VOLUMETRIC EVALUATION OF BASKETBALL THROWS

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This study aims to provide an evaluation, through some simplifications, for the most forgiving throw positions in basketball throughout the playing field. Throws are modeled as differential equations and then solved numerically. These numerical solutions are checked against major events such as backboard or rim collision, score etc. and continue (after a bounce) or terminate accordingly. A volumetric approach have been undertaken to summarize these throws into a positional graph which is also presented.

KEYWORDS: basketball, modeling, field throw, bank shot, direct shot

INTRODUCTION: Although basketball is a highly dynamical game in which shots are mostly taken in spite of players of the opposing team, knowing the positions which are more forgiving to throw errors may give a player (or in turn, a team) the edge necessary to win the match. Because of that, analysis of throws without taking other players into account remains important. Several approaches have been researched to accomplish this. Tran and Silverberg (2008), and Okubo and Hubbard (2015) focuses on throwing kinematics in their papers. Among the previous studies, another point of focus have been free throws: Seppala-Holtzman (2012), Tran and Silverberg (2008), Hamilton and Reinschmidt (1997), Okubo and Hubbard (2006), and Maymin et al. (2012). Silverberg (2013) studies bank shots and compares them to direct shots. Another point of interest have been jump shots: Okazaki et al. (2015).
Silverberg et al. (2003) formulates the equations governing the motion of a basketball, and models the shooter as a probabilistic input to the system, resulting in shooter's probability of making a given shot. Although their work sheds light on positional evaluation of shots, it does not give a total view on all field shots. This study aims to accomplish that, though analytically instead of statistically.

METHODS: A computer model for basketball throws have been programmed using MATLAB (The Mathworks Inc., 2014) software, evaluated by solving the differential equations

$$\frac{dx(t)}{dt} = v(t), \tag{1}$$

$$\frac{dv(t)}{dt} = \begin{cases} a(t) - k\ ||v(t)||^2 \frac{v(t)}{||v(t)||}, & \text{if } v(t) \neq 0 \\ a(t), & \text{otherwise} \end{cases} \tag{2}$$

$$\frac{da(t)}{dt} = 0 \tag{3}$$

numerically (with initial conditions specified by gravity, throw velocity and position) via an ode solver function (namely, 'ode45') of MATLAB which is based on an explicit Runge-Kutta (4,5) formula, detecting when major events (e.g. backboard or rim collision, successful throw, ground

hit) occur and act accordingly. The program simulated throws for various velocity and position values. Considering the field as a grid of $0.5m \times 0.5m$ squares, the simulation has been run for all the grid intersections in the right half of the field, since our assumption that the rim is in the center of the field laterally allows us to make use of the symmetry of the field. All throws are assumed to be from a height of $2m$.
The field used in our model follows FIBA (2014), short of backboard-rim bridge, which is not accounted for in the simulations. Also the points where the horizontal distance to the rim is less than $1m$ as well as any point at the back of the rim line are not taken into account.
The method employed is inspired by Freitas (2014), and improves upon that by modeling in $3D$, hence generalizing the idea of surface area to a volume. At a given point in the field, we consider the velocities those which result in a scoring throw. Letting these velocity vectors define a 3D solid, the volume of this solid would be an indicator of the 'forgiveness' of that position to variations in velocity, i.e. player error. In our approach, a scoring throw represents a cube with $0.5 \times 0.5 \times 0.5 = 0.125m^3$ of volume, so calculating the volume of the solid corresponds to simply counting the number of successful throws and then multiplying that by 0.125. Since the exact volume is irrelevant to our purposes, we will simply count the number of scoring throws. Representations of two such solids constructed from velocities can be seen at Figure 1.

