

**A Appendix****A.1 Appendix: Auxiliary Tables and Figures**

Table 2

*Overview Data Sets with Grouped Data*

Data set	Country	Measure	No. of Brackets
European Social Survey (ESS)	Multiple Countries	Family Income	10
Microcensus (MZ)	Germany	Individual Income	24
Socio-Economic Panel Study (SOEP, 1988)	Germany	Household Wealth	7-9
Household Economic Survey (HES)	New Zealand	Investment Income	18
Current Population Survey (CPS)	USA	Family Income	16
General Social Survey (GSS)	USA	Family Income	26
		Personal Earnings	26
American National Election Studies (ANES)	USA	Family Income	22
Annual American Time Use Survey (ATUS)	USA	Weekly Earnings	Only top coded

Table 3  
MCIB and other approaches

Type	Method	Author(s)	Procedure	Pro's/Con's
<i>Non-Parametric</i>	Midpoint Estimation	Hout (2004)	midpoint <sub>b</sub> = $(L_b + U_b)/2$	Pro's: Easy to estimate Con's: Does not address within-bin variance; no imputation options
	Robust Pareto Midpoint Estimator (RPME)	Von Hippel, Scarpino, and Holas (2016)	Similar to Midpoint Estimation, but assigns arithmetic mean of a Pareto distribution to top-bracket as well as constraints and varies the Pareto parameter $\alpha$ and provides the median, geometric and harmonic mean as alternative midpoints for the top bin	Pro's: Easy to estimate and robust to small sample sizes Con's: Does not address within-bin variance; not very accurate if the number of bins is low; no imputation option
	CDF Interpolation	Von Hippel, Hunter, and Drown (2017)	Fits interpolated CDFs to grouped data (counts in grouped and continuous distribution are equal for each bin); top bin: Pareto, rectangular, or exponential	Pro's: Values of the fitted distribution represent the number of observations observed within each bin Con's: no imputation option
<i>Parametric</i>	Random Empirical Distribution Imputation (REDI)	King (2022)	Performs cold-deck imputation from a same bin of reference data set to impute continuous measures	Pro's: Imputes real metric values for each observation from the grouped distribution Con's: Can not be used without a suitable reference data set
	Multimodel Generalized Beta Estimator (MGBE)	McDonald and Ransom (1979), McDonald and Xu (1995), and Jenkins (2012)	Fits various generalized beta functions to the grouped data and then uses the distribution with the best fit or averages estimates across models	Pro: Does not assume one specific distribution to adequately fit the data Cons: Slow estimation and only provides specific moments of the distribution (i.e., no imputation option)
	Mean Constrained Integration over Brackets (MCIB)	Jargowsky and Wheeler (2018)	Approximates densities (with varying density functions) across bins and integrates over bins to estimate moments of the distribution	Pro: Utilizes more information from the grouped distribution and addresses within-bin variance Cons: Only estimates specific moments of the distribution

