SOFTWARE FOR MATHEMATICAL MODELLING AND COMPUTER SIMULATION OF SPORTS MOVEMENTS AND TRAINING APPARATUS

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To make the analysis of interaction of a sportsman with an apparatus we must develop the mathematical model and then we must show the realization of this method using a computer. Then the most importanta spect of this problem is the degree of detailization of the model and, as a result, it's adequacy to the types of movement under study. The development of software for the model includes several questions, which are closely connected with each other, i. e. : selection of the mathematical model, registration of the experimental data and its loading into the computer, analysis of the parameters of the model with regard to the experimental data, variation of the parameters and functions with the aim of obtaining movements with the given properties (characteristics), presenting of the results of calculation in a form of diagrams, tables and kinematic schemes. These very aims determine the structure of software, destined for selection and analysis of the adequate models, determining the interaction of the sportsman and apparatus. At present it seems that the simple approach is the approach based on the use of dynamics for the system of solids (Wittenburg J. , 1977). Let us suppose that the segments of the sportsman's body and the levers of the simulators (apparatus) are all solids interacting via generalised forces, which represent the forces and moments, depending on parameters and time. Hence, our problem is limited to the analysis of the closed kinematic chain with the non-stationary inner connections and external (probably non-stationary) interactions. Let us use the traditional approach based on the introduction of the reaction ties, and let us write down the equations of movement for the ramified chain with n-segments (in the Newton-Euler form).

The radius-vector r of the fulcrum is directed from the inertional basis to the conditional fulcrum of the system (it may be a real fulcrum, e.g. ankle joint in the phase with one fulcrum). Geometrical and mass-inertial characteristics (GMIC) of the system in i-s mobile basis m_i $|\zeta_i|$, ρ_{ci}, ζ_{ci} (mass, length position of the centre of masses and the central tensor of inertia, respectively) are given or are calculated for the segments either using appropriate formulas or directly (for the apparatus or simulator). Note that the problem of recalculation of GMIC is quite useful in software in the case of changing ties (on/off), and at the same time one must store information about the system with the maximum number of links as the basic information. The value of parameter n is essentially non-linear and determines the difficulty of the algorithms of calculation (integration of the equation of movements). This is why, in the phase of processing of the experimental data, as a rule, we try to eliminate "unnecessary" degrees of freedom. As a value for the reciprocal mobility of the links we recommend the use of the linear regressional estimation:

$$\tilde{q}_i(t) = \Omega_i t + q_i^0 , \quad i = \overline{1,n} , \tag{1}$$

where q_i^0, Ω_i represents the mean value during the period of observation T of the value of the relative shift and speed (velocity). After calculation:

$$S_i^2 = \frac{1}{k-2}\sum_{j=1}^{k}\left(\tilde{q}_i(t_j) - q_i(t_j)\right)^2 + \left(\Omega_i T\right)^2 , \quad i = \overline{1,n} , \quad T = t_k - t_1 \tag{2}$$

we get the measure of the relative mobility of the links.

The sequence of the connection of segments in the model we shall determine in the structural matrix μ with the dimensions (n x n) consisting of zeros and ones and having a triangular form, if the numeration of the segments is arranged in the direction of increasing from the conditional fulcrum. The interlinks pivot connections represent the highest interest from the point of view of parametrization, however for deducting of the equations of movement let us consider that in each pivot point there exists an inter-segment moment M_i, then i-s body of the absolute moment U_i, is determined from the formula:

$$\mu^T \bar{U} = \bar{M} \quad , \tag{3}$$

where $\bar{U} = \{ \underline{U}_i \}^T$, $\bar{M} = \{ \underline{M}_i \}^T$, and T is for transposition.

Let r_i vectors connecting the pivot points (or directed to the centre of mass for end segment), $\bar{\omega} = \{\omega_i\}^T$ is the column or absolutely angular velocities, then the dynamics equation for the system of apparatus-sportsman is

$$\begin{cases} \sum\limits_{j=1}^{n} \underline{\underline{A}}_{ij} \cdot \dot{\underline{\omega}}_j + \underline{c}_i \times \ddot{\underline{r}} = \sum\limits_{j=1}^{n} (\underline{\omega}_j \underline{B}_{ij} - \underline{\omega}_j \times \underline{\underline{A}}_{ij}) \cdot \underline{\omega}_j + \underline{c}_j \times \underline{g} + \underline{U}_i \ , \quad i = \overline{1,n} \ , \\ \sum\limits_{j=1}^{n} \dot{\underline{\omega}}_j \times \underline{c}_j + M^c \ddot{\underline{r}} = \sum\limits_{j=1}^{n} (\underline{c}_j \underline{\omega}_j^2 - \underline{\omega}_j (\underline{\omega}_j \cdot \underline{c}_j)) + \underline{R} + M^c \underline{g} \ , \end{cases} \tag{4}$$

