Cumulativity without plural projection[1]

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Abstract. It has been proposed that the part structures of denotations of plurals ‘project’ to the denotations of expressions including those plurals (e.g., Gawron and Kehler 2004, Kubota and Levine 2016, Schmitt 2019/2020). If such a *plural projection* is possible, not only plural DPs but also expressions *including* those plural DPs denote pluralities (e.g., *type the two recipes* denotes a plurality {TYPE(recipe1),TYPE(recipe2)} instead of a singularity {TYPE({recipe1,recipe2})}). One piece of support for plural projection comes from Schmitt’s (2019) observation about a type of *cumulativity*. In this paper, I examine a wider range of such cumulativity, and show that a source of cumulativity in the literature (e.g., Krifka 1989, Kratzer 2007) can capture all the relevant cumulativity data *without* plural projection while an analysis with plural projection can capture only a proper subset of those data. Therefore, this paper concludes that the relevant cumulativity does not support the need of plural projection.

Keywords: plurality, cumulativity, plural projection

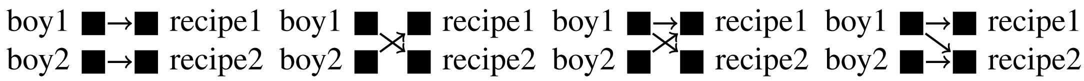
1. Introduction

This paper addresses the meanings of expressions including a plural DP (e.g., *type **the two recipes***), through an investigation into the semantic derivation of *cumulativity*. Cumulativity is a phenomenon where sentences with multiple plurals like (1) receive particular ‘weak’ truth conditions (e.g., Kroch 1974, Scha 1981).

(1) [The two boys]$_{PL1}$ typed [the two recipes]$_{PL2}$

Sentence (1) has weak truth conditions; it is true if one of the *cumulative scenarios* in (2) is true in the evaluation world. For example, (1) is true when one of the two boys typed one of the two recipes and the other boy typed the other recipe. Throughout this paper, I assume that *the two boys* refers to boy1 and boy2, and that *the two recipes* refers to recipe1 and recipe2.

(2) **Cumulative scenarios in which (1) can be true**[2]



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[2]In some bipartite graphs in (2), boy1 and boy2 are both related to recipe1 (or recipe2) as in the left bottom graph. This refers to a scenario where boy1 and boy2 *each* typed recipe1 *as a whole*, which is odd here due to the nature of how typing happens (unless it means each of them typed *what’s written in recipe1*. Also note that there is no such oddness in sentences like: *the two boys **saw** the two recipes*). But it should be noted that even if we interpret the left bottom graph as the scenario where boy1 typed *a part of recipe1* and boy2 typed *the rest of recipe1* and recipe 2, sentence (1) is still true. I will call such scenarios the *sub-atomic cumulative scenarios*. See Section 3.

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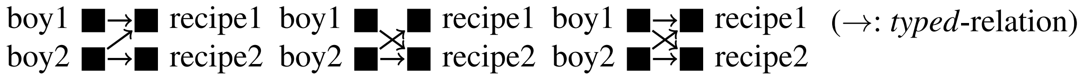


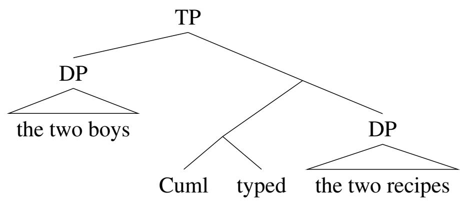
I call the relations between *the two boys* and *the two recipes* in the cumulative scenarios in (2) the *cumulations* or *cumulative typed-relations*, and I call the weak truth conditions in question the *cumulative truth conditions*, which can be defined as in (3b). For instance, the cumulative truth conditions of (1) are the following: (1) is true iff each of the two boys typed at least one of the two recipes, and each of the two recipes was typed by at least one of the two boys.

(3) a. A P B, where A and B are arguments of the predicate P.
b. 1 iff $\forall x \in S_A[\exists y \in S_B[P(y)(x)=1]] \wedge \forall y \in S_B[\exists x \in S_A[P(y)(x)=1]]$,
where S_A and S_B are sets of objects that $[\![A]\!]$ and $[\![B]\!]$ consist of intuitively.
(adapted from Schmitt 2019, 7)

Cumulative truth conditions have been argued to arise from a grammatical, *compositional source* such as a covert operator on predicates (Beck and Sauerland 2000). (4) shows the definition of such an operator *Cuml*, which encodes cumulative truth conditions.

(4) Cuml (P)(B)(A) = 1 iff $\forall x \in S_A[\exists y \in S_B[P(y)(x)=1]] \wedge \forall y \in S_B[\exists x \in S_A[P(y)(x)=1]]$
where S_A and S_B are sets of objects that $[\![A]\!]$ and $[\![B]\!]$ consist of intuitively.
(adapted from Beck and Sauerland 2000, 351)

Cuml takes a binary predicate P and its co-arguments, returning 1 iff there is a cumulative P-relation between the co-arguments. For example, under this approach, sentence (1) has the LF in (5a), and the cumulative truth conditions in (5b).

(5) a.


b. $[\![TP]\!]$ = 1 iff $\forall x \in \{boy1,boy2\}[\exists y \in \{recipe1,recipe2\}[\text{TYPED}(y)(x)=1]] \wedge$
$\forall y \in \{recipe1,recipe2\}[\exists x \in \{boy1,boy2\}[\text{TYPED}(y)(x)=1]]$

In this way, compositional source can derive cumulative truth conditions of ‘simple’ sentences like (1). However, Schmitt (2019) observes that compositional source does not straightforwardly derive cumulative truth conditions of so-called *flattening examples* on its own. (6) exemplifies flattening examples, which feature a plural including another plural.

(6) The two boys made Abe **[type [the two recipes]$_{PL2}$ and create this blog]$_{PL1}$**.
(adapted from Schmitt 2019, 27)

Schmitt (2019) observes that descriptively sentence (6) is true if there is a cumulative *made*-relation between the set of individuals and the set of propositions in (7). In other words, the sentence is true if each of the two boys is in the *made*-relation with one of the three propositions, and each of the three proposition is in the *made*-relation with one of the two boys.

(7) {boy1,boy2} - {ABE-TYPE-RECIPE1,ABE-TYPE-RECIPE2,ABE-CREATE-BLOG}

Schmitt (2019) demonstrates that compositional source cannot straightforwardly derive such cumulative truth conditions based on the two sets in (7) on its own; for instance, if they take the meaning of *and* to be non-Boolean following Link (1983) among others, the complement of *made* in (6) can denote a *singleton* set like {ABE-TYPE-RECIPE1$\wedge$ABE-TYPE-RECIPE2$\wedge$ABE-CREATE-BLOG}, and Cuml cumulatively relates this singleton set with {boy1,boy2} via *made*. As a result, it is wrongly predicted that sentence (6) is true only if *each* of the two boys made Abe type recipe1 and recipe2, and create the blog.[3]

To capture flattening examples among others, Schmitt (2019) proposes to base compositional source on the compositional process called the *plural projection*. Plural projection effectively enables the complement of *made* in (6) to denote the set of three propositions in (7), and serves to derive the cumulative truth conditions of (6), as elaborated in Section 2.

As hinted by Schmitt (2020), however, I observe that a source of cumulativity, which I call the *inferential source* (e.g., Krifka 1989, Kratzer 2007), can capture flattening examples using an event semantics. In (6), for instance, inferential source effectively derives the two sets in (8) (NB: e_1, e_2, and e_3 in (8) correspond to the three propositions in the set in (7)), and enables (6) to be true if there is a cumulative *made*-relation between those two sets, as elaborated in Section 4-5. Thus, inferential source is sufficient to capture the flattening example in (6).