Table 4  
Further MCIB estimations: net income

stat	original	4 brackets	diff	8 brackets	diff	12 brackets	diff	16 brackets	diff	20 brackets	diff	24 brackets	diff
90	mean	2,654 (7)	2,808 (10)	5.77 (0.46)	2,790 (9)	5.12 (0.45)	2,830 (9)	6.63 (0.45)	2,811 (10)	5.90 (0.45)	2,792 (8)	2,763 (8)	4.09 (0.45)
	sd	1,748	2,525 (19)	44.46 (1.22)	2,211 (15)	26.48 (0.97)	2,347 (18)	34.30 (1.17)	2,237 (17)	27.98 (1.14)	2,168 (15)	2,043 (12)	16.91 (0.86)
	median	2,250	2,330 (52)	3.53 (-10.09)	2,250 (76)	0.02 (-9.17)	2,238 (94)	-0.52 (-8.51)	2,269 (104)	0.85 (-8.12)	2,248 (130)	2,256 (188)	0.26 (-4.96)
	p1	320	518 (260)	-9.74 (-11.25)	382 (763)	-5.13 (-2.14)	473 (857)	-1.68 (1.41)	528 (832)	0.39 (0.44)	559 (856)	578 (822)	2.26 (0.09)
	p10	820	1,301 (3.75)	-3.73 (-11.86)	1,413 (701)	0.48 (-4.47)	1,428 (1,441)	1.06 (1.55)	1,423 (1,441)	0.86 (0.27)	1,417 (823)	1,413 (840)	0.49 (0.76)
	p25	1,400	3,500 (4.02)	19.58 (5.17)	3,529 (3,529)	1.11 (1.11)	3,572 (3,572)	2.71 (3.94)	3,586 (4,955)	3.23 (-1.69)	3,599 (4,988)	3,568 (5,081)	2.56 (3.05)
	p75	3,500	5,000 (6.988)	37.26 (9.14)	6,659 (9,215)	24.81 (8.12)	6,840 (9,200)	31.65 (7.53)	6,720 (9,209)	27.13 (7.87)	6,630 (9,222)	6,452 (9,192)	17.02 (7.27)
	p90	5,000	9,000 (0.80)	199.97 (0.00)	12,294 (0.00)	124.10 (0.00)	13,381 (0.00)	165.06 (1.66)	12,564 (2,698)	134.26 (1.63)	12,076 (2,699)	11,106 (2,780)	79.35 (4.74)
	Correlation		0.79 (0.00)		0.85 (0.00)		0.84 (0.00)		0.86 (0.00)		0.87 (0.00)		0.89 (0.00)
95	mean	2,654	2,684 (9)	1.11 (0.00)	2,724 (8)	2.60 (0.40)	2,698 (7)	1.66 (0.38)	2,698 (8)	1.63 (0.39)	2,699 (8)	2,780 (9)	4.74 (0.43)
	sd	1,748	2,123 (14)	21.49 (0.00)	1,951 (10)	11.61 (0.75)	1,920 (10)	9.87 (0.75)	1,916 (11)	9.62 (0.75)	1,885 (10)	2,210 (18)	26.47 (1.14)
	median	2,250	2,352 (50)	4.53 (-10.17)	2,287 (76)	1.65 (-9.17)	2,233 (90)	-0.78 (-8.68)	2,272 (107)	0.96 (-8.02)	2,259 (105)	2,245 (169)	-0.21 (-5.71)
	p1	320	518 (250)	-10.09 (-12.09)	381 (762)	-5.17 (-2.17)	455 (855)	-2.36 (1.30)	538 (827)	0.74 (0.26)	516 (824)	601 (844)	3.14 (0.89)
	p5	518	1,249 (3.566)	-5.70 (-11.86)	1,442 (701)	0.57 (-4.47)	1,417 (1,441)	0.63 (1.55)	1,431 (1,441)	1.17 (0.27)	1,382 (823)	1,434 (840)	1.29 (0.76)
	p10	820	3,500 (4.02)	19.58 (5.17)	3,523 (3,529)	1.42 (1.11)	3,538 (3,572)	1.42 (3.94)	3,529 (4,955)	1.08 (-1.69)	3,514 (4,988)	3,623 (5,081)	4.62 (3.05)
	p25	1,400	5,000 (6.988)	37.26 (9.14)	6,247 (9,215)	6.14 (8.12)	5,036 (9,200)	1.35 (7.25)	5,006 (10,201)	0.22 (45.23)	5,082 (9,747)	4,915 (12,679)	-3.22 (138.61)
	p75	3,500	9,000 (0.80)	94.25 (0.00)	10,229 (0.00)	46.32 (0.00)	10,123 (0.00)	42.31 (2.04)	10,201 (2,694)	45.23 (1.49)	9,747 (2,668)	9,747 (2,700)	138.61 (1.71)
	p90	5,000	7,58 (8)	7.58 (0.00)	2,700 (8)	1.71 (0.38)	2,709 (7)	2.04 (0.36)	2,694 (7)	1.49 (0.36)	2,668 (7)	2,700 (7)	1.71 (0.37)
	Correlation		0.86 (0.00)		0.93 (0.00)		0.94 (0.00)		0.95 (0.00)		0.95 (0.00)		0.99 (0.00)
99	mean	2,654	2,856 (8)	7.58 (0.00)	2,700 (8)	1.71 (0.38)	2,709 (7)	2.04 (0.36)	2,694 (7)	1.49 (0.36)	2,668 (7)	2,700 (7)	1.71 (0.37)
	sd	1,748	2,127 (8)	21.71 (0.00)	1,887 (9)	7.99 (0.68)	1,858 (8)	6.32 (0.63)	1,812 (8)	3.70 (0.60)	1,794 (9)	1,815 (8)	3.84 (0.63)
	median	2,250	2,393 (51)	6.36 (-10.14)	2,289 (70)	1.72 (-9.42)	2,188 (91)	-2.73 (-8.63)	2,286 (104)	1.60 (-8.13)	2,260 (108)	2,278 (175)	1.25 (-5.45)
	p1	320	518 (254)	-9.96 (-11.86)	352 (701)	-6.27 (-4.47)	453 (859)	-2.47 (1.47)	520 (868)	0.07 (1.81)	537 (823)	592 (840)	2.78 (0.76)
	p5	518	1,263 (3.529)	-5.17 (-11.86)	1,435 (701)	1.34 (-4.47)	1,441 (1,441)	1.55 (1.55)	1,407 (1,441)	0.27 (0.27)	1,411 (823)	1,419 (840)	0.72 (0.76)
	p10	820	4,020 (4.02)	19.58 (5.17)	3,529 (3,529)	1.11 (1.11)	3,572 (3,572)	2.71 (3.94)	3,586 (4,955)	3.23 (-1.69)	3,599 (4,988)	3,568 (5,081)	2.56 (3.05)
	p25	1,400	5,964 (6.988)	37.23 (9.14)	6,534 (9,215)	20.11 (8.12)	6,415 (9,200)	15.62 (7.53)	6,448 (9,209)	5.57 (7.87)	6,042 (9,222)	6,193 (9,192)	7.27 (7.22)
	p75	3,500	9,243 (0.80)	9.14 (0.00)	9,215 (0.00)	8.12 (0.00)	9,200 (0.00)	7.53 (1.66)	9,209 (2,698)	7.87 (1.63)	9,222 (2,699)	9,192 (2,780)	7.22 (4.74)
	p90	5,000											
	Correlation		0.80 (0.00)		0.93 (0.00)		0.97 (0.00)		0.98 (0.00)		0.98 (0.00)		0.99 (0.00)

