INTRODUCTION

The performance of maximal effort human jumping is an important aspect of many sporting skills. Some have used descriptive models to explain the segmental coordination pattern used by elite jumpers (Bobbert et al., 1988), while others have turned to forward dynamics models in which the kinematics of jumping are simulated (Pandy et al., 1991; van Soest et al., 1992). One interesting class of simulations are optimization models, in which the kinetic and kinematic profiles that generate an optimal movement outcome are predicted by optimizing an objective function associated with successful completion of the motion. Vertical jumping studies have become popular in part because the objective function is unambiguous - the desired outcome is maximal jump height. In addition, optimal control models using ‘bang-bang’ control are relevant due to the maximal effort nature of these jumps (Pandy et al., 1991).

Underlying these optimization studies is a basic question of interest to movement scientists: How does the CNS coordinate the multi-segmented musculoskeletal system to successfully complete a motion? Jensen et al. (1991) examined segmental kinematics in maximal effort jumping under different task conditions in an attempt to describe features inherent in successful jumping. Subjects jumped at varying take-off angles, ranging from a vertical jump to a standing long jump (forward, horizontal). Using the timing of leg joint reversals as descriptive markers, a general proximal-to-distal coordination pattern was shown. The present investigation was undertaken to extend the scope of this ‘task analysis’ through optimization models. As a first step in this modeling scheme we constructed a simple model with extensor joint torque actuators. In this paper we report the basic coordination associated with different jumping tasks by examining the pattern of joint torque onset times.

METHODOLOGY

The model comprised 4 linked rigid segments (foot, shank, thigh and HAT). Segmental lengths and other anthropometric values (moments of inertia, masses, location of mass centers) were derived from the literature (Winter, 1990). Pin joints represented points of articulation between segments, with the metatarsal constrained as a pin joint with the ground. All joints were rotational in nature and the segmental angles \( \theta \) were referenced to the vertical.

The equations of motion were expressed in the form:

\[ A(\theta)\dot{\theta} + B(\theta, \dot{\theta}) = F \]

where \( A \) is the inertia matrix, \( B \) is the vector of transient terms (i.e. radial acceleration forces), \( \theta \) and \( \dot{\theta} \) are the angular positions and velocities, and \( F \) are the net joint mo-
ments representing the torque contributions of the ankle plantarflexors, knee extensors and hip extensors. The movement of the model was simulated by integrating this set of ordinary differential equations given the initial conditions (segmental angles) and the times of onset of the three controlling torques. Integration was performed numerically using the LSODI software routines (Hindmarsh, 1980) implemented on a 80486 computer. This numerical integrator is based on backward differentiation formulae which are useful for the solution of stiff equations, and it has been shown to be both accurate and robust (Winslow, 1990). The model was validated by performing simulations in which the kinematics could be predicted theoretically through conservation of energy, and in limited cases of non-conservative movements (constant torque about a single joint).

Model kinematics were driven by torque patterns defined by torque-velocity relations (Alexander, 1989):

\[
T = T_0 \left( \omega_v - \omega \right) / (\omega_v + G \omega)
\]

where \( \omega_v \) represents maximal shortening velocity (\( \approx 15 \) rad/s), \( G \) represents a curvature factor in the \( T - \omega \) relation (\( = 3 \)), and \( T_0 \) represents the maximal isometric torque value of each joint (\( = 225, 173 \) and \( 228 \) Nm for the ankle, knee and hip). Further physiological relevance was imparted by realistic time constants that controlled the rate at which each torque reached maximum (20 ms), and by rejecting trials in which the heel was driven below the ground during the jump (heel constraint). All jumps began from a static posture with torques set to zero. Each jumping performance was dictated by the onset times of the three torque generators. The optimization algorithm (Figure 1) searched for the pattern of activation onset times for the 3 torque generators which maximized an objective function (\( \phi \)), using a multidimensional downhill simplex method (Press et al., 1988).

\[
\phi = \beta_1 V + \beta_2 H
\]

where \( \beta_1 \) and \( \beta_2 \) are weighting coefficients, and \( V \) and \( H \) are maximal vertical and forward displacement of the model center of mass, respectively. The choice of jumping task was specified by the weighting coefficients \( (\beta_1, \beta_2) \), with \( (1, 0) \) representing a vertical jump and \( (0,1) \) representing a forward jump. The objective function was evaluated at the point of take-off, using standard equations of projectile motion to...
RESULTS AND DISCUSSION

The first test of the model concerned the effect of initial conditions on the optimization of both vertical ($\beta_1, (1,0)$) and forward jumping ($\beta_1, (0,1)$). With the set of initial conditions shown in Figure 2a ($\theta_0 = \{1.0, -0.7, 1.1, -0.8\}$), the model’s maximum height was 1.586 m. From this initial posture, the optimal forward jump displayed a maximal displacement of only 1.049 m. However, by changing the initial posture ($\theta_0 = \{0.8, -0.8, 0.8, -1.0\}$) the optimal forward jump was improved to 1.789 m (Figure 2b), but the optimal vertical jump was only 1.325 m.

