DEFINITION OF THE ACTUAL ANGULAR VELOCITY OF REAL-LIVE BODY MOTIONS

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INTRODUCTION

By appointed moments of development in sport science always the problem of airborn movements is considered. For rigid bodies we have a complex but well-known theory to solve problems of these movements, but for living subjects these problems must be solved by a special and more complex approach. Especially we must consider the inner forces.

In the case of sporting movements, we have often difficulty to find serviceable parameters for simulation. The number of parameters and boundary values of three-dimensional models is beyond 20 and the number of rule-functions beyond 10. In practice we see, that it takes weeks to find the right functions. Therefore, trying to simulate a complex movement, it is necessary to analyse it before.

But just the analysis shows some difficulties. For example, what means the vector of angular velocity of a system, composed of several limbs, which are able to move one against another? This question is also important for methods of training. It is clear, that this vector is not directly measurable. The analysis of film or videopictures and subsequent computing must help.

We want to clarify some problems which occurred by determination of angular velocity of body-movements. We suppose that the time histories of position in space of all segments would be find out.

DEFINITION

In sporting practice the term of twist-axis or somersault-axis are usual. But they are not exactly defined (especially with crooked position!). Further it is known, that an athlete is able to influence his moment of inertia and angular velocity only by moving his trunk and limbs and to take every position in space (Fig.1).

How to define under these conditions the vector of angular velocity of a human body?
The physically only excellent cartesian system of coordinates is the system of principal axes of inertias of the whole body. We can compute them out of the tensor of inertia of the whole body together with the principal moment of inertia for any position. This system is fixed at the center of mass and works as initial system to compute the angular velocity.

Like in the case of the rigid bodies the EULER's theorem allows to define in only a single way the twist angle and the axis of rotation of the body at any small interval of time. Then, the angular velocity is the first derivative of the twist angle. In this way the angular velocity can be exactly defined and computed.

PROBLEMS

The tensor of inertia is given by the formula

$$A= \int (r^t I - r^t r)\, dm$$

(Fig.2). The matrix A is symmetric, its elements are named a_{ij}; $i,j = 1,2,3$. The system of principal axes of inertness is to be distinguished by disappering of the elements a_{ij} besides the diagonal of A. Mathematically such a system always exists, for A is symmetric. The principal moments of inertia I_1, I_2, I_3 are the (real) roots of the cubic equation

$$\mathrm{Det}\ (A - l*I) = 0.$$

Now arise a problem of ranging the eigenvalues l_i. Two eigenvalues can not be different in the case of equality (Fig.3). Therefore, we have to find methods to define the time histories of the eigenvalues.

First method:
We take the principal moment of inertia from a moment t and compute a approximate eigenvalues from the moment t+Dt by the formula

$$l_i(t+Dt) \quad l_i(t) + Dt \frac{dl_i}{dt}$$

We have

$$\frac{dl_k}{dt} = \Sigma\, l_k{}^{ij} \frac{da_{ij}}{dt}$$

and, for example

$$l_i{}^{ii} = \frac{a_{jj}a_{kk} - a_{jk}{}^2 + l_1{}^2 - l_1(a_{jj} - a_{kk})}{(l_1 - l_2)(l_1 - l_3)}$$

i, j, k = 1, 2, 3 and several.
the approximate eigenvalues l_i (t+Dt) help us to range them and to define the system of principal axes. The method is workable if principal moments of inertia differ more then 0.05 kgm. The derivates of the elements a_{ij} must be computed numerically.

Second method:
We define in a common sense the system of principal axes inertness at the beginning of analysis and assume we have computed the system of any time t. Now, there are able 24 several cartesian systems at the time t+Dt, which only differ in directions and orientations. Under these 24 systems we find the one which minimizes the sum of scalar products of the axes at and t+Dt subsequently (Fig. 4) and so on till the end of time interval.

The second method is based on the principle of continuity: if the elements a_{ij} of the symmetric matrix A differ only slightly, the eigenvalues differ small too.

It is easy to see, that the algorithm only works in the case if the twist angle does not exeed 45 degrees (Fig.5). Therefore, the frequency of filmcamera must answer theis purpose. Our computing programm bases on the second method and works without error.

Figures 1-5 are taken from "Training und Wettkampf", Sportverlag Berlin, 1990.

SUMMARY

The twist or the somersault axes of a human body are not defined exactly. We note that an athlete is able only with inner strength to take every position in space under free fall conditions. It is impossible to describe the rotative velocity in the case, only the angular momentum and the principal moments of inertia of the body are known. We give a definition of the vector of angular velocity for human body movements as alteration in time of the cartesian system of principal axes of inertness.