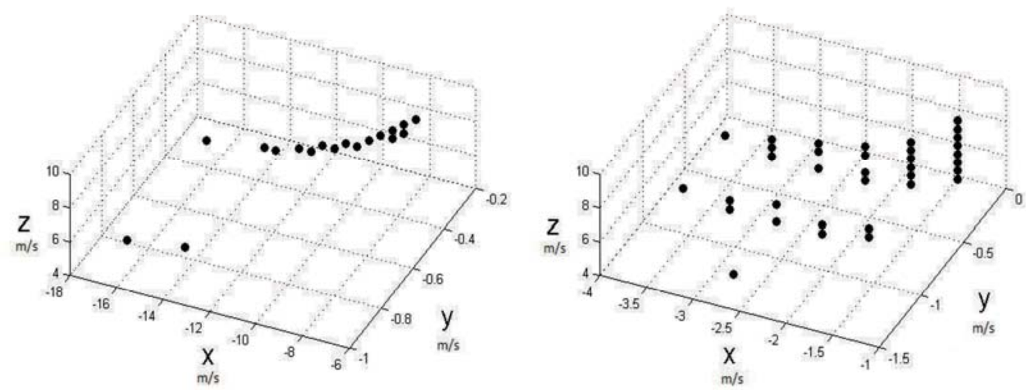


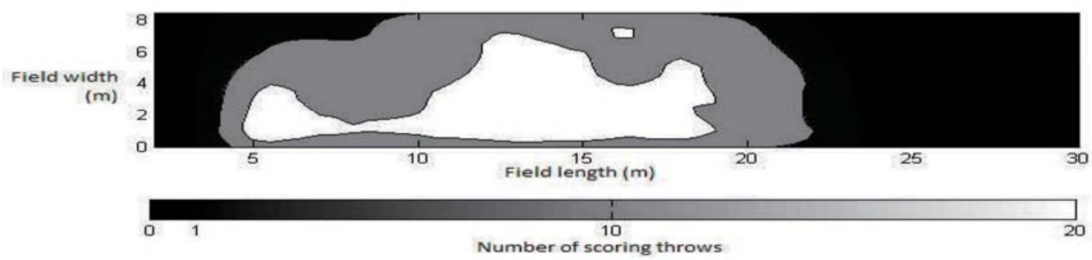
Figure 1: Examples of two solids for a bank shot (left) and a direct shot (right). Each point corresponds to a velocity of a scoring throw from a fixed point in the field. Together, those points constitute solids which we consider their volumes as an indicator of "forgiveness" of the position. Z axis corresponds to the vertical component of the velocity, whereas Y and X axes correspond to components along field width and length, respectively.

Simulations consisted of two sets; one for bank shots, and the other for rim shots. For bank shots, the field is scanned for throws hitting backboard at its horizontal mid point from side to side and vertical mid point from the top to the rim center, offset by rim radius so that the ball touches the backboard at that point. For rim shots, the field scanned for throws passing through the center of the rim. For both of these sets, simulation was given an error tolerance of $1mm$.
After scanning the field for initial throw velocities for both bank shots and rim shots, we scan each position for scores by changing this initial velocity (and by iteration any resulting scoring velocity) by $0.5m/s$ in each direction. That is, if v is the velocity of such a throw, then the neighbor vectors $v \pm (0, 0, 0.5)$, $v \pm (0, 0.5, 0)$, $v \pm (0.5, 0, 0)$ of these throws are also checked for scoring throws.
Our model takes air friction and both rim and backboard bounces into account, but does not account for the ball slipping at the rim nor backspin. When ball hits the backboard, it immediately bounces back without any compression.
After running the simulation until there are no more successful throws with less than $30m/s$

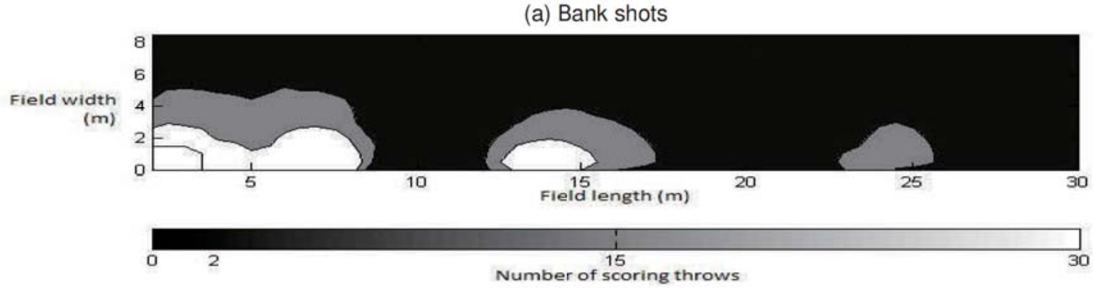
speed and after applying convolution filter, we pass onto analyzing them.

