ADVANCED SOLUTION OF AORTA BLOOD FLOW PROPERTIES UNDER BODY ACCELERATION FORCES

S. Abdalla Wassf Isaac
Physics and mathematics Engineering Department,
Faculty of Engineering, Port – Said, Egypt.

A mathematical model for blood (Newtonian fluid) flow with body acceleration force has been investigated. The body acceleration due to the movement of a tractor and the blood pressure gradient of thoracic aorta of a man have been considered. The actual geometric forms of pressure gradient and body acceleration force have been translated into mathematical forms by Fourier series. The geometrical forms of the total driving forces have been obtained for different frequencies of both forces. The exact solutions of velocity, flow rate, wall shear, etc. have been obtained in terms of ber and bei function. It is found that the geometrical form of the total driving force F_T gives a very useful information about the velocity, flow rate and wall shear profiles.

KEY WORDS: body acceleration, mathematical model, total driving force, properties of blood.

INTRODUCTION: Human body is subjected to body vibrations in many situations. These body forces could be intentional or unintentional. Some of the examples are swinging kids in cradle for sleep or for pleasure, making subject to lie down on vibrating tables as a therapy for heart disease Starr and Noordergraaf (1967), travel in road vehicles (car, bus, motor bicycle, truck, tractor etc.), in water vehicles, in air vehicles and fast body movements in sports activity (playing tennis, bowling in cricket, gymnastics etc.). The information about the vibrations provided by eye does not give quantitative measure of its severity. It is therefore desirable to analyse the effects of different types of vibrations on different parts of body. Such a body of knowledge could be useful in the diagnosis and therapeutic treatment of some health problems (joints pain, vision loss, and vascular disorders due to the vibrations), the design of protective pads and machines in laying down the standards for the body exposure to different types of vibrations Griffin (1990). The aim of this paper is two fold: the first one is to obtain results for blood flow with body acceleration force with actual forms of inputs give some idea about the details required for such work; and the second one is to compare the results of the actual and approximate (used in literature) inputs.

METHODS: The pressure gradient profile ($\partial p / \partial z - t$) in thoracic aorta of a man, given by Milnor (1989), is shown in fig. 1. This geometrical form can be expressed in a mathematical form by a Fourier series (Kreyszig (1991)) as

$$-\frac{\partial p}{\partial z} = A_0 + \sum_{n=1}^{9} \left[A_n \cos(n\, w_p\, t) + B_n \sin(n\, w_p\, t) \right] , \quad t \geq 0. \tag{1}$$

where *p is pressure* and a function of *t and z*, *z* is axial distance, *t* is time, and $w_p = 2\pi F_p$, F_p is heart pulse frequency.

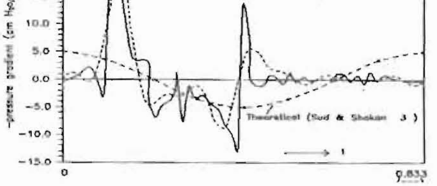

Figure 1. A comparison of experimental (Milnor) and theorical profiles of the pressure gradient in thoracic aorta of a man (R=1.17 cm, z=7.3 cm, z – distance between two points, pressure gradient=(p_1-p_2)/z).

The coefficients A_n and B_n have been obtained, and the corresponding geometrical form shown in fig. 1. The body acceleration force G for the movement of a tractor has been considered by Griffin (1990). The geometrical form of this force given in fig. 2.

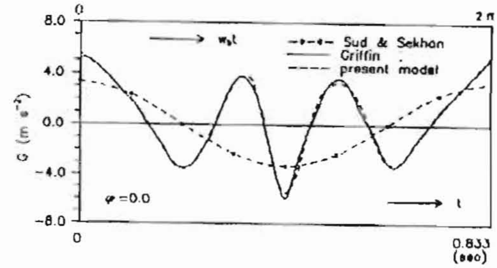

Figure 2. A comparison of body force (Gz=G) profiles.