The relative magnitude of onset times varied substantially between the two optimal jumps. The maximal height jump from the first posture was accomplished with torque onsets of 0.02791, 0.01810 and 0.04674 seconds for the ankle, knee and hip, respectively. For the maximal forward distance jump from the second posture, the pattern of torque onsets was 0.05257, 0.04248 and 0.07913 s (a-k-h). In both of these maximal jumps, the order of onset times was knee, ankle and hip, and therefore did not follow a proximal-to-distal sequence.

The second set of simulations involved further alteration of the jumping task, implemented by changing the relative values of $\beta_1$ and $\beta_2$. This caused the model to produce jumps involving both height and distance, and therefore was analogous to the experimental work of Jensen et al. (1991). For each of these tasks the initial position was set at $\theta_0 = \{0.9, -0.7, 0.9, -0.8\}$. From this initial position, the onset times of the joint torques were ordered ankle, hip and knee for ten of thirteen jumping tasks (Figure 3). In Figure 3 the $\beta$ number on the x axis ranges from 1 ($\beta_1 (1,0)$, purely vertical), through 7 ($\beta_1, (1,1)$, equal vertical and forward), to 13 ($\beta_1, (0,1)$, purely forward).

As the task changed from purely vertical to purely forward, the onset times were delayed. For the $\beta$ numbers from 1 to 5, the onset times tended to remain in the same proportion. However, at higher $\beta$ numbers (above 5, indicating a greater forward component) there were more variable onset combinations. This trend is most evident at $\beta$ numbers greater than 10, when there is a switch in the order of onset for the hip and knee torques.
During this set of simulations it was discovered that near-optimal solutions existed for some task weighting conditions. These near-optimal jumps were generated with different combinations of torque onset times than in the optimal jumps, and are associated with local maxima in the solution set. Figure 4 illustrates the torque onset times for 4 such local maxima from a vertical jump.

These torque onset time combinations produced jump heights of 1.481, 1.502, 1.507 and 1.513 m. This result is contrary to the findings of Pandy et al. (1991) and van Soest et al. (1992), who report globally optimal solutions with no occurrence of local maxima in vertical jump simulations. There are several possible reasons for the near-optimal
solutions, including our use of simple, single joint extensor torque actuators rather than more specific muscle models representing both one- and two-joint extensor or flexor muscles, our use of differing initial conditions, or the existence of solutions laying close to the boundary of our ankle constraint.

The final set of simulations were designed to illustrate the effect of relative torque magnitude among the three joints. This was accomplished by altering the knee torque magnitude in 5 Nm steps away from the 173 Nm value used in the standard model (range 148 - 198 Nm). The hip and ankle torques were not changed. These new model parameters were invoked for maximal height jumps, with the initial position set at $\theta_e = \{0.9, -0.7, 0.9, -0.8\}$. As would be expected, the model jump height increases with higher knee extensor torque values, with a difference of 5.8 cm between the lowest and highest torque values (Figure 5). However, the relative times of torque onset that produced these jumps changed as the knee extensors got 'stronger'. At low knee torque values the order of onset was knee - ankle - hip; for knee torques between 163 and 178 the order was ankle - knee - hip; while at the highest knee torques the order was ankle - hip - knee. The 'plateaus' seen in Figure 5 near these transition points indicate that local maxima may be responsible for these alterations in torque onset order.

![Figure 5. Effect of relative knee torque.](image)

CONCLUSION

The simulation results illustrate the dependence of the optimal solution for a given task on initial conditions. Although altering the jumping task or relative muscular knee strength resulted in variations for torque onset times, the interpretation of these changes is clouded by the existence of near-optimal solutions. Future work will explore the use of a simulated annealing algorithm (Kirkpatrick et al., 1983) to find truly optimal solutions. However, it is possible that humans maximize jumping performance by staying within one series of local maxima solutions. Physiological characteristics of the neuromuscular system may constrain the optimal solution to one of these local maxima sets. The addition of more realistic force actuators that represent both one- and two-joint muscles may help to clarify this question.
REFERENCES