where $M^c = \sum\limits_{i=1}^{n} m_i$ - the total mass of the system. $\underline{R}$ - is the main vector of the external forces influencing upon the system, the gravity force M^c_g is distinguished into the separate component in the Newton equations and in the form of $c_i x g$ enters the equations of Euiler. Vectors U_i represent the main moments acting upon the i-s body of the system, i.e. the moments of the concentrated external forces must be included into U_i. In equation (3) we use common designations for scalar, vector and double products and the tensors are marked with two dots underneath, while the points above are for differentiation by time (t). Let us write down the designations used in (4):

$$\underline{\underline{A}}_{ij} = \begin{cases} \underline{\underline{A}}_{2ij} \mu_{ij} m_i + \underline{\underline{A}}_{1ij} \sum\limits_{k=i+1}^{n} \mu_{ki} \mu_{kj} m_k \ , & j < i \\ \underline{\underline{J}}_i + m_i \underline{\underline{A}}_{3i} + \underline{\underline{A}}_{1ii} \sum\limits_{k=i+1}^{n} \mu_{ki} m_k \ , & j = i \\ \underline{\underline{A}}^*_{ji} & , j > i \end{cases} \tag{5}$$

$$\underline{\underline{A}}_{1ij} = \underline{\underline{E}} (\underline{r}_i \cdot \underline{r}_j) - \underline{r}_j \underline{r}_i \ , \quad \underline{\underline{A}}_{2ij} = \underline{\underline{E}} (\underline{r}_j \cdot \underline{\rho}_{cj}) - \underline{\rho}_{cj} \underline{r}_i \ ,$$

$$\underline{\underline{A}}_{3i} = \underline{\underline{E}} \, \underline{\rho}_{ci}^2 - \underline{\rho}_{ci} \underline{\rho}_{ci} \ , \quad \underline{\underline{A}}^*_{1ji} = \underline{\underline{A}}_{1ij} \ , \quad \underline{\underline{A}}^*_{2ji} \neq \underline{\underline{A}}_{2ij}$$

E - is the single tensor, * - means conjugation,

$$\underline{B}_{ij} = \begin{cases} \mu_{ij} m_i (\underline{\rho}_{ci} \times \underline{r}_j) + (\underline{r}_i \times \underline{r}_j) \sum\limits_{k=i+1}^{n} \mu_{ki} \mu_{kj} m_k \ , & j < i \\ 0 & , j = i \\ -\underline{B}_{ji} & , j > i \end{cases} \tag{6}$$

$$\underline{c}_i = m_i \underline{\rho}_{ci} + \underline{r}_i \sum\limits_{k=i+1}^{n} \mu_{ki} m_k \quad , \qquad i,j = \overline{1,n} \quad .$$

To evaluate the integral properties of the system let us write down the expressions for the total mechanical energy and for the kinematic moment relatively to the inertial basis:

$$
\begin{aligned}
E &= T + \Pi = \bar{\omega}^T \cdot \bar{\bar{G}} \cdot \bar{\omega} + M^c \dot{\underline{R}}_c^2 / 2 - M^c \underline{R}_c \cdot \underline{g} \,, \\
\underline{H} &= I \bar{\bar{G}} \cdot \bar{\omega} + M^c \underline{R}_c \times \dot{\underline{R}}_c \,,
\end{aligned}
\tag{7}
$$

here we use the determinations I = (1,...,1) - is the matrix - line with the length of ***n*****, consisting of ones; G - is the matrix of dimension (n x n), consisting of tensors**

$$
\underline{\underline{G}}_{ij} = \underline{\underline{A}}_{ij} - \left(\underline{\underline{E}} (\underline{c}_i \cdot \underline{c}_j) - \underline{c}_j \underline{c}_i \right) / M^c \,, \qquad i,j = \overline{1,n} \,,
\tag{8}
$$

and the expressions for movement of the velocity and acceleration of the centre of masses of the system are given below:

$$
\begin{aligned}
&\underline{R}_c = \underline{R}_c^{(0)} = \underline{r} + \sum_{i=1}^{n} \underline{c}_i / M^c \,, \quad \dot{\underline{R}}_c = \underline{R}_c^{(1)} = \dot{\underline{r}} + \sum_{i=1}^{n} \underline{\omega}_i \times \underline{c}_i / M^c \,, \\
&\ddot{\underline{R}}_c = \underline{R}_c^{(2)} = \ddot{\underline{r}} + \sum_{i=1}^{n} \left(\dot{\underline{\omega}}_i \times \underline{c}_i + \underline{\omega}_i (\underline{\omega}_i \cdot \underline{c}_i) - \underline{c}_i \underline{\omega}_i^2 \right) / M^c \,.
\end{aligned}
\tag{9}
$$