This can be expressed in Fourier series form (Kreyszig (1991)) as

$$G = a_0 + \sum_{n_1=1}^{9} \left[a_{n_1} \cos(n_1 \omega_b t + \phi) + b_{n_1} \sin(n_1 \omega_b t + \phi) \right], \quad t > 0 \tag{2}$$

Where $\omega_b = 2\pi F_b$, F_b is body force frequency, and ϕ is its phase difference with heart pulse. The coefficients a_{n_1} and b_{n_1} have been determined. Let G_z be the body force in general. The profiles of total driving force F_T, given by

$$F_T = -\frac{\partial p}{\partial z} + G_z \tag{3}$$

for different frequencies and amplitudes of G_z, are shown in fig. 3.

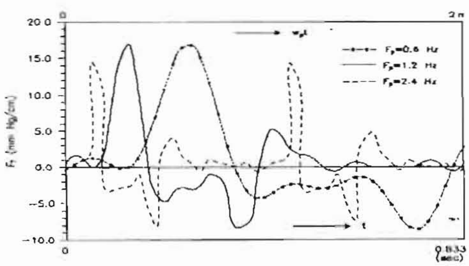

Figure 3. Variation ot total force F_T with pressure gradient frequency F_P (φ=0.0, F_b=1.2 Hz, R=1.17 cm and G_z=0.1G).

Assuming the flow of blood as laminar and one dimensional; blood to be incompressible and Newtonian fluid; and the tube walls to be rigid and very long compared to its diameter, the flow governing equations can be written as

$$\rho \frac{\partial u_z}{\partial t} = \rho a_0 + \rho \sum_{n_1=1}^{9} \left[a_{n_1} \cos(n_1 w_b t + \phi) + b_{n_1} \sin(n_1 w_b t + \phi) \right] + A0$$

$$+ \sum_{n=1}^{9} \left[A_n \cos(n\, w_p\, t) + B_n \sin(n\, w_p\, t) \right] + \mu_f \left(\frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r}\frac{\partial u_z}{\partial r} \right) \qquad (4)$$

where ρ *and* μ_f are the density and viscosity of blood respectively, u_z is the axial component of velocity and r is the radial co-ordinate. The initial and boundary conditions for the problem are Sud and Sekhon (1985)

$$u_z(r,0) = \frac{(R^2 - r^2)(A_0 + \sum_{n=1}^{9} A_n)}{4\mu_f} \qquad (5a)$$

$u_z(r,t) = 0,$ at r = R (5b) & $u_z(0,t)$ is finite at r =0, (5c) where R is the tube radius.

Using Laplace transform technique and Bessel functions theory Kreyszig (1991), we have obtained an exact analytic solution (u_z) of eqn. (4) with initial and boundary conditions (5), which is a purely real term with ber & bei { Spanier and Oldham (1987)}. Now the other fluid dynamically important quantities (flow rate Q, wall shear T_w, apparent viscosity η_a and flow resistance R_f) can be given by

$$Q = 2\Pi \int_0^R r u_z(r,t)\, dr \qquad (6) \qquad\qquad T_w = \mu_f \frac{\partial u_z(r,t)}{\partial r} \uparrow_{r=R} \qquad (7)$$

$$\eta_a = \frac{-\Pi R^4 (\partial p/\partial z)}{8Q} \qquad (8) \qquad\qquad R_f = (-\partial p/\partial z)/Q \qquad (9)$$

