailment correctly predicts that the target sentence in (7) should be judged false at a world like w_1 in (17), where both guides did see Carl. However, it wrongly predicts that the sentence should be judged *true* at a world like w_1' in (18), where just one of the guides saw Carl. Since the sentence is clearly false at w_1', as well, it must carry a stronger entailment that *no* guide saw Carl, as noted earlier in (9).[3]

(16) **Problem 1: predicted entailment (too weak)**
$\neg\forall y \leq_{AT} G$ [saw(y, Carl)]

(17) **Facts at w_1**
a. Guide 1 → Amy, Carl
b. Guide 2 → Bani, Carl

(18) **Facts at w_1'**
a. Guide 1 → Amy
b. Guide 2 → Bani, Carl

The internal alternatives, on the other hand, result in inferences which are *too strong*. Neither (14a) nor (14b) is entailed by the cumulative proposition in (8), so *only* would negate both to yield the entailments in (19): that not every guide saw Amy, and that not every guide saw Bani. These are unproblematic in a world where one guide saw Amy and the other Bani, as in w_2 in (20). The target sentence is intuitively true, and both entailments in (19) are verified. Yet, a problem arises in a world like w_2' in (21), where one guide saw Amy, and the other saw both Amy and Bani. The sentence is predicted to be *false*, because the entailment in (19a) fails. Yet, the sentence is still true. (7) does not intuitively place any upper limit on how many of the guides saw Amy or Bani. The desideratum, then, is to derive no negative inferences at all about Amy and Bani from the internal alternatives, contrary to the prediction.

(19) **Problem 2: predicted entailments (too strong)**
a. $\neg\forall y \leq_{AT} G$ [saw(y, Amy)]
b. $\neg\forall y \leq_{AT} G$ [saw(y, Bani)]

(20) **Facts at w_2**
a. Guide 1 → Amy
b. Guide 2 → Bani

(21) **Facts at w_2'**
a. Guide 1 → Amy
b. Guide 2 → Amy, Bani

Hence, the predicate approach readily derives the cumulative presupposition of (7) from **, but ** then propagates through the focus alternatives, resulting in universal statements which yield problematic inferences when negated in the assertive component of *only*.

[3]The presentation of Problem 1 would be affected if additional alternatives were considered, but the core problem would remain, given Katzir's algorithm for alternative computation. See the Appendix for discussion.

2.2. Sharpening the puzzle

Before discussing our proposed solution, it will be useful to sharpen the puzzle further. As we will discuss in detail in the next section, both Problems 1 and 2 would resolve if *only* negated *existential* alternatives, rather than universal. A clue in this direction comes from the form of the actually observed exclusive entailment. The target inference that *no* guide saw Carl, repeated in (22), takes the form of a negative existential statement.

(22) **Recall: target assertion**
$\neg\exists y \leq_{AT} G$ [saw(y, Carl)]

The core tension, then, is between the presupposition of the *only* sentence, and its assertion. In the presupposition, cumulativity is observed, which involves *universal* inferences, and at the same time, the negated alternatives pattern as *existential* in the assertion. How can the presupposition and assertion be reconciled with one another?

As previewed, our strategy will be to follow not the predicate approach, but the implicature approach, and to establish that the requisite cut between presupposition and assertion follows for free in that perspective. The LF, recall, will be (23). Because the plurals in the scope of *only* are interpreted as existential in the implicature approach, the focus alternatives will also be existential, leading to the target assertion. Later in the composition, Exh introduces cumulativity as an implicature just into the presupposition.

(23) **Skeletal structure: Exh $>$ *only* $>\exists$**
[Exh [only [[$Op_{\exists}$ PL] [PRED [$Op_{\exists}$ PL]]]]]

We present the analysis in two steps. We first show how the target assertion derives, and then consider the contribution of Exh and present a derivation of the cumulativity presupposition based on mechanisms independently needed for implicatures generally.

3. Step 1: Existential alternatives

Following Magri (2014) and Bar-Lev (2018, 2020), we take plurals to be associated with a core existential meaning. In analyzing (7), we for now focus on the fragment of the LF in (24a). ** is removed from the predication and, in the composition we assume, each plural merges with a covert existential distributivity operator, $Op_{\exists}$. As defined in (24b), $Op_{\exists}$ applies to an entity, and existentially quantifies over its atoms. The complement of *only* no longer carries any universal entailments, but rather expresses just the existential meaning that *a* guide saw Amy *or* Bani, as in (25). That constitutes the revised prejacent of *only*.

(7) The guides only saw [Amy and Bani]$_F$.

(24) **LF for (7): *only* $>\exists$ (partial)**
a. [only [[$Op_{\exists}$ [the guides]] [saw [$Op_{\exists}$ [Amy and Bani]$_F$]]]]
b. $[\![Op_{\exists}]\!] = \lambda X\,.\,\lambda f_{et}\,.\,\exists x \leq_{AT} X$ [f(x)]

(25) **Prejacent of *only***
$\exists y \leq_{AT} G$ [$\exists x \leq_{AT}$ A+B [saw(y,x)]]

Now, what alternatives does *only* quantify over? Replacing the object conjunction with *Amy* and *Bani* respectively yields the structures in (26) for the internal alternatives, and the external alternative is derived from the structure in (27), with *Carl* as the object.