RESULTS: Figure 2 shows the result of simulations. First two contour graph denotes the number of successful throws for bank shots (Figure 2a) and direct shots (Figure 2b). Figure 2c is the difference between bank shots and direct shots fed into signum function, i.e. it specifies the places where there are more successful bank shots found than direct shots (Bs $>$ Ds), an equal quantity of such shots (Bs = Ds), and more successful direct shots found than bank shots (Bs $<$ Ds).

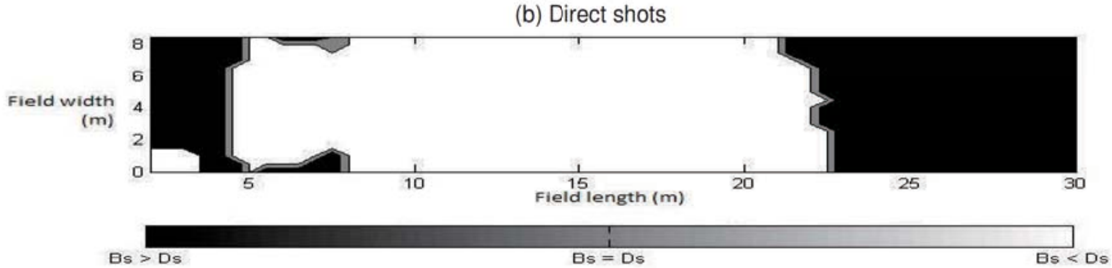
In Figure 2, x axis is the field length where the rim center is at 1.8055 according to FIBA (2014), and y axis is the field width. Because of the symmetry of throws, left side (w.r.t. player facing the backboard) is omitted.



(a) Bank shots



(b) Direct shots



(c) Difference

Figure 2: Top two images summarize the number of successful bank shots (a) and direct shots (b) throughout the field. (c) summarizes the difference between the number of successful bank shots and direct shots throughout the field: Right side of the scale (white) corresponds to more direct shots, whereas left side (black) corresponds to more bank shots. Note the differences between scales of direct shots and bank shots.

DISCUSSION: Since our rather big step sizes ($0.5m$ for grid, and $0.5m/s$ for velocity) led to a noisy graph, we have used a convolution filter as described.

Although for a simulation of such a big step size the ball slip might be negligible, but nevertheless our negligence of ball slip and especially backspin might have skewed the results too. Some other studies like Okubo and Hubbard (2010), Liu et al. (2006), Okubo and Hubbard (2004), Silverberg et al. (2003) gives insight to those factors.

Silverberg (2013) argues that the bank shot can be extremely advantageous over the direct shot, and this study opens a parenthesis to pinpoint the locations for that advantage. Direct shots are regularly applicable overall, yet a bank shot is more preferable than a direct shot at some specific locations, namely near the rim and too far to the rim as shown in Figure 2c.
Our naive volumetric approach i.e. counting the number of successful throws, disregards the shape of the solid and hence considers a long, thin pole and a thick cube as the same, yet as a measure of forgiveness that is not optimal to say the least. Hence the approach of Seppala-Holtzman (2012), i.e. fitting a rectangle between boundaries might be replicated as fitting a maximal sphere inside the solid.
This study did not take into account arguably the most important factor, the player. In that sense, a probabilistic method as in Silverberg et al. (2003) might be utilized to further elaborate our findings.

CONCLUSION: This study evaluated the basketball field by essentially counting the number of successful shots taken from each point, both for bank shots and for direct shots. By comparing said shots, we have found that both bank shots and direct shots have their preferable positions when it comes to scoring. A player who is aware of the forgiveness of throws throughout the field may alter their throw mechanics accordingly, and such a coach may position the players better, and even change the game strategy according to this knowledge, especially when further information on both their and opposing team players' capabilities are available.

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