RESULTS AND DISCUSSIONS : We have considered an actual from of pressure gradient in thoracic aorta of a man (fig. 1, Milnor (1989)). The mathematical form of pressure gradient has been obtained and given by eqn. (1), the first 19 terms of Fourier series have been considered. This is in contrast to one or two terms considered in the literature [(1985), (1986), (1990) (1991), and (1995)], Abdalla wassf Isaac (1996) has been considered actual forms of pressure gradient. A comparison of mathematical form with actual form shows a very good agreement between the two, error is of the order of 3% (fig. 1). We have considered a body accelerative force due to the movement of a tractor (fig. 2, Griffin (1990)). The mathematical form (eqn. (2)) for the body force profile, given by Griffin (1990), has been obtained by using Fourier analysis Kreyszig (1991). We have taken first 19 terms of Fourier series. A comparison of the profile from the mathematical form (eqn. (2) and fig.2) and the actual geometrical form is shown in fig. 2. The error is of the order 2%. A vector sum of body acceleration force and pressure gradient gives the direction and magnitude of total driving force F_T. The F_T *-time* profiles for different frequencies and magnitudes of body force and pressure gradient are shown in fig. 3. Usually this profile has been ignored in literature but suggested in ref. (1996). It is observed that this is an important and useful profile. This helps in deciding the time for which the fluid dynamic quantities may be computed. Also looking at its shape, one could predict flow behaviour to some extent qualitatively. Two variations of velocity seem to be important. The usual variation of velocity with radial distance r from the tube axis for a chosen time (u_z - r) and the new variation of velocity with time at a given radial distance (u_z - *t*). This is in contrast with one variation considered in the literature, Abdalla wassf Isaac (1996) has been studied (u_z - *r*) and (u_z - *t*) profiles. It is also observed that from of u_z – *t* profiles are almost similar to F_T - *t* profiles at all radial distances. Hence F_T - *t* profiles an important source of information and prediction. Perhaps, this may be due to linear nature of the analysis. A comparison of the profiles of blood flow rate (without and with body force) with pressure gradient and total force are be done. The velocity gradients (wall shear) is influenced by the frequency of the dominant force ($|\partial p/\partial z|$ or $|G_z|$). As the frequency of the stronger force increases, the magnitude of the wall shear and its frequency increases. The increases in the wall shear magnitude could give rise to shear injury of the arterial wall and the deformation in the shape of wall cells; whereas, an increase in wall shear frequency could cause the wall muscle fatigue, leading to loss of viscoelasticity of arterial wall. Flow resistance (R_f) profile have bean got. Usually this profile has been ignored in the literature, Milnor (1989) has mentioned about the flow resistance, Abdalla wassf Isaac (1996) has been obtained this profile. We observe that with the change of F_b, this profile does not change significantly. Further, it is noticed that a change in heart pulse frequency F_p significantly influences the amplitude of the flow resistance.

CONCLUSIONS: In this paper, total driving force profiles (F_T – t, fig. 3) have been obtained. These profiles give some idea, in advance, about the velocity, flow rate and wall shear profiles. Hence, these profiles appear to be important profiles. The influence of body forces is two fold. The first one is, it changes the fluid driving force profile (fig. 3). Hence, it provides a mechanism for controlling the fluid driving force which in turn Controls the form of the profiles of velocity, flow rate and wall shear. The second one is, to shift velocity profile form u = 0 axis, which alters the negative velocity duration. This in turn gives a control on back flow regions duration and size. The time and size of back flow region play an important role in many arterial diseases Young (1979).

REFERENCES:

Abdalla Wassf Isaac, A. S. (1996) A Study of Pulsatile Flow of Blood in the Presence of External Periodic Excitations, Ph. D. thesis, I. I. T., bombay, India

Chaturani, P. and Palanisamy, V. (1990) *Biorheology*, **27**, 747 .

Chaturani, P. and Palanisamy, V.(1991)*Int. J. Engng. Sci.*, **29**, 113 .

Chaturani, P. and Wassf Isaac, A.S.A. (1995*) Int. J. Engng. Sci.*, **33**, 1807.

Griffin, M. J. (1990) Handbook of Human Vibrations, New York .

Kreyszig, E.(1991) Advanced Engineering Mathematics, *Wiley Eastern Ltd.*, Bombay.

Milnor, W. R. (1989) Hemodynamics, *Williams and Wilkins*, Baltimore.

Spanier, J. and Oldham, K. B. (1987) An Atlas of Functions, *Academic Press*, Washington.

Starr, I. and Noordergraaf, A. (1967) Ballistocardiography in Cardiosvascular Research. *Lippincott*, New York .

Sud, V. K. and Sekhon, G. S. (1985) *Bull. Math. Biol.*, **47**, 35 .

Sud, V. K. and Sekhon, G. S. (1986) *J. Biomech.*, **19**, 929.

Young , D. F. (1979) *J. Biomech. Engng.* Trans. ASME., **101**, 157.

Acknowledgement: This work is supported by the Egyptian Ministry of Higher Education, Egyptian Government, through Physics and Mathematics Engineering Department; Faculty of Engineering (Port - Said), Suez Canal University.