Compiled by authors based on SOEP v37. The table provides several descriptive statistics of the net income distribution at the household level in 2017, with non-negative and non-zero values. The original distribution is taken directly from the SOEP data. For the MCIB distribution, the data are artificially grouped into 4, 8, 12, 16, 20, and 24 brackets with n upper cut-off limits set to the 90th, 95th, and 99th percentiles. Variables are winsorized at the 0.5th and the 99.5th percentiles, respectively. The difference in percent is mean-adjusted for the percentile estimates. Bootstrapped standard errors are in parentheses based on 500 bootstrap weights.

Table 5  
Further MCIB estimations: Gross wealth

stat	original	4 brackets	diff	8 brackets	diff	12 brackets	diff	16 brackets	diff	20 brackets	diff	24 brackets	diff	
90	mean	271,144 (2,167)	-6.08 (1.00)	256,154 (1,785)	-5.53 (1.00)	257,430 (1,875)	-5.06 (1.07)	258,549 (1,845)	-4.65 (0.95)	259,555 (1,869)	-4.27 (1.04)	258,872 (1,724)	-4.53 (0.98)	
	sd	416,787 (5,360)	-14.60 (3.769)	353,276 (3,854)	-15.24 (3.138)	350,500 (3,821)	-15.90 (3.47)	352,597 (3,705)	-15.40 (3.32)	356,341 (3,954)	-14.50 (3.49)	352,671 (3,557)	-15.38 (3.36)	
	median	156,000	-24.90	153,682	-1.49	155,957	-0.03	156,217	0.14	155,539	-0.30	156,286	0.18	
	p1	400	0.28	599	0.07	470	0.03	505	0.04	470	0.03	408	0.00	
	p5	2,300	6.060	3,002	3.002	2,506	0.08	2,627	0.12	2,615	0.12	2,615	0.12	
	p10	5,000	12.144	5,957	3.35	5,215	0.08	5,483	0.18	5,380	0.14	5,140	0.05	
	p25	23,000	30.444	22,285	-0.26	22,710	-0.11	22,517	-0.18	23,670	0.25	22,914	-0.03	
	p75	350,000	359,418	3,47	348,992	-0.37	349,294	-0.26	348,701	-0.48	348,985	-0.37	350,073	0.03
	p90	610,987	611,035	0.02	611,236	0.09	611,035	0.02	611,035	0.02	611,035	0.02	611,035	0.02
	p95	900,000	858,816	-15.19	862,207	-13.94	853,804	-17.04	859,379	-14.98	863,346	-13.52	856,021	-16.22
p99	2,300,000	1,903,808	-146.12	1,916,268	-141.52	1,873,696	-157.22	1,891,589	-150.63	1,879,604	-155.05	1,891,589	-150.63	
Correlation		0.71 (0.00)	0.72 (0.01)	0.71 (0.01)	0.73 (0.01)	0.71 (0.01)	0.71 (0.01)	0.73 (0.01)	0.71 (0.01)	0.71 (0.01)	0.71 (0.01)	0.72 (0.01)	0.71 (0.01)	
95	mean	271,144 (2,167)	-1.85 (0.00)	263,372 (1,997)	-2.87 (1.10)	262,943 (1,992)	-3.02 (1.01)	262,715 (1,925)	-3.11 (1.06)	262,038 (1,849)	-3.36 (1.04)	263,865 (1,883)	-2.68 (1.03)	
	sd	416,787 (5,360)	-9.47 (4.137)	372,996 (4,231)	-10.51 (1.56)	372,168 (4,187)	-10.71 (1.41)	369,335 (4,098)	-11.39 (1.50)	364,718 (3,845)	-12.49 (1.48)	374,756 (4,182)	-10.08 (1.51)	
	median	156,000	-25.36	153,018	-1.91	155,241	-0.49	156,608	0.39	156,297	0.19	158,143	1.37	
	p1	400	0.38	608	0.08	550	0.06	527	0.05	465	0.02	370	-0.01	
	p5	2,300	7.068	3,048	3.048	2,811	0.19	2,507	0.08	2,324	0.01	2,182	-0.04	
	p10	5,000	14.211	3.40	6,044	0.38	5,706	0.26	5,383	0.14	5,118	0.04	5,380	0.14
	p25	23,000	35.329	4.55	22,824	-0.07	22,662	-0.12	23,160	0.06	23,211	0.08	22,492	-0.19
	p75	350,000	384,360	12.67	355,237	1.93	350,726	0.27	348,177	-0.67	351,177	0.43	349,668	-0.12
	p90	610,987	641,795	11.36	615,552	1.68	609,677	-0.48	604,628	-2.35	611,033	0.02	610,379	-0.22
	p95	900,000	902,976	1.10	902,976	1.10	904,168	1.54	903,572	1.32	903,572	1.32	902,976	1.10
p99	2,300,000	2,012,113	-106.17	1,992,490	-113.41	2,018,785	-103.71	1,998,967	-111.02	1,948,854	-129.51	2,039,206	-96.18	
Correlation		0.79 (0.00)	0.82 (0.00)	0.83 (0.00)	0.83 (0.00)	0.83 (0.00)	0.83 (0.00)	0.83 (0.00)	0.84 (0.00)	0.84 (0.00)	0.84 (0.00)	0.83 (0.00)	0.83 (0.00)	
99	mean	271,144 (2,167)	44.80 (0.00)	308,097 (2,489)	13.63 (1.25)	286,454 (2,222)	5.65 (1.15)	279,900 (2,160)	3.23 (1.11)	275,587 (2,152)	1.64 (1.14)	274,098 (2,049)	1.09 (1.12)	
	sd	416,787 (5,360)	560,663 (3,552)	34.52 (4.362)	470,641 (4,362)	12.92 (1.81)	438,157 (4,713)	5.13 (1.75)	425,791 (4,724)	2.16 (1.71)	418,098 (4,825)	413,861 (4,666)	-0.70 (1.69)	
	median	156,000	-24.39	153,383	-1.68	154,540	-0.94	156,770	0.49	155,753	-0.16	157,207	0.77	
	p1	400	1,455	0.39	583	0.07	557	0.06	459	0.02	523	0.05	448	0.02
	p5	2,300	7,700	1.99	2,946	0.24	2,781	0.18	2,287	0.00	2,504	0.08	2,305	0.00
	p10	5,000	15,350	3.82	5,873	0.32	5,686	0.25	5,185	0.07	5,293	0.11	5,299	0.11
	p25	23,000	38,187	5.60	21,850	-0.42	22,633	-0.14	23,264	0.10	22,635	-0.13	22,784	-0.08
	p75	350,000	574,400	82.76	357,395	2.73	349,779	-0.08	349,721	-0.10	350,081	0.03	347,865	-0.79
	p90	610,987	1,243,345	233.22	862,724	92.84	611,760	0.29	622,505	4.25	610,125	-0.32	611,167	0.07
	p95	900,000	1,601,356	258.67	1,338,008	161.54	1,173,164	100.75	1,072,275	63.54	990,202	33.27	931,063	11.46
p99	2,300,000	2,300,179	0.07	2,300,179	0.07	2,307,808	2.88	2,300,179	0.07	2,300,179	0.07	2,300,179	0.07	
Correlation		0.72 (0.00)	0.86 (0.00)	0.91 (0.00)	0.93 (0.00)	0.91 (0.00)	0.93 (0.00)	0.93 (0.00)	0.95 (0.00)	0.95 (0.00)	0.95 (0.00)	0.96 (0.00)	0.96 (0.00)	