Let us write down two more correlations which may be used both for direct calculation of its components and for control of result of numerical calculations. These correlations represent the changes in the moment of the impulse and of the total energy of the whole system

$$
\begin{aligned}
\dot{\underline{H}} &= \underline{r} \times \underline{R} + \underline{R}_c \times M^c \underline{g} + \underline{M}_1 \,, \\
\dot{E} &= \dot{\underline{r}} \cdot \underline{R} + \bar{\Omega}^T \cdot \bar{M} = \dot{\underline{r}} \cdot \underline{R} + \bar{\omega}^T \cdot \bar{U} \,,
\end{aligned}
\tag{10}
$$

where $\bar{\Omega} = \{ \underline{\Omega}_i \}^T$ is matrix-column, containing relative angular velocities. Finishing with the analysis of general equations and correlations let us note, that some segments in the system may be considered as fictitious, and this fact ensures the possibility to model the arbitrary intersegmental connections.

The numerical realization of the equations of movements (3) and their accompanying correlations makes it possible to consider a number of problems concerning processing of the result of registration of movements, modelling new movements and the given properties of the apparatus (simulator). On the basis of this problem is the assumption about the adequacy of the resulting model sportsman-apparatus and of the real properties of the system. For creation of the adequate model we suggest to consider the adequacy of the model to the apparatus and the sportsman separately. In both cases the degree of proximity of the mathematical model and the real object may be considered based on the results of the most simple experiments which are registered, for instanse with a help of shooting a film at high speed and of tensor and accelerometric measurements. The principle difference of the system lies in the fact that the interval generalized forces, realizing the movements of the apparatus (simulator), as a rule are of the stationary nature and they may be described with the system of parameters (characteristics of damping, and springs, coefficients of friction, etc.), unlike the non-stationary moments, created by muscular efforts of a sportsman. From the point of view of calculations the problems are limited to the calculating of the parameters of the internal links of the simulator (apparatus) and to finding out the characteristics of the generalized forces for a sportsman versus time. Let us suppose that in the results of the experiment we obtained the time dependencies of the generalized movements. It is obvious that the most economical (in the general case) seems to be a variant of estimation of these dependencies with the help of the single parametrical cubical smoothing splines (Reinsch C.H. 1967). It makes it possible to consider the generalised values, changing in a comparatively wide range, determined by the registered information about the movement. Variations of the parameters from smoothing makes it possible to obtain infinite sets of curves of the generalized shifts from the interpolation to straight lines, drawn according to the method of least squares. For the optimum estimation of the smoothed parameters values we suggest a minimizing the function of nonconformity of the calculated and additionally measured kinematic and force values. In the case of movement with single support and in the presence of a forceplate for the support reaction, forces this function might be of the following form:

$$J_K = \sum_{p=1}^{K} \int_{t_0+t_p^*}^{t_m-t_p^*} \left(\underline{R}_c^{(p-1)}(t) - \tilde{\underline{R}}_c^{(p-1)}(t) \right) \cdot \underline{\underline{\Delta}}_p \left(\underline{R}_c^{(p-1)}(t) - \underline{R}_c^{(p-1)}(t) \right) dt, \quad K=\overline{1,3} \tag{11}$$

where t_0 , t_m - the starting and finishing moments of time, t_p - fragment intervals of consideration do not take into account the errors of approximation on the ends of the interval, the tensor, whose matrix is diagonal and contains the weight coefficients , determining the degree of input of different criteria. Besides the parameters of smoothing, all other system parameters enter into the function of (II)-type, namely GMIC (for a sportsman), parameters of the intersegmental connections for an apparatus - simulator. As a rule, the resourse of the computer is limited due to this fact. We suggest to range all the parameters in accordance with the possible input into the J values during their variation, for example in the range of the errors of measurements. We think a great help in such a case will be the procedure allowing one to receive the interaction between the problem of experimental data (choosing of the adequate model) and the problem of modelling of given systems with their following integration with high accuracy of the movements equations and "input of noise" of the results of integration (distortion of the phase picture, e.g. with the help of an evenly distributed error of the given amplitude). Along with this we can vary the distribution of GMIC, distort frequency of skills and other parameters in the system. In the result of such studies the procedure of minimization (II) may be carried out in several consecutive stages. For given calculations, described below, we used the procedure of the combined search for the minimum, and this procedure consists of the method of pseudo-accidental search with the use of the evenly distributed sequence (LP search) and of the Nelder-Mead's method (J.A. Nelder, R.Mead, 1965) - the method of the deformed polyhedron.