(26) **Internal alternatives**
a. [[$Op_{\exists}$ [the guides]] [saw [$Op_{\exists}$ Amy]]]
b. [[$Op_{\exists}$ [the guides]] [saw [$Op_{\exists}$ Bani]]]

(27) **External alternative**
[[$Op_{\exists}$ [the guides]] [saw [$Op_{\exists}$ Carl]]]

Each structure contains the existential operator, and therefore expresses an existential meaning. (26a) says that *a* guide saw Amy, and likewise for the other alternatives, as shown. With the prejacent of *only* existential, the focus alternatives are existential in kind.

(28) **Internal alternatives**
a. $\exists y \leq_{AT} G$ [saw(y, Amy)]
b. $\exists y \leq_{AT} G$ [saw(y, Bani)]

(29) **External alternative**
$\exists y \leq_{AT} G$ [saw(y, Carl)]

The two problems raised in the preceding section are now readily solved to derive the target assertion. Problem 1 resolves immediately. Before, negating the universal external alternative resulted in a too weak inference that *not every* guide saw Carl. But, with the alternatives now existential, negating the external alternative delivers precisely the observed strong inference that *no* guide saw Carl, as flagged in (30).

(30) **Problem 1 solved: Predicted entailment (target)**
$\neg\exists y \leq_{AT} G$ [saw(y, Carl)]

At first glance, though, there does still seem to be a problem with the internal alternatives. Because the existential internal alternatives in (28) are weaker than their universal counterparts in (14), negating them should yield *even stronger* negative inferences about Amy and Bani. Negating (28a) and (28b) would yield the unwanted inferences that *no* guide saw Amy, as in (31a), and that *no* guide saw Bani, as in (31b).

(31) **New problem 2: Predicted entailments (too strong?)**
a. $\neg\exists y \leq_{AT} G$ [saw(y, Amy)]
b. $\neg\exists y \leq_{AT} G$ [saw(y, Bani)]

Yet, the new Problem 2 can be avoided with an independent revision to *only*. The inferences in (31) are *so* strong that, together, they would contradict the prejacent of *only*. The prejacent in (25), recall, says that a guide did see Amy or Bani. Accordingly, if the internal alternatives were negated, the LF in (24a) would presuppose that a guide saw Amy or Bani and, at the same time, assert that no guide saw either. Fox (2007) proposed that *only* avoids creating such contradictions when it computes its assertion. The lexical entry is updated in (32).

(32) **Defining *only* (final version)**
$[\![\text{only}]\!](C) = \lambda p\,.\,\lambda w : p(w)\,.\,\boxed{\forall q \in IE(C)(p)\,[\,\neg q(w)\,]}$

Rather than negating any alternative non-weaker than the prejacent, *only* is restricted to negate only those alternatives that are *innocently excludable*. The innocently excludable alternatives are, in effect, those which can be jointly negated without contradicting the prejacent, as per (33). The internal alternatives are *not* innocently excludable, and so will not be negated. No negative inferences about Amy and Bani are derived — solving Problem 2.

(33) **Defining innocent exclusion**
$IE(C)(p) = \cap \{\, C' \subseteq C : C'$ is a maximal subset of C s.t.
$\{\, \neg q : q \in C' \,\} \cup \{p\}$ is consistent $\}$

If *only* has access to existential alternatives, then, the assertion is correctly predicted. Just the external alternative is innocently excludable, and negating it derives the target inference that no guide saw Carl. With the assertion in place, the next step is to re-introduce the universal inferences characteristic of cumulativity into the presupposition.

4. Step 2: Re-sourcing cumulativity

By itself, the prejacent presupposition of *only* no longer captures cumulativity. As noted, if (7) has the LF in (24a), the presupposed prejacent conveys just the existential meaning that *a* guide saw Amy *or* Bani, as repeated in (34). The desideratum is to strengthen the presupposition so that cumulativity is derived. As we will see, the necessary strengthening can be achieved based on independent mechanisms if cumulativity is in general analyzed as a grammatical implicature, following Bar-Lev (2018, 2020) and Chatain (2022).

(34) **Recall: Prejacent of *only* (presupposed)**
$\exists y \leq_{AT} G\,[\,\exists x \leq_{AT} A{+}B\,[\,saw(y,x)\,]\,]$

As a point of departure, it is useful to bring out a key empirical parallel. It has been observed that, when *only* occurs with a scalar item in its scope, implicatures do not arise in the negated alternatives — but are attested in the presupposition. Baseline data can be constructed with *free choice*. A basic case of free choice, first without *only*, is (35a). The disjunction is strengthened to yield the stronger conjunctive inference in (35b) as an implicature.

(35) **Free choice disjunction**
a. Amy can have cake or ice cream.
b. *Implicature:* Amy can have cake *and* she can have ice cream.