Compiled by authors based on SOEP v37. The table provides several descriptive statistics of the gross wealth distribution at the household level in 2017, with non-negative and non-zero values. The original distribution is taken directly from the SOEP data. For the MCIB distribution, the data are artificially grouped into 4, 8, 12, 16, 20, and 24 brackets with upper cut-off limits set to the 90th, 95th, and 99th percentiles. Variables are winsorized at the 0.5th and the 99.5th percentiles, respectively. The difference in percent is mean-adjusted for the percentile estimates. Bootstrapped standard errors are in parentheses based on 500 bootstrap weights.

Table 6  
Original data compared to MCIB estimations: Reduced datasets

	net household income			gross household wealth		
	(orig. 50%)	(MCIB 50%)	(Δ%)	(orig. 20%)	(MCIB 20%)	(Δ%)
mean	2,656 (10)	2,656 (11)	0.03 (0.55)	2,679 (16)	2,681 (17)	0.08 (0.85)
sd	1,751 (11)	1,817 (12)	3.76 (0.92)	1,812 (19)	1,882 (22)	3.83 (1.59)
median	2,260 (320)	2,248 (72)	-0.51 (-9.35)	2,200 (320)	2,212 (90)	0.55 (-8.60)
p1	530	375	-5.85	500	384	-4.34
p10	828	769	-2.21	828	763	-2.41
p25	1,400	1,393	-0.28	1,400	1,411	0.41
p75	3,500	3,519	0.71	3,500	3,580	2.98
p90	5,000	4,938	-2.32	5,000	5,010	0.37
p95	6,000	6,042	1.59	6,000	6,108	4.05
p99	9,000	9,310	11.66	9,500	9,820	11.94
N	9,850	9,850		3,939	3,939	

