A DYNAMIC SIMULATION OF HIGH BAR MOVEMENTS WITH BAR STRAIN

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The kip movements of two expert gymnasts are simulated using a simple rigid three-link model with taking into account dynamic strain of the bar and the measured time history of joint angles. We show that after some parameter identification this model can reproduce the measured kip movements of the gymnasts and the bar strain to some extent. Moreover, we discuss the identified bar parameters that are not entirely consistent to a range of parameters expected from the standard specification of the Federation of International Gymnastics (FIG).

KEY WORDS: gymnastics, simulation, high bar, kip movement

INTRODUCTION:
It is important for athletes to learn how well they utilize not only their body dynamics but also dynamical properties of exercise equipments/apparatus interacting with their body. The Federation of International Gymnastics (FIG 1996) provides the norms of gymnastic apparatus. However, expert gymnasts say that the slight difference of the apparatus property can affect their performance particularly for advanced techniques, even if the change is within the norm. Some detailed biomechanical model is necessary to assess various factors influencing performance (Hiley & Yeadon 2005).

On the other hand many researchers have been investigated the optimality of the movements in various fields. Especially, the hand reaching is a well-discussed movement for which many criteria have been proposed (Todorov 2004). Recently, such framework is expected to be applied to understanding skilled athletic movements, e.g. for high bar movements (Yamasaki et al. 2006). For solving optimal problem under some criterion, simple dynamical model is desirable in terms of the computational costs and parameter identification. Thus, we study a simple dynamical model of the kip movements of expert gymnasts on the high bar with taking into account the dynamic strain of the bar.

METHOD:

Data Collection: Subjects of the experiment were two male gymnasts (RI and YY), who had thirteen and ten years of gymnastic carrier, respectively. Each subject with joint markers performed eight trials of kip movements on the high bar in his usual manner. The kip movements in the sagittal plane were recorded by a high-speed video camera (FOR-A, VFC-300) at 90 fps.

Data Analysis: The positions of the horizontal bar, shoulder joint, hip joint and ankle joint were measured by using a digitizing software (DKH, FrameDIAS II) with the help of the markers on each joint. The markers on the bar were stuck out sideways. The position data were low-pass filtered by a third-order Butterworth filter with cut-off frequency of 5 Hz. The bar positions and the joint angles were defined as shown in Figure 1. For each subject, averaged data of all the trials in sync at the final time when $\theta_1$ crosses $\pi/2$ were used as the measured data, $x_0^E(t)$, $y_0^E(t)$, $\theta_1^E(t)$, $\theta_2^E(t)$, and $\theta_3^E(t)$.

Figure 1: The Model of High Bar Movements
Model: The kip motion of the expert gymnast is simply assumed to be modelled as the rigid 3-link (Figure 1), where no active torque acts on the contact between the bar and the hand and active torques act on the shoulder and hip joints. The friction between the hands and the bar is modelled by the viscous friction $b\dot{\theta}$ where $b$ is the friction coefficient. The movement of the bar in the sagittal plane is modelled as a point mass $m_0$ that is connected via a spring with a stiffness coefficient $K$ and a damper with a viscous coefficient $B$ to an origin of the 2D space (Hiley & Yeadon 2005, Yamamoto et al. 2007). The motion equations of the model are written in the form:

$$M(X)\ddot{X} + H(X, \dot{X}) + G(X) + D(X, \dot{X}) = F$$  \hspace{1cm} (1)

where $X = [x_0, y_0, \theta_1, \theta_2, \theta_3]^T$ is the vector of generalized coordinates (5x1), $M$ is the inertial force matrix (5x5), $H$ is the centrifugal and Coriolis force vector (5x1), $G$ is the gravitational force vector (5x1), and $D(X, \dot{X}) = [Kx_0 + Bx_0, Ky_0 + By_0, b\dot{\theta}_1, 0, 0]^T$ is the visco-elastic force to the bar and the friction between the hands and the bar. The force inputs to the motion equations are only torques at the shoulder and hip joints: $F = [0, 0, 0, \tau_2, \tau_3]^T$.

See also Yamamoto et al. (2007) for details of the motion equations.

Forward-Dynamics-like Simulation (Van Den Bogert et al. 1989, Hiley & Yeadon 2005): Time histories of joint angles of shoulder and hip are prescribed by the fifth order spline function $\eta(t)$ (2x1) that approximate the measured joint angles $[\theta_2^E(t), \theta_3^E(t)]$. The angular velocities and accelerations are also prescribed by their differentiation $\dot{\eta}(t)$ and $\ddot{\eta}(t)$. Under these constraints, the dynamics corresponding to the remaining degrees of freedom $\xi := [x_0, y_0, \dot{\theta}_1]^T$ is written in the form:

$$\frac{d}{dt} \begin{bmatrix} \xi \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} \dot{\xi} \\ f(\xi, \dot{\xi}, \eta(t), \dot{\eta}(t), \ddot{\eta}(t)) \end{bmatrix}$$  \hspace{1cm} (2)

This equation can be numerically integrated with the measured joint angles $\eta(t)$ and initial value $\xi(t_0)$ fitted to the measured data.