Alxatib (2014, 2020) discusses counterpart data with *only*, such as (36), which is naturally interpreted as in (37). *Only* negates alternatives of the form *x can have cake or ice cream*, with the disjunction interpreted literally, to derive the assertion in (37b). But, free choice is still observed in the presupposition in (37a). Hence, free choice is derived as a *presuppositional implicature* in data with *only* (for related cases, see e.g. Gajewski & Sharvit 2012, Spector & Sudo 2017, Marty & Romoli 2021, Del Pinal et al. 2021).

(36) Only Amy_F can have cake or ice cream.

(37) a. **Presupposition (free choice)**
Amy can have cake and she can have ice cream

b. **Assertion (no free choice)**
No one else can have cake or ice cream.

The pattern in (37) with free choice closely mirrors what we have observed with cumulativity: that, in an example with *only*, cumulativity arises just in the presupposition. If cumulativity is an implicature, parallel to free choice, cumulativity will be introduced into the presupposition in our case by whatever mechanism yields free choice in (36).

In the remainder of this section, we will start by considering how cumulativity can be unified with free choice as an implicature in data without *only* (in Section 4.1), and then we will return to the *only* data where the implicature is presuppositional (in Section 4.2).

4.1. Unifying cumulativity and free choice

To start, then, what is the mechanism for free choice strengthening in the basic case, repeated in (38a)? Fox (2007) and Bar-Lev & Fox (2020) credit free choice to a covert exhaustivity operator. For illustration, we will follow the implementation in Bar-Lev & Fox (2020). (38a) is assigned the LF in (38b), with Exh taking scope over the modal and disjunction.

(38) **LF for free choice: *only* $>\vee$**
a. Amy can have cake or ice cream.
b. [Exh [can [Amy have cake or ice cream]]]

To derive free choice, Bar-Lev & Fox propose that Exh is formulated as in (39), based not only on *innocent exclusion*, but also *innocent inclusion*. Exh asserts that all innocently excludable alternatives to its prejacent are false, and asserts not only its prejacent, but all alternatives which are innocently includable. In effect, the innocently includable alternatives are those that are not innocently excludable, and can be jointly asserted to be true without contradiction, as in (40). Free choice is derived as an implicature based on innocent inclusion.

(39) **Defining Exh**
$[\![\text{Exh}]\!](\text{C}) = \lambda\text{p} \,.\, \lambda\text{w} \,.\, \forall\text{q} \in \text{IE(C)(p)} \,[\, \neg\text{q(w)} \,] \wedge \boxed{\forall\text{q'} \in \text{II(C)(p)} \,[\, \text{q'(w)} \,]}$

(40) **Defining innocent inclusion**
$\text{II(C)(p)} = \cap\{\, \text{C'} \subseteq \text{C} : \text{C'}$ is a maximal subset of C s.t.
$\{\text{r} : \text{r} \in \text{C'}\} \cup \{\text{p}\} \cup \{\neg\text{q} : \text{q} \in \text{IE(C)(p)}\}$ is consistent $\}$

The prejacent of Exh in (38b) conveys that it is allowed for Amy to have at least one of cake or ice cream, as in (41). Assuming that the individual disjuncts are alternatives to the disjunction, the alternatives that Exh quantifies over will include (42a) and (42b). For our purposes, it will be sufficient to consider just these two alternatives.

(41) **Prejacent of Exh**
◊ [have(Amy, cake) ∨ have(Amy, ice cream)]

(42) **Crucial alternatives**
a. ◊ [have(Amy, cake)] (II)
b. ◊ [have(Amy, ice cream)] (II)

To assess the contribution of Exh, the first step is to determine whether the alternatives are innocently excludable — and they are not. Negating (42a) and (42b) would yield entailments that Amy is not allowed to have cake and that she is not allowed to have ice cream, and these jointly contradict the prejacent in (41). The alternatives are, however, innocently includable, and Exh will therefore assert them, deriving the component entailments of free choice: that Amy *is* allowed to have cake and that she *is* allowed to have ice cream.

As the derivation of free choice illustrates, innocent inclusion is a mechanism by which a weak existential meaning (like disjunction) can be strengthened to derive a strong universal implicature (in the case of free choice, conjunctive). Bar-Lev (2018, 2020) proposes that the weak existential meaning of plurals can likewise be strengthened to yield observed universal inferences via innocent inclusion-based implicatures.

Continuing to set aside *only*, consider the sentence in (1), from the outset of the paper, and its associated LF under the implicature analysis, repeated in (43). The predication conveys just the familiar existential proposition that a guide saw Amy or Bani, per (44). That constitutes the prejacent of Exh, which can strengthen (44) to deliver cumulativity.

(1) The guides saw Amy and Bani.