	net household income			gross household wealth		
	(orig. 50%)	(MCIB 50%)	(Δ%)	(orig. 20%)	(MCIB 20%)	(Δ%)
mean	2,656 (10)	2,656 (11)	0.03 (0.55)	2,679 (16)	2,681 (17)	0.08 (0.85)
sd	1,751 (11)	1,817 (12)	3.76 (0.92)	1,812 (19)	1,882 (22)	3.83 (1.59)
median	2,260 (320)	2,248 (72)	-0.51 (-9.35)	2,200 (320)	2,212 (90)	0.55 (-8.60)
p1	530	375	-5.85	500	384	-4.34
p10	828	769	-2.21	828	763	-2.41
p25	1,400	1,393	-0.28	1,400	1,411	0.41
p75	3,500	3,519	0.71	3,500	3,580	2.98
p90	5,000	4,938	-2.32	5,000	5,010	0.37
p95	6,000	6,042	1.59	6,000	6,108	4.05
p99	9,000	9,310	11.66	9,500	9,820	11.94
N	9,850	9,850		3,939	3,939	

Inequality Measures						
Mean Log Dev	0.22 (0.00)	0.28 (0.00)	26.30 (1.58)	0.23 (0.00)	0.28 (0.00)	22.58 (2.52)
Theil	0.20 (0.00)	0.22 (0.00)	9.92 (1.22)	0.21 (0.00)	0.22 (0.00)	9.05 (2.07)
Gini	0.35 (0.00)	0.36 (0.00)	3.48 (0.56)	0.35 (0.00)	0.37 (0.00)	3.18 (0.94)
COV	0.66 (0.00)	0.68 (0.00)	3.72 (0.70)	0.68 (0.01)	0.70 (0.01)	3.75 (1.25)
Rel. Mean Dev	0.25 (0.00)	0.26 (0.00)	3.10 (0.59)	0.26 (0.00)	0.26 (0.00)	3.01 (0.97)
Correlation		0.96 (0.00)		0.96 (0.00)		

Compiled by authors based on SOEP v37. The table provides several moments of the net income and gross wealth distribution at the household level in 2017, with non-negative and non-zero values. The original distribution is taken directly from the SOEP data. For the MCIB distribution, the data are artificially grouped into 10 brackets with an upper cut-off limited to the 98th percentile. Variables are winsorized at the 0.5th and the 99.5th percentiles, respectively. The difference in percent is mean-adjusted for the percentile estimates. Bootstrapped standard errors are in parentheses, based on 500 bootstrap weights. The table provides the same setting as Table 1 in the main text, only with randomly reduced datasets with only 50 and 20 percent of the observations.

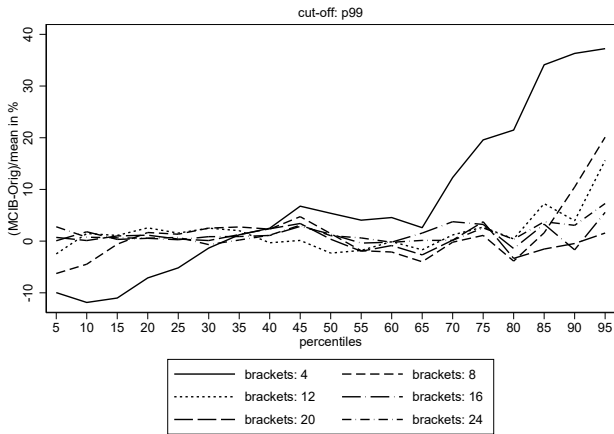


Figure 7. p99 cut-off: household net income. Compiled by authors based on SOEP v37. The panel shows the mean-adjusted difference in percent between the original distribution and the MCIB approximation (y-axis) along percentiles of households' net income (x-axis) in 2017. The panel depicts differences for various numbers of brackets. Variables are winsorized at the 0.5th and the 99.5th percentile, respectively.

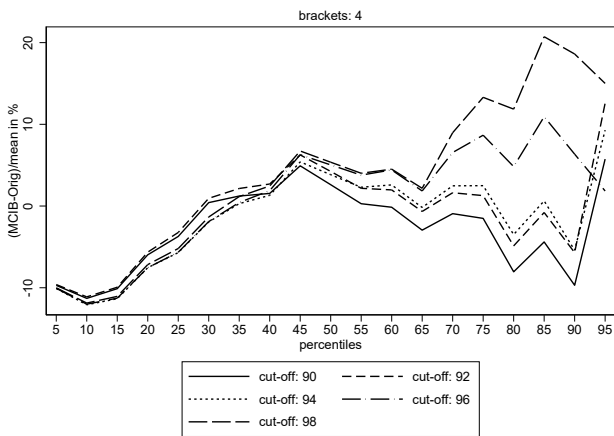


Figure 8. 4 brackets: household net income. Compiled by authors based on SOEP v37. The panel shows the mean-adjusted difference in percent between the original distribution and the MCIB approximation (y-axis) along percentiles of households' net income (x-axis) in 2017. The panel depicts differences for various cut-off limits. Variables are winsorized at the 0.5th and the 99.5th percentile, respectively.

