MATHEMATICAL MODEL OF ENERGY COST IN SPRINT RUNNING

Thiago Corrêa Duarte, Elisandra Furlan de Lima Campos, Carlos Coutinho, Roberto Lampert Ribas, Leonardo Alexandre Peyré-Tartaruga, Jefferson Fagundes Loss

Exercise Research Laboratory, Federal University of Rio Grande do Sul, Porto Alegre, Brazil

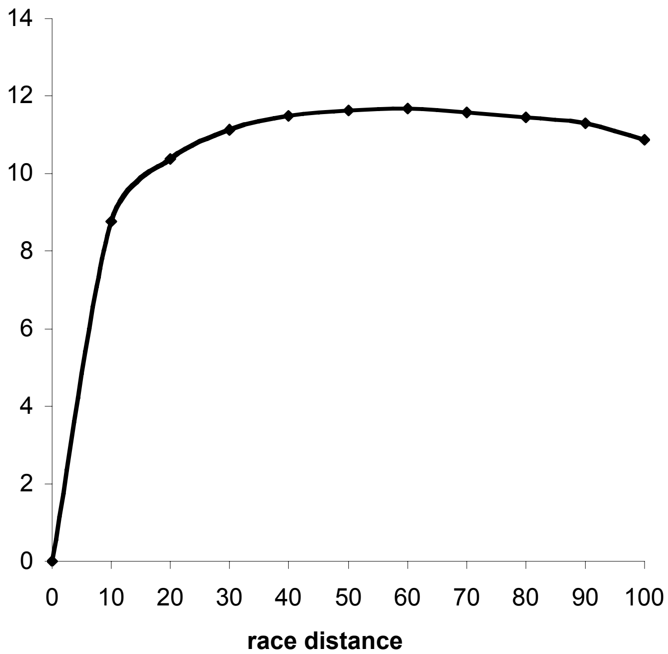
The aim of this study was to analyze energetic cost and metabolic power during the 100m races of Athletics (data from the World Athletic Championship 1997) using a new energetic approach. We used the instantaneous speed at every 10 metres of the race, due to the acceleration characteristics and in order to predict the energetic cost throughout the construction of a mathematical model adapted from Di Prampero et al. (2005). The inputs used in the model were the 'equivalent body mass', 'equivalent slope', instantaneous speeds and proportionality constant to each 10cm of events. The average energy cost was 6.78 J.Kg^{-1}.m^{-1}.

KEY WORDS: 100 metres, sprint running, energetic cost.

INTRODUCTION:

The first important mathematical model on running performance was presented by A.V. Hill (1925). Besides his contributions to muscle physiology, A.V. Hill offered a possible explanation for the relation between the distances and the times (performance) of athletic events. Since then, many studies have presented mathematical formulations in order to determine the volume of metabolic energy expended (for example, Arsac and Locatelli, 2002, Di Prampero, 2003; 2005, Ward-Smith, 1999). On the other hand, studies devoted to the mechanics of sprint running (Cavagna, Komarek & Mazzoleni, 1971, Peyré-Tartaruga, Coertjens, Black, Tartaruga, Ribas & Kruel, 2003), mainly related to the mechanical cost, are rather scarce.

Figure 1: Average instantaneous speeds during the race.



The mechanical cost and work of locomotion have been studied extensively at constant speeds (Cavagna, Saibene & Margaria, 1963, 1964, Cavagna, 1975, Minetti, Ardigò & Saibene, 1994, Willems, Heglund & Cavagna, 1995). Recently, a new energetic approach for sprint running has been presented by Di Prampero, Fusi, Sepulcri, Morin, Belli & Antonutto (2005). This model uses the inclinations of the body in relation to the ground due to forward accelerations. However, they analyzed only the first 30m of the race. The aim of the present study was to estimate the energetic cost and metabolic power of the whole 100m race from the forward instantaneous speeds at each 10m in elite sprint runners.

METHOD:

The original data used are from the World Championship of IAAF, 1997. The sprint runners are Ato Boldon, Donovan Bailey, Maurice Greene, Tim Montgomery and Frank Frederiks.

The equation model is described by Di Prampero, Fusi, Sepulcri, Morin, Belli & Antonutto (2005). In short: The input variables are the equivalent slope (ES), equivalent body mass (EM), forward speed (V) and proportionality constant (k). The equation was reported firstly by Minetti, Moia, Roi, Susta & Ferretti (2002) and adapted to sprint running by Di Prampero, Fusi, Sepulcri, Morin, Belli & Antonutto (2005). Matlab vs. 5.3 software was used to run the model. To interpolate the speed points at each 10m a spline function was used, and the instantaneous speeds and accelerations were estimated at each 10cm.