(8) {boy1,boy2} - {e_1,e_2,e_3}, where:
e_1: Abe typed recipe1. **e_2**: Abe typed recipe2. **e_3**: Abe created the blog.

One crucial observation in this paper is that inferential source is needed to capture a wider range of flattening data anyway even if one assumes plural projection. Consider (9).

(9) The two boys made Abe [type **[the ramen recipe]**$_{SG}$ and create this blog]$_{PL}$.

Sentence (9) differs from the flattening example in (6) only in that the object of *type* is a singular DP. To distinguish these two types of flattening examples, I call sentences like (9) and sentences like (6) the *SG flattening* and the *PL flattening*, respectively.

Throughout this paper, I assume that *the ramen recipe* refers to ramen(recipe) which consists of noodle(recipe) and broth(recipe), and that noodle and broth correspond to recipe1 and recipe2 constituting *the two recipes*. Given this assumption, descriptively, the SG flattening in (9) is true if there is a cumulative *made*-relation between the two sets in (10), just like the cumulative truth conditions of the PL flattening in (6).

(10) {boy1,boy2} - {ABE-TYPE-NOODLE, ABE-TYPE-BROTH, ABE-CREATE-BLOG}

Importantly, given that noodle and broth correspond to recipe1 and recipe2, the SG flattening in (9) and the PL flattening in (6) always receive the same truth values in the same scenarios. Despite this similarity, however, it will be shown that compositional source with plural projection can capture the PL flattening in (6), but *not* the SG flattening in (9). In contrast, it will be

[3]For a more detailed investigation, see Schmitt (2019).

shown that inferential source can capture both of those flattening data. Therefore, this paper argues that inferential source is sufficient to capture all the flattening data in the literature and thus flattening data do not support the need of plural projection. In supporting this argument, this paper also sheds light on the importance of addressing data like (9) with a *singular* DP when formulating an analysis of cumulativity in general. Moreover, this paper also shows that adopting inferential source brings about another good result in explaining why certain lexical items and cumulative scenarios affect the truths/naturalness of some sentences.

In what follows, Sections 2–3 first show in turn that compositional source with plural projection captures PL flattening but *not* SG flattening. Sections 4–5 then show in turn that inferential source captures SG flattening and PL flattening. Section 6 then discusses lexical/contextual effects on the naturalness of sentences in cumulative scenarios. Finally, Section 7 concludes.

2. Plural projection helps compositional source capture PL flattening

This section introduces Schmitt's (2019) analysis of how plural projection enables compositional source to capture PL flattening. First, Schmitt (2019) assumes not only individual pluralities (11a) (e.g., Link 1983), but also higher-order pluralities like predicate pluralities (11b) (e.g., Gawron and Kehler 2004) and proposition pluralities (11c).

(11) a. $[\![$the two boys$]\!]$ = {boy1,boy2}
b. $[\![$sing and dance$]\!]$ = {SING,DANCE} (SING := $\lambda x_e.\lambda w_s$.x sings in w)
c. $[\![$This boy sings and that boy dances$]\!]$ = {SING(boy1),DANCE(boy2)}

Schmitt (2019) also assumes a new compositional rule to enable plural projection, i.e., the compositional process of projecting the part structures of the denotations of plurals to the denotations of expressions including those plurals. For instance, if the DP *the two recipes* denotes a plurality {recipe1,recipe2}, a VP including the DP like *type the two recipes* also denotes a plurality {TYPE(recipe1),TYPE(recipe2)} instead of a singularity like {TYPE({recipe1,recipe2})} (12) & (13). Note that in (12), the two sets compose in a point-wise fashion, and the part structure of {recipe1,recipe2} projects to {TYPE(recipe1),TYPE(recipe2)} in the sense that the derived set involves two members as {recipe1,recipe2} does.

(12) **Correct derivation:**

{TYPE(recipe1),TYPE(recipe2)}

{TYPE} {recipe1,recipe2}

(13) **Incorrect derivation:**

{TYPE({recipe1,recipe2})}

{TYPE} {recipe1,recipe2}

Now, compositional source can capture the PL flattening in (14).

(14) The two boys [[Cuml made] [$_{TP}$ Abe [$_{\&P}$ type the two recipes and create this blog]]].

On the assumption that *and* takes its argument sets and returns their union, &P in (14) denotes {TYPE(recipe1),TYPE(recipe2),CREATE(blog)}. Then, this set composes with {abe}, and TP denotes {TYPE(recipe1)(abe),TYPE(recipe2)(abe),CREATE(blog)(abe)}. Finally, Cuml takes *made*, the set denoted by TP, and {boy1,boy2}, deriving the cumulative truth conditions saying that (14) is true iff there is a cumulative *made*-relation between the two sets mentioned above. In this way, plural projection enables compositional source to capture PL flattening.[4]

3. Plural projection does *not* help compositional source capture SG flattening

This section demonstrates that plural projection does *not* enable compositional source to capture SG flattening. This will be demonstrated based on the assumption that plural projection and Cuml can access the proper parts of denotations of *plurals*, but not *singulars*. In other words, even if the singular *the ramen recipe* consists of a noodle recipe and a broth recipe, it effectively denotes a singularity {ramen}, and plural projection and Cuml can access only ramen, and not the proper parts of ramen (e.g., noodle and broth). With this assumption in mind, consider the SG flattening in (15).

(15) The two boys [[Cuml made] [Abe type the ramen recipe and create this blog]].

Given the above assumption, plural projection enables (15) to be true if there is a cumulative *made*-relation between the two sets in (16a), and not between the two sets in (16b).

(16) a. {boy1,boy2} - {TYPE(**ramen**)(abe),CREATE(blog)(abe)}
b. {boy1,boy2} - {TYPE(**noodle**)(abe),TYPE(**broth**)(abe),CREATE(blog)}

Thus, compositional source with plural projection enables (15) to be true only if at least boy1 or boy2 made Abe type the *whole* ramen recipe. However, (15) is true, for example, if boy1 made Abe type only the noodle recipe, and boy2 made Abe type only the broth recipe, and create this blog. Therefore, compositional source with plural projection fails to capture SG flattening.

In what follows, I call cumulations like (16b), where the proper parts of an atom (e.g., noodle and broth of ramen) exist as members of a set, the *sub-atomic cumulations*. Given this, I also call the cumulations like (16a) the *atomic cumulations*, and use *cumulations* as an umbrella term for atomic and sub-atomic cumulations. Likewise, I will adopt the same type of distinctions for terms like *cumulativity* and *cumulative scenarios*.