The example of calculation described further is performed with the electronic computer of IBM PC/AT using a program package in FORTRAN. The package permits one to realize research of analogs containing up to 16 units and is organized as several interacting problems which reflect the structure described at the beginning of the article. Figures 1-3 represent results of the calculation of test movements of six-elements (jump down to the strain gauge platform which further repulses upwards).

Track smoothing parameters and inertial mass characteristics were selected under condition of minimal function of type (9) taking into account complimenting strain measurements. The graphs of behaviour of horizontal and vertical (R_x , R_y) components of support reactions by the results of optimisation as represented in the Figure 2. It the same figure the behaviour of the complete energy (E) curve is represented. Figure 3 represents the curves of alteraction of interunit moments in the ankle, knee-joint and hip joint (M_1, M_2, M_3).

The character of curve behaviour and their values correspond to the executed movement and permit to appneciate power consumption and amplitude values of interunit moments.

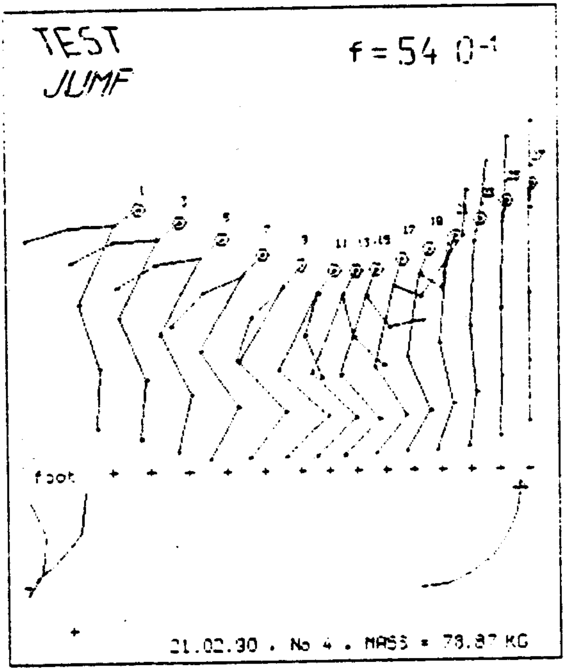

Figure 1:

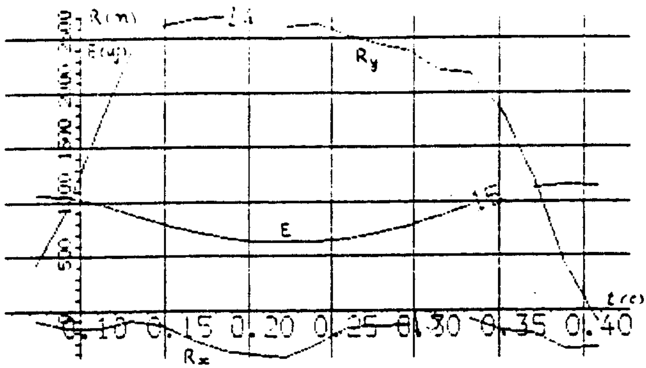

Figure 2:

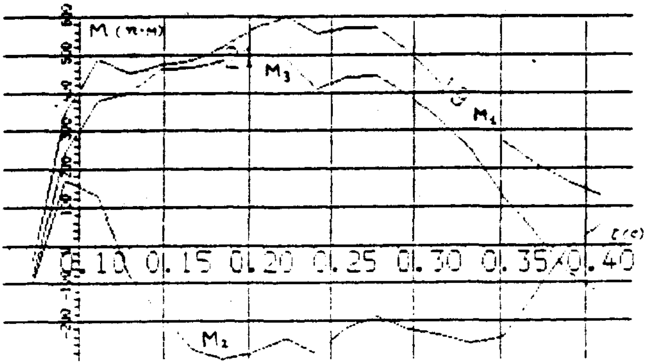

Figure 3:

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