Table 7  
Original data compared to MCIB estimations with 10 brackets with adjusted brackets settings

(1)	(2)	(3)	(4)	(5)	(6)
gross household wealth	(orig.)	(MCIB) (orig. brackets)	(Δ%)	(MCIB) (adj. brackets)	(Δ%)
mean	271,144	274,028	0.01	266,816	-
	(2,118)	(2,007)	(0.01)	(1,970)	(0.01)
sd	416,787	401,723	-	385,375	-
	(5,625)	(4,531)	(0.02)	(4,643)	(0.02)
median	156,000	155,722	0.00	154,612	-
					0.01
p1	400	533	0.00	538	0.00
p5	2,300	2,728	0.00	2,915	0.00
p10	5,000	5,420	0.00	6,100	0.00
p25	23,000	23,042	0.00	227,66	0.00
p75	350,000	346,950	-	359,176	0.03
			0.01		
p90	610,987	672,557	0.23	607,267	-
					0.01
p95	900,000	997,407	0.36	884,713	-
					0.06
p99	2,300,000	2,133,898	-	2,113,580	-
			0.61		0.69
N	12,698	12,698		12,698	
Inequality Measures					
Mean Log Dev	1.14	1.14	-	1.10	-
	(0.01)	(0.01)	0.01	(0.01)	0.04
Theil	0.73	0.72	-	0.70	-
	(0.01)	(0.01)	0.01	(0.01)	0.04
Gini	0.62	0.62	0.00	0.62	-
	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)
COV	1.54	1.47	-	1.44	-
	(0.01)	(0.01)	0.05	(0.01)	0.06
Rel. Mean Dev	0.45	0.46	0.01	0.45	-
	(0.00)	(0.00)	(0.01)	(0.00)	(0.01)
Correlation		0.91		0.94	
		(0.00)		(0.00)	

Compiled by authors based on SOEP v37. The table provides several moments of the gross wealth distribution at the household level in 2017, with non-negative and non-zero values. The original distribution is taken directly from the SOEP data. For the MCIB distribution, the data are artificially grouped in 10 brackets with an upper cut-off limited to the 98th percentile. Column (3) and (4) show the original approximation and column (5) and (6) show the estimates with adjusted bracket size. Variables are winsorized at the 0.5th and the 99.5th percentiles, respectively. The difference in percent is mean-adjusted for the percentile estimates. Bootstrapped standard errors are in parentheses, based on 500 bootstrap weights.

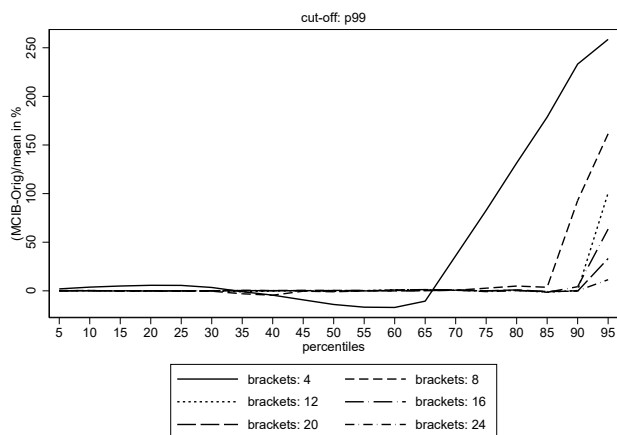


Figure 9. p99 cut-off: household gross wealth. Compiled by authors based on SOEP v37. The panel shows the mean-adjusted difference in percent between the original distribution and the MCIB approximation (y-axis) along percentiles of households' gross wealth (x-axis) in 2017. The panel depicts differences for various numbers of brackets. Variables are winsorized at the 0.5th and the 99.5th percentile, respectively.

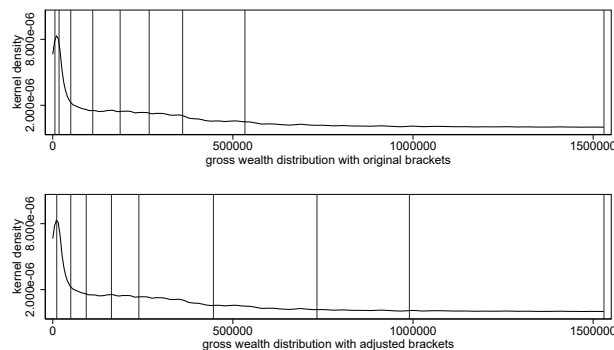


Figure 11. Kernel density plots with brackets settings. Compiled by authors based on SOEP v37. The panels show the brackets along the Epanechnikov kernel density function for the setting with 10 brackets (the last one is not depicted here) and a cut-off at the 98th percentile. The upper panel shows the original bracket setting. The lower panel shows a setting with approximately linear density functions within each bracket.