The crucial assumption in the above analysis is that singulars like *the ramen recipe* effectively denotes singularities like {ramen}, and plural projection and Cuml can access only the members of the singularities like ramen and not their proper parts like noodle and broth. The rest of this section supports this assumption. First, consider (17). Sentence (17a) is felicitous and it is true if there is a cumulation between the subject and the predicate conjunction. For instance, (17a) is true in the scenario where the noodle recipe is completely correct and the broth recipe is completely wrong. In contrast, sentence (17b) sounds contradictory; the sentence indicates

[4] The analysis presented here is the one minimally different from Beck and Sauerland's analysis with Schmitt's (2019) idea about plural projection. In Schmitt's (2019) original analysis, the new compositional rule for plural projection can itself derive cumulativity *without* Cuml. For instance, in (14), the matrix predicate derives a predicate plurality {MADE(TYPE(recipe1)(abe)),MADE(TYPE(recipe2)(abe)),MADE(CREATE(blog)(abe))}, and the compositional rule 'cumulatively composes' the set with {boy1,boy2}. I presented the analysis in the text to make it clearer what exactly is required for Beck and Sauerland's analysis of cumulativity to capture PL flattening.

that *the whole ramen recipe* is both completely correct and completely wrong.[5]

(17) a. **The two recipes** are completely correct and completely wrong.
b. #**The ramen recipe** is completely correct and completely wrong.
(adapted from Paillé 2020, 846)

If *the ramen recipe* can denote {noodle,broth} as *the two recipes* can, and Cuml can access noodle and broth, then (17b) should sound as natural as (17a), contrary to fact. On the other hand, if *the ramen recipe* denotes {ramen}, it 'cumulatively composes' with {COMPLETELY-CORRECT,COMPLETELY-WRONG}. Then, (17b) is predicted true iff *the whole ramen recipe* is both completely correct and completely wrong, which is contradictory as desired.[6]

Such a contrast as in (17a-b) is not limited to copular sentences. Consider (18): (18a) is true in the cumulative scenario, but (18b) is false, which is again not expected if *the ramen recipe* can denote {noodle,broth}, and Cuml can access noodle and broth, as in (18a).[7]

(18) [Scenario: Boy1 circled only the noodle recipe with a pen. Boy2 circled only the broth recipe with a pen. So the whole ramen recipe is not circled.]
a. The two boys circled **the two recipes**. b. The two boys circled **the ramen recipe**.

Likewise, it is unexpected that (19b) is false unlike (19a) if plural projection can access noodle and broth of *the ramen recipe*, because on that assumption, Cuml cumulatively relates {boy1,boy2} with {ABE-CIRCLE-NOODLE,ABE-CIRCLE-BROTH} via *made*, just like in (19a).

(19) [Scenario: Boy1 made Abe circle only the noodle recipe with a pen. Boy2 made Abe circle only the broth recipe with a pen. So the whole ramen recipe is not circled.]
a. The two boys made Abe circle **the two recipes**.
b. The two boys made Abe circle **the ramen recipe**.

It should be noted that while (17b), (18b), and (19b) are false, sentences can be true in sub-atomic cumulative scenarios in principle, as in (20). Note that (20b) is true in the sub-atomic cumulative scenario unlike (18b). Compositional source with plural projection does not straightforwardly capture the *lexical* difference in truth between (18b) and (20b), either.[8]

[5] De Vries (2017) observes a similar contrast between plurals and collective DPs: ***The women** are short and tall.* vs. #***The committee** is short and tall.*

[6] Viola Schmitt (p.c.) observes that sentence (17b) may not be infelicitous because Cuml cannot access the parts of denotations of singulars. She bases this observation on infelicitous examples like: #*the two recipes are each completely correct and each completely wrong.* The subject of this sentence can denote a plurality like {noodle,broth}, and yet the sentence sounds unnatural. So she raises the possibility that Cuml can actually access the parts of denotations of singulars, and that (17b) sounds unnatural for a reason why her sentence above sounds unnatural. However, even if this is right, we still need an account of why the *contrast* between (17a) and (17b) exist if Cuml can ever access the parts of denotations of *the ramen recipe*.

[7] In fact, (18b) seems to be false in *any* sub-atomic cumulative scenarios due to a lexical property of *circled*. In contrast, other sentences like (20b) and (21a) with different predicates *can* be true in such scenarios.

[8] Note that sentences *without* any singular DPs can also show sub-atomic cumulativity. For instance, the sentence *the two boys typed the 50 ramen recipes* is true if there is a sub-atomic cumulation between the two *plurals*, as the sentence is true in the following scenario: there are 50 ramen recipes each of which consists of a noodle recipe and a broth recipe. Boy1 typed all the noodle recipes. Boy2 typed all the broth recipes.

(20) [Scenario: Boy1 typed the noodle recipe. Boy2 typed the broth recipe.]
a. The two boys **typed** the two recipes. b. The two boys **typed** the ramen recipe.

Compositional source does not straightforwardly capture the *contextual* difference in truth of (21a) between (21b) and (21c) either if *the ramen recipe* can denote the plurality {noodle,broth}, and Cuml can access noodle and broth. Note that sentence (21a) is true in (21b) but not in (21c).

(21) a. The two boys ruined **the ramen recipe**.
b. **Scenario where (21a) is *true*:** Boy1 ruined the noodle recipe by adding too much salt. Boy2 ruined the broth recipe by adding too much sugar.
c. **Scenario where (21a) is *false*:** Boy1 ruined the noodle recipe by adding too much salt. Boy2 ruined the broth recipe by adding too much sugar. But the whole ramen recipe is not ruined; the badness of salt and sugar canceled each other out.

(21b) differs from (21c) in that the whole ramen recipe is likely to be ruined in (21b) but it is clearly not ruined in (21c). Given this, compositional source can capture the contrast between (21b) and (21c) if we assume that (21a) violates some presupposition in (21c); for instance, each member of the complement set of *ruined* has to be ruined. But such an analysis is valid only if *the ramen recipe* denotes {ramen} and not {noodle,broth}, because the noodle recipe and the broth recipe are ruined in (21c). It should also be noted that the contextual effect in (21) disappears if we replace *the ramen recipe* in (21a) with *the two recipes*. This is not easily explained either if Cuml can access noodle and broth of *the ramen recipe*.

In this way, plural projection and Cuml does not seem to access the parts of denotations of singulars, and this assumption is typologically plausible as well; there exist other operators like Cuml that cannot access the parts of denotations of singulars. Consider (22).

(22) a. Each of **the car parts** was painted. b. #Each of **the car** was painted.
(adapted from Schwarzschild 1996, 164)

In (22a), *each* can quantify over the parts of denotation of *the car parts*. In contrast, in (22b), it cannot quantify over the parts of denotation of *the car*, and thus, (22b) is infelicitous. In this way, it is attested that *each* cannot access the parts of denotations of singulars like Cuml.[9]

To sum up, compositional source with plural projection cannot easily capture SG flattening on the assumption that plural projection and Cuml cannot access the parts of denotations of singulars. In the next section, I will demonstrate that inferential source can capture SG flattening.

[9] Schwarzschild (1996) addresses a possibility that *each syntactically* selects a plural as its first argument and thus sentence (22b) is ungrammatical due to *the car* lacking a plural morphology. However, he undermines such a view based on the facts that *each* cannot combine with non-count plurals (1b).

(1) a. The funds were ill-gotten. b. #Each of the funds was ill-gotten. (Schwarzschild 1996,165)

4. Inferential source captures SG flattening

This section demonstrates how inferential source captures SG flattening step by step. SG flattening involves three crucial components in (23a-c). I first demonstrate how inferential source captures those three components, and then turn to the SG flattening in (23d).

(23) a. **Sub-atomic cumulativity**
The two boys typed **[the ramen recipe]**$_{SG}$.
b. **Cumulativity in sentences with a predicate conjunction**
The two boys typed the ramen recipe **and** created this blog.
c. **Cumulativity in causative sentences**
The two boys **made** Abe type the ramen recipe.
d. **SG flattening**
The two boys made Abe [type [the ramen recipe]$_{SG}$ and create this blog]$_{PL}$.

4.1. Sub-atomic cumulativity

This section demonstrates that inferential source can capture the sub-atomic cumulativity in (23a). First, adopting an event semantics (Davidson 1967), I assume that while the domains of DPs and eventualities (i.e., events or states (Bach 1986)) involve both singularities and pluralities (24a-b), the domain of predicates just involves predicates like TYPE (24c).