(43) **LF for (1): Exh $>$ ∃**
[Exh [[$Op_{\exists}$ [the guides]] [saw [$Op_{\exists}$ [Amy and Bani]]]]]

(44) **Prejacent of Exh**
$\exists y \leq_{AT} G$ [$\exists x \leq_{AT}$ A+B [saw(y,x)]]

We assume that Exh associates with both the subject plural and the object plural within its scope, and quantifies over *subdomain* alternatives. We take it that the subdomain alternatives based on the subject and the object can be computed independently from one another. To illustrate concretely, the subdomain alternatives based on the object are as in (45). The object plurality in the prejacent, Amy+Bani, is replaced by one of its atomic sub-parts, Amy in (44a) and Bani in (44b). At the same time, the subject plurality, G, is held constant. (44a) says that a guide saw Amy, and (44b) says that a guide saw Bani.

(45) **Sub-domain alternatives (object)**
a. $\exists y \leq_{AT} G$ [saw(y, Amy)] (II)
b. $\exists y \leq_{AT} G$ [saw(y, Bani)]] (II)

These alternatives are not innocently excludable. Negating (45a) and (45b) would introduce entailments that no guide saw Amy and that no guide saw Bani, which jointly contradict the prejacent in (44). But, crucially, they are innocently includable, and asserting them yields the

first component of cumulativity. (45a) and (45b) together entail that each of Amy and Bani was seen by a guide, as in (46). The object existential in the prejacent is strengthened to a wide scope universal in the derived inference.

(46) **Universal inference derived**
(45a) $\wedge$ (45b) $\Leftrightarrow$ $\forall$x $\leq_{AT}$ A+B [$\exists$y $\leq_{AT}$ G [saw(y,x)]]

The second universal inference stems from the subdomain alternatives based on the subject, given in (47). Now, the subject plurality, G, is replaced by one of its atomic sub-parts, while the object plurality is held constant with its value in the prejacent, Amy+Bani. (47a) says that Guide 1 saw Amy or Bani, and (47b) says that Guide 2 saw Amy or Bani.

(47) **Sub-domain alternatives (subject)**
a. $\exists$x $\leq_{AT}$ A+B [saw(Guide 1, x)]] (II)
b. $\exists$x $\leq_{AT}$ A+B [saw(Guide 2, x)]] (II)

Once again, these alternatives are not innocently excludable, but are innocently includable, and asserting them yields the entailment in (48), that each guide saw Amy or Bani. Here, the subject existential in the prejacent is strengthened to a wide scope universal. Taking the two inferences in (46) and (48) together, cumulativity is captured in full.

(48) **Universal inference derived**
(47a) $\wedge$ (47b) $\Leftrightarrow$ $\forall$y $\leq_{AT}$ G [$\exists$x $\leq_{AT}$ A+B [saw(y,x)]]

To obtain cumulativity, it is important that just one of the subject and the object is replaced at a time in the sub-domain alternatives. If both were replaced at once, the alternative set would include additional propositions, such as (49). More generally, Exh would quantify over the set of all propositions saw(y,x), where y is a sub-part of G and x is a sub-part of Amy+Bani. All such alternatives would be innocently includable, and asserting them would convey that *each* guide saw *each* of Amy and Bani. This reading, involving double distributivity, may be available in (1), but is stronger than the target cumulative reading.

(49) **Additional alternatives?**
saw(Guide 1, Amy)

The derivation we have presented departs in detail from the ones in prior work, which assume different ways of restricting the alternative set to avoid double distributivity.[4] The preceding path, however, is sufficient for illustration. What is crucial is that cumulativity is credited to the same mechanism as free choice. Both are implicatures, computed by Exh from a weak existential input on the basis of innocent inclusion. Now that unification is in place, we are ready to return to the core data with *only*.

[4]For discussion, see Bar-Lev (2018), Appendix A, and Bar-Lev (2020), Section 8.2. Chatain (2022) also argues for an implicature analysis of cumulativity, and puts forward a derivation where there would be two instances of exhaustification in (1), together leading to the two universal inferences.

4.2. Strengthening in the presupposition

To re-iterate, the common desideratum for cumulativity and free choice in the *only* data is for strengthening to take place within the presupposition. Because cumulativity has the same source as free choice, cumulativity will be introduced into the presupposition in (50) through the same mechanisms independently proposed for free choice in (51).

(50) a. The guides only saw [Amy and Bani]$_F$.
b. *Presupposition:* the guides saw Amy and Bani between them

(51) a. Only Amy$_F$ can have cake or ice cream.
b. *Presupposition:* Amy can have cake and she can have ice cream

Following Alxatib (2020), we adopt the LF in (52) for the free choice case. Here, the covert Exh is introduced with highest scope, above *only*.[5] The crucial feature of this configuration is that Exh takes scope over a presupposition trigger, since *only*, as defined in (32), triggers a presupposition that its prejacent is true. The question is, then: what does Exh contribute when its prejacent is presuppositional?

(52) **LF for (36): Exh $>$ *only* $>$ $\vee$**
[$\boxed{\text{Exh}}$ [only [can [Amy$_F$ have cake or ice cream]]]]

Alxatib argues that Exh itself triggers a presupposition, one which is obtained by exhaustifying the *presupposition* of its prejacent relative to the *presuppositions* of alternatives (after Magri 2009, Gajewski & Sharvit 2012, Marty 2017, Marty & Romoli 2021). For exposition, we can take Exh to encode the definedness condition in (53) (Alxatib's 'simple' formulation). Dom applies to a proposition to return its presupposition.

