ARCHER-BOW-ARROW SYSTEM ADJUSTMENT IN THE VERTICAL PLANE

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A significant part of bow tuning is vertical adjustment that strives for a zero angle of attack of the arrow. This requires laborious procedures through a lengthy and tedious trial and error phase. A mechanical and mathematical model of bow and arrow geometry in the vertical plane in braced and drawn situations has been constructed and investigated. An asymmetrical scheme, rigid beams, concentrated elastic elements, and non-stretchable string are the main features of the model. Numerical results of the computer simulation are presented in tabular and graphical form. This facilitates the use of the methods by archers and coaches.

KEY WORDS: archery, bow tuning, modelling, computer simulation.

INTRODUCTION: At first blush, there is no problem to obtain a zero angle of attack of an arrow. A modern sport bow consists of the two equal limbs. If the bow is symmetric of an arrow in the vertical plane a string force should coincide with the longitudinal axis of the arrow. Therefore the arrow should move along its longitudinal axis. Unfortunately this is impossible because of two reasons. The first one arises from the Archer Paradox phenomenon (Kooi & Sparenberg, 1997). The issue is that an archery arrow has a flexible shaft that bends and buckles because a column like force and a force impulse to its tail when a string slides from the fingers. To obtain for the arrow a free way around the handle, archers use procedures named "horizontal" adjustment. There are several investigations on practical and theoretical fields that elucidate lateral deflection of archery arrows. The main idea is to match the arrow with the bow, i.e. to have a period of lateral bending equal to the time of flight of the arrow in the bow area (Zanevskyy, 2001). The arrow passes the handle and moves to a target without any collision. The second reason why the string force may not coincide with the arrow's longitudinal axis is the fact, that the bow cannot be a symmetrical system of the arrow axis in the vertical plane in any case. The hand holding the bow and the arrow cannot exist simultaneously in the same axis of symmetry (Figure 1). Because of this obvious fact, the arrow is situated somewhere above or below the hand holding the bow in the middle of the handle. Hence, an arrow-bow system is asymmetric in the vertical plane. Although the asymmetry is small (~3%) it causes a significant angle of attack about 0.1 rad (Zanevskyy, 2000). Hence, to obtain a zero angle of attack, an archer must tune the nocking-point position or/and the initial position of limbs, i.e. the angles of the limbs with the handle without a string, and the rest height. There are common recommendations regarding these parameters in Owners Manuals proposed by bow's manufacturers, and in archery books and journals. For example, the nocking height has to be 6 - 10 mm (Bajer *et al*, 1982), a difference between the limb angles has to produce a difference between the distances from the ends of the handle to the braced string (tiller) 1/8" - 3/8" (Hoyt/Easton USA, 1993), or1/3" - 1/2" (Stuart, 1989), i.e. 3 - 13 mm. As we can notice, the ranges of the recommended parameters are too wide. They are used for preliminary or coarse adjustment of a bow. There are two experimental methods used for tuning the bow in the vertical plane: the Bare Shaft Method developed in by Max Hamilton and the Eliason method (Bajer *et al.*, 1982). These methods allow variation of only one parameter – the nocking height. The initial set-up of limb angles relative to the handle and the rest height just have not been taken into consideration in the frame of the methods. The aim of this research is to create an analytical method to mathematically determine an optimal combination of bow parameters to ensure a zero angle of attack of an arrow launched from a string.

METHODS: A schematic model of a bow and arrow system has been presented in Figure 2. There are two positions depicted, namely the braced "1" and fully drawn "2" bow. The index "B" is applied to mark parameters in the braced position. It is assumed that a non-stretchable string contacts a straight limb at the tip point only. A Cartesian co-ordinate system x-y is pinned to a bow handle. The x-axis is perpendicular to the handle and divides it in half.

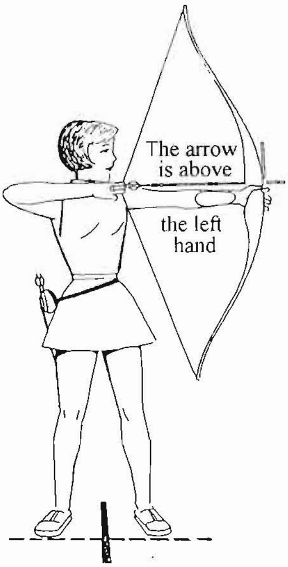

Figure 1. A drawn bow is asymmetrical in the vertical plane in any case (The drawing has been borrowed from: Bajer *et al.*, 1982)