(24) a. $D_e = \{\{ada\}, \{bea\}, ..., \mathbf{\{ada,bea\}}, ...\}$ b. $D_s = \{\{e_1\}, \{e_2\}, ..., \mathbf{\{e_1,e_2\}}, ...\}$
c. $D_{eest} = \{\text{TYPE}, \text{CREATE}, ...\}$, where $\text{TYPE} = \lambda\chi_e.\lambda\psi_e.\lambda\varepsilon_s.\text{type'}_\varepsilon(\psi,\chi)$
(χ, ψ and ε: a set of individuals/eventualities)

Under an event semantics, predicates take an extra variable over eventualities compared to the predicates in the more traditional analyses, as exemplified in (25). The predicate in (25) reads: ε is an eventuality of each member of χ being typed as a result of each member of ψ doing something. It should be noted that what each member of ψ does does not need to be typing; the members of ψ just cause each member of χ to be typed.[10]

(25) $[\![\text{type}]\!] = \lambda\chi_e.\lambda\psi_e.\lambda\varepsilon_s.\text{type'}_\varepsilon(\psi,\chi)$

At this moment, sentences denote functions from eventualities to truth values, as in (26).

(26) a. The boy typed the ramen recipe. b. $[\![(26a)]\!] = \lambda\varepsilon_s.\text{type'}_\varepsilon(\{boy1\},\{ramen\})$

To enable sentences to denote truth values, I assume a phonologically null operator '$\exists\varepsilon$', which existentially quantifies over an eventuality variable. Given this, (26a) has the LF in (27a) and the truth conditions in (27b): (26a) is true iff there is an eventuality ε of the ramen recipe being typed as a result of boy1 having done something.

[10]This assumption will be of use to analyze the so-called *collectivity*. See Section 7.

(27) a. [$\exists\varepsilon$[The boy typed the ramen recipe]]
b. $[\![(27a)]\!] = 1$ iff $\exists\varepsilon$[typed'$_{\varepsilon}$({boy1},{ramen})]

Now, we turn to (28a) and (28b) repeated from (23a). Note that both sentences are true in the cumulative scenario.

(28) [Scenario: Boy1 typed the noodle recipe. Boy2 typed the broth recipe.]
a. The two boys typed **the two recipes**. b. The two boys typed **the ramen recipe**.

Sentence (28a) and (28b) have the truth conditions in (29a) and (29b).

(29) a. $[\![(28a)]\!] = 1$ iff $\exists\varepsilon$.[typed'$_{\varepsilon}$({boy1,boy2},{noodle,broth})]
b. $[\![(28b)]\!] = 1$ iff $\exists\varepsilon$.[typed'$_{\varepsilon}$({boy1,boy2},{ramen})]

(29a) says (28a) is true iff there is an eventuality ε of each of the noodle recipe and the broth recipe being typed as a result of boy1 and boy2 each having done something. Thus, under our analysis with inferential source, sentences do *not* have cumulative truth conditions unlike analyses with compositional source. Instead, sentences have weaker truth conditions compatible with cumulative scenarios. For instance, the truth conditions in (29) are satisfied in the scenario in (28). First, in (28), there are two typing eventualities (30a-b). To simplify the notation, I omit existential operator $\exists_{\{e\}}$ when discussing eventualities in a scenario. As a result, (30a-b) can be simplified as in (30c-d).

(30) **Scenario**
a. $\exists_{\{e_1\}}$[typed'$_{\{e_1\}}$({boy1},{noodle})] b. $\exists_{\{e_2\}}$[typed'$_{\{e_2\}}$({boy2},{broth})]
c. typed'$_{\{e_1\}}$({boy1},{noodle}) d. typed'$_{\{e_2\}}$({boy2},{broth})

(30c) says there exists an eventuality of the noodle recipe being typed as a result of boy1 having done something, and (30d) makes a similar existential claim. Given these, we can naturally assume that the following inference, which I call the *cumulative inference*, is valid (31). The inference says that if there are eventualities {e_1} and {e_2} in (30c-d), there is also a union of those eventualities {e_1,e_2}, namely an eventuality of each of the noodle recipe and the broth recipe being typed as a result of boy1 and boy2 each having done something (c.f., Krifka 1989).

(31) **Cumulative inference**
typed'$_{\{e_1\}}$({boy1},{noodle}) & typed'$_{\{e_2\}}$({boy2},{broth}) $\rightarrow$
typed'$_{\{e_1,e_2\}}$({boy1,boy2},{noodle,broth})

Notice that {e_1,e_2} in (31) serves as a witness for the existential claim about ε in (29a). Thus, (28a) is correctly predicted true in the atomic cumulative scenario. In this way, inferential source can capture atomic cumulativity.

The truth conditions of (28b) (i.e., (29b)) are also satisfied with another inference which can be naturally assumed to be valid in the scenario of (28b). But before introducing the second inference, I would first like to note that I will represent the mereological structures of individuals with sums, if necessary. For instance, *the ramen recipe* denotes {ramen} which is equivalent to {noodle+broth} on our assumption that the ramen recipe consists of a noodle recipe and a broth recipe. It should be noted however that {ramen}/{noodle+broth} is *not* equivalent to {noodle,broth}, which can be denoted by expressions such as *the two recipes* that consists of a noodle recipe and a broth recipe.[11] Given this assumption, I present the second inference in (32), which I call the *parts-whole inference*; if (i) there is $\{e_1,e_2\}$ in (31), (ii) {noodle+broth} can be assumed to be equivalent to {ramen} *in the given scenario* ($=_c$: contextual equivalence), and (iii) the ramen recipe can be naturally assumed to be typed in the scenario, then we can naturally assume with our world knowledge about typing that $\{e_1,e_2\}$ has {ramen} as its theme. In (32), TH is a function that takes an eventuality and returns a set of themes in the eventuality.

(32) **Parts-whole inference**
typed'$_{\{e_1,e_2\}}$({boy1,boy2},{noodle,broth}) & {noodle+broth} $=_c$ {ramen} &
{ramen} $\in$ TH(typed'$_{\{e_1,e_2\}}$) $\rightarrow$ typed'$_{\{e_1,e_2\}}$({boy1,boy2},{**ramen**})

Notice that $\{e_1,e_2\}$ in (32) serves as a witness for the existential claim about ε in (29b). Thus, (28b) is correctly predicted true in the sub-atomic cumulative scenario. In this way, inferential source can capture sub-atomic cumulativity.

In the rest of this section, I will support the need of the second and third premises in (32). First, the second premise in parts-whole inference (e.g., '{noodle+broth} $=_c$ {ramen}' in (32)) is needed to explain that (28b) can be false in scenarios like (33).

(33) **Scenario where (28b) is *false*:** Boy1 typed a noodle recipe. Boy2 typed a broth recipe. But those recipes do not constitute any ramen recipe. Boy3 typed the ramen recipe.

In the scenario of (33), there are $\{e_1\}$ and $\{e_2\}$ in (30c-d), and the cumulative inference in (31) is also valid. However, the parts-whole inference in (32) is not valid in the scenario of (33). This is because {noodle+broth} is not equivalent to {ramen} in (33) (while the other two premises in (32) are satisfied). In this way, the premise about contextual equivalence in parts-whole inference is needed to explain why sentence (28b) is false in (33) while it is true in the scenario of (28b).

Next, I will show that the third premise in parts-whole inference (e.g., '{ramen} $\in$ TH(ruined'$_{\{e_1,e_2\}}$)' in (32)) is also needed to explain data like (34), repeated from (21) in Section 3. Remember that (34a) is true in (34b), but not in (34c).