(53) **Presupposition of Exh**
$w \in \text{Dom}([\![\text{Exh}]\!](C)(p))$ only if $\forall q \in \text{IE}(\{\text{Dom}(q) : q \in C\})(\text{Dom}(p))$ [$\neg q(w)$] $\wedge$
$\forall q' \in \text{II}(\{\text{Dom}(q) : q \in C\})(\text{Dom}(p))$ [$q'(w)$]

Free choice is then derived in the same way as in the preceding subsection — except that the computation now takes place within the presupposition. Due to *only*, Exh's complement, isolated in (54a), presupposes the proposition expressed by *only*'s complement, that it's allowed for Amy to have at least one of cake or ice cream, as in (54b).

(54) **Prejacent of Exh**
a. [only [can [Amy$_F$ have cake or ice cream]]]
b. $\Diamond$ [have(Amy, cake) $\vee$ have(Amy, ice cream)] $= \text{Dom}([\![(54a)]\!])$

The alternatives which arise from simplifying the disjunction are given by (55a) and (55b). Because Exh scopes over *only*, the alternatives each contain *only*, and thus presuppose (56a) and (56b). Given (53), the global presupposition results from exhaustifying (54b) relative to a set which includes (56a) and (56b). As before, (56a) and (56b) are not innocently excludable,

[5] Even though *only* precedes the subject DP in the surface string, *only* is shown as a propositional operator in (52). The discussion would not be substantially affected if *only* composed directly with the DP instead.

but are innocently includable, and the global presupposition will therefore entail them. The strengthened presupposition, entailing (56a) and (56b), conveys free choice.

(55) **Crucial alternatives**

a. [only [can [Amy$_F$ have cake]]]

b. [only [can [Amy$_F$ have ice cream]]]

(56) **Presuppositions of alternatives**

a. $\Diamond$ [have(Amy, cake)] $= \text{Dom}([\![(55a)]\!])$; II

b. $\Diamond$ [have(Amy, ice cream)] $= \text{Dom}([\![(55b)]\!])$; II

The presuppositional cumulativity inference in (7) is obtained in parallel. We arrive at the complete LF for our case in (57), which adds a wide scope Exh to the fragment from Step 1. With Exh taking scope over *only*, innocent inclusion-based strengthening will again take place within the presupposition. The complement of Exh in (58a) carries just *only*'s weak prejacent presupposition that a guide saw Amy or Bani, as repeated in (58b).

(57) **LF for (7): Exh $>$ *only* $>$ $\exists$ (final)**

[$\boxed{\text{Exh}}$ [only [[Op$_\exists$ [the guides]] [saw [Op$_\exists$ [Amy and Bani]$_F$]]]]]

(58) **Prejacent of Exh**

a. [only [[Op$_\exists$ [the guides]] [saw [Op$_\exists$ [Amy and Bani]$_F$]]]]

b. $\exists y \leq_{AT} G$ [$\exists x \leq_{AT}$ A+B [saw(y,x)]] $= \text{Dom}([\![(58a)]\!])$

Whereas *only* associates with the focused object, Exh associates with the subject and object plurals, and quantifies over independently computed sub-domain alternatives, just as in the preceding subsection. In the current configuration, the sub-domain alternatives based on the object are given by (59a) and (59b), each containing *only*. Within the alternatives, *only* yields the prejacent presuppositions in (60a) and (60b). Relative to (58b), (60a) and (60b) are, again, not innocently excludable, but are innocently includable, and the global presupposition will therefore come to entail that each of Amy and Bani was seen by a guide.

(59) **Sub-domain alternatives (object)**

a. [only [[Op$_\exists$ [the guides]] [saw [Op$_\exists$ Amy$_F$]]]]

b. [only [[Op$_\exists$ [the guides]] [saw [Op$_\exists$ Bani$_F$]]]]

(60) **Presuppositions of alternatives**

a. $\exists y \leq_{AT} G$ [saw(y, Amy)] $= \text{Dom}([\![(59a)]\!])$; II

b. $\exists y \leq_{AT} G$ [saw(y, Bani)] $= \text{Dom}([\![(59b)]\!])$; II

In a parallel way, the sub-domain alternatives based on the subject are given by (61a) and (61b), and within these, *only* triggers the presuppositions in (62). These are again innocently includable relative to (58b), and the global presupposition thus comes to carry the additional entailment that each guide saw Amy or Bani, deriving cumulativity in full.[6]

[6]While Exh affects the presupposition in (57), it will have no substantial impact on the assertion. With respect to the assertion, each of the alternatives is either weaker than the prejacent, hence not innocently excludable, or its

(61) **Sub-domain alternatives (subject)**

a. [only [[$\text{Op}_{\exists}$ Guide 1] [saw [$\text{Op}_{\exists}$ [Amy and Bani]$_F$]]]]

b. [only [[$\text{Op}_{\exists}$ Guide 2] [saw [$\text{Op}_{\exists}$ [Amy and Bani]$_F$]]]]

