ON THE TWO – PHASE MODAL FOR A PULSATILE FLOW UNDER PERIODIC BODY ACCELERATION

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A particle – fluid suspension model is applied to the problem of pulsatile blood flow through a circular tube under the influence of body acceleration. With the help of finite Hankel and Laplace transforms, analytic expressions for axial velocity for fluid and particle phase, fluid acceleration, wall shear stress and instantaneous flow rate have been obtained. It as observed that the solution can be used for all feasible values of pulsatile and body acceleration (as, riding a tractor, operating a jack hammer andsudden and fast movements of body during gymnastics and sports activities). Using physiological data, the following qualitative and quantitative results have been obtained. The maximum of the axial velocity and fluid acceleration shifts from the axis of the tube to the vicinity of the tube wall as the tube diameter increases. The effect of C on the velocity and acceleration are non-uniform. The effect of C on τ_b are again non- uniform.

KEY WORDS: finite Hankel, Laplace transforms, pulsatile, acceleration, velocity

INTRODUCTION: The theoretical and experimental studies of blood flow phenomena in the vessels of the mammalian circulatory system has been the main object of scientific research for over hundred and fifty year. The study is a complex one due to the complicated structure of blood and circulatory system but the investigations have proved to by very useful for the development of pathological patterns in mammalian physiology, for the diagnosis of cardiovascular diseases and for other clinical purposes. In the numerous important contributions made to understand the behavior of blood when it flows through the vessels of the circulatory system study by (1952) and (1964), blood has been considered as a single phase homogeneous Newtonian viscous fluid. This approach does not account for the presence of red cells in blood. Experimental study by (1972) on blood flow indicate that blood can no longer be treated as a single phase homogeneous viscous fluid when the diameter of the blood vessel is smaller than 1000 μ. It is surprising to note that the individuality of the red cells (of diameter 8 μ) is important in even such large vessels (with diameter up to 100 cells diameter). Skalak (1972) observed that in capillary blood vessels whose diameter (4 – 10 μ) are equal or smaller than that of a red blood cell, an accurate description of the flow requires consideration of the red blood cells as discrete particles. Thus, in dealing with the problem of microcirculation also, the individuality of the red blood cells can't be ignored. Therefore, for realistic description of blood flow, it is perhaps more appropriate to treat blood as a two-phase fluid, that is, a suspension of red cells in plasma. It is with in view, we consider the two-phase model of blood flow in the present investigation. Several studies on blood flow assuming blood as a fluid- particle system have been reported by (1979). Prolonged exposures to accelerative disturbances that are common in normal life (for instance, while landing and taking off of aircrafts, riding a tractor, operating a jack hammer and sudden and fast movements of body during gymnastics and sports activities) may lead to health problems like headache, abdominal pain, loss of vision and increased pulse rate even though human body can adapt to changes as (1974). It is possible that dangerous of body acceleration and pressure gradient of blood flow may be responsible for such health problems. It is, therefore, desirable to set a standard for short and long term exposures of human being to such acceleration. If the response of the human system to such accelerations is understood properly, the controlled accelerations can be used for therapeutic treatments, development of new diagnostic tools and for better designing of protective pads as (1973).

Due to physiological importance of body acceleration, many mathematical models have been proposed for pulsatile flow blood with body acceleration by (1995) and (1986) by considering blood as a Newtonian, non- Newtonian, power law, caisson and micro polar fluid.

METHODS: Using two-phase model of blood, we shall investigate the pulsatile flow of blood with periodic body acceleration through a rigid circular tube of a radius R. Since whole blood is a complex mixture, an attempt to analyze the system in an exact manner is very difficult. Therefore, we make a number of simplifications based on the properties of blood and the flow situation under consideration. The basic assumptions of this investigation are the following: 1. Blood is considered as a two-phase fluid, that is, suspension of red cells in plasma which is a Newtonian incompressible fluid. 2. The red cell is a rigid neutrally buoyant spherical particle, the specific gravity of the cell is about 1.1 and that of plasma is about 1.03 such that the effect of gravity on blood flow is very small (i.e. setting tendency of erythrocytes is negligible). 3. Cell-cell interaction is neglected. 4. Interaction between two phases is according to Stokes drag law. 5. The volume fraction occupied by the red cells is taken as a constant. 6. Brownian motion of red cells is neglected. 7. The flow is axis-symmetric and the velocity components $u_f = u_f(r,t)$, $u_p = u_p(r,t)$, denote fluid and particle phase axial velocities.