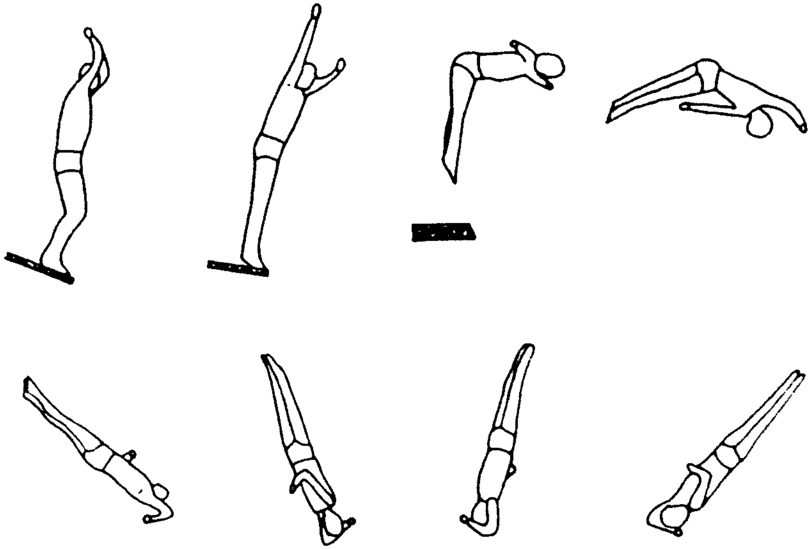

Figure 1: Example of divers movement

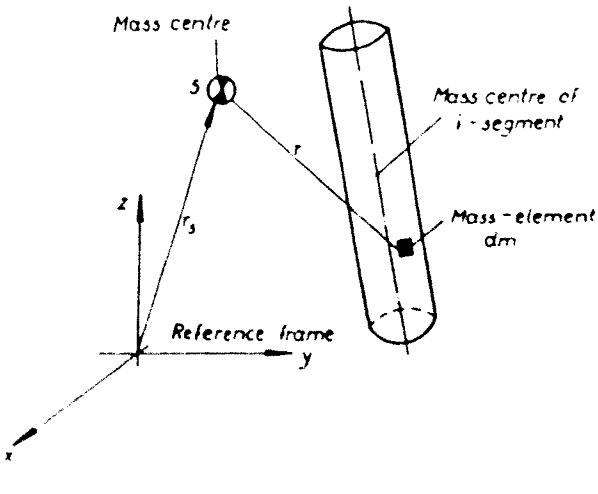

Figure 2: Terms for computing the tensor of inertia

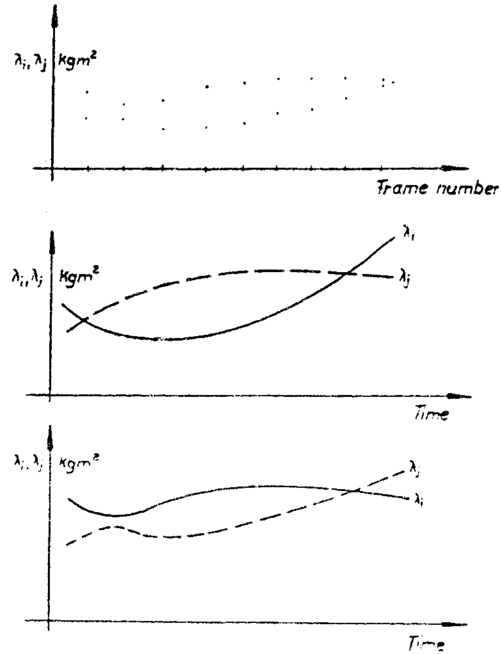

Figure 3: Principal movements of inertia and two several time histories of them

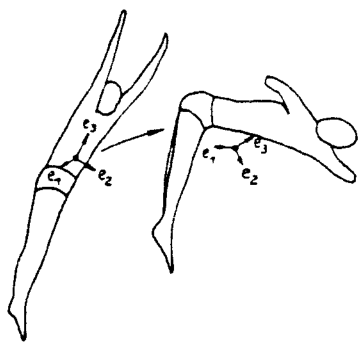

Figure 4: System of principal **axes** e_1, e_2, e_3 on t and t+Δt

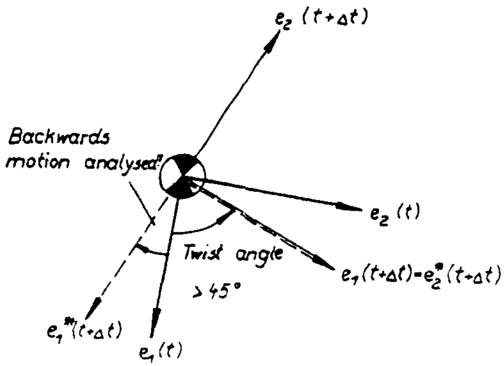

Figure 5: Wrong defined system of principal axes e_1^*, e_2^*, on $t+\Delta t$; right is $e_1(t+\Delta t)$, $e_2(t+\Delta t)$
(axis e_3 normal to the plane)