(62) **Presuppositions of alternatives**

a. $\exists x \leq_{AT}$ A+B [saw(Guide 1, x)]] $= \text{Dom}([\![(61a)]\!])$; II

b. $\exists x \leq_{AT}$ A+B [saw(Guide 2, x)]] $= \text{Dom}([\![(61b)]\!])$; II

Hence, the weak existential presupposition that *only* itself triggers within the complement of Exh is strengthened by Exh to capture cumulativity in the global presupposition. Presupposed cumulativity follows from the same mechanisms as presupposed free choice.[7]

4.3. Further prediction

Presuppositional implicatures can be observed not just with *only*, but with a range of other presupposition triggers. As noted by Gajewski & Sharvit (2012), whenever an operator is downward monotonic in its assertion, but not its presupposition, implicatures arise just in the presupposition. The parallel between cumulativity and free choice replicates in further data. Configurations with *negative factives* offer a case in point. Consider first the baseline in (63), due to Marty & Romoli (2021), which involves free choice. (63) is naturally interpreted to presuppose (64a) and assert (64b). Free choice is observed just in (64a).

(63) Amy is unaware that we can have cake or ice cream.

(64) a. **Presupposition (free choice)**
We can have cake and we can have ice cream

b. **Assertion (no free choice)**
Amy doesn't know either that we can have cake or ice cream.

Marty & Romoli propose the LF in (65), with Exh taking scope over *unaware*. A lexical entry for *unaware* is in (66). *Unaware* operates on the literal meaning of the disjunction to assert (64b). Being factive, *unaware*, like *only*, also introduces a presupposition that its prejacent is true. In (65), then, *unaware* triggers the presupposition that it is allowed that we have at least one of cake or ice cream. By exhaustifying that relative to the presuppositions of alternatives, Exh yields presuppositional free choice — as in the earlier derivation with *only*.[8]

negation is already entailed by the presupposition that Exh introduces. We omit details for space.

[7]Rather than taking *only* to co-occur with Exh, Bar-Lev & Fox (2020) re-define the lexical entry for *only* itself to compute II-based implicatures within the presupposition. Note, however, that in (36), the disjunction giving rise to presuppositional free choice is non-focused, as is the subject plurality in (7), which plays a role in the computation of cumulativity. Alxatib (2020) shows that, in general, problematic readings would be derived if *only* could associate with non-focused material (see his Section 3.3). This provides one motivation to source presuppositional implicatures with *only* to a separate Exh.

[8]Del Pinal et al. (2021) have recently suggested a different LF for (63), where Exh takes scope under *unaware*. In their analysis, Exh is defined to assert just its prejacent, while implicatures are always presupposed. Exh's presupposition projects to capture (64a), while *unaware* operates on Exh's assertion to derive (64b). We must leave it to future work to consider an extension of their analysis to data with *only*.

(65) **LF for (63): Exh $>$ *unaware* $>$ $\vee$**
[$_{TP}$ Exh [Amy is unaware that [$_{TP}$ can [we have cake or ice cream]]]]

(66) $[\![\text{unaware}]\!] = \lambda p\,.\,\lambda x\,.\,\lambda w : p(w)\,.\,\neg\text{Believe}(p)(x)(w)$

If cumulativity is an implicature, cumulativity should likewise arise just in the presupposition when *unaware* takes scope over plurals. We observe that the prediction is borne out in (67), which is naturally interpreted as in (68). (68a) involves cumulativity, while in (68b), the core existential meaning of the predication is detected. The sentence is true in a scenario where, for instance, the grandparents sat in separate chairs, and Amy does not know that either grandparent sat in either chair. The expected LF is (69), and the target reading results.

(67) Amy is unaware that her grandparents sat in those chairs.

(68) a. **Presupposition (cumulativity)**
Her grandparents sat in those chairs between them.

b. **Assertion (existential)**
Amy doesn't know that either grandparent sat in either chair.

(69) **LF for (67): Exh $>$ *unaware* $>$ $\exists$**
[$_{TP}$ Exh [Amy is unaware that [$_{TP}$ [Op$_{\exists}$ her gpts] [sat in [Op$_{\exists}$ those chairs]]]]]

The puzzle we observed with *only* — that cumulativity arises in the presupposition, but not the assertion — thus fits within a broader pattern. Presuppositional cumulativity inferences seem to track the distribution of presuppositional implicatures.

4.4. Taking stock

Overall, for our central case in (7), the implicature analysis in (70) has reconciled universal inferences in the presupposition with the negation of existential alternatives in the assertive component. Because cumulativity is severed from the predication, *only* can operate on the core existential meaning to derive the target assertion, as discussed in Step 1. Cumulative inferences are then added by Exh into the presupposition, as in Step 2. A presuppositional implicature follows for free in light of baseline data with free choice.

(7) The guides only saw [Amy and Bani]$_F$.