Parameter Identification: The body parameters of the model were estimated according to statistical anthropometric data (Ae et al. 1992 and Kouchi & Mochimaru 2002) and some measured data from the subjects (Yamasaki et al. 2006 for the values). The other parameters $m_0, B, K$, and $b$ were adjusted by minimizing mean squared error:

$$E_s(\xi^E, \xi^E) = \int_{t_f}^{t_i} (\xi^E - \xi^E)^TW(\xi^E - \xi^E) dt$$  \hspace{1cm} (3)

where $\xi^E$ is the measured data and $W = \text{diag}(1, 1, 10^{-3})$.

RESULTS:
Numerical integration of Equation (2) was conducted by using MATLAB ode45 function. Figures 2 and 3 show the time changes of the simulation variables (solid lines) compared to the measured data (dashed lines), for subject RI and YY, respectively. For shoulder and hip joints ($\theta_2$ and $\theta_3$), twenty knots of splines, shown by the small circles, were evenly spaced in time, so that the measured data and the splines are hardly distinguishable in the figures.
The different onsets of shoulder extention and hip flexion between the subjects indicate that the subjects performed the kip movement in different manner. However, the upper three pannels of both figures 2 and 3 show small difference between the model's and measured movement. Thus, despite its simplicity, the model could reproduce the subject-dependent kip movement and the bar strain to some extent.

![Figure 2: Simulation Result (subject RI)](image1)

![Figure 3: Simulation Result (subject YY)](image2)

**DISCUSSION:**

Table 1 shows the results of parameter fitting, i.e., minimization of Equation (3), conducted by MATLAB fminsearch function. Note that a part of the identified parameters are different between the subjects, especially in $m_0$, although the subjects used the same apparatus in the experiment.

<table>
<thead>
<tr>
<th>subject</th>
<th>$m_0$ [kg]</th>
<th>$B$ [Ns/m]</th>
<th>$K$ [N/m]</th>
<th>$b$ [Ns/m]</th>
<th>$E_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>RI</td>
<td>$4.78 \times 10^1$</td>
<td>$7.35 \times 10^2$</td>
<td>$2.85 \times 10^4$</td>
<td>$4.79$</td>
<td>$9.81 \times 10^{-5}$</td>
</tr>
<tr>
<td>YY</td>
<td>$3.86 \times 10^1$</td>
<td>$7.63 \times 10^2$</td>
<td>$2.80 \times 10^4$</td>
<td>$4.57$</td>
<td>$5.83 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

The model parameters of the bar ($m_0$, $B$ and $K$) can be verified with a part of the specification test of FIG. Considering only the vertical movement of the bar without the gymnast, the motion equation of the model is a simple second order linear system. Then, by comparison with a static traction stress test (test A in FIG 1996) we obtain inequality:
\[
\frac{m_0g + 2180}{0.1} \leq K \leq \frac{m_0g + 2220}{0.08} \tag{A1}
\]
Moreover, from an oscillation-damping test (test C in FIG 1996) we obtain inequalities:
\[
1.25 \leq \frac{2m \log_2 2}{B} \leq 4.9, \quad 3.0 \leq \frac{1}{2\pi} \sqrt{\frac{K - B^2}{m}} \leq 3.25 \tag{C1, C2}
\]
where \( m \) is \( m_0 + 60 \). It is easy to check that the bar parameters in Table 1 satisfy (A1) but do not satisfy (C1) and (C2) for both RI and YY.
The identified parameters have possibilities to compensate error of modelling or the other body parameters. Regarding modelling error, the above discrepancy between the bar parameters and the FIG specification seems to be due to unmodelled body dynamics, for instance, small bending of the elbow, knee, or trunk, or visco-elastic property around the shoulder (Hiley and Yeadon, 2005). The magnitude of the effect of such unmodelled dynamics would be subject-dependent, e.g., as shown by \( m_0 \) in Table 1.

CONCLUSION:
A simple rigid three-link model with dynamic strain of the bar was constructed for simulating the kip movements of two expert gymnasts. Despite its simplicity, this model could reproduce the measured movements to some extent. However, to separate the effects of the apparatus and the body characteristics, more detailed model would be necessary.

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