The assumption 1 may be reasonable as far as the theological properties of blood are concerned. Blood cells are actually biconcave discoid shaped highly flexible particles. Thus, by assumption 2, one is limited in not being able to account for cell shape and the deformation it undergoes during shear flow. However, the cell deformability is not significant at low shear rates (as (1970)). The effect of cell- cell interaction (assumption 3) is felt important only at high concentration of cells and this makes our study limited to dilute suspension. The results using the constant concentration of red cells in a suspension (assumption 5) are exactly true in the limit of flow concentration. Since Brownian motion is significant only for very small particles such as protein molecules as (Lightfoot 1974), the assumption 6 seems reasonable.

In view of the above assumption, appropriate equations (neglecting the body forces and body couples) describing the flow of plasma – cell system and governing the pulsatile blood flow through a rigid cylindrical tube with periodic body acceleration are given by (1979) and (1995),

$$\rho_f(1-C)\frac{\partial u_f}{\partial t} = \rho_f(1-C)\,a_0\cos(\omega_b t+\phi)+(1-C)$$
$$(A_o + A_1\cos\omega_p t)+\mu_f\left(\frac{\partial^2 u_f}{\partial r^2}+\frac{1}{r}\frac{\partial u_f}{\partial r}\right)+CF_0(u_p-u_f) \quad (1)$$

$$\rho_p\frac{\partial u_p}{\partial t} = \rho_p C\,a_0\cos(\omega_b t+\phi)+C(A_o+A_1\cos\omega_p t)$$
$$+CF_0(u_p-u_f) \quad (2)$$

where C is the volume fraction occupied by the red cells, ρ_f and ρ_p are actual densities of fluid and particles (the fluid phase density ρ_f (1 – C) and the particle phase density is ρ_pC), μ_f represents the mixture viscosity, (and in the case of blood, μ_f is the viscosity of plasma and thus is independent of red blood cells concentration C), the force exerted by spherical rigid particles (for a dilute suspension) of uniform size upon fluid is given by $CF_0(u_p-u_f) = 6\pi\delta n_p\mu_f(u_p-u_f)$, n_p and δ are the number density and radius of the particles and $F_0 = 9\,\mu_f/2\delta^2$, the pressure gradient $-\partial p/\partial z$ and body acceleration g are given by

$$-\frac{\partial p}{\partial z} = A_0 + A_1\cos\omega_p t, \quad t \geq 0 \qquad (3)$$

$$g = a_0(\cos\omega_b t+\phi), \quad t \succ 0 \qquad (4)$$

where $\omega_p = 2\pi f_p$, f_p is the pulse frequency, A_0 and A_1 are pressure gradient of a steady flow and amplitude of oscillatory part, a_0 is the amplitude of body acceleration, $\omega_b = 2\pi f_b$, f_b is body acceleration frequency, ϕ is the phase angle of g with respect to heart action (pressure gradient) and t is the time. The initial and boundary conditions are ((1985) and (1996)),

$$u_f(r,0) = \frac{(R^2 - r^2)(A_0 + A_1)}{4\mu_f}, \qquad u_p(r,0) = \frac{(R^2 - r^2 + 4\mu_f / F_0)(A_0 + A_1)}{4\mu_f},$$

$$u_f(r,0) = 0, \text{ at r = R}, \qquad u_f(r,t) \text{ and } u_p(r,t) \text{ is finite at r = 0} \qquad (5)$$

The solution of equations (1) and (2) and using the initial and boundary conditions (5) are obtained by the consecutive use of Laplace and Hankel transforms, defined as

$$\bar{f}(\xi_i, s) = \int_0^\infty e^{-st} dt \int_o^R r J_0(r\xi_i)\, f(r,t)\, dr \qquad (6)$$

where ξ_i are the roots of the equation $J_0(R\xi_i) = 0$, tilde and bar stand for the Hankel and Laplace transforms respectively. The inverse of the joint transform is given by

$$f(r,t) = \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} e^{st} \frac{2}{R^2} \sum_{i=1}^{\infty} \bar{f}(\xi_i, s) \frac{J_0(r\xi_i)}{J_1^2(R\xi_i)}\, ds \qquad (7)$$

where $\gamma \geq 0$ and the summation is taken over all positive roots of $J_0(R\xi_i) = 0$, J_0 and J_1 are Bessel functions of the first kind. The application of (6) on partial differential equations (1), (2) and the initial and boundary conditions (5) leads to a system of algebraic equations. The inversion of the solutions of these algebraic equations using (7) gives the properties of blood (the velocities u_f, u_p, the volume flow rate Q, the fluid acceleration F and wall shear stress τ_ω).