(70) **Recall: final LF: Exh $>$ *only* $>$ $\exists$**
[Exh [only [[Op$_{\exists}$ [the guides]] [saw [Op$_{\exists}$ [Amy and Bani]$_F$]]]]]

It bears note that the analysis might also furnish a second LF for (7). While Exh attaches above *only* in (70), Exh could in principle attach beneath *only*, as well. Given the lexical entry for Exh in (39) above, (71) would yield the same meaning as the predicate analysis did with *only* scoping over **. Exh would occur in the focus alternatives, and result in universal statements, leading to an off target assertion. So, while the implicature analysis does generate the target reading, there is a potential over-generation concern.

(71) **Conceivable LF: *only* > Exh > ∃**
[only [Exh [[$Op_{\exists}$ [the guides]] [saw [$Op_{\exists}$ [Amy and Bani]$_F$]]]]]

LFs like (71) are, however, expected to be unavailable, again based on free choice data. Recall the baseline in (36). If Exh could scope under *only*, as in (72), (36) would have a parse on which the focus alternatives express free choice. Instead of asserting that no one other than Amy can have cake *or* ice cream, (72) would convey just that no one else can have cake *and* can have ice cream. Since continuing (36) with, for instance, *#But, Bill can have cake* is deviant (with *or* unstressed), the scope order must be marginalized. This might fit with a broader pattern where Exh is restricted in downward monotonic environments, since *only* is downward monotonic in its assertion (cf. Fox & Spector 2019).

(36) Only Amy$_F$ can have cake or ice cream.

(72) **Conceivable LF: *only* > Exh > ∨**
[only [Exh [can [Amy$_F$ have cake or ice cream]]]]

5. A solution from trivalence?

We have shown that the interaction between cumulativity and *only* follows directly under the implicature analysis, but not the predicate analysis. Still, could the predicate analysis be revised to capture the interaction? Recall that in the predicate analysis as we presented it, ** occurs in the scope of *only* and introduces universal inferences into both the presupposition and the focus alternatives, resulting in *only* deriving incorrect truth conditions. Our solution replaced universal alternatives with the existential ones in (73) and (74).

(73) **Internal alternatives (not IE)**
a. $\exists y \leq_{AT} G$ [saw(y, Amy)]
b. $\exists y \leq_{AT} G$ [saw(y, Bani)]

(74) **External alternative (IE)**
$\exists y \leq_{AT} G$ [saw(y, Carl)]

We proposed to derive such existential alternatives by removing ** from the scope of *only* and by taking the core plural predication to be existential. The effect of replacing universal with existential alternatives could, however, be achieved within a predicate analysis if the lexical semantics of ** is suitably revised. What is needed is a *trivalent* meaning for ** which specifies non-complementary conditions for truth and falsity (cf. Gajewski 2005, Križ 2015, Bar-Lev 2020, Chatain 2020). The necessary entry is formulated in (75).

(75) **Trivalent entry for **
$[\![**]\!] = \lambda f_{e,et} . \lambda X . \lambda Y .$

$$\begin{cases} 1 & \text{if} \quad \forall y \leq_{AT} Y [\, \exists x \leq_{AT} X [\, f(y,x) \,] \,] \wedge \ \forall x \leq_{AT} X [\, \exists y \leq_{AT} Y [\, f(y,x) \,] \,] \\ 0 & \text{if} \quad \neg [\, \exists y \leq_{AT} Y [\, \exists x \leq_{AT} X [\, f(y,x) \,] \,] \,] \\ \# & \text{otherwise} \end{cases}$$

The revised entry for **, like the classical one, encodes cumulative universal inferences in the truth condition, but the falsity condition is now a negated existential statement. This entry

enables a revised predicate analysis, as *only* can now scope over **, as in the LF repeated in (76), and have compositional access to both universal and existential components. The intuited presupposition and assertion are derived in tandem, provided the entry for *only* references truth in its presupposition and falsity in its assertion, as in (77), and provided that exclusion in the definition of IE likewise references falsity, as in (78).

(76) **Recall: predicate analysis: *only* > ****
[only [[the guides] [**saw [Amy and Bani]$_F$]]]

(77) **Defining *only* (IE-based)**
$[\![\text{only}]\!](C) = \lambda p\,.\,\lambda w : \underline{p(w)=1}\,.\,\forall q \in IE(C)(p)\,[\,\underline{q(w)=0}\,]$

(78) $IE(C)(p) = \cap\,\{\,C' \subseteq C : C'$ is a maximal subset of C s.t.
$\{[\lambda w.\ \underline{q(w)=0}] : q \in C'\} \cup \{\lambda w\,.\,p(w)=1\}$ is consistent $\}$

Given the trivalent entry for **, *only* in the LF in (76) yields a cumulative presupposition, in virtue of the universal inferences in its prejacent's *truth* condition, highlighted in (79). At the same time, as highlighted in (80) and (81), the focus alternatives express negated existentials in their *falsity* condition. Given the trivalent revisions of *only* and IE, the external alternative is innocently excludable and yields the intended negative existential inference, while the internal alternatives are not innocently excludable and yield no strengthening.