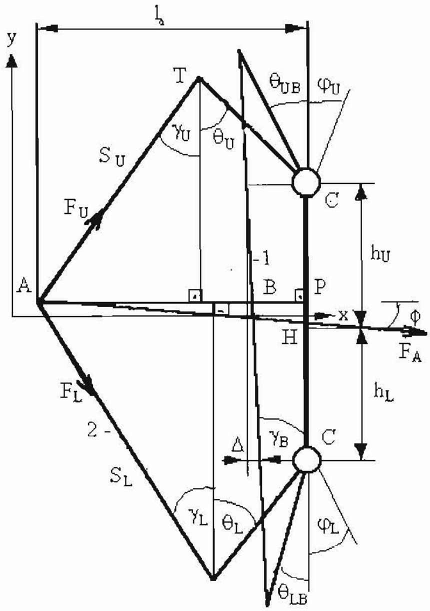

Figure 2. A schematic model of a bow and arrow system in the vertical plane: "1" is a bow in the braced situation and "2" – drawn situation.

Geometric and force equations of the bow in the fully drawn position are:

$$l_a = l\sin\theta_U + S_U\sin\gamma_U\,;\; l_a = l\sin\theta_L + S_L\sin\gamma_L\,; \tag{1}$$

$$y_A = \frac{h}{2} + l\cos\theta_U - S_U\cos\gamma_U\,;\; y_A = S_L\cos\gamma_L - l\cos\theta_L - \frac{h}{2}\,; \tag{2}$$

$$c(\theta_U + \varphi_U) = F_U l\sin(\theta_U + \gamma_U)\,;\; c(\theta_L + \varphi_L) = F_L l\sin(\theta_L + \gamma_L)\,; \tag{3}$$

$$F_x = F_U\sin\gamma_U + F_L\sin\gamma_L\,;\; F_y = F_U\cos\gamma_U - F_L\cos\gamma_L\,;\; F_A = \sqrt{F_x^2 + F_y^2}\,; \tag{4}$$

$$tg\phi = -\frac{F_y}{F_x}\,; \qquad tg\phi = \frac{y_A + (h_U - h_L)/2}{l_a}, \tag{5}$$

where l_a is a length of an arrow which defines a full draw of a bow; l is a length of a limb; S_U, S_L are length of upper and lower branches of a string; $\theta_U, \gamma_U, \theta_L, \gamma_L$ are angles between limbs and the string branches and a handle; $h = h_U + h_L$ are the length of a handle and its upper and lower parts divided by the line of force acting with an archer's hand; F_A, F_x, F_y are a force acting to the string by an other archer's hand and its projection to the axes of co-ordinate; ϕ is an angle between the line of action of this force and the x-axis; c is virtual stiffness of a limb; F_U, F_L are tensile forces in string's branches; φ_U, φ_L are angles of limb set-up, i. e. without a string.

The equations for the braced bow can be written as: (see Figure 2):

$$l(\cos\theta_{UB} + \cos\theta_{LB}) + h = S\cos\gamma_B\,;\; l(\sin\theta_{UB} - \sin\theta_{LB}) = S\sin\gamma_B\,;$$

$$F_B l\sin(\theta_{UB} - \gamma_B) = c(\theta_{UB} + \varphi_U)\,;\; F_B l\sin(\theta_{LB} + \gamma_B) = c(\theta_{LB} + \varphi_L)\,. \tag{6}$$

Co-ordinate of the nocking-point for the bow with a braced string, a string height and a nocking height are (Figures 3, 4):

$$y_B = \Delta S - \tfrac{1}{2} l(\cos\theta_{LB} - \cos\theta_{UB}); \; l_B = l \sin \tfrac{1}{2}(\theta_{UB} + \theta_{LB}); \; \Delta N \equiv NB = y_B - y_P + l_b \gamma_B . \quad (7)$$

A plunger position that produces zero angle of attack of an arrow has been determined with co-ordinate (see Figure 2):

$$y_P = y_B - \frac{(y_A - y_B) l_b}{l_a - l_b}, \quad (8)$$

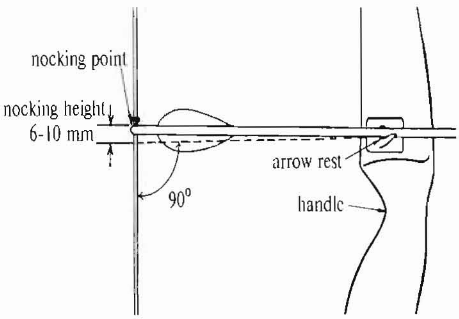

Figure 3. Recommendations regarding a nocking height (The drawing has been borrowed from: Bajer *et al.*, 1982).