(34) a. The two boys ruined **the ramen recipe**.
b. **Scenario where (34a) is *true*:** Boy1 ruined the noodle recipe by adding too much salt. Boy2 ruined the broth recipe by adding too much sugar.
c. **Scenario where (34a) is *false*:** Boy1 ruined the noodle recipe by adding too much salt. Boy2 ruined the broth recipe by adding too much sugar. But the whole ramen recipe is not ruined; the badness of salt and sugar canceled each other out.

First, regardless of the scenarios, (34a) has the truth conditions in (35).

[11] Representing singularity/plurality and mereological structure in different ways enables a straightforward explanation of why sentences like (17b) are infelicitous while sentences like (17a) are felicitous. See (43) below, where it is explained how inferential source captures the contrast in felicity between (17a) and (17b).

(35) $[\![(34a)]\!] = 1$ iff $\exists\varepsilon[\text{ruined'}_{\varepsilon}(\{\text{boy1,boy2}\},\{\text{ramen}\})]$

Also, in the scenario of both (34b) and (34c), there are two ruining eventualities in (36a). Given those eventualities, we can assume the presence of $\{e_1,e_2\}$ via cumulative inference (36b).

(36) a. **Scenario**
i. $\text{ruined'}_{\{e_1\}}(\{\text{boy1}\},\{\text{noodle}\})$ ii. $\text{ruined'}_{\{e_2\}}(\{\text{boy2}\},\{\text{broth}\})$
b. **Cumulative inference**
(36a-i) $\wedge$ (36a-ii) $\rightarrow \text{ruined'}_{\{e_1,e_2\}}(\{\text{boy1,boy2}\},\{\text{noodle,broth}\})$

So far we can assume the same eventualities in the scenarios of both (34b) and (34c). But a crucial difference arises when we consider the parts-whole inference in (37).

(37) **Parts-whole inference**
(36b) $\wedge$ $\{\text{noodle+broth}\} =_c \{\text{ramen}\} \wedge \{\text{ramen}\} \in \text{TH}(\text{ruined'}_{\{e_1,e_2\}})$
$\rightarrow \text{ruined'}_{\{e_1,e_2\}}(\{\text{boy1,boy2}\},\{\text{ramen}\})$

The inference in (37) is valid in the scenario of (34b), and $\{e_1,e_2\}$ in the conclusion of (37) serves as a witness for the existential claim about ε in (35). In contrast, (37) is *not* valid in the scenario of (34c), because $\text{TH}(\text{ruined'}_{\{e_1,e_2\}})$ does not involve {ramen}; that is, the whole ramen recipe is *not* ruined in (34c). In this way, the third premise in parts-whole inference is needed to explain why (34a) is true in (34b) but not in (34c).[12]

Lastly, the third premise in parts-whole inference is also needed to capture the lexical difference in the availability of cumulativity between (38) and (39), which we observed in Section 3.

(38) [Scenario: Boy1 circled only the noodle recipe with a pen. Boy2 circled only the broth recipe with a pen. So the whole ramen recipe is not circled.]
a. The two boys circled **the two recipes**. b. The two boys circled **the ramen recipe**.

(39) [Scenario: Boy1 typed the noodle recipe. Boy2 typed the broth recipe.]
a. The two boys **typed** the two recipes. b. The two boys **typed** the ramen recipe.

In (38), while sentence (38a) with a plural object is true in the cumulative scenario, sentence (38b) with a singular object is false. This is expected by our analysis because parts-whole inference is relevant only in (38b), and it is *not* valid in the scenario; the whole ramen recipe is not circled in (38b), so the third premise in parts-whole inference is not satisfied, preventing the parts-whole inference being valid in (38b).[13] In contrast to (38b), (39b) (as well as (39a)) is true in the cumulative scenario. This is also expected because the parts-whole inference is valid in (39b) as the whole ramen recipe is typed in the scenario. In this way, the third premise in parts-whole inference serves to capture the lexical dependency of cumulativity as well.

[12]Remember that the sentence *the two boys ruined* ***the two recipes*** is true in (34c). So the falsity of (34a) in (34c) is due to the singularity of *the ramen recipe*. This fact is also compatible with our analysis because the falsity of (34a) in (34c) is captured by a premise of parts-whole inference, which is needed to analyze the *sub-atomic* cumulativity in (34a) but it is *not* needed to analyze *atomic* cumulativity in the above example with *the two recipes*.
[13]The contrast between (19a) and (19b) can be explained in the same way.

4.2. Cumulativity in sentences with a predicate conjunction

This section shows that inferential source can capture cumulativity in (40); the sentence is true if there is a cumulation between the two plurals, as it is true in the cumulative scenario given.

(40) [Scenario: Boy1 typed the ramen recipe. Boy2 created this blog.]
The two boys typed the ramen recipe and created this blog.

One way of deriving cumulativity in (40) is to attribute them to the meaning of *and*. Assuming a version of non-Boolean *and* defined in (41a) (c.f., Lasersohn 1995), we can assume that sentence (40) has the truth conditions in (41b).[14]

(41) a. $[\![\text{and}]\!] = \lambda f_{<e,st>}.\lambda g_{<e,st>}.\lambda\chi_e.\lambda\varepsilon_s.\exists\chi_1,\chi_2,\varepsilon_1,\varepsilon_2[\chi = \chi_1\cup\chi_2 \ \&\ \varepsilon = \varepsilon_1\cup\varepsilon_2 \ \&\ f(\chi_1)(\varepsilon_1) \ \&\ g(\chi_2)(\varepsilon_2)]$

b. $[\![(40)]\!] = 1$ iff $\exists\chi_1,\chi_2,\varepsilon,\varepsilon_1,\varepsilon_2[\{\text{boy1,boy2}\} = \chi_1\cup\chi_2 \ \&\ \varepsilon = \varepsilon_1\cup\varepsilon_2 \ \&\ \text{typed'}_{\varepsilon_1}(\chi_1,\{\text{ramen}\}) \ \&\ \text{created'}_{\varepsilon_2}(\chi_2,\{\text{blog}\})]$

In the scenario of (40), there are one typing eventuality and one creating eventuality (42).

(42) **Scenario**

a. typed'$_{\{e_1\}}$({boy1},{ramen}) b. created'$_{\{e_2\}}$({boy2},{blog})

$\{e_1\}$, $\{e_2\}$, $\{e_1,e_2\}$, {boy1}, and {boy2} serve as witnesses for the existential claims about ε_1, ε_2, ε, χ_1, and χ_2 in (41b). Thus, sentence (40) is correctly predicted true. In this way, inferential source captures cumulativity in (40).

Likewise, inferential source can capture cumulativity in (43a), repeated from Section 3. Crucially, it can also capture the infelicity of (43b).

(43) a. **The two recipes** are completely correct and completely wrong.

b. #**The ramen recipe** is completely correct and completely wrong.

(adapted from Paillé (2020), 846)

Sentence (43b) has the truth conditions in (44). The sentence is correctly predicted to be infelicitous because χ_1 and χ_2 are both {ramen}, and the sentence is predicted to mean that the ramen recipe is both completely correct and completely wrong, which is contradictory.[15]

(44) 1 iff $\exists\chi_1,\chi_2,\varepsilon,\varepsilon_1,\varepsilon_2[\underline{\mathbf{\{ramen\}} = \chi_1\cup\chi_2} \ \&\ \varepsilon = \varepsilon_1\cup\varepsilon_2 \ \&\ \text{cw'}_{\varepsilon_1}(\chi_1) \ \&\ \text{cc'}_{\varepsilon_2}(\chi_2)]$,
where cw' and cc' are the predicates completely.wrong' and completely.correct'.