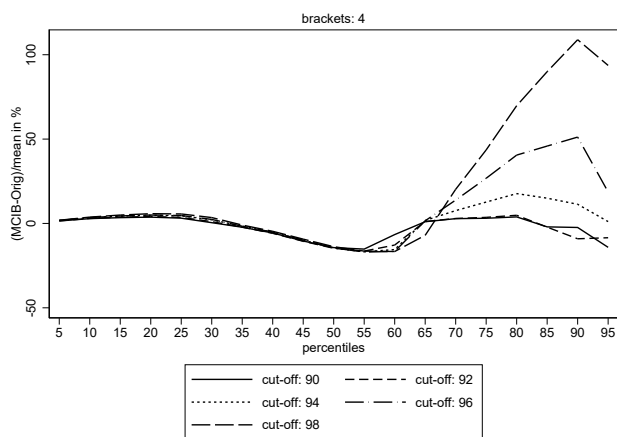


Figure 10. 4 brackets: household gross Wealth. Compiled by authors based on SOEP v37. The panel shows the mean-adjusted difference in percent between the original distribution and the MCIB approximation (y-axis) along percentiles of households' gross wealth (x-axis) in 2017. The panel depicts differences for various cut-off limits. Variables are winsorized at the 0.5th and the 99.5th percentile, respectively.

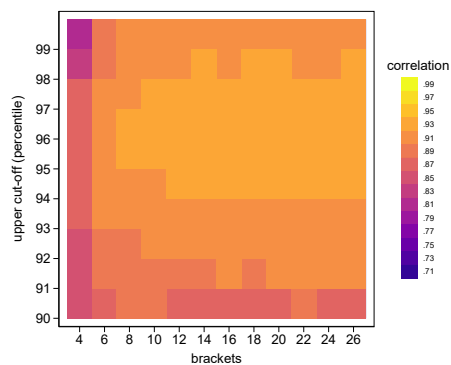


Figure 12. Correlation between original and simulated data by number of brackets and upper cut-off percentiles. Compiled by authors based on SOEP v37. The figure shows the correlation between the original distribution and the MCIB approximation based on equal width brackets. The y-axis depicts different upper cut-off limits; the x-axis different numbers of brackets. Variables are winsorized at the 0.5th and the 99.5th percentiles, respectively.

## A.2 Simulations

Our analysis relies on German survey data, but can our findings be generalized to other countries? We provide suggestive evidence that the results are relatively stable for different types of distributions and that the cut-off becomes more important if the distribution is more unequal. For the simulation, we take the following three steps: First, we estimate the Generalized Beta of the Second Kind (GB2) parameters  $a$ ,  $b$ ,  $p$ ,  $q$  based on the income distribution from our main analysis. This approach is based on Jenkins (2009), estimating the following probability density function using maximum likelihood:

$$f(y) = \frac{ay^{ap-1}}{b^{ap}B(p, q)[1 + (y/b)^a]^{p+q}}, \quad (3)$$

where parameters  $a$ ,  $b$ ,  $p$ ,  $q$  are each positive,  $B(p, q)$  represents the Beta function. Estimating the parameters on household income using the SOEP data, the parameters take the values  $a = 2.46$ ,  $b = 4048.35$ ,  $p = .76$ ,  $q = 1.64$ .

Second, we manipulate the  $a$  parameter to simulate different distributions, leaving  $b$ ,  $p$ ,  $q$  constant. Increasing (decreasing) the  $a$  parameter leads to lower (higher) levels of inequality, here measured by the Gini coefficient. We manipulate the  $a$  parameter so that the simulated distributions exhibit Gini coefficients of 0.2, 0.4, and 0.6 (indicating low, medium and higher inequality). The corresponding  $a$  parameters are provided in Table 8. The univariate kernel estimates of the distributions are provided in Figure 13.

Third, we ran the same methodology as in the main analysis on the simulated distributions. Using different brackets and cut-off specifications, we artificially group the data, and impute a metric distribution based on the grouped information. The correlations are provided in Figure 14.

The simulations show that our baseline rule from the main analysis, more than six brackets and an upper cut-off at the 95th holds for typical income distributions, i.e., simulations 1 and 2. A more equal distribution than in the basic case, as shown in the upper right panel of Figure 14, does not necessarily require fewer brackets to receive a similar level of correlation as in our baseline approach. However, the cut-off

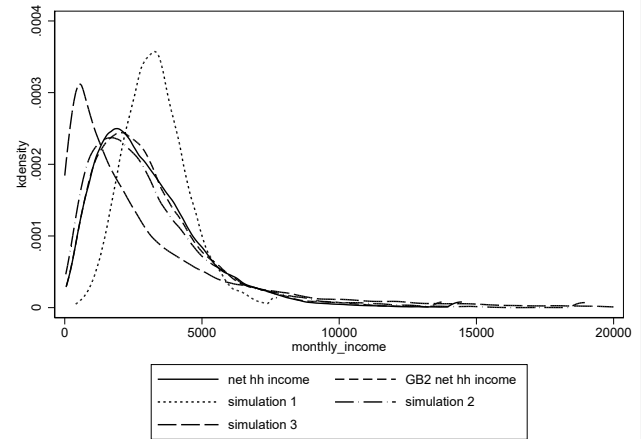


Figure 13. Net household income: kernel density functions. Compiled by authors based on SOEP v37. The figure shows the kernel density estimates of the net household income variable, its GB2 approximation, and three simulations. Estimates are based on the Epanechnikov function. For readability, the figure is cut-off at the x-axis at €20,000.