The energetic cost of sprint running (C_{sr}) is defined as,

$$C_{sr} = (155.4ES^5 - 30.4ES^4 - 43.3ES^3 + 46.3ES^2 + 19.5ES + 3.6)EM + kV^2 \quad (1)$$

where the ES is equivalent slope, EM is equivalent normalized body mass, k is proportionality constant and V is forward speed. The metabolic power is determined as follows:

$$P_{met} = C_{sr} * V$$

RESULTS:

The maximum speed was 11.8m.s^{-1}, located between 58.6m and 61.7m of the race, corresponding to 6.14s and 6.41s, respectively (Figure 1).

The average values of energetic cost decreased during the race, yielding minimum values of 2.26 J.kg^{-1}.m^{-1} (Figure 2).

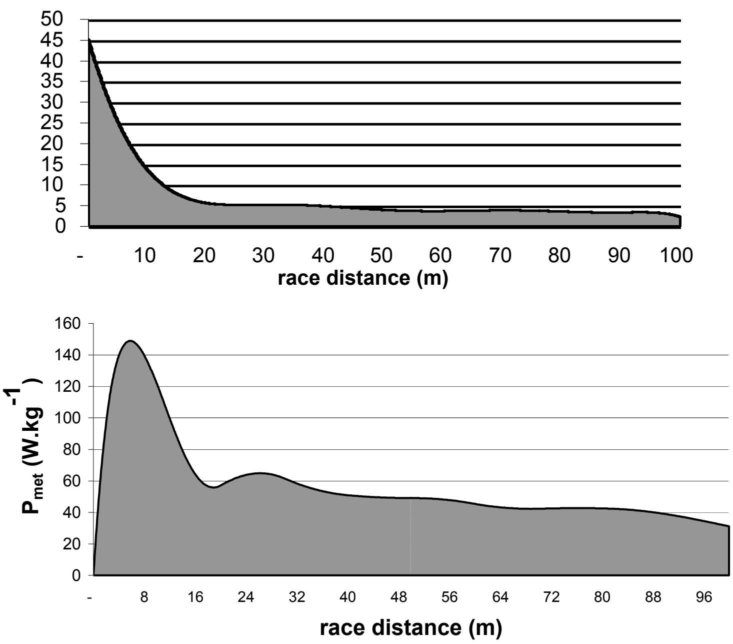


Figure 2. Average values of energetic cost and power during the race.

As shown in table 1, the average of the C_{sr} reveals a small, representative value of 15.47% of the maximum C_{sr}. The average P_{met} is 29.4% of the maximum C_{sr} throughout the race.

DISCUSSION:

The purpose of this study was to estimate energy cost and metabolic power of the 100m race. In order to test this model, we use data provided by IAAF, specifically, instantaneous speeds at each 10m of the race. Data was obtained from the final of the 1997 World Championship, held in Athens/Greece. The model developed in the present study seems satisfactory to determine the energy cost of sprint running.

Table 1 Mean and peak energy cost and metabolic power of the 100m race.

Mean		Peak	
C_{sr}	P_{met}	C_{sr}	P_{met}
$(J \cdot kg^{-1}.m^{-1})$	$(W \cdot kg^{-1})$	$(J \cdot kg^{-1} \cdot m^{-1})$	$(W \cdot kg^{-1})$
6.78 ± 0.088	57.52 ±1.17	43.81 ± 1.89	149.02 ± 2.31

The C_{sr} is related mainly to body slopes and the variation of the speed during the race. Di Prampero's experiment used 12 male non-competitive runners. They found out an average C_{sr}= 10.7 ± 0.59 $J.kg^{-1}.m^{-1}$ and average P_{met} = 4.66 ± 61.0 $W.kg^{-1}$. The maximum C_{sr} was equal to 43.8 $J.kg^{-1}.m^{-1}$ and the maximum metabolic power in the present study was 61.67% greater than reported on experiment of Di Prampero.
The construction of the model allows an innovative approaching to study the mechanical determinants of the sprint races. Optimization models could be used to study the compromise between the cost of mass transport and the amount of muscle mass.

CONCLUSION:

The maximum energy power reported in the present study was 62% greater than in recreational sprint runners. The present modellistic approach opens a field of possibilities and applications for the analysis of sprint running.

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