RESULTS AND DISCUSSION : The flow investigations have been carried out by computing the values of the flow variables at a particular site in cardiovascular system. The variation of the axial velocity u_f of the fluid phase, fluid phase acceleration F, wall shear stress τ_ω and instantaneous flow rate Q with f_b, a_0, tube radius R and volume fraction occupied by particles C have been studied. It is observed that the variation in body acceleration amplitude a_0 brings in qualitative as well as quantitative changes in velocity profiles in wide tubes whereas the changes of velocity profiles in narrow tubes are only quantitative in nature. When pressure gradient and body acceleration are in phase, in small diameter tubes (arteriole and coronary), body acceleration influences the velocity near the axis more than near the wall and the maximum velocity is observed near the axis.

The effects of volume fraction occupied by the particles C on velocity in the case of arterioles is that as C increases, the velocity increases, though the increase is very small.

Variation of fluid phase acceleration with tube radius and $\bar{t}$ has been shown for different values of parameters. The body acceleration frequency f_b plays an important role in fluid phase acceleration. It increases the amplitude of fluid phase acceleration as f_b increases. It is observed that the amplitude of fluid phase acceleration for flows with body acceleration is several times more than without body acceleration. As the tube diameter increases, maximum value of fluid phase acceleration shifts towards the wall. Also, body acceleration influences fluid phase acceleration in the vicinity of the wall in the case of a wider artery. The effect of C on fluid phase acceleration F in flow without body acceleration are qualitatively quite interesting, though they are small in magnitude. In the case when C = 0, the results agree with the Newtonian model as (1991).

Variation of wall shear stress τ_w with various parameters has been studied. The variation of wall shear amplitude τ_b due to body acceleration with tube diameter in cardiovascular system is interesting under prevailing physiological conditions (Table 1).

Table 1 Variation of wall shear amplitude with f_b and C due to body acceleration.

	Arteriole		Coronary		Femoral	
f_b	C = 0.0	0.6	0.0	0.6	0.0	0.6
1.2	0.380887	0.396427	5.820159	5.837594	5.557310	4.627643
1.8	0.380886	0.396426	4.967295	4.912663	4.346281	3.385057
2.4	0.380885	0.396424	4.304852	4.228501	3.304852	2.718919

The effect of the frequency f_b of body acceleration on the instantaneous flow rate. It is observed that in the arteriole, the smaller the frequency of excitation, the greater is the peak flow rate, even though a_0 is kept constant. The amplitude of flow rate Q_b due to body acceleration decreases as the tube diameter decreases (Table 2) and Q_b decreases as f_b increases in all the tubes.

Table 2 Variation of flow rate amplitude with f_b and C.

	Arteriole $Q_b \times 10^{-10}$		Coronary $Q_b \times 10^{-3}$		Femoral $Q_b \times 10^{-3}$	
f_b	C = 0.0	0.6	0.0	0.6	0.0	0.6
1.2	0.414697	0.431616	0.407181	0.404399	8.342375	7.613072
1.8	0.414696	0.431615	0.337677	0.327998	5.764000	5.230962
2.4	0.414694	0.431612	0.282483	0.270759	4.414064	4.015571

CONCLUSION: An analytical method for solving the pulsatile flow of blood with periodic body acceleration by considering blood as a two-phase fluid, that is, a suspension of cells in plasma which is a Newtonian fluid is presented. Fluid and particle phase velocities are determined and further steady pulsatile velocities with body acceleration for both cases are deduced. Expressions for fluid phase acceleration, wall shear stress and instantaneous flow rate have been obtained and using physiological data, the following observations have been made. Body acceleration effects in narrow and wide tubes are qualitatively different and they are more important in wide tubes quantitatively. Body acceleration frequency appears to be a strong parameter influencing the flow qualitatively and quantitatively and the variations in the flow variables are further influenced by (effect of C) the presence of the particles. Thus, it may concluded from the observations made on the previous section that in cardiovascular system, the presence of red cells in blood influences and produces both qualitatively and quantitatively changes in the flow variables and hence its effects on blood flow are important in both narrow and wide tubes.

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