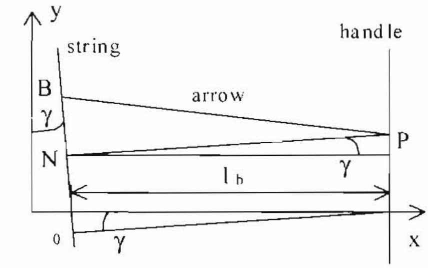

Figure 4. Schematic diagram for determination of a nocking height: B_0 is the nearest point of a string to the handle centre; N is the nearest point of a string to the plunger.

RESULTS AND DISCUSSION: Using the mathematical model (1)-(8), a simulation experiment has been provided regarding an optimal combination of bow and arrow parameters for their tuning in the vertical plane onto a zero angle of attack of an arrow. The method of simple iterations to solve systems of linear algebraic and transcendental equations and the method of dividing a section in half when iterations disperse were used. The model and methods have been tested during the tuning of a modern sport bow and an arrow with parameters (Hoyt/Easton USA, 1993): h=0,68 m; l=0,52 m; S=1,62 m; $\varphi = 0{,}083$ rad, c=129 Nm; l_a=0,72 m. The simulation covered multiple parameters and a wide range of values. Using instructions of bow's producers as well as feel and intuition of top archers and coaches, a four-factor variation was applied. A difference between the length of lower and upper branches of a string was up to 100 mm, a nock-point height – to 12 mm, a plunger height – to 40 mm, and a difference in distances between upper and lower ends of a handle to a string (Δtiller) – to 11 mm. The principal numerical results are summarised in Table 1. The range of the nocking-point height is near constant. It varies slightly with a full range of limb angles difference. In the real range of difference between a length of lower and upper branches of a string ($2\Delta S = S_L - S_U = 20 - 100$ mm), the nocking-point height is from 2 to 20 mm and a plunger height is from 6 to 33 mm. For the common nocking-point height from 6 to 10 mm (Bajer *et al.*, 1982), the difference between the length of branches is from 60 to 80 mm. For a common difference of tiller from 3 to 13 mm (Hoyt/Easton USA, 1993; Stuart, 1989), a difference between angles of lower and upper non-braced limbs $2\Delta\varphi = \varphi_U - \varphi_L$ should be from 0.004 to 0.017 rad. The lines of the Table matched the mentioned intervals and a minimum suitable plunger height $y_p = 20 - 30$ mm have been shaded in a grey colour. A plunger height about 30-40 mm is essential for free motion of the arrow above a hand holding a handle. We can see from the results in the Table that the previously stated condition requires a difference between a length of the string's branches not smaller than 80 mm. Notice, these parameters are almost independent of the difference between limb angles, but the latter causes direction of an arrow flight.

CONCLUSION: Common recommendations with respect to bow tuning in the vertical plane are rather approximate and the main accent has been made in empirical methods to consolidate a bow's internal parameters. These methods are based on the laborious procedures through a tedious and complicated trial and error phase that consumes effort and time. Bow and arrow system could not be symmetrical in the vertical plane in any case because an arrow and a hand holding a bow handle can not exist simultaneously in the axis of symmetry as an arrow should get a zero angle of attack.

Table 1. Calculated results to tune a bow for a zero angle of attack of an arrow (mm).

Tiller:	$\Delta=0$ **mm;**		$\Delta\phi=0$ **Mrad**		**Tiller:**	$\Delta=4$ **mm;**		$\Delta\phi=5$ **mrad**	
ΔS	**Ya**	**Yb**	**Yp**	**ΔN**	**ΔS**	**Ya**	**Yb**	**Yp**	**ΔN**
10	18	10	6	5	10	9	8	7	2
20	37	20	13	9	20	28	18	13	6
30	55	30	19	12	30	46	28	20	9
40	73	40	25	16	40	64	38	26	13
50	91	50	32	20	50	82	48	33	17
Tiller:	$\Delta=7$ **mm;**		$\Delta\phi=10$ **mrad**		**Tiller:**	$\Delta=11$ **mm;**		$\Delta\phi=15$ **mrad**	
ΔS	**Ya**	**Yb**	**Yp**	**ΔN**	**ΔS**	**Ya**	**Yb**	**Yp**	**ΔN**
10	0	5	8	-1	10	-9	3	8	-4
20	19	15	14	3	20	10	13	14	0
30	37	25	20	6	30	28	23	21	4
40	55	35	26	10	40	46	33	27	7
50	73	45	33	14	50	64	43	33	11

To obtain a zero angle of attack, a nocking-point of a drawn bow, a nocking-point of a braced bow, and a plunger point should lie on the same straight line. In the case of a zero angle of attack, the nocking height is influenced slightly by the difference between the lengths of string branches but significantly depends on the difference between the initial angles of the limbs. The more the angles differ the smaller is the angle of an arrow with a line normal to the handle, and the smaller becomes the asymmetry of the drawn bow.

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