could be slightly lower for a more equal distribution providing slightly higher correlations for lower cut-offs. Interestingly, the smaller variance does not improve the approximation, and we observe a slightly smaller correlation for 10 to 18 brackets. Increasing the Gini coefficient to 0.4 (Simulation 2, lower left panel in Figure 14) also does not change the baseline rule. However, setting lower cut-offs leads to more imprecise estimates. Finally, simulation 3 (lower right panel in Figure 14) would require more brackets and a later cut-off to receive precise estimates. Nevertheless, a Gini coefficient of 0.6 is relatively high for an income distribution and points toward the baseline rule measured for the wealth distribution. We summarize from this simulation exercise that our baseline estimates seem to hold for typical levels of dispersion of income and, potentially, wealth and, as discussed in the main analysis, the more unequal the distribution is, the more important the right cut-off point.

Table 8  
Simulations based on different  $a$  parameters from the GB2 estimates

Variable	GB2 $a$ estimate	Gini coefficient
net household income	n/a	0.35
GB2 net household income	2.46	0.36
GB2 Simulation 1	4.68	0.20
GB2 Simulation 2	2.17	0.40
GB2 Simulation 3	1.30	0.60

Compiled by authors based on SOEP v37. The table shows the estimated  $a$  parameter based on the GB2 distribution if applicable, and the Gini coefficient.



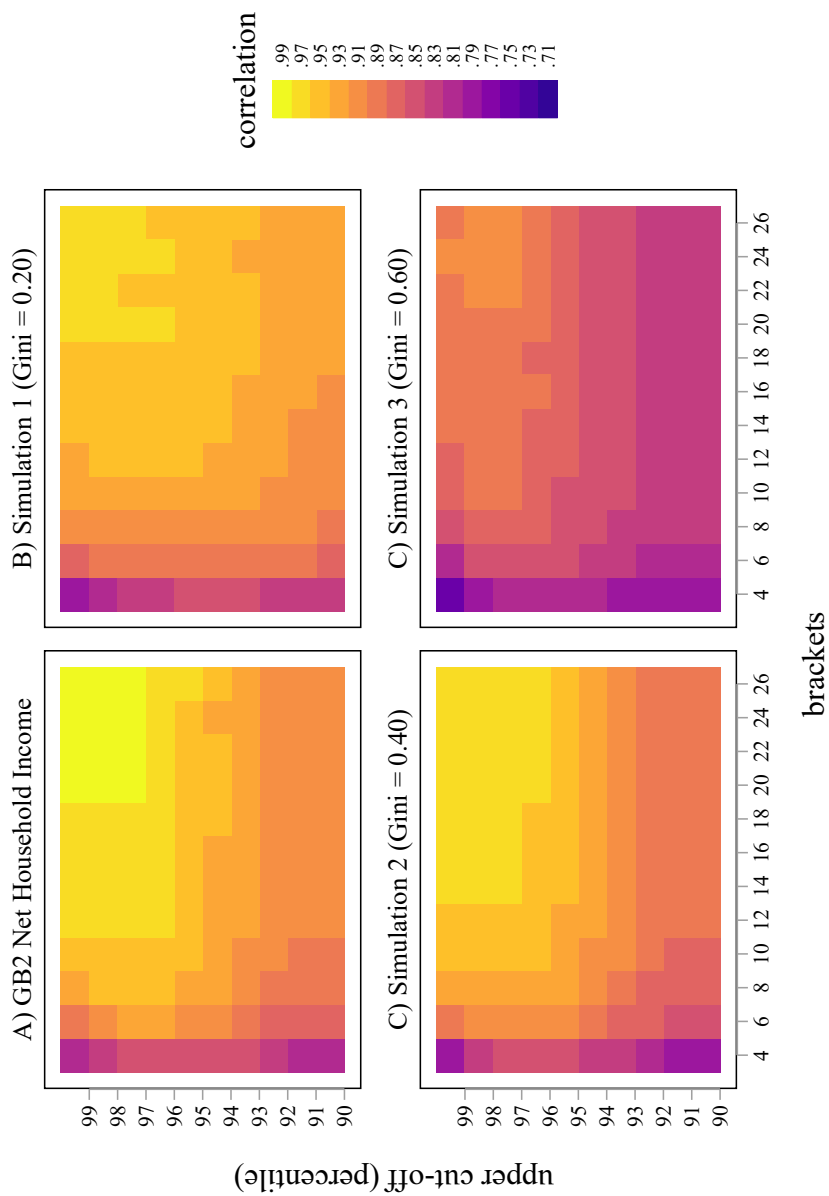


Figure 14. Correlation between original and simulated data by number of brackets and upper cut-off percentiles. Compiled by authors based on SOEP v37. The figure shows the correlation between the original distribution and the MCIB approximation for household net income and the three simulations. The y-axis depicts different upper cut-off limits; the x-axis different numbers of brackets. Variables are winsorized at the 0.5th and the 99.5th percentiles, respectively.