**Appendix 1**

Proof of the variance  from Eq. (13):

Let EP, E, and VP, V, respectively, denote the expectations and variances over the probability sampling method P and the non-randomized questioning design of our family ( = , , , , ). Then, the theoretical variance of  can be written as

 

For the first of these two summands,

 

from Eq. (8) applies. For the second summand, with Ik being a (1/0)-variable indicating the membership of respondent k in sample s and ,

 

applies. Therein,

 

Hence,

 

and

 .

Proof of the unbiasedness of  from Eq. (14) for  from Eq. (13):

The expected value of

 

is calculated by



The first of the two expectations E within the brackets of this term yields



Moreover,

 

applies. Hence with EP(Ik · Il) = kl, the expectation over the sampling method P is given by



applies. With

 

and ,

 ,

which completes the proof of the unbiasedness of .

**Appendix 2**

Proof of the variance  from Eq. (16):

For SI sampling in Eq. (8),

 

applies (cf., for instance, Särndal et al. 1992, p. 70). Inserting dk = 1/k = N/n in the second sum of Eq. (13) gives the result of the second sum of Eq. (16).

Proof of the unbiasedness of  from Eq. (17) for  from Eq. (16):

For SI sampling with fixed n, for the first summand in Eq. (14),

 

applies (cf., for instance, Särndal et al. 1992, p. 45). With , and kl = kl  k · l, this results in

 

For the second summand of Eq. (14), that is

 ,

with dk = N/n,

 

applies, which completes the proof of Eq. (17).

**Appendix 3**

To show the direct relation between the variance  from Eq. (13) and the measures of privacy protection from Section 4.2, from Eqs. (18) and (19), the parameters  and  can be expressed as functions of  and . For  > 0,



and



applies. For  < 0, this results in



and

.

Inserting these expressions into the parameters  and  of the second component of the variance expression (13) shows that the additional variance depends solely on the privacy protection offered by the respective model. Questioning designs with combinations of the design parameters that lead to the same privacy protection levels have equal variances.