In this way, inferential source captures cumulativity in sentences with a predicate conjunction.

[14]Another way of capturing cumulativity in (40) is to assume that the predicate conjunction denotes a doubleton set {TYPED-RAMEN,CREATED-BLOG} and it cumulatively composes with {boy1,boy2} with a version of Cuml.

[15]If we represent both plurality and mereological structure with sets, then {ramen} in (44) can be replaced by {noodle,broth}, and (43b) is wrongly predicted felicitous. Thus, it is useful to use the different notations, sets and sums, for plurality and mereological structure respectively; this way, {ramen} is *not* equivalent to {noodle,broth}, and {ramen} in (44) *cannot* be replaced by {noodle,broth}. As a result, inferential source can capture the infelicity of (43b) as well as the felicity of (43a).

4.3. Cumulativity in causative sentences

This section shows that inferential source can capture cumulativity in causative sentence (45). Descriptively, (45) is true if there is a cumulative *made*-relation between {boy1,boy2} and the set of propositions {ABE-TYPE-NOODLE,ABE-TYPE-BROTH}, as (45) is true in the scenario.

(45) [Scenario: Boy1 made Abe type the noodle recipe. Boy2 made Abe type the broth recipe.]
The two boys made Abe type the ramen recipe.

I assume *made* has the denotation in (46a), which has an effect of linking a causing eventuality ε with a caused eventuality ε'. Given this assumption, (45) has the truth conditions in (46b).

(46) a. $[\![\text{made}]\!] = \lambda\rho_{st}.\lambda\chi_e.\lambda\varepsilon_s.\exists\varepsilon\text{'}.[\text{made}_\varepsilon(\chi,\varepsilon\text{'}) \ \&\ \rho(\varepsilon\text{'})]$
b. $[\![(45)]\!] = 1$ iff $\exists\varepsilon,\varepsilon\text{'}.[\text{made}_\varepsilon(\{\text{boy1,boy2}\},\varepsilon\text{'}) \ \&\ \text{type'}_{\varepsilon'}(\{\text{abe}\},\{\text{ramen}\})]$

In the given scenario, there are two typing eventualities and two causing eventualities (47).

(47) **Scenario**
a. type'$_{\{e'_1\}}$({abe},{noodle})
b. type'$_{\{e'_2\}}$({abe},{broth})
c. made'$_{\{e_1\}}$({boy1},{e'$_1$})
d. made'$_{\{e_2\}}$({boy2},{e'$_2$})

Based on {e'$_1$} and {e'$_2$}, the cumulative inference derives {e'$_1$,e'$_2$} (48a). Then, based on {e'$_1$,e'$_2$} and the fact that {noodle+broth} is equivalent to {ramen} and the ramen recipe is typed in the scenario, the parts-whole inference derives the typing eventuality whose theme is {ramen} (48b). Lastly, based on {e$_1$} and {e$_2$}, the cumulative inference derives {e$_1$,e$_2$} (48c).

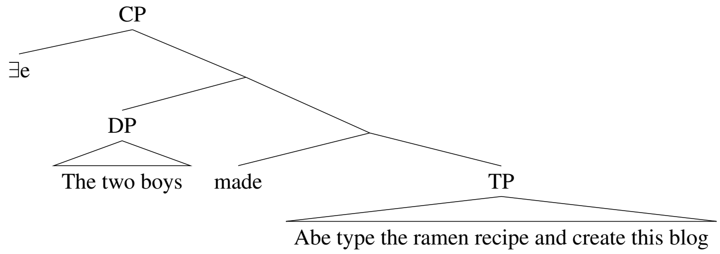
(48) a. **Cumulative inference**
(47a) $\wedge$ (47b) $\rightarrow$ type'$_{\{e'_1,e'_2\}}$({abe},{noodle,broth})
b. **Parts-whole inference**
(48a) $\wedge$ {noodle+broth} $=_c$ {ramen} $\wedge$ {ramen} $\in$ TH(type'$_{\{e'_1,e'_2\}}$)
$\rightarrow$ type'$_{\{e'_1,e'_2\}}$({abe},{ramen})
c. **Cumulative inference**
(47c) $\wedge$ (47d) $\rightarrow$ made'$_{\{e_1,e_2\}}$({boy1,boy2},{e'$_1$,e'$_2$})

{e$_1$,e$_2$} and {e'$_1$,e'$_2$} in (48) serve as witnesses for the existential claims about ε and ε' in (46b). In this way, inferential source can capture cumulativity in causative sentences.

4.4. SG flattening

Without any further assumption, inferential source can capture SG flattening (49). Remember that descriptively (49) is true if there is a cumulative *made*-relation between {boy1,boy2} and the set of propositions {ABE-TYPE-NOODLE,ABE-TYPE-BROTH,ABE-CREATE-BLOG}.

(49) [Scenario: Boy1 made Abe type the noodle recipe. Boy2 made Abe type the broth recipe, and create this blog.]
The two boys made Abe [type **[the ramen recipe]**$_{SG}$ and create this blog]$_{PL}$.



The crucial steps in the derivation of (49) are given below.

(50) a. $[\![\mathrm{TP}]\!] = \lambda\varepsilon'_s.\exists\varepsilon'_1,\varepsilon'_2[\varepsilon'=\varepsilon'_1\cup\varepsilon'_2\&\ \mathrm{type}'_{\varepsilon'_1}(\{\mathrm{abe}\},\{\mathrm{ramen}\})\ \&$
$\mathrm{create}'_{\varepsilon'_2}(\{\mathrm{abe}\},\{\mathrm{blog}\})]$

b. $[\![\mathrm{CP}]\!] = \exists\varepsilon,\varepsilon'.[\mathrm{made}'_{\varepsilon}(\{\mathrm{boy1},\mathrm{boy2}\},\varepsilon')\ \&\ \exists\varepsilon'_1,\varepsilon'_2[\varepsilon'=\varepsilon'_1\cup\varepsilon'_2\ \&$
$\mathrm{type}'_{\varepsilon'_1}(\{\mathrm{abe}\},\{\mathrm{ramen}\})\ \&\ \mathrm{create}'_{\varepsilon'_2}(\{\mathrm{abe}\},\{\mathrm{blog}\})]]$

In the given scenario, there are six eventualities (51).

(51) **Scenario**

a. $\mathrm{type}'_{\{e'_1\}}(\{\mathrm{abe}\}\{\mathrm{noodle}\})$
b. $\mathrm{type}'_{\{e'_2\}}(\{\mathrm{abe}\}\{\mathrm{broth}\})$
c. $\mathrm{create}'_{\{e'_3\}}(\{\mathrm{abe}\}\{\mathrm{blog}\})$
d. $\mathrm{made}'_{\{e_1\}}(\{\mathrm{boy1}\},\{\mathrm{e'}_1\})$
e. $\mathrm{made}'_{\{e_2\}}(\{\mathrm{boy2}\},\{\mathrm{e'}_2\})$
f. $\mathrm{made}'_{\{e_3\}}(\{\mathrm{boy2}\},\{\mathrm{e'}_3\})$

Based on $\{e'_1\}$ and $\{e'_2\}$, the cumulative inference derives $\{e'_1,e'_2\}$ (52a). Then, based on $\{e'_1,e'_2\}$ and the fact that {noodle+broth} is equivalent to {ramen} and the ramen recipe is typed in the scenario, the parts-whole inference in (52b) is valid. Lastly, based on $\{e_1\}$, $\{e_2\}$, and $\{e_3\}$, the cumulative inference derives $\{e_1,e_2,e_3\}$ (52c).