(79) **Trivalent prejacent**

$$\begin{cases} 1 & \text{if} \quad \boxed{\forall y \leq_{AT} G\,[\,\exists x \leq_{AT} A{+}B\,[\,f(y,x)\,]\,] \wedge\ \forall x \leq_{AT} A{+}B\,[\,\exists y \leq_{AT} G\,[\,f(y,x)\,]\,]} \\ 0 & \text{if} \quad \neg[\,\exists y \leq_{AT} G\,[\,\exists x \leq_{AT} A{+}B\,[\,f(y,x)\,]\,]\,] \\ \# & \text{otherwise} \end{cases}$$

(80) **Internal alternatives (not IE)**

a. $$\begin{cases} 1 & \text{if} \quad \forall y \leq_{AT} G\,[\,f(y, \text{Amy})\,] \\ 0 & \text{if} \quad \boxed{\neg\exists y \leq_{AT} G\,[\,f(y, \text{Amy})\,]} \\ \# & \text{otherwise} \end{cases}$$

b. $$\begin{cases} 1 & \text{if} \quad \forall y \leq_{AT} G\,[\,f(y, \text{Bani})\,] \\ 0 & \text{if} \quad \boxed{\neg\exists y \leq_{AT} G\,[\,f(y, \text{Bani})\,]} \\ \# & \text{otherwise} \end{cases}$$

(81) **External alternative (IE)**

$$\begin{cases} 1 & \text{if} \quad \forall y \leq_{AT} G\,[\,f(y, \text{Carl})\,] \\ 0 & \text{if} \quad \boxed{\neg\exists y \leq_{AT} G\,[f\,(y, \text{Carl})\,]} \\ \# & \text{otherwise} \end{cases}$$

How does the trivalency-based predicate analysis compare to the implicature analysis that we have pursued in this paper? Both coincide in their effects for sentences with multiple plurals, and it seems hard to discriminate between them with reference to this empirical domain in isolation. However, the two analyses come apart when considering the broader pattern that *only* participates in. We have emphasized above that under the implicature analysis, cumulative readings under *only* are derived from the same mechanisms as free choice readings with *only*, both relying on a weak meaning of *only*'s prejacent that yields a presupposition strengthened via innocent inclusion. By not linking cumulativity to free choice, the revised predicate analysis seems to us to be missing a generalization.

6. Conclusion

We have presented new evidence for the implicature approach to cumulativity based on the observation that cumulativity patterns like an implicature in the way it interacts with the focus operator *only*. The implicature approach correctly allows for exclusive inferences from *only* to access the plural predication's weak existential core, and it correctly predicts, given baseline implicature data, that cumulative universal inferences arise as a presupposition.

Appendix

In assessing the predictions for (7) throughout the paper, we have confined our attention to only those focus alternatives which result from replacing the object conjunction (*Amy and Bani*) with a simplex DP (*Amy*, *Bani*, *Carl*). Katzir's algorithm for alternative computation, however, would yield additional conjunctive alternatives. If a full alternative set is considered, Problem 1 for the original (bivalent) predicate analysis is more limited.

Consider again the LF in (82), and suppose that ** has its classical lexical entry in (4). The focus alternatives would include the conjunctions in (83), which result from replacing just one conjunct in the prejacent with *Carl*. In addition to the original external alternative, *only* would negate these conjunctive alternatives to derive the entailments in (84).

(82) **Recall: predicate analysis: *only* > ****
[only [[the guides] [**saw [Amy and Bani]$_F$]]]

(83) a. [[the guides] [**saw [Amy and Carl]]]
b. [[the guides] [**saw [Bani and Carl]]]

(84) a. $\neg[\![$**saw$]\!]$(Amy+Carl)(G)
b. $\neg[\![$**saw$]\!]$(Bani+Carl)(G)

These entailments will have the welcome effect of rendering the LF in (82) *false* in the world w_1' — repeated in (85) from Section 2.1 — a world where a guide saw Carl. This is because in w_1', the two guides *did* see Amy and Carl between them, falsifying the entailment in (84a). However, the additional negative entailments are still too weak, as they do allow for (7) to be true in certain scenarios where a guide saw Carl. For example, (7) will come out true in the world w_1'' in (86), a world with *three* guides, where one guide saw Carl in addition to Bani, one only saw Amy, and one only saw Bani.

(85) **Facts at w_1'**
a. Guide 1 → Amy
b. Guide 2 → Bani, Carl

(86) **Facts at w_1''**
a. Guide 1 → Amy
b. Guide 2 → Bani
c. Guide 3 → Bani, Carl

For (7) to correctly come out false in (86), *only* would have to negate the alternative in (87a), to derive the further entailment in (87b). However, this alternative involves replacing *Amy and Bani* in (82) with a conjunction that is structurally more complex and, accordingly, is not generated by Katzir's algorithm. Given Katzir's algorithm, the traditional predicate analysis cannot fully capture the target inference that no guide saw Carl.

(87) a. [[the guides] [**saw [Amy and Bani and Carl]]]
b. $\neg[\![$**saw$]\!]$(Amy+Bani+Carl)(G)

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