(52) a. **Cumulative inference**
(51a) $\wedge$ (51b) $\rightarrow \mathrm{type}'_{\{e'_1,e'_2\}}(\{\mathrm{abe}\}\{\mathrm{noodle,broth}\})$

b. **Parts-whole inference**
(52a) $\wedge$ $\{\mathrm{noodle+broth}\} =_c \{\mathrm{ramen}\} \wedge \{\mathrm{ramen}\} \in \mathrm{TH}(\mathrm{type}'_{\{e'_1,e'_2\}})$
$\rightarrow \mathrm{type}'_{\{e'_1,e'_2\}}(\{\mathrm{abe}\}\{\mathrm{ramen}\})$

c. **Cumulative inference**
(51d) $\wedge$ (51e) $\wedge$ (51f) $\rightarrow \mathrm{made}'_{\{e_1,e_2,e_3\}}(\{\mathrm{boy1},\mathrm{boy2}\},\{\mathrm{e'}_1,\mathrm{e'}_2,\mathrm{e'}_3\})$

$\{e_1,e_2,e_3\}$, $\{e'_1,e'_2\}$, $\{e'_3\}$, and $\{e'_1,e'_2,e'_3\}$ in (51)-(52) serve as witnesses for the existential claims about ε, ε'_1, ε'_2, and ε' in (50b). In this way, inferential source captures the SG flattening in (49) by effectively relating {boy1,boy2} and $\{e'_1,e'_2,e'_3\}$ cumulatively.

5. Inferential source captures PL flattening *for free*

This section demonstrates that inferential source developed in previous sections captures the PL flattening in (53) *for free*. Remember that descriptively (53) is true if there is a cumulative *made*-relation between {boy1,boy2} and the set of propositions {ABE-TYPE-RECIPE1,ABE-TYPE-RECIPE2,ABE-CREATE-BLOG}, as the sentence is true in the cumulative scenario.

(53) [Scenario: Boy1 made Abe type recipe1. Boy2 made Abe type recipe2 and create the blog.]
The two boys made Abe [type **[the two recipes]**$_{PL2}$ and create this blog]$_{PL1}$.

Under the proposed analysis with inferential source, (53) has the truth conditions in (54).

(54) $[\![(53)]\!]$ = 1 iff $\exists\varepsilon,\varepsilon'$.[made'$_{\varepsilon}$({boy1,boy2},$\varepsilon'$) & $\exists\varepsilon'_1,\varepsilon'_2[\varepsilon'=\varepsilon'_1\cup\varepsilon'_2$ & type'$_{\varepsilon'_1}$({abe},**{recipe1,recipe2}**) & create'$_{\varepsilon'_2}$({abe},{blog})]]

Notice that the truth conditions of the PL flattening in (54) differs from those of the SG flattening in (50b) only in that the theme of ε'_1 is {recipe1,recipe2} instead of {ramen}. So on the assumption that recipe1 and recipe2 in (54) are equivalent to noodle and broth that constitute ramen in (50b), $\{e_1,e_2,e_3\}$, $\{e'_1,e'_2\}$, $\{e'_3\}$, and $\{e'_1,e'_2,e'_3\}$ in (51)-(52) serve as witnesses for the existential claims about ε, ε'_1, ε'_2 and ε' in (54). Crucially, $\{e'_1,e'_2\}$ needed here is the one derived immediately after the cumulative inference in (52a), and the parts-whole inference in (52b) is not needed to analyze the PL flattening in (53). Thus, the truth of the PL flattening in (53) can be confirmed with *one less inference* than the SG flattening in (49). Therefore, inferential source captures the PL flattening in (53) *for free*, and I conclude that flattening data do not support the need of plural projection.

6. Atomic cumulativity and lexical/contextual dependency

In the previous sections, we observed that some lexical items (e.g., *circle* vs. *type*) and some scenarios (e.g., the scenario of the 'ruined' example in (34)) affect the truths of sentences in *sub-atomic* cumulative scenarios. In this section, I will discuss similar phenomena with respect to *atomic* cumulativity. It will be shown that adopting inferential source has another beneficial result in explaining those phenomena. First, we address a lexical effect on the truths of sentences in atomic cumulative scenarios. Consider (55). While sentence (55a) with a *plural* relative clause head is true, sentence (55b) with a *singular* relative clause head is false in the atomic cumulative scenario.

(55) [Scenario: Sue knows man1 who likes country1. Amy knows man2 who likes county2.]
a. Sue and Amy know **men** who like the two countries.
b. Sue and Amy know **a man** who likes the two countries.
(adapted from Beck and Sauerland 2000, 365)

Compositional source with plural projection does not capture the falsity of (55b) without additional assumptions; it wrongly enables (55b) to be true in scenarios like the one in (55) where there is a cumulative *know*-relation between {sue,amy} and {A-MAN-WHO-LIKES-COUNTRY1, A-MAN-WHO-LIKES-COUNTRY2}. On the other hand, inferential source can straightforwardly explain the falsity of (55b) as well as the truth of (55a). First, (55a) and (55b) have the LF in (56a) and the truth conditions in (56b-iii); the difference between (55a) and (55b) lies in whether *(a) man* or *men* is used in the LF/truth conditions.

(56) a. [$_{CP_2}$ [$_{DP}$ **[a man/men]** OP 1 $\exists\varepsilon$ who t_1 likes the two countries] [$_{CP_1}$ 2 $\exists\varepsilon$' Sue and Amy know t_2]]

b. **Crucial steps in the derivation of (56a)**

i. $[\![\text{DP}]\!] = \lambda f.\exists\chi[\underline{\textbf{man'/men'}}(\chi) \wedge \exists\varepsilon[\text{like'}_{\varepsilon}(\chi,\{\text{country1,country2}\})] \wedge f(\chi)]$

ii. $[\![\text{CP}_1]\!] = \lambda\chi.\exists\varepsilon\text{'[know'}_{\varepsilon'}(\{\text{sue,amy}\},\chi)]$

iii. $[\![\text{CP}_2]\!] = 1 \text{ iff } \exists\chi[\underline{\textbf{man'/men'}}(\chi) \wedge \exists\varepsilon[\text{like'}_{\varepsilon}(\chi,\{\text{country1,country2}\})] \wedge$
$\exists\varepsilon\text{'[know'}_{\varepsilon'}(\{\text{sue,amy}\},\chi)]]$

In the scenario of (55), the following cumulative inferences are valid.

(57) **Cumulative inferences**

a. $\text{know'}_{\{e'_1\}}(\{\text{sue}\},\{\text{man1}\}) \wedge \text{know'}_{\{e'_2\}}(\{\text{amy}\},\{\text{man2}\}) \rightarrow$
$\text{know'}_{\{e'_1,e'_2\}}(\{\text{sue,amy}\},\{\text{man1,man2}\})$

b. $\text{like'}_{\{e_1\}}(\{\text{man1}\},\{\text{country1}\}) \wedge \text{like'}_{\{e_2\}}(\{\text{man2}\},\{\text{country2}\}) \rightarrow$
$\text{like'}_{\{e_1,e_2\}}(\{\text{man1,man2}\},\{\text{country1,country2}\})$

Lastly, following Link (1983) among others, I assume that the extensions of *singular* count nouns are sets of *singularities*. So in (55b), the extension of *man* involves {man1} and {man2}, but *cannot* involve pluralities like {man1,man2} (58a). On the other hand, the extensions of *plurals* can involve pluralities. So in (55a), the extension of *men* involves {man1} and {man2}, and from those two sets, we can also assume {man1,man2} to exist in the extension via cumulative inference (i.e., men'({man1}) $\wedge$ men'({man2}) $\rightarrow$ men'({man1,man2})) (58b).

(58) a. **Extension of man'**: {{man1}, {man2}, ...}

b. **Extension of men'**: {{man1}, {man2}, {man1,man2},...}

As for (55a), {man1,man2}, $\{e_1,e_2\}$, and $\{e'_1,e'_2\}$ in (57) and (58b) serve as witnesses for the existential claims about χ, ε, and ε' in (56b-iii) with men'. Thus, (55a) is correctly predicted true. In contrast, as for (55b), there is no element that can correspond to χ in (56b-iii); {man1,man2} is in a part of the extensions of know' and like' (57), but it is not in the extension of man' (58a). Thus, the existential claims in (56b-iii) with man' are not valid, and (55b) is false. In this way, inferential source captures the lexical effect on the truths of (55a) and (55b).

Next, we address a contextual effect on the coherence of sentences in atomic cumulative scenarios. Consider (59). While sentence (59a) sounds coherent in (59b), it sounds incoherent in

(59c); it sounds like commenting on the impossible situation where *both* Abe and Ken win in the final match.[16]

(59) a. If Abe and Ken win, Ada and Ai will be happy.

b. **Scenario where (59a) sounds *coherent***: Abe and Ken will fight in different boxing semi-final matches. If Abe wins, Ada will be happy. If Ken wins, Ai will be happy.

c. **Scenario where (59a) sounds *incoherent***: Abe will fight against Ken in a boxing final match. If Abe wins, Ada will be happy. If Ken wins, Ai will be happy.

Compositional source with plural projection enables (59a) to be true if there is a cumulative *if*-relation between {ABE-WIN,KEN-WIN} and {ADA-WILL-BE-HAPPY,AI-WILL-BE-HAPPY} (e.g.,Schmitt 2019). Thus, the approach correctly predicts that (59a) sounds coherent in (59b), but it does not capture the incoherence of (59a) in (59c). On the other hand, inferential source straightforwardly captures its incoherence as well as the coherence of (59a) in (59b). First, (59a) has the LF in (60a) and the truth conditions in (60b-ii). Note that here I have adopted an intensional semantic framework (e.g., Lewis 1970) where linguistic expressions are directly mapped to their intensions except for the words whose extensions are constant across worlds like proper names and functional items (e.g., *if*, *the*, *every*). For example, CP_1 in (60a) denotes (60b-i) which reads: there is an eventuality ε of Abe and Ken each winning *in the world w*.

(60) a. $[_{CP_3}[$ if $[_{CP1}$ $\exists\varepsilon$ Abe and Ken wins$]]$ $[_{CP_2}$ $\exists\varepsilon$' Ada and Ai will be happy$]]$

b. **Crucial steps in the derivation of (60a)**

i. $[\![CP_1]\!] = \lambda w.\exists\varepsilon[\text{win'}^{w}_{\varepsilon}(\{\text{abe,ken}\})]$

ii. $[\![CP_3]\!] = 1$ iff $\forall w[\exists\varepsilon[\text{win'}^{w}_{\varepsilon}(\{\text{abe,ken}\})] \rightarrow \exists\varepsilon\text{'}[\text{happy'}^{w}_{\varepsilon'}(\{\text{ada,ai}\})]]$

$[\![CP_3]\!]$ reads: (60a) is true iff in all the worlds where there is an eventuality of Abe and Ken each winning, there is an eventuality of Ada and Ai each being happy. Importantly, there cannot be such a winning eventuality in (59c) because only one boxer can win in the final match. So the conditional statement in (60b-ii)/(59a) is vacuously true in (59c), and I take this as the cause of the incoherence of (59a) in (59c). In contrast, (59a) does not have such a problem in (59b) because both Abe and Ken can win in different semi-final matches. Also, in (59b), we can assume $\{e_1,e_2\}$ and $\{e\text{'}_1,e\text{'}_2\}$ in (61) via cumulative inference, and those eventualities correspond to ε and ε' in (60b-ii). Thus, the scenario in (59b) is compatible with the truth conditions in (60b-ii), so the sentence is correctly predicted to sound coherent in (59b).

(61) a. $\text{win'}^{w}_{\{e_1\}}(\{\text{abe}\}) \wedge \text{win'}^{w}_{\{e_2\}}(\{\text{ken}\}) \rightarrow \text{win'}^{w}_{\{e_1,e_2\}}(\{\text{abe,ken}\})$

b. $\text{happy'}^{w}_{\{e'_1\}}(\{\text{ada}\}) \wedge \text{happy'}^{w}_{\{e'_2\}}(\{\text{ai}\}) \rightarrow \text{happy'}^{w}_{\{e'_1,e'_2\}}(\{\text{ada,ai}\})$

To sum up, inferential source can provide a straightforward explanation of why the truths/coherence of sentences in atomic cumulative scenarios can be affected by some lexical items and some scenarios where the sentences are used.

[16] See also Winter (2000) and Beck and Sauerland (2000) for other observations about atomic cumulations in conditional sentences.

7. Conclusion

This paper demonstrated that flattening examples, which seem *prima facie* to support the need of plural projection, do not actually support it. First, Section 2 briefly introduced how plural projection enables compositional source of cumulativity to capture PL flattening. Section 3 then demonstrated that compositional source with plural projection does not capture SG flattening. After that, Section 4 showed that a source of cumulativity, which I call the inferential source (e.g., Krifka 1989, Kratzer 2007), can capture SG flattening and more generally sub-atomic cumulativity. It was shown that this approach assumes that sentences do not have cumulative truth conditions but very weak truth conditions that are compatible with cumulative scenarios via inferences. What is worth noting here is that such truth conditions are compatible with the so-called *collective scenarios* as well. Consider (62a) and its truth conditions in (62b) which reads: (62a) is true iff there is an eventuality ε of the box being moved as a result of boy1 and boy2 each having done something. There is such an eventuality in the collective scenario of (62). Thus, our approach with inferential source correctly predicts (62a) is true.

(62) [Scenario: The two boys moved the box together. None of them moved anything alone.]
a. The two boys moved the box. b. $[\![(62a)]\!] = 1$ iff $\exists\varepsilon[\text{moved}_{\varepsilon}(\{\text{boy1,boy2}\},\{\text{box}\})]$

After showing that inferential source is needed to capture SG flattening, I showed in Section 5 that it also captures PL flattening for free. Thus, Section 5 concluded that inferential source is sufficient to capture all the flattening examples discussed in the literature and thus flattening examples do not support the need of plural projection. Through this discussion, it also turned out that an event semantics plays an important role in deriving the meanings of flattening examples. Also, it became clearer that we should address cumulativity with a *singular* DP when we formulate an analysis of cumulativity. Lastly, in Section 6, it was also presented that inferential source can provide a straightforward explanation about why the truths/coherence of sentences in atomic cumulative scenarios are affected by some lexical items and some scenarios where the sentences are used. In this way, inferential source can capture a wide range of both true/felicitous/coherent and false/infelicitous/incoherent data in cumulative scenarios. But it remains to be seen how generally cumulative and parts-whole inferences are valid; for instance, (i) are there more predicates like *circle* that always make parts-whole inference invalid? (ii) While Beck and Sauerland (2000) propose that Cuml takes a *syntactically derived* predicate like [$\lambda x \lambda y$.y **wants to marry** x] (instead of a lexical predicate) as its first argument in some sentences, do cumulative and parts-whole inferences also apply to such predicates in some sentences? Examining such questions is an important task for future